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Recent advance in nonlinear aeroelastic analysis and control of the aircraft

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Abstract A review on the recent advance in nonlinear aeroelasticity of the aircraft is presented in this paper. The nonlinear aeroelastic problems are divided into three types based on different research objects, namely the two dimensional airfoil, the wing, and the full aircraft. Different nonlinearities encountered in aeroelastic systems are discussed firstly, where the emphases is placed on new nonlinear model to describe tested nonlinear relationship. Research techniques, especially new theoretical methods and aeroelastic flutter control methods are investigated in detail. The route to chaos and the cause of chaotic motion of two-dimensional aeroelastic system are summarized. Various structural modeling methods for the high-aspect-ratio wing with geometric nonlinearity are discussed. Accordingly, aerodynamic modeling approaches have been developed for the aeroelastic modeling of nonlinear high-aspect-ratio wings. Nonlinear aeroelasticity about high-altitude long-endurance (HALE) and fight aircrafts are studied separately. Finally, conclusions and the challenges of the development in nonlinear aeroelasticity are concluded. Nonlinear aeroelastic problems of morphing wing, energy harvesting, and flapping aircrafts are proposed as new directions in the future.

1. Introduction

Aeroelasticity is the field of study that deals with the interaction of structural, inertia, and aerodynamic forces. In classic aeroelastic theory, linear assumptions are made for the aerodynamics and the structures, and the aeroelastic problem reduces to the solution of a set of linear equations that can be easily solved.[1] However, when the airspeed increases to high subsonic or transonic speeds, the assumption usually leads to results with insufficient accuracy. One example is the transonic dip that linear aerodynamics cannot predict. Flow separation and shock oscillation phenomena are also beyond the capability of classic aeroelasticity.[2] On the other hand, nonlinearities arising from aeronautic structures have attracted much more investigations. With structural nonlinearities, aeroelastic system may exhibit a variety of phenomena such as LCO and chaotic vibration. An extensive review of the analysis of structural nonlinearities for airfoil section may be found in Ref.[2] HALE aircraft has been developed rapidly over the last decade for various applications, including military reconnaissance, science research and telecommunication service. One significant feature of HALE

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aircraft is the high-aspect-ratio wing design concept, which makes the wing have high lift-to-drag ratio and lightweight flexible structure and operate with large deformation. So in the aeroelastic analysis of high-aspect-ratio flexible aircraft, geometrical nonlinearity has to be taken into account due to the large wing deformation. Flutter instability can jeopardize aircraft structure and its performance. Efforts have been made to reduce or delete the negative effect of nonlinearities on aeroelastic system. A great deal of research activity devoted to flutter control of aeroelastic system has been accomplished.[3]

Since the 1950s, there have been a great amount of literature available in the fields of nonlinear aeroelasticity.[4–6] Lee et al.[2] presented an exhaustive review on the bifurcation and chaos of two-dimensional airfoil in 1999. Dowell et al.[5] provided a critique of the results obtained via various methods using as a framework correlations between theory and experiment or alternative theoretical models in 2003. A collaborative research program was launched in UK in 2002. One of the two groups focus the work on nonlinear aeroelastics for fixed wing aircraft to develop useable methodologies to model and predict nonlinear aeroelastic behavior of complete aircraft, and to investigate the effect of nonlinearities on aeroelastic behavior. Ref.[6] provides a summary of the nonlinear aeroelastic project that includes the essential details of the methods and approaches that have been used, and exploitation of the methods for practical industrial application. In Ref.[2], only two degree-of-freedom (DOF) airfoil is discussed. With the emergence of more results in the fields of nonlinear aeroelasticity, it is necessary to make a comprehensive review on the advance in nonlinear aeroelasticity, especially in recent ten years.

In this paper, the advancement of recent years and challenges of nonlinear aeroelasticity in aircraft are summarized. Nonlinear aeroelastic problems of the two dimensional airfoil, the high-aspect-ratio wing, and the full aircraft are included. The rest of this paper is organized as follows. Section 2 presents the nonlinear aeroelastic problems of two dimensional airfoils. Structural and aerodynamic nonlinearities, methods used for aeroelastic analysis, nonlinear aeroelastic behaviors, and flutter active control approaches are introduced. In Section 3, structural and aerodynamic modeling for the aeroelasticity of high-aspect-ratio wing with geometrically nonlinear deformation is discussed. Section 4 shows the nonlinear aeroelasticity of HALE and fight aircraft. Conclusions and outlook are drawn and presented in Section 5.

2. Nonlinear aeroelasticity of airfoils

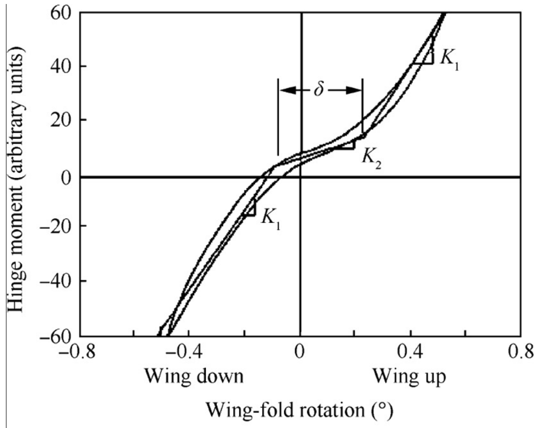
The two-dimensional airfoil could be used to represent the motion of wing in longitudinal direction with satisfactory precision. As the aeroelastic model of airfoil has only two or three DOFs, it is convenient to study the mechanism and methodology of nonlinear aeroelasticity. This section will introduce the nonlinearities in aeroelastic system, research techniques, nonlinear aeroelastic behavior and the aeroelastic control methods. In fact, the nonlinearities and aeroelastic behavior of airfoils may occur in other aeroelastic systems of the wings or aircraft.

