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One-loop effective potential from higher-dimensional AdS black holes

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Abstract

We study the quantum effects in a brane-world model in which a positive constant curvature brane universe is embedded in a higher-dimensional bulk AdS black hole, instead of the usual portion of the AdS_5. By using zeta regularization, in the large mass regime, we explicitly calculate the one-loop effective potential due to the bulk quantum fields and show that it leads to a non-vanishing cosmological constant, which can definitely acquire a positive value.

1. Introduction

The study of quantum effects in the brane-world has been a subject of quite some activity recently. In particular, the one-loop effective potential for bulk quantum fields has been calculated, for the case when the bulk space is the 5-dimensional AdS and the brane is flat [1]. This is one of the directions which relates the AdS/CFT correspondence [2] with the brane-world paradigms [3]. (It is interesting to note that the one-loop potential obtained from the bulk space may be useful in the development of the holographic renormalization group (RG) in the AdS/CFT set up [4–6].) Among the applications of a calculation of this type, one can envisage radion stabilization and the derivation of an induced cosmological constant. Subsequently, this kind of calculation has been generalized to the case when the brane is de Sitter space [7–9], which is an interesting example since this setup seems to be the one corresponding to our observable universe. The main technical ingredient in our calculation, which allows to carry it explicitly to the end, will be zeta-function regularization [10,11].

In view of the recent developments on cosmological models, it seems interesting to generalize the study of quantum effects due to bulk fields to different sorts of bulk spaces. Indeed, if our universe is a brane-world of any kind, it is unclear a priori what is the right bulk space. For example, it is already known that brane gravity trapping occurs in an AdS 5-dimensional black hole [12–15] in just the same way as in the Randall–Sundrum model [3].

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Moreover, in the AdS/CFT correspondence, the case of a bulk AdS black hole represents a different phase of the same theory and there is the exciting connection that a transition between an ordinary bulk AdS and a bulk AdS black hole corresponds to the confinement–deconfinement transition in the dual CFT [16]. Thus, it appears quite natural to expect that, at some epoch or other of its evolution, our brane universe may have been embedded into a bulk AdS black hole.

The purpose of this Letter is—by the way of a calculable explicit example—to give consistency to the ideas above. With this aim, we consider the physical de Sitter brane universe to be embedded into a higher-dimensional AdS black hole. We show that, by using zeta-regularization techniques, one can explicitly calculate the one-loop effective potential due to the bulk quantum fields. For the sake of simplicity, a bulk scalar will be here considered, but the calculation could be easily extended to other cases. The one-loop effective potential that we shall obtain will then be compared with the potential found in the case when the bulk is a pure AdS space. Such one-loop effective potential may be actually responsible for the dominant contribution to the brane cosmological constant during some period of the evolution of our brane-world. One should note on passing that applications of the one-loop effective potential from a bulk AdS black hole to inducing the correct hierarchy does not look so interesting, since such kind of background does not seem at present to be able to provide a natural solution of this problem [17].

To begin with, as a background bulk space, we may consider an asymptotically AdS generic black hole solution, in $D = 2 + d$ dimension, with Euclidean time τ and extra radial coordinate r. The metric reads [18–21]

$$ds^2 = g_{MN}\, dx^M\, dx^N = A(r)\, d\tau^2 + \frac{1}{A(r)} dr^2 + r^2\, d\Sigma_d^2, \tag{1}$$

where $M, N = 1, \dots, D$, Σ_d is the constant curvature brane space–time and

$$A(r) = k + \frac{r^2}{\ell^2} - \frac{r_0^{d+1}}{\ell^2 r^{d-1}}\,. \tag{2}$$

Here $k = 0, 1, -1$ for Minkowski, de Sitter and anti-de Sitter space, respectively, ℓ is related to the bulk cosmological constant, and r_0 is a typical length parameter, which depends on the mass of the black hole and on the bulk Newton constant. In the non-extremal case, the function $A(r)$ has a simple zero at $r = r_H$, which is the minimum (positive) value admissible for r. For the particular case $k = 0$, one trivially has $r_H = r_0$.

2. Near-horizon approximation

As is well known, one-loop calculations in a generic black-hole background are hard, if not impossible, to perform exactly. One is compelled to make use of some approximation. Here, we will investigate quantum one-loop effects in the so called near-horizon approximation. This one proves to be a good approximation to the exact problem in hand as far as the black hole mass—in our case the length parameter r_H—is sufficiently large. In this situation the bulk black hole space–time becomes a Rindler-like space–time and the statistical mechanics of the black holes can be investigated in detail (for a further discussion see, for example [22]).

