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Determination of large diameter bored pile's effective length based on Mindlin's solution

Zhijun Zhou [a,*], Duanduan Wang [a], Lipeng Zhang [a], Weisi Ma [a,b]

[a] School of Highway, Chang'an University, Xi'an 710064, China
[b] Department of Civil Engineering, University of Illinois at Urbana-Champaign, Urbana-Champaign, IL 61820, USA

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ABSTRACT

The calculation equation of large diameter bored pile's effective length is connected with its distribution of pile shaft resistance. Thus, there is a great difference between the calculation results under the different distributions of pile shaft resistance. Primarily, this paper summarizes the conceptualized mode of pile shaft resistance under the circumstance that the soil surrounding the piles presents different layer distributions. Secondly, based on Mindlin's displacement solution and in consideration of the effect of pile diameter, the calculation equation is optimized with the assumption that the pile shaft resistance has a parabolic distribution. The influencing factors are analyzed according to the calculation result of effective pile length. Finally, combined with an engineering example, the calculation equation deduced in this paper is analyzed and verified. The result shows that both the Poisson ratio of soil and pile diameter have impacted the effective pile length. Compared with the Poisson ratio of soil, the effect of pile diameter is more significant. If the pile diameter remains the same, the effect of the Poisson ratio of soil to the effective pile length decreases as the ratio of pile elastic modulus and soil share modulus increases. If the Poisson ratio of soil remains the same, the effect of the pile diameter to the effective pile length increases as the ratio of pile elastic modulus and soil share modulus increases. Thus the optimized calculation result of pile effective length under the consideration of pile diameter effect is more close to the actual situation of engineering and reasonably practicable.

1. Introduction

In order to satisfy the requirement of bearing capacity, the design of highway bridge usually chooses a larger pile length, which would cause a certain waste. A larger number of theoretical and experimental researches show that a large diameter bored pile has an effective length when the pile diameter is fixed. Under the action of vertical load, large

* *Corresponding author*. Tel.: +86 29 85355787.
E-mail addresses: zlp@chd.edu.cn (Z. Zhou), wangduanduan11@163.com (D. Wang), 772797602@qq.com (L. Zhang), mwsxuan@hotmail.com (W. Ma).
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diameter bored pile would produce compression and relative displacement of piles and soil. In the process of load transferring and spreading to the surrounding soil, the bearing capacity of single pile would not increase after a certain limit. At this point, the length of pile is called effective pile length (Lai and Yang, 2007; Zhang, 2009).

There are several methods to calculate the effective pile length, such as the method of controlling ultimate bearing capacity (Shu and Huang, 2001; Wang and Jia, 2001; Yang and Liang, 2006), the method of controlling pile top settlement (Dai et al., 2012; Zhang et al., 2014; Zhou et al., 2007), the method of controlling pile stiffness (Leung et al., 2010; Tong et al., 2012), and using the finite element method (Chen et al., 2012; Wang et al., 2013; Yang et al., 2014). However, different calculation methods lead to discrepancies among calculation results. Aiming to control the ultimate bearing capacity, the rationality of theoretical calculation result mainly depends on the assumption of pile shaft resistance distribution, which is different in different soil layers. In general, the calculation of effective pile length is based on the assumption that pile shaft resistance is triangular distribution or rectangular distribution, but the actuality differs from these assumptions, which do not reasonably reflect on the effective pile length truly and reasonably. Ding (2005) and Shu and Wang (2001) assumed that the distribution of pile shaft resistance was inverted triangle and deduced the following formula under that assumption.

$$l_e = \frac{3s_{a0}E_pA}{Q_a} \tag{1}$$

$$l_e = (4.7 \sim 5.2)r_0\sqrt{\frac{E_p}{E_s}} \tag{2}$$

where l_e is the effective pile length, s_{a0} is settlement under the working load, Q_a is working Load, E_p is the elastic modulus of pile, E_s is the compression modulus of soil, A is section area of pile, and r_0 is radius of pile.

Sun (2008) assumed that the distribution of pile shaft resistance was double fold triangle, and deduced the following formula of effective pile length based on this assumption.

$$l_e = (4.1 \sim 4.5)r_0\sqrt{\frac{E_p}{E_s}} \tag{3}$$

But the measured data of field test shows that pile shaft resistance does not change linearly with the pile embedded depth.

