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A simply constructed third-order modifications of Newton's method

Changbum Chun

School of Liberal Arts, Korea University of Technology and Education, Cheonan City, Chungnam 330-708, Republic of Korea

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Abstract

In this paper, we present a simple and easily applicable approach to construct some third-order modifications of Newton's method for solving nonlinear equations. It is shown by way of illustration that existing third-order methods can be employed to construct new third-order iterative methods. The proposed approach is applied to the classical Chebyshev–Halley methods to derive their second-derivative-free variants. Numerical examples are given to support that the methods thus obtained can compete with known third-order methods.

MSC: 65H05; 65B99

Keywords: Newton's method; Iterative methods; Nonlinear equations; Order of convergence; Chebyshev–Halley methods; Root-finding methods

1. Introduction

Solving nonlinear equations is one of the most important problems in numerical analysis. To solve nonlinear equations, iterative methods such as Newton's method are usually used. Throughout this paper we consider iterative methods to find a simple root α, i.e., $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, of a nonlinear equation $f(x) = 0$ that uses f and f' but not the higher derivatives of f.

Newton's method for the calculation of α is probably the most widely used iterative methods defined by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{1}$$

It is well known [14] that this method is quadratically convergent.

In recent years, many modifications of the Newton method that do not require the computation of second derivatives have been developed and analyzed, see [1–4,7–10,12–15] and references therein.

One classical third-order modification of Newton's method is given [12] by

$$x_{n+1} = x_n - \frac{f(x_n) + f(y_n)}{f'(x_n)}, \tag{2}$$

E-mail address: cbchun@kut.ac.kr.

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where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{3}$$

this notation will be used throughout.

Another third-order variant of Newton's method appeared in [15] where rectangular and trapezoidal approximations to the integral in Newton's theorem

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t)\,\mathrm{d}t \tag{4}$$

were considered to rederive Newton's method and to obtain the cubically convergent method

$$x_{n+1} = x_n - \frac{2f(x_n)}{f'(x_n) + f'(y_n)}, \tag{5}$$

respectively.

Frontini and Sormani [4] considered the midpoint rule for the integral of (4) to obtain the third-order method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'((x_n + y_n)/2)}. \tag{6}$$

In [8], Homeier derived the following cubically convergent iteration scheme:

$$x_{n+1} = x_n - \frac{f(x_n)}{2}\left(\frac{1}{f'(x_n)} + \frac{1}{f'(y_n)}\right) \tag{7}$$

by applying Newton's theorem to the inverse function $x = f(y)$ instead of $y = f(x)$.

There exists another third-order method often called Newton–Steffensen method [13] given by

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f'(x_n)[f(x_n) - f(y_n)]}. \tag{8}$$

More recently, new third-order methods have been proposed and analyzed in [3] by applying a modified finite difference approximation to Osada's result [11], two of which are given by

$$x_{n+1} = x_n - \frac{3}{2}\frac{f(x_n)}{f'(x_n)} + \frac{1}{2}\frac{f(x_n)f'(y_n)}{f'(x_n)^2}, \tag{9}$$

and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{1}{2}\frac{f'(x_n) - f'(y_n)}{f(x_n) + f'(x_n)}\frac{f(x_n)}{f'(x_n)}, \tag{10}$$

respectively.

It has been shown that the above-mentioned methods are comparable and can be competitive to Newton's method in the performance and efficiency. Now that many efficient third-order methods have appeared in open literature, it would be desirable to have a simple approach to make full use of them in devising new modifications of Newton's method, this is the main purpose of this paper. By way of illustration, we show that any pair of existing third-order iteration formulas may be used to construct new third-order methods. The methods thus obtained are proven to be third-order convergent, and several numerical examples are given to support that they can be competitive in performance with other known third-order methods.