In fact, defining the new coordinates, ρ and θ, by means of

$$r = r_H + \frac{A'(r_H)}{4}\rho^2, \qquad \tau = \frac{2}{A'(r_H)}\theta, \tag{3}$$

one gets

$$ds^2 \sim \rho^2\, d\theta^2 + d\rho^2 + r_H^2\, d\Sigma_d^2. \tag{4}$$

As usual, in order to avoid the conical singularity at $\rho = 0$, one has to require the coordinate θ to be periodic with a period of 2π. This corresponds to a period β of the Euclidean time τ given by the Hawking condition

$$\beta = \frac{4\pi}{A'(r_H)}. \tag{5}$$

In this way, the space–time becomes locally $R_2 \times \Sigma_d$, this is to say, the Euclidean version of a Rindler-like space–time.

Consider then the action for a scalar with scalar-gravitational coupling in the bulk, e.g.,

$$\mathcal{S} = \frac{1}{2}\int d^D x\,\sqrt{g}\big[g^{MN}\partial_M\phi\partial_N\phi + m^2\phi^2 + \xi R\phi^2\big], \tag{6}$$

m being the mass, ξ the constant coupling with gravitation, and R the scalar curvature of the whole manifold. The latter action can be rewritten as

$$\mathcal{S} = \frac{1}{2}\int d^D x\,\sqrt{g}\phi L\phi, \quad L = L_D + M^2 = L_2 + L_d + M^2, \tag{7}$$

where $L_D = L_2 + L_d$ is a Laplacian-like operator on the D-dimensional manifold, while

$$L_2\phi \equiv -\frac{1}{\rho^2}\partial_\theta^2\phi - \frac{1}{\rho}\partial_\rho(\rho\partial_\rho\phi) \tag{8}$$

is the flat Laplacian in 2 dimensions,

$$L_d\phi \equiv \left[-\nabla_d^2 + \frac{(d-1)^2}{4r_H^2}\right]\phi \tag{9}$$

is a Laplacian-like operator (with a particular non-minimal coupling) on the constant curvature space–time Σ_d and, finally,

$$M^2 = m^2 + \xi R - \frac{(d-1)^2}{4r_H^2} = m^2 + \frac{1}{r_H^2}\left[\xi d(d-1) - \frac{(d-1)^2}{4}\right] \tag{10}$$

is a constant term. For computational reason, we have added and subtracted in expressions (9) and (10) the constant term $(d-1)^2/(4r_H^2)$.

3. One-loop effective potential

Since we are interested in the effective potential, we have to compute the heat-kernel trace and then, via the Mellin transform, the zeta function corresponding to the operator L. In fact, in the zeta-function regularization scheme, the one-loop contribution to the effective potential is given by

$$V^{(1)} = -\frac{\zeta'(0|L/\mu^2)}{2V_D} = -\frac{\zeta'(0|L) + \log\mu^2\zeta(0|L)}{2V_D}, \tag{11}$$

V_D being the volume of the manifold and μ a free parameter, which one has to introduce for dimensional reasons. It must be fixed by renormalization. Following Ref. [23], we write the effective potential in the form

$$V_{\text{eff}} = V_r(\mu) + V^{(1)}(\mu), \tag{12}$$

where $V_r(\mu)$ is the renormalized vacuum energy. The effective potential is a physical observable and for this reason it cannot depend on the choice of the arbitrary scale parameter μ. This means that it has to satisfy the renormalization condition [23]

$$\mu\frac{dV_{\text{eff}}}{d\mu} = 0. \tag{13}$$

From the latter equation we determine $V_r(\mu)$ up to an integration constant $V_r(\mu_0)$, which we choose to be vanishing, and thus obtain in this way the renormalization point μ_0. After such an operation one finally gets the renormalized effective potential in the form

$$V_{\rm eff}=-\frac{\zeta'(0|L/\mu_0^2)}{2V_D}=-\frac{\zeta'(0|L)+\log\mu_0^2\zeta(0|L)}{2V_D}. \tag{14}$$