Wang and Chen (2011) deduced the calculation formula of effective pile length with the assumption that the distribution of pile shaft resistance was parabolic.

$$l_e = d\sqrt{\frac{5E_p(3-2\mu)(1+\mu)}{4E_s}} \tag{4}$$

where μ is Poisson ratio of soil, d is pile diameter.

In Eq. (4), the pile diameter (d) is mainly used in the calculations of elastic compression and the compression coefficient of soil around pile, but it does not reflect the influence of pile diameter. The pile bearing capacity not only depends on the natures of soil around pile, but also the geometric dimensioning of pile. The influence of pile diameter cannot be ignored when the pile diameter is more than 0.8 m. Thus, the calculation result accuracy of large diameter bored pile length needs further validation.

Based on the research of Wang and Chen (2011) and Liu et al. (2014b), this paper considers the effect of pile diameter and deduces the calculation equation of pile effective length with the assumption that the pile shaft resistance distribution is parabolic. Then, the influencing factors are analyzed and the calculation result is verified by the engineering example.

2. Conceptualized mode of pile shaft resistance

Due to the influencing factors such as the distribution of soil around pile and the modulus ratio of pile and soil, the distribution of pile shaft resistance is polytropic and complicated (Qiu et al., 2014). According to statistics, the conceptualized mode of pile shaft resistance along pile can be divided into 6 kinds (Liu et al., 2014a), which are shown in Fig. 1.

The isosceles trapezoidal distribution of pile shaft resistance often occurs for long piles in hard soil layer. When the strength of soil around the pile increases gradually from top to tip of the pile, pile shaft resistance would play fully with the increase of depth. On the contrary, the distribution of pile shaft resistance appears inverted trapezoidal when the strength of soil around pile changes weakly. If the soil around pile is interaction layer of soft and hard sand, or clay, the upper soil is comparatively weak. Then, because of the pile compression, pile shaft resistance of the lower soil layers plays laggingly and increases gradually. At this time the distribution of pile shaft resistance usually presents olive shape. For the middle long pile or long pile, if soil around the pile is hard gravel soil, sandy soil or cohesive soil, pile shaft resistance of lower soil cannot play fully. This moment pile shaft resistance usually presents lantern shape. Garlic shaped pile shaft resistance is more common in the condition that pile side soil is soft at the central and mutation into hard at the lower. Pile shaft resistance of middle long pile or long pile in hard–soft–hard soil usually presents peak–valley shape.

It can be seen that the distribution forms of pile shaft resistance are various and complex. In practical engineering, the distribution of pile shaft resistance is not linear along the whole or part of pile body (Liang et al., 2015; Limkatanyu et al., 2009). For the olive and lantern shaped pile shaft resistance distribution, the practical distribution curve is closer to parabola, which conforms to the hypothesis of pile shaft resistance in this article.

3. Calculation of effective pile length

The value of shaft resistance is assumed equal to 0 at the top of pile and the effective pile length firstly increases and then decreases along the pile. There is no displacement at the point of effective pile length. The distribution of pile shaft resistance is shown in Fig. 2, and satisfied the following Eq. (5) at the depth z.