2. Development of methods and convergence analysis

For the sake of simplicity and illustration, let us consider the third-order methods defined by (7) and (8), respectively. To derive the methods, we approximately equate the correcting terms of both methods to obtain the following approximate expression:

$$\frac{f(x_n)^2}{f'(x_n)[f(x_n)-f(y_n)]} \approx \frac{f(x_n)}{2}\left(\frac{1}{f'(x_n)}+\frac{1}{f'(y_n)}\right), \tag{11}$$

this gives a new approximation

$$f'(y_n) \approx \frac{f'(x_n)[f(x_n)-f(y_n)]}{f(x_n)+f(y_n)}. \tag{12}$$

We then apply the approximation (12) to any other iterative method depending on $f'(y_n)$, which will be shown by way of illustration with some known third-order methods.

Using (12) in (5), we obtain the known third-order method given in (2)

$$x_{n+1} = x_n - \frac{f(x_n)+f(y_n)}{f'(x_n)}. \tag{13}$$

Using (12) in (9) and (10), we obtain new modifications of Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)+2f(y_n)}{f(x_n)+f(y_n)}\frac{f(x_n)}{f'(x_n)} \tag{14}$$

and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f(x_n)f(y_n)}{[f(x_n)+f(y_n)][f(x_n)+f'(x_n)]}, \tag{15}$$

respectively.

We can substitute the new approximation (12) in other third-order methods depending on $f'(y_n)$ to find more modifications of Newton's method.

It has been shown that the Maple package can be successfully employed to rederive error equations of iterative methods, that is, to find their order of convergence (see [1] for details), the methods (14) and (15) in this case are found to be third-order as shown in the following theorem.

Theorem 2.1. *Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f: I \to \mathbf{R}$ for an open interval I. If x_0 is sufficiently close to α, then the methods defined by* (14) *and* (15) *have third-order convergence and satisfy the error equations*

$$e_{n+1} = 3c_2^2 e_n^3 + O(e_n^4), \tag{16}$$

and

$$e_{n+1} = c_2(1+3c_2)e_n^3 + O(e_n^4), \tag{17}$$

respectively, where $e_n = x_n - \alpha$ and $c_k = f^{(k)}(\alpha)/k!f'(\alpha)$.

Repeating the above process with other pairs of third-order methods, we can find other approximations to $f'(y_n)$. For example, if we approximately equate the correcting terms of the methods defined by (5) and (7), we obtain

$$\frac{2f(x_n)}{[f'(x_n)+f'(y_n)]} \approx \frac{f(x_n)}{2}\left(\frac{1}{f'(x_n)}+\frac{1}{f'(y_n)}\right), \tag{18}$$

this gives another new approximation to $f'(y_n)$

$$f'(y_n) \approx \frac{f'(x_n)^2}{2f'(x_n)-f'(y_n)}. \tag{19}$$

Using (19) in (9) and (10), we obtain new modifications of Newton's method

$$x_{n+1} = x_n - \frac{3}{2}\frac{f(x_n)}{f'(x_n)} + \frac{1}{2}\frac{f(x_n)}{2f'(x_n) - f'(y_n)} \tag{20}$$

and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{1}{2}\frac{f(x_n)[f'(x_n) - f'(y_n)]}{[f(x_n) + f'(x_n)][2f'(x_n) - f'(y_n)]}, \tag{21}$$

respectively.

By the help of the Maple package, it can also be shown that the methods (20) and (21) are both third-order convergent.

Theorem 2.2. *Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f: I \to \mathbf{R}$ for an open interval I. If x_0 is sufficiently close to α, then the methods defined by* (20) *and* (21) *are third-order convergent and satisfy the error equations*

$$e_{n+1} = [4c_2^2 + \tfrac{1}{2}c_3]e_n^3 + \mathrm{O}(e_n^4), \tag{22}$$

and

$$e_{n+1} = [c_2 + 4c_2^2 + \tfrac{1}{2}c_3]e_n^3 + \mathrm{O}(e_n^4), \tag{23}$$

respectively, *where $e_n = x_n - \alpha$ and $c_k = f^{(k)}(\alpha)/k!f'(\alpha)$.*

The classical Chebyshev–Halley methods [5] which improve Newton's method are given by