In the approximation we are considering, L_2 and L_d commute, thus (here t is the heat-kernel parameter)

$$\mathrm{Tr}\,e^{-tL}=\mathrm{Tr}\,e^{-tL_2}\,\mathrm{Tr}\,e^{-tL_d}e^{-tM^2}. \tag{15}$$

Let us put the branes at say $\rho=\rho_1$ and $\rho=\rho_2$ (with $\rho_1>\rho_2$). Then the eigenfunctions and the eigenvalues of L_2 are given by

$$-L_2\phi=\lambda^2\phi,\quad \phi=\mathrm{e}^{in\theta}\big(\alpha_n J_n(\lambda\rho)+\beta_n N_n(\lambda\rho)\big),\ \lambda=\lambda_n\ (n=0,\pm1,\pm2,\ldots). \tag{16}$$

Here J_n and N_n are Bessel and the Neumann functions, respectively. If, for simplicity, we impose Dirichlet boundary condition at the branes, that is

$$\phi(\rho_1)=\phi(\rho_2)=0, \tag{17}$$

then, the eigenvalues λ_n are implicitly given by the equation

$$J_n(\lambda_n\rho_1)N_n(\lambda_n\rho_2)=J_n(\lambda_n\rho_2)N_n(\lambda_n\rho_1). \tag{18}$$

For simplicity, now let us compute the effective potential for the scalar field in the bulk with only one brane. In fact, it is not difficult to realize that the two-brane case does not add any additional physical insight (although the calculation is rather more involved). We put the brane at $\rho=\rho_0$, the bulk space being defined by $\rho<\rho_0$. Thus, it turns out that the eigenfunctions and eigenvalues of L_2 are given by

$$L_2\phi=\lambda^2\phi,\quad \phi=\mathrm{e}^{in\theta}J_n(\lambda\rho),\ \lambda=\lambda_{n,k}=\frac{j_{n,k}}{\rho_0}\ (n=0,\pm1,\pm2,\ldots). \tag{19}$$

Here $j_{n,k}$ are the zeros of the Bessel functions $J_n(x)$: $J_n\left(j_{n,k}\right)=0$.

4. Large mass expansion

As it always happens in a situation of this kind, it is not possible to do the calculation exactly in a closed way, valid in the whole range of parameters of the problem. For different domains of the parameters a different expansion must be chosen. Here, we choose to perform an expansion for large values of the constant term M^2 (with respect to the renormalization point μ_0^2),[1] since this one corresponds to the most interesting (and natural) situation from the physical viewpoint. In this situation the zeta function acquires the form

$$\zeta(s|L)\sim\sum_{r=0}^{\infty}K_r(L_D)\frac{\Gamma(s+(r-D/2))}{\Gamma(s)}M^{D-r-2s}. \tag{20}$$

The latter expression directly follows from the Mellin-like transform

$$\zeta(s)=\frac{1}{\Gamma(s)}\int_0^\infty dt\,t^{s-1}\,\mathrm{Tr}\,e^{-tL}, \tag{21}$$

[1] Which will set up, all the way from now, the unit in which M^2 is to be measured.

by using the heat-kernel expansion

$$\operatorname{Tr} e^{-tL} \sim \sum_{r=0}^{\infty} e^{-tM^2} K_r(L_D) t^{(r-D)/2}, \tag{22}$$

$K_r(L_D)$ being the Seeley–De Witt coefficients corresponding to the operator L_D.

A direct computation gives

$$\zeta(0|L) = K_D(L) = \sum_{n \leqslant D \text{ even}} \frac{(-1)^{n/2} K_{D-n}(L_D) M^n}{(n/2)!}, \tag{23}$$

while

$$\begin{aligned} \zeta'(0|L) = {} & \sum_{n \leqslant D \text{ even}} \frac{(-1)^{n/2} K_{D-n}(L_D) M^n}{(n/2)!} \left[-\log M^2 + \gamma + \psi\left(1 + \frac{n}{2}\right) \right] \\ & + \sum_{n \leqslant D \text{ odd}} \Gamma\left(-\frac{n}{2}\right) K_{D-n}(L_D) M^n + O\left(\frac{1}{M}\right), \end{aligned} \tag{24}$$

γ being the Euler–Mascheroni constant and ψ the digamma function (logarithmic derivative of the Γ function).