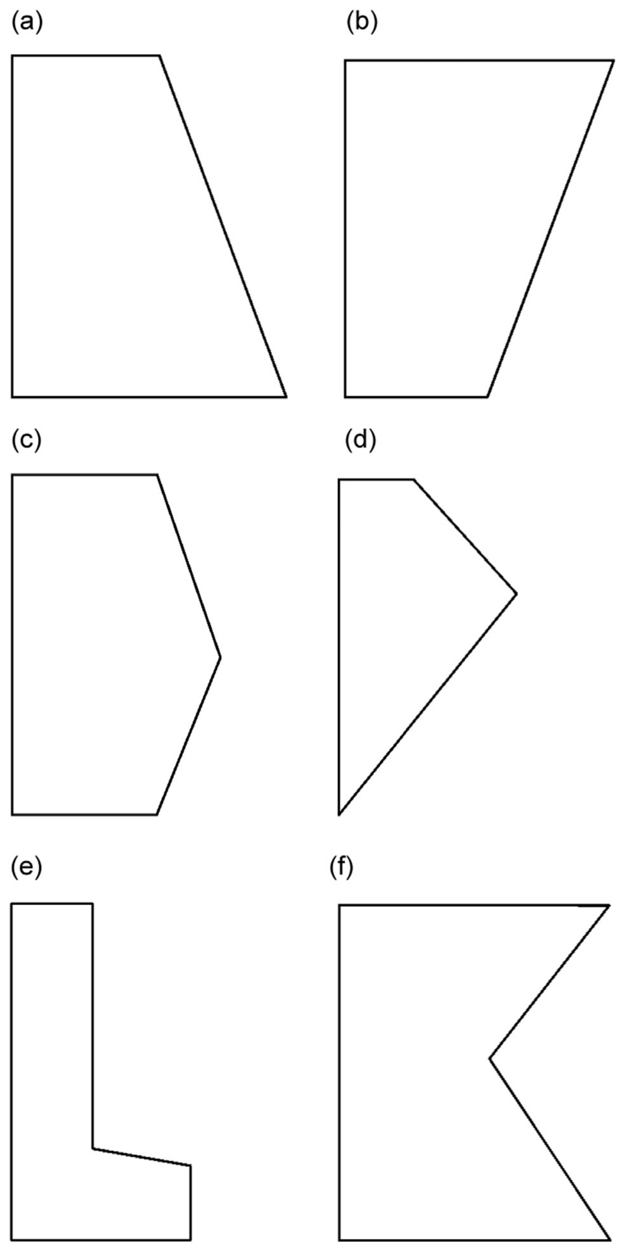

Fig. 1 – Conceptualized mode of shaft resistance along pile. (a) Isosceles trapezoidal distribution. (b) Inverted trapezoidal distribution. (c) Olive shaped distribution. (d) Lantern shaped distribution. (e) Garlic shaped distribution. (f) Peak–valley shaped distribution.

$$\tau(z) = \frac{12P}{\pi d l_e^4} z(z-l_e)^2 \tag{5}$$

where $\tau(z)$ is pile shaft resistance at the depth of z, P is the load on pile top, z is embedded depth of pile. In Fig. 2, L is the pile length and is equal to l_e.

Then pile shaft resistance at the embedded depth z is shown as follow

$$Q_s = \pi d \int_0^z \tau(z)dz = \pi d \int_0^z \frac{12P}{\pi d l_e^4} z(z-l_e)^2 dz$$
$$= \frac{12P}{l_e^4}\left(\frac{1}{4}z^4 - \frac{2}{3}z^3 l_e + \frac{1}{2}z^2 l_e^2\right) \tag{6}$$

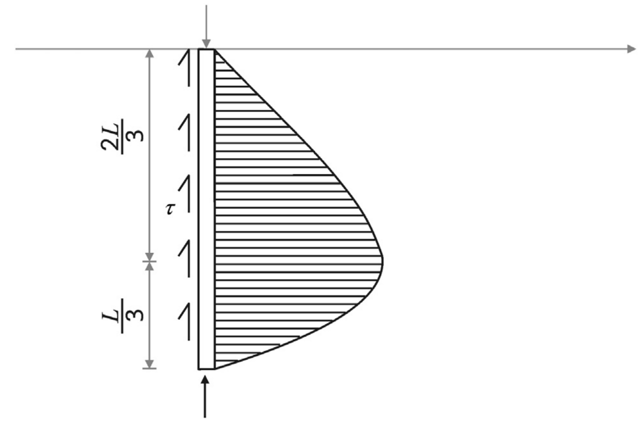

Fig. 2 – Pile shaft resistance distribution along the pile.

where Q_s is pile shaft resistance around the pile from the top to the depth of z.

The elastic compression produced by pile micro element under the load is shown as follow

$$ds = -\frac{P(z)}{E_p A}dz \tag{7}$$

The pile top settlement of large diameter bored pile can be simply described by pile elastic compression and pile tip settlement.

$$S_p = S_{ps} + S_{pb} \tag{8}$$

where S_p is pile top settlement, S_{ps} is pile elastic compression, and S_{pb} is pile tip settlement.

Since the pile tip settlement $S_{pb} = 0$, then

$$S_p = S_{ps} \tag{9}$$

Then the pile top settlement is shown as follow

$$S_p = \int_0^{l_e} \frac{P(z)}{E_p A} dz = \int_0^{l_e} \frac{P_0 - Q_s}{E_p A} dz$$
$$= \frac{1}{E_p A}\int_0^{l_e}\left[P - \frac{12P}{l_e^4}\left(\frac{1}{4}z^4 - \frac{2}{3}z^3 l_e + \frac{1}{2}z^2 l_e^2\right)\right]dz = \frac{2Pl_e}{5E_p A} \tag{10}$$

where $P(z)$ is axial force of pile shaft at the depth z.