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}\frac{L_f(x_n)}{1 - \beta L_f(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \tag{24}$$

where

$$L_f(x_n) = \frac{f''(x_n)f(x_n)}{f'(x_n)^2}. \tag{25}$$

This family is known to converge cubically, and includes, as particular cases, the classical Chebyshev's method ($\beta = 0$), Halley's method ($\beta = \frac{1}{2}$) and super-Halley method ($\beta = 1$) (see [5–7,14] for more details). It is observed that the methods depend on the second derivatives in computing process, this making its practical utility restricted rigorously, so that the methods that do not require the computation of second derivatives would be desired. In what follows the idea presented in the above is applied to derive second-derivative-free variants of Chebyshev–Halley methods.

To derive a second-derivative-free approximation to $f''(x_n)$ in (24), let us consider any third-order method requiring $f''(x_n)$, for example, the Cauchy method [14] defined by

$$x_{n+1} = x_n - \frac{2}{1 + \sqrt{1 - 2L_f(x_n)}}\frac{f(x_n)}{f'(x_n)}, \tag{26}$$

where $L_f(x_n)$ is defined by (25), and any second-derivative-free third-order method, for example, the method of Weerakoon–Fernando defined by (5).

Approximately equating the correcting terms of the methods (5) and (26), we have

$$\frac{1 + \sqrt{1 - 2L_f(x_n)}}{2}f'(x_n) \approx \frac{f'(x_n) + f'(y_n)}{2}. \tag{27}$$

Using (27) we can approximate after simplifying

$$f''(x_n) \approx \frac{f'(x_n)^2 - f'(y_n)^2}{2f(x_n)}. \tag{28}$$

Using (28) in (24), we obtain a new one-parameter family of methods free from second derivative

$$x_{n+1} = x_n - \frac{1}{2} \cdot \frac{(5-2\beta) f'(x_n)^2 + (2\beta - 1) f'(y_n)^2}{(2-\beta) f'(x_n)^2 + \beta f'(y_n)^2} \cdot \frac{f(x_n)}{f'(x_n)}. \tag{29}$$

For the methods defined by (29), we have

Theorem 2.3. *Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f: I \to \mathbf{R}$ for an open interval I. If x_0 is sufficiently close to α, then the methods defined by* (29) *have third-order convergence for any $\beta \in \mathbf{R}$, and satisfy the error equation*

$$e_{n+1} = [\tfrac{1}{2} c_3 + (3 - 2\beta) c_2^2] e_n^3 + \mathrm{O}(e_n^4), \tag{30}$$

where $e_n = x_n - \alpha$ and $c_k = f^{(k)}(\alpha)/k! f'(\alpha)$.

Proof. Let α be a simple zero of f. Consider the iteration function F defined by

$$F(x) = x - \frac{1}{2} \cdot \frac{(5-2\beta) f'(x)^2 + (2\beta - 1) f'(y(x))^2}{(2-\beta) f'(x)^2 + \beta f'(y(x))^2} \cdot \frac{f(x)}{f'(x)}, \tag{31}$$

where $y(x) = x - f(x)/f'(x)$.

In view of an elementary, tedious evaluation of derivatives of F, we employ the symbolic computation of the Maple package to compute the Taylor expansion of $F(x_n)$ around $x = \alpha$ (see [1] for details). We find after simplifying that

$$x_{n+1} = F(x_n) = \alpha + [\tfrac{1}{2} c_3 + (3 - 2\beta) c_2^2] e_n^3 + \mathrm{O}(e_n^4), \tag{32}$$

where $e_n = x_n - \alpha$ and $c_k = f^{(k)}(\alpha)/k! f'(\alpha)$. Thus,

$$e_{n+1} = [\tfrac{1}{2} c_3 + (3 - 2\beta) c_2^2] e_n^3 + \mathrm{O}(e_n^4), \tag{33}$$

which indicates that the order of convergence of the methods defined by (29) is at least 3. This completes the proof. □