The heat-kernel coefficients $K_n(L_D)$ can be given in terms of $K_i(L_2)$ and $K_j(L_d)$. In fact, since

$$\operatorname{Tr} e^{-tL_D} = \operatorname{Tr} e^{-tL_2} \operatorname{Tr} e^{-tL_d} \sim \sum_{n=0} K_n(L_D) t^{(n-D)/2} \tag{25}$$

and

$$\operatorname{Tr} e^{-tL_2} \sim \sum_{i=0} K_i(L_2) t^{(i-2)/2}, \qquad \operatorname{Tr} e^{-tL_d} = \sum_{j=0} A_j(L_d) t^{j-d/2}, \tag{26}$$

one gets

$$K_n(L_D) = \sum_{i+2j=n} K_i(L_2) A_j(L_d). \tag{27}$$

Here we have used the notation $A_j = K_{2j}$ since on a compact manifold without boundary, Σ_d, all K_j coefficients with odd index vanish. The $A_n(L_d)$ coefficients depend on the horizon manifold and, in principle, they can be computed in terms of geometric invariants. The other coefficients $K_n(L_2)$ are associated with the Dirichlet Laplacian on the disk with radius ρ_0. For dimensional reasons,[2] they have the form

$$K_i(L_2) = d_i \rho_0^{2-i}, \tag{28}$$

where d_i are numerical coefficients which can be evaluated with the help of the techniques developed in Ref. [24]. The explicit values of the ones we shall use in the following read

$$\begin{aligned} & d_0 = \frac{1}{4}, \qquad d_1 = -\frac{\sqrt{\pi}}{4}, \qquad d_2 = \frac{5}{12}, \qquad d_3 = \frac{\sqrt{\pi}}{128}, \qquad d_4 = \frac{347}{5040}, \\ & d_5 = \frac{25}{192\sqrt{\pi}} + \frac{37\sqrt{\pi}}{16384}, \qquad d_6 = \frac{602993}{5765760}. \end{aligned} \tag{29}$$

Eqs. (23) and (24) are valid in arbitrary dimensions. For cosmological applications, however, it is interesting to specify for the $D = 6$ case, which corresponds to a 4-dimensional brane as horizon manifold. From Eqs. (14), (23)

[2] Note that the dimensions of the R_i take care of the corresponding powers of M.

and (24), for $D=6$ and in the large-M limit, we obtain

$$V_{\rm eff}=\frac{1}{2V_D}\bigg[\frac{8\sqrt{\pi}}{15}K_1(L_6)M^5-\frac{4\sqrt{\pi}}{3}K_3(L_6)M^3+2\sqrt{\pi}K_5(L_6)M \\ -\frac{K_0(L_6)M^6}{6}\left(\log\frac{M^2}{\mu_0^2}-\frac{11}{6}\right)+\frac{K_2(L_6)M^4}{2}\left(\log\frac{M^2}{\mu_0^2}-\frac{3}{2}\right) \\ -K_4(L_6)M^2\left(\log\frac{M^2}{\mu_0^2}-1\right)+K_6(L_6)\log\frac{M^2}{\mu_0^2}\bigg]+O\left(\frac{1}{M}\right). \tag{30}$$

In the following we shall analyze in detail the case $k=1$, that is, the de Sitter brane, which is most promising from the cosmological viewpoint. The simplest case $k=0$, that is the Minkowski brane, will be directly obtained in the limit of vanishing curvature.

5. The de Sitter brane case

As anticipated, the de Sitter case ($k=1$) may be very interesting for cosmological applications. The results that we obtain for this case are the following.

The heat-kernel coefficients for the operator $-\nabla^2+9/4r_H^2$ on the 4-dimensional sphere can be taken, for instance, from Ref. [11]. The non-vanishing ones read

$$A_0=\frac{V_4}{16\pi^2},\qquad A_1=-\frac{V_4}{64\pi^2r_H^2},\qquad A_2=\frac{V_4}{16\pi^2r_H^4}. \tag{31}$$

From Eqs. (27) and (28) and setting $V_6=\pi\rho_0^2V_4$, it follows that

$$K_n(L_6)=\frac{V_6}{16\pi^3}\left(\frac{d_n}{\rho_0^n}-\frac{d_{n-2}}{4r_H^2\rho_0^{n-2}}+\frac{d_{n-4}}{r_H^4\rho_0^{n-4}}\right). \tag{32}$$