Based on Mindlin's solution shown in Fig. 3, the vertical displacement caused by any point $K(x_0, y_0, z_0)$ in the semi-infinite elastic space under the concentrated load, which is forced at depth z, is shown as Eq. (11) (Mindlin, 1936).

$$w_K = \frac{N}{16\pi G(1-\mu)}\left[\frac{3-4\mu}{R_1} + \frac{8(1-\mu)^2 - (3-4\mu)}{R_2} + \frac{(z_0-z)^2}{R_1^3}\right.$$
$$\left. + \frac{(3-4\mu)(z_0+z)^2 - 2z_0 z}{R_2^3} + \frac{6z_0 z(z_0+z)^2}{R_2^5}\right] \tag{11}$$

where w_K is the displacement of the point K, G is share modulus of soil, r and z_0 are the vertical distances from point K to the z axis and the x axis, N is the force at the point O. In Eq. (11)

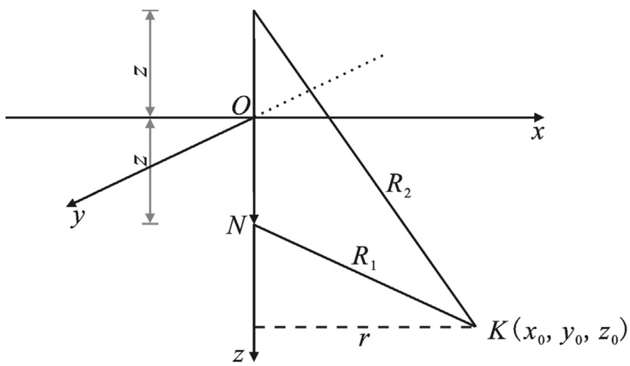

Fig. 3 – Calculation diagram of Mindlin's solution.

$$R_1 = \sqrt{r^2 + (z_0 - z)^2}$$

$$R_2 = \sqrt{r^2 + (z_0 + z)^2}$$

As shown in Fig. 4, a micro element dsdz at the surface of pile side is analyzed, and the pile shaft resistance on the micro element is shown as follow

$$F = \tau(z)\,\mathrm{ds}\mathrm{dz} \tag{12}$$

where F is the pile shaft resistance of the hole micro element, ds, dz, dθ are width, height and angle of micro element of pile body.

In polar coordinates, Eq. (12) can be expressed as Eq. (13).

$$F = \tau(z)\frac{d}{2}\mathrm{d}\theta\mathrm{dz} = \frac{6P}{\pi l_e^4} z(z - l_e)^2 \mathrm{d}\theta\mathrm{dz} \tag{13}$$

Take the pile shaft resistance of Eq. (13) into Eq. (11), and make integral operation in the whole range of pile side. At this time, F is equivalent to N. Then the vertical displacement at point K in foundation caused by pile shaft resistance can be obtained.

$$S_p = 2\int_0^{\pi}\int_0^{l_e} w_K \mathrm{d}\theta\,\mathrm{dz} \tag{14}$$

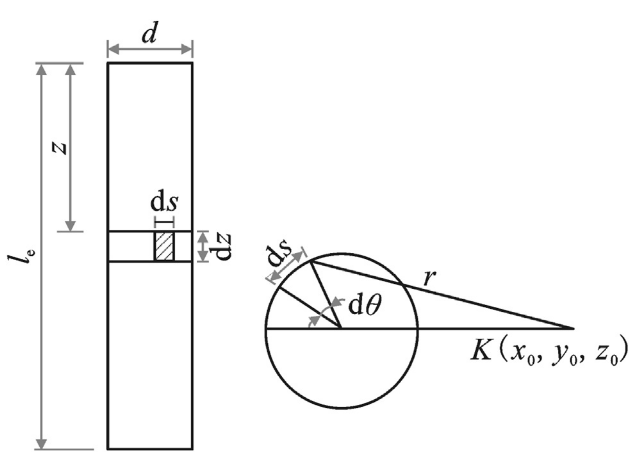

Fig. 4 – Integral schematic diagram of pile shaft resistance.