The family (29) includes, as particular cases, the following ones:

For $\beta = 0$, we obtain a new third-order variant of Chebyshev's method

$$x_{n+1} = x_n - \frac{1}{4} \left[5 - \frac{f'(y_n)^2}{f'(x_n)^2} \right] \cdot \frac{f(x_n)}{f'(x_n)}. \tag{34}$$

For $\beta = \frac{1}{2}$, we obtain a new third-order variant of Halley's method

$$x_{n+1} = x_n - \frac{4 f'(x_n) f(x_n)}{3 f'(x_n)^2 + f'(y_n)^2}. \tag{35}$$

For $\beta = 1$, we obtain a new third-order variant of super-Halley's method

$$x_{n+1} = x_n - \frac{1}{2} \cdot \frac{3 f'(x_n)^2 + f'(y_n)^2}{f'(x_n)^2 + f'(y_n)^2} \cdot \frac{f(x_n)}{f'(x_n)}. \tag{36}$$

For $\beta = \frac{3}{2}$, we obtain a new third-order method

$$x_{n+1} = x_n - \frac{2[f'(x_n)^2 + f'(y_n)^2]}{f'(x_n)^2 + 3 f'(y_n)^2} \cdot \frac{f(x_n)}{f'(x_n)}. \tag{37}$$

For $\beta = 3$, we obtain a new third-order method

$$x_{n+1} = x_n - \frac{1}{4} \cdot \left[3 + \frac{f'(x_n)^2}{f'(y_n)^2} \right] \cdot \frac{f(x_n)}{f'(x_n)}. \tag{38}$$

In a similar fashion, the proposed approach can be continuously applied to produce various types of approximations to the other terms of iterative methods that we have considered such as $f(y_n)$, $f'(\frac{1}{2}(x_n + y_n))$ and $f''(x_n)$, which can in turn be used to derive some third-order iterative methods. It is observed that the midpoint method (6) can be obtained by using the midpoint value $f'(\frac{1}{2}(x_n + y_n))$ instead of the arithmetic mean of $f'(x_n)$ and $f'(y_n)$ in the method of Weerakoon and Fernando (5). It is worth mentioning that this aspect was observed in [10], and applied to Homeier's method (7) to obtain a modification of Newton's method. However, it should be emphasized that their result is just a special case of our idea presented in this contribution.

3. Numerical examples

All computations were done using MAPLE using 64 digit floating point arithmetics (Digits := 64). We accept an approximate solution rather than the exact root, depending on the precision (ε) of the computer. We use the following stopping criteria for computer programs: (i) $|x_{n+1} - x_n| < \varepsilon$, (ii) $|f(x_{n+1})| < \varepsilon$, and so, when the stopping criterion is satisfied, x_{n+1} is taken as the exact root α computed. For numerical illustrations in this section we used the fixed stopping criterion $\varepsilon = 10^{-15}$.

We present some numerical test results for various cubically convergent iterative schemes in Table 1. Compared were the method defined by (2) (PM), the method of Weerakoon and Fernando defined by (5) (WF), the midpoint rule defined by (6) (MP), Homeier's method defined by (7) (HM), the method of Kou et al. (KM) [9], and the methods (14) (CM1), (15) (CM2), (20) (CM3), and (21) (CM4) introduced in the present contribution. We used the test functions and display the approximate zeros x_* found up to the 28th decimal places

$$\begin{aligned}
&f_1(x) = x^3 + 4x^2 - 10, \quad x_* = 1.3652300134140968457608068290,\\
&f_2(x) = \sin^2 x - x^2 + 1, \quad x_* = 1.4044916482153412260350868178,\\
&f_3(x) = x^2 - e^x - 3x + 2, \quad x_* = 0.2575302854398607604553673049 4,\\
&f_4(x) = \cos x - x, \quad x_* = 0.7390851332151606416553120876 7,\\
&f_5(x) = (x-1)^3 - 1, \quad x_* = 2,\\
&f_6(x) = xe^{x^2} - \sin^2 x + 3\cos x + 5, \quad x_* = -1.2076478271309189270094167584,\\
&f_7(x) = \sin x - x/2, \quad x_* = 1.8954942670339809471440357381,\\
&f_8(x) = (x^3 + 4x^2 - 10)^2, \quad x_* = 1.3652300134140968457608068290.
\end{aligned}$$

As convergence criterion, it was required that the distance of two consecutive approximations δ for the zero was less than 10^{-15}. Also displayed are the number of iterations to approximate the zero (IT), the value $f(x_*)$ and the computational order of convergence (COC).