Now, using Eq. (30) we are able to write the final result under the explicit form

$$V_{\rm eff}=\frac{1}{\rho_0^6}\bigg[\frac{d_1(M\rho_0)^5}{60\pi^{5/2}}-\frac{(M\rho_0)^3}{24\pi^{5/2}}\left(d_3-\frac{d_1\rho_0^2}{4r_H^2}\right)+\frac{M\rho_0}{16\pi^{5/2}}\left(d_5-\frac{d_3\rho_0^2}{4r_H^2}+\frac{d_1\rho_0^4}{r_H^4}\right) \\ -\frac{d_0(M\rho_0)^6}{192\pi^3}\left(\log\frac{M^2}{\mu_0^2}-\frac{11}{6}\right)+\frac{(M\rho_0)^4}{64\pi^3}\left(d_2-\frac{d_0\rho_0^2}{4r_H^2}\right)\left(\log\frac{M^2}{\mu_0^2}-\frac{3}{2}\right) \\ -\frac{(M\rho_0)^2}{32\pi^3}\left(d_4-\frac{d_2\rho_0^2}{4r_H^2}+\frac{d_0\rho_0^4}{r_H^4}\right)\left(\log\frac{M^2}{\mu_0^2}-1\right)+\frac{1}{32\pi^3}\left(d_6-\frac{d_4\rho_0^2}{4r_H^2}+\frac{d_2\rho_0^4}{r_H^4}\right)\log\frac{M^2}{\mu_0^2}\bigg] \\ +O\left(\frac{1}{M}\right). \tag{33}$$

6. The flat brane case

The one-loop effective potential for the simplest case $k=0$ (Minkowski) can be obtained from Eq. (30) by observing that

$$K_n(L_6)=\frac{\rho_0^2V_4d_n}{16\pi^2\rho_0^n}=\frac{V_6d_n}{16\pi^3\rho_0^n}, \tag{34}$$

or, more simply, it can be obtained by taking the flat limit $r_H \to \infty$ in Eq. (33). We get

$$V_{\mathrm{eff}} = \frac{1}{\rho_0^6}\left[\frac{d_1(M\rho_0)^5}{60\pi^{5/2}} - \frac{d_3(M\rho_0)^3}{24\pi^{5/2}} + \frac{d_5 M\rho_0}{16\pi^{5/2}} - \frac{d_0(M\rho_0)^6}{192\pi^3}\left(\log\frac{M^2}{\mu_0^2} - \frac{11}{6}\right) + \frac{d_2(M\rho_0)^4}{64\pi^3}\left(\log\frac{M^2}{\mu_0^2} - \frac{3}{2}\right)\right.$$
$$\left. - \frac{d_4(M\rho_0)^2}{32\pi^3}\left(\log\frac{M^2}{\mu_0^2} - 1\right) + \frac{d_6}{32\pi^3}\log\frac{M^2}{\mu_0^2}\right] + O\left(\frac{1}{M}\right). \tag{35}$$

7. Concluding remarks

Performing a numerical analysis of the result above, we arrive to the conclusion that such quantity—which should be identified essentially with the cosmological constant—is generically non-zero and can acquire positive and negative values, depending on the specific choice of the parameters. The same is seen to happen for a dS brane in the bulk we are considering, Eq. (33). This has now to be compared with the Casimir effect for a dS brane in the AdS bulk: in that case the cosmological constant is always zero [7]. A more detailed analysis shows actually, that for the range of values for M and ρ_0 of physical interest, which have been under discussion in the recent literature: (i) all the series we have here converge very quickly (this already happens for $\rho_0 > 10^{-10}$ cm), and (ii) the value of the induced cosmological constant that we obtain from them is positive, as needed to explain the observed acceleration in the expansion of the universe. In this preliminary analysis it is too soon to discuss about a numerical matching with the observational values.[3] Similarly, it can be shown from Eq. (33), that the effective potential corresponding to the case of a de Sitter brane is also non-zero, and can be made positive too, providing an enlarged number of interesting situations.

Summing up, from this simple examples we have here considered it becomes already clear that, in order to induce a 4-dimensional cosmological constant in a brane-world universe, it turns out that an AdS black-hole bulk with a one-brane configuration is far more attractive than the pure AdS bulk. We have here dealt only with some simple examples but, as has been pointed out already, the present calculation can be extended, without essential trouble, to more realistic situations and, thus, it seems to open a number of different, very promising possibilities.

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[3] See e.g., [25] for alternative and complementary approaches.

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