The dot of pile top section is seen as the Mindlin coordinate dot (0, 0, 0), and the soil on the border between pile and soil is seen as the research object, so $x_0 = d/2$, $y_0 = 0$, $z_0 = 0$. These are brought into the Eq. (14) with the Eq. (11) together, and the following equation can be obtained.

$$R_1 = R_2 = \sqrt{\frac{d^2}{4} + z^2}$$

The Eq. (14) can conduct the following result through integral calculation.

$$S_p = \frac{P}{\pi G l_e^4}\left[(1+\mu)\int_0^{l_e}\frac{z(z-l_e)^2}{(d^2/4+z^2)^{1/2}}\mathrm{dz} + \int_0^{l_e}\frac{z^3(z-l_e)^2}{(d^2/4+z^2)^{3/2}}\mathrm{dz}\right] \tag{15}$$

Assuming the integral factors of I_1 and I_2 described as Eqs. (16) and (17).

$$I_1 = \int_0^{l_e}\frac{z(z-l_e)^2}{(d^2/4+z^2)^{1/2}}\mathrm{dz} \tag{16}$$

$$I_2 = \int_0^{l_e}\frac{z^3(z-l_e)^2}{(d^2/4+z^2)^{3/2}}\mathrm{dz} \tag{17}$$

In the integral factors, the integral calculation to z is in fact to pile length. Then pile diameter would change to the following form.

$$d = \frac{1}{\lambda}z \tag{18}$$

where λ is the pile aspect ratio.

Taking Eq. (18) into the Eqs. (16) and (17), the integral factors can be simplified.

$$I_1 = \int_0^{l_e}\frac{z(z-l_e)^2}{z\sqrt{1/(4\lambda^2)+1}}\mathrm{dz} = \frac{2\lambda l_e^3}{3\sqrt{4\lambda^2+1}} = I_1' l_e^3 \tag{19}$$

$$I_2 = \int_0^{l_e}\frac{z^3(z-l_e)^2}{z^3[1/(4\lambda^2)+1]^{3/2}}\mathrm{dz} = \frac{8\lambda^3 l_e^3}{3(4\lambda^2+1)\sqrt{4\lambda^2+1}} = I_2' l_e^3 \tag{20}$$

The influence coefficients of integral factors can be assumed to I_1' and I_2'.

$$I_1' = \frac{4\lambda}{\sqrt{4\lambda^2+1}}$$

$$I_2' = \frac{8\lambda^3}{(4\lambda^2+1)\sqrt{4\lambda^2+1}}$$

Bringing the simplified integral factors into the Eq. (15), Eq. (21) can be obtained.

$$S_p = \frac{P}{\pi G l_e}\left[(1+\mu)I_1' + I_2'\right] \tag{21}$$

In order to facilitate the further calculation of the Eq. (21), the influence coefficients I_1' and I_2' under the different aspect ratios are calculated. The calculation results are listed in Table 1.

Table 1 – Influence coefficients under different aspect ratios.

λ	I_1'	I_2'
10	0.332917446	0.332087228
15	0.333148302	0.332778548
20	0.333229215	0.333021077
25	0.333266687	0.333133433
30	0.333287047	0.333194493
35	0.333299325	0.333231319
40	0.333307295	0.333255224
45	0.333312759	0.333271614
50	0.333316668	0.333283340
55	0.333319560	0.333292015
60	0.333321760	0.333298614
65	0.332917446	0.333303750
70	0.333148302	0.333307825

It can be seen that the values of the influence coefficients of integral factors under the different aspect ratios have very little change. Then the result of $I_1' = I_2' = 1/3$ can be obtained approximately. So the pile top displacement expressed by Eq. (21) can be further simplified.

$$S_p = \frac{(\mu+2)P}{3\pi G l_e} \tag{22}$$

Because the pile shaft resistance on pile top is 0, there is no relative displacement between pile and soil on the top. According to the displacement compatibility equation, Eqs. (10) and (22) are combined as Eq. (23).

$$\frac{2Pl_e}{5E_pA} = \frac{(\mu+2)P}{3\pi G l_e} \tag{23}$$

So the effective pile length can be simply calculated.

$$l_e = \frac{d}{2}\sqrt{\frac{5(\mu+2)E_p}{6G}} \tag{24}$$

The Eq. (24) is the bored pile effective length with the assumption that pile shaft resistance is distributed as parabola.