The test results in Table 1 show that for most of the functions we tested, the methods introduced in the present presentation have at least equal performance compared to the other third-order methods, and can also compete with Newton's method. It can be observed that for the function f_8 having a repeated zero, all of the third-order methods under consideration show linear convergence even if the initial guess $x_0 = 1.4$ is rather close to zero as in Newton's method, which is well known.

We also present some numerical test results for various Chebyshev–Halley methods, their variants and the Newton method in Table 2. Compared were the Newton method (NM), Chebyshev's method (CHM), Halley's method (HM), super-Halley's method (SHM), the variant of Chebyshev's method defined by (34) (VCHM), the variant of Halley's method defined by (35) (VHM) and the variant of super-Halley's method defined by (36) (VSHM). All computations were done using MAPLE using 128 digit floating point arithmetics (Digits := 128). Displayed in Table 2 is the number of iterations (IT) required such that $|f(x_n)| < 10^{-32}$.

Table 1
Comparison of various cubically convergent iterative schemes

	IT	$f(x_*)$	δ
$f_1, x_0 = 1.5$			
PM	4	0	2.99e − 29
WF	4	0	3.37e − 32
MP	4	0	3.36e − 33
HM	4	0	4.4e − 43
KM	4	0	8.82e − 31
CM1	4	−1.3e − 62	3.3e − 27
CM2	4	0	1.57e − 24
CM3	4	0	1.19e − 25
CM4	4	−1.3e − 62	−1.39e − 23
$f_2, x_0 = 2.0$			
PM	5	1.3e − 63	1.39e − 33
WF	5	−2.0e − 63	6.02e − 42
MP	5	−2.0e − 63	7.11e − 41
HM	4	−2.0e − 63	1.08e − 24
KM	5	−2.0e − 63	5.29e − 31
CM1	5	1.3e − 63	1.89e − 29
CM2	5	−2.0e − 63	9.23e − 25
CM3	5	1.3e − 63	7.11e − 27
CM4	5	1.3e − 63	3.55e − 23
$f_3, x_0 = 2$			
PM	5	0	3.23e − 42
WF	5	1.0e − 63	1.62e − 34
MP	4	1.0e − 63	3.95e − 24
HM	5	0	9.33e − 43
KM	5	1.0e − 63	5.51e − 29
CM1	5	−1.0e − 63	3.81e − 38
CM2	5	0	6.06e − 30
CM3	5	1.0e − 63	1.23e − 27
CM4	5	1.0e − 63	2.12e − 31
$f_4, x_0 = 1.7$			
PM	4	0	1.75e − 24
WF	4	1.0e − 64	1.04e − 21
MP	4	−3.32e − 61	1.45e − 20
HM	4	−5.02e − 59	9.64e − 20
KM	5	0	2.36e − 33
CM1	4	0	4.91e − 23
CM2	4	−6.67e − 50	4.77e − 17
CM3	4	9.99e − 60	3.33e − 20
CM4	4	−1.88e − 57	1.14e − 19
$f_5, x_0 = 3.5$			
PM	6	0	1.84e − 28
WF	6	0	3.28e − 37
MP	6	0	1.26e − 42
HM	5	0	1.46e − 24
KM	6	0	2.50e − 35
CM1	6	0	8.19e − 23
CM2	6	1.11e − 49	2.1e − 17
CM3	6	1.32e − 54	4.72e − 19
CM4	6	0	1.04e − 43
$f_6, x_0 = -2$			
PM	7	−4.0e − 63	3.34e − 35
WF	7	−4.0e − 63	3.11e − 44
MP	6	−4.0e − 63	2.12e − 23
AM	6	−4.0e − 63	4.35e − 45

Table 1 (*continued*)

	IT	$f(x_*)$	δ
HM	6	$-4.0e-63$	$2.57e-32$
KM	6	$-4.0e-63$	$8.87e-34$
CM1	7	$-4.0e-63$	$3.63e-26$
CM2	7	$-4.0e-63$	$9.5e-33$
CM3	7	$-4.05e-56$	$5.84e-20$
CM4	7	$-4.0e-63$	$9.96e-24$
$f_7, x_0 = 2.3$			
PM	4	$-7.63e-52$	$1.12e-17$
WF	4	$-3.0e-64$	$1.13e-21$
MP	4	$-1.39e-59$	$3.64e-20$
HM	4	$-3.0e-64$	$2.22e-38$
KM	4	$-3.70e-46$	$8.27e-16$
CM1	4	$-1.01e-46$	$5.0e-16$
CM2	5	$-3.0e-64$	$1.07e-39$
CM3	4	$-5.0e-64$	$8.32e-44$
CM4	5 8	$-3.0e-64$	$4.62e-38$
$f_8, x_0 = 1.4$			
PM	33 (COC: 1.21)	$2.73e-29$	$5.27e-16$
WF	30 (COC: 1.18)	$8.46e-30$	$3.52e-16$
MP	30 (COC: 1.18)	$8.25e-30$	$3.48e-16$
HM	24 (COC: 1.12)	$4.55e-30$	$3.87e-16$
KM	33 (COC: 1.21)	$2.63e-29$	$5.18e-16$
CM1	35 (COC: 1.22)	$4.94e-29$	$6.39e-16$
CM2	35 (COC: 1.22)	$5.01e-29$	$6.43e-16$
CM3	37 (COC: 1.24)	$2.61e-29$	$4.33e-16$
CM4	37 (COC: 1.24)	$2.64e-29$	$4.35e-16$

Table 2
Comparison of various Chebyshev–Halley type methods and Newton's method

$f(x)$	IT						
	NM	CHM	HM	SHM	VCHM	VHM	VSHM
$f_1, x_0 = 2$	7	5	5	5	5	5	5
$f_1, x_0 = 1$	7	5	5	5	5	5	5
$f_2, x_0 = 2.3$	8	6	6	5	6	6	5
$f_2, x_0 = 2$	8	6	5	5	6	5	5
$f_3, x_0 = 0$	6	4	4	4	5	4	4
$f_3, x_0 = 1$	6	5	5	5	5	5	5
$f_4, x_0 = 1.7$	7	5	5	5	5	5	5
$f_4, x_0 = 1$	6	5	5	5	5	4	4
$f_5, x_0 = -1$	8	5	5	5	7	6	5
$f_5, x_0 = -1.5$	8	6	5	6	6	6	6
$f_6, x_0 = 3.5$	8	5	5	5	6	6	5
$f_6, x_0 = 3.4$	10	7	6	5	8	7	7
$f_7, x_0 = 1.6$	7	5	5	5	6	5	5
$f_7, x_0 = 2$	6	5	4	4	5	5	4

The results presented in Table 2 show that for most of the functions we tested, the variants introduced in the present presentation have equal performance as compared to the corresponding classical methods that do require the computation of second derivatives, and also converge more rapidly than Newton's method.

4. Conclusions

In this work we presented a simple approach to construct some modifications of Newton's method from known third-order methods and some second-derivative variants of the Chebyshev–Halley methods. It has been proved that they are third-order convergent. Some of the obtained methods were compared in performance to the other known third-order methods and the classical Chebyshev–Halley methods, and it was observed that they have at least equal performance. Our approach may be continuously applied to obtain as many new methods, not just restricted to third-order by doing exactly the same way as we did in this contribution.

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