4. Analysis of the influence coefficient of effective pile length

The influence coefficients mainly include Poisson ratio of soil, pile diameter, elastic modulus of pile, pile and soil modular ratio and friction coefficient between pile and soil for effective length of large diameter bored pile (Goit et al., 2014; Guo, 2009). According to the formula of effective pile length, this paper mainly focuses on the influences of Poisson ratio and pile diameter.

The value of effective pile length under different Poisson ratios of soil when the value of pile diameter is 1.3 m ($d = 1.3$ m) is shown in Fig. 5. The value of effective pile length under the different pile diameters when the value of the Poisson ratio of soil is 0.3 ($\mu = 0.3$) is shown in Fig. 6. The abscissa is the 10^{-4} times of the ratio of pile elastic modulus E_p and soil share modulus G. It is clear to see that effective pile length is positively correlated with Poisson ratio of soil around the pile and pile diameter. When pile diameter and Poisson ratio of soil are constant, effective pile length is increasing with the increase of $(E_p/G) \times 10^{-4}$. The influence of pile diameter on effective pile length is more significant, while the influence of the Poisson ratio is less. For example, if the Poisson ratio of soil increased from 0.2 to 0.5, effective pile length would increase 2.25%, 2.15% and 2.06%, respectively, as calculated by $(E_p/G) \times 10^{-4} = 0.5$.

It is clear that effective pile length and the Poisson ratio of soil are positive correlation. The growth rate of effective pile length increases gradually with the increase of the Poisson ratio of soil, but it seems not obvious, even the value is slightly falling. Also taking $(E_p/G) \times 10^{-4} = 0.5$ as an example, if pile diameter increased from 1.0 m to 1.8 m, effective pile length would increase 20.00%, 16.67%, 14.48% and 12.51%, respectively. Obviously, effective pile length increases gradually with the increase of pile diameter, but its growth rate reduce gradually. The reason is that the ultimate bearing capacity of single pile increases larger with pile diameter increasing, which would lead to the pile tip resistance developing at a larger proportion.

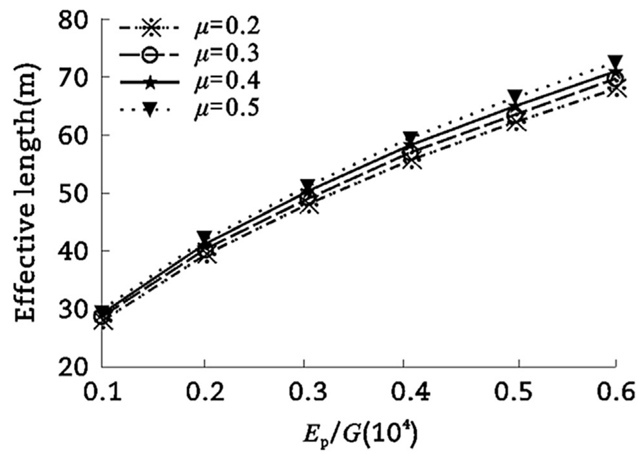

Fig. 5 – Relationship between effective pile length and Poisson ratio.

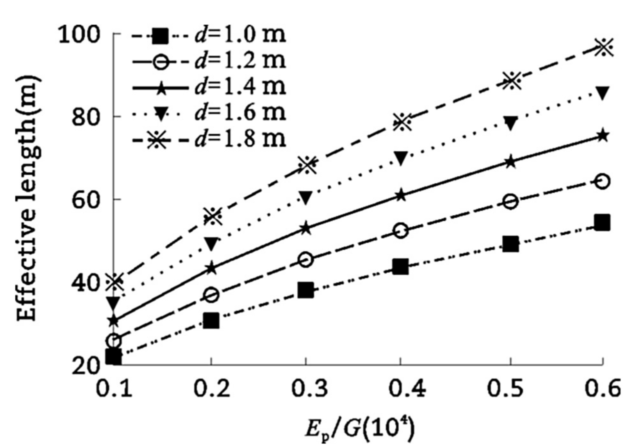

Fig. 6 – Relationship between effective pile length and pile diameter.

In a word, effective pile length is proportional to both Poisson ratio of soil and pile diameter. Compared to the Poisson ratio of soil, the influence of pile diameter on effective pile length is more significant. Thus, the influence of pile diameter cannot be ignored in the calculation of effective pile length.

5. Analysis of engineering example

There is a test pile in test area of an airport expressway, which is marked as S1. The pile diameter and length is 1.3 m and 44 m, respectively. The pile is a classified large diameter pile. The pore-forming is done by auger drill, and the pile body is poured by C25 concrete. The strata crossed by the test pile is mainly made of medium sand and coarse sand, which are mixed with a mild clay layer. The distribution curve of pile shaft resistance can be obtained through a vertical static load test on test pile, which is shown in Fig. 7.

According to the strata condition of the field test area, the calculation parameters of effective pile length are aggregated in Table 2. The effective pile length can be obtained by bringing these parameters into the Eq. (24).

The calculation result basically fits with the test result. The value of effective pile length can be obtained if the calculation parameters are applied to the Eqs. (1)–(4). The values of effective pile lengths are 69.70, 85.12, 74.32, and 71.65 m, respectively. Compared with the calculation result of Eq. (24), the differences between these values and the test piles are too large. It can be seen that the calculation Eq. (24) of effective pile length deduced by this paper is more reliable and reasonable.

Table 2 – Summary of calculation parameters of effective pile length.

Parameter	Pile diameter (m)	Poisson ratio of soil	Elastic modulus of pile (MPa)	Share modules of soil (MPa)
Value	1.3	0.27	2.8×10^4	14.2
Result	$l_e = 39.70$ m			

6. Conclusions

(1) The distribution of pile shaft resistance is assumed to be parabolic, and the influencing factors are considered. Based on the Mindlin's displacement solution, the calculation formula of large diameter bored pile's effective length is derived. The way how the Poisson ratio and pile diameter influence the effective pile length and the changing trend are analyzed. The Poisson ratio and pile diameter are positively correlated with effective pile length, and the effect of pile diameter is more significant. Thus, the influence of pile diameter cannot be ignored.

(2) Comparing with the test data, the calculation result of pile shaft resistance considering the effect of pile diameter fits well with the test data. It is more reasonable than other calculation methods. Because the calculation parameters are less, the formula is more application in practical engineering calculation.

(3) The accuracy of calculation results is related to the pile–soil mechanical model and its assumptions. In order to further improve the calculation accuracy, the

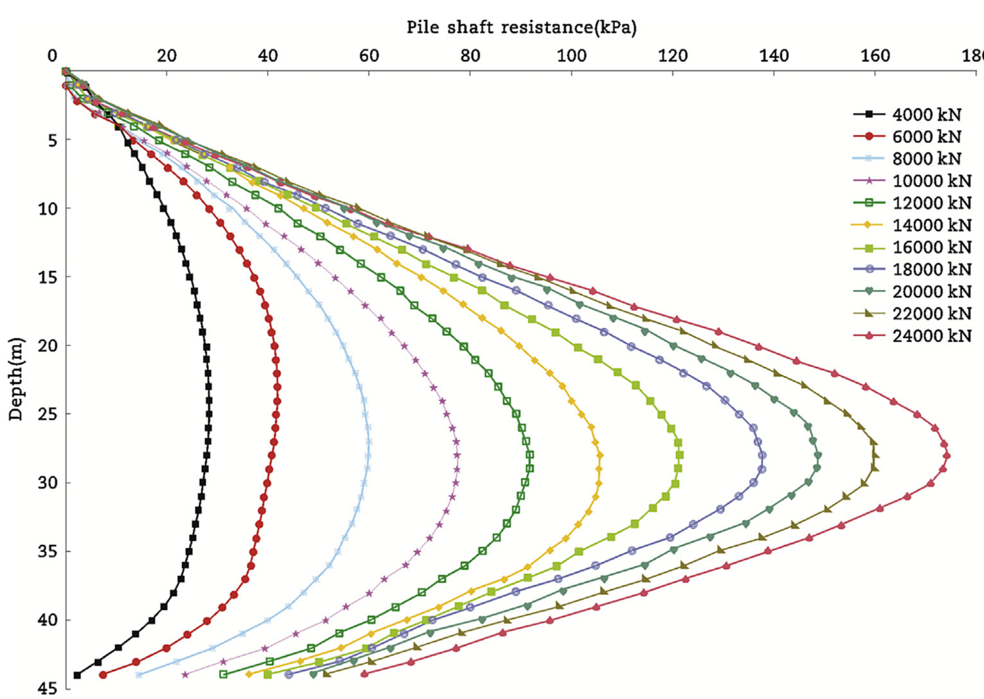

Fig. 7 – Distribution curve of S1 pile shaft resistance.

quantity of filed test and the research of pile–soil interaction should be strengthen.

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