2.1. Nonlinearities in aeroelasticity

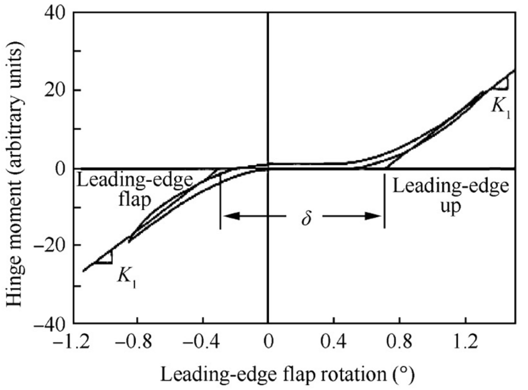
Nonlinearities in aeroelasticity include structural nonlinearities and aerodynamic nonlinearities. In some situations, such as shock waves in transonic flows, wing tip vortices, and dynamic stall, aerodynamic nonlinearities have to be taken into account. Nonlinear aerodynamic effects are more difficult to analyze because the fluid motion is governed by equations where analytical solutions are practically non-existent. A fully nonlinear aerodynamic code to solve the Euler equations coupled with a structural model in a two-dimensional flow case has been developed by Djayapertapa and Allen[7] and Djayapertapa et al.[8] However, the use of full computational fluid dynamic (CFD) and computational structural dynamics (CSD) coupled codes in design loops is very time-consuming, particularly when considering an active control system. Additionally, the lack of visibility of the full nonlinear equations in state space form makes various forms of control law design and stability analysis extremely difficult if not impossible. A reduced order model (ROM) of the Euler code was then created,[9] and the full nonlinear and reduced order aerodynamic models in control law design were compared. With an equivalent accuracy to the CFD method, the ROM requires a computational time that is much more comparable with traditional linear methods.

Conversely, structural nonlinearities could arise from worn hinges of control surfaces, loose control linkages, and material behavior as well as various other sources. Aging and combat aircraft that carry heavy external stores are more concerned with the effects associated with nonlinear structures. Structural nonlinearities may be classified as being either distributed or concentrated. In general, distributed structural nonlinearities are governed by elasto dynamic deformations that affect the whole structure. Concentrated nonlinearities, on the other hand, act locally and are commonly found in control mechanisms or in the connecting parts between wing, pylon, engine or external stores.[2] The concentrated nonlinearities can be classified basically into three types: cubic nonlinearity, freeplay nonlinearity, and hysteresis nonlinearity. Time delay is another nonlinearity that should be considered in the problem of aeroelastic control.[10] The three classical structural nonlinearities have been investigated by many researchers, but they are not exactly the same with the relationships from experimental test.

Two examples of the hysteresis nonlinear relationship from ground testing data are shown in Fig. 1, K_1 and K_2 are the stiffness of spring, δ is the amount of freeplay. But they were ignored and simplified as bilinear and freeplay nonlinearities in their investigation. With the unsteady aerodynamic model from vortex lattices formulated in a continuous time domain and expressed in a dimensionless form, a combined nonlinearity of freeplay and cubic stiffening was investigated by Zhao and Hu.[11] In order to improve the nonlinear model, rational polynomial approximation could be used to describe the nonlinear relationship. And for hysteresis nonlinearity, the item of velocity has to be included. The work of Li et al.[12]showed that the aeroelastic system with rational polynomials in pitch exhibits almost exactly the same dynamic responses, such as convergence and LCO, with what were seen in the system with freeplay or hysteresis nonlinearities. The theoretical analysis of the energy transformation shows that the switching point in the bilinear or hysteresis nonlinearity has no effect on the aeroelastic response of the system, which was also verified by numerical examples.[13] Nonlinear relationship in aeroelastic system should be tested from the actual aircraft structures. Until now, there is not enough data from experimental test. Rational polynomial model has advantages to describe the nonlinear relationship in aeronautical structures. And the



(a) Wing-fold hinge



(b) Leading-edge flap

Fig. 1 Nonlinearities from testing data.[2]

aeroelastic system with rational polynomial is continuous and derivable, so it can be solved by using various methods including theoretical approaches.

2.2. Research techniques

The techniques used for nonlinear aeroelasticity include theoretic method, numerical method, wind tunnel experimental and flight tests. In recent years, various theoretic approaches have been developed to investigate aeroelastic systems with nonlinearities. The nonlinear aeroelastic performance of an airfoil with a strong cubic nonlinear restoring force was studied by using harmonic balance method.[14] The results show a good agreement with those obtained from numerical simulations. However, the secondary bifurcation can be captured by including terms at least up to the ninth harmonic in the series representation for the pitch and plunge motions. Based on the harmonic balance method, an approach was proposed for estimating the equivalent damping associated with portions of the aerodynamics of aeroelastic system with cubic stiffness.[15] The equivalent damping matrix of aeroelastic system can be determined analytically. Liu and Dowell extended the harmonic balance method to study the aeroelastic airfoil including a control surface with a freeplay nonlinearity.[16] A high dimensional harmonic balance method was employed for the aeroelastic airfoil to investigate the amplitude and the frequency of the LCO.[17] The center manifold theory was applied by Liu et al.[18] to study an aeroelastic airfoil with cubic nonlinearity. Analytical formulas for predicting the frequencies and amplitudes of LCO are obtained. But this method is limited for the aeroelastic system where the second derivative exists and is continuous. So the center manifold theory does not apply for the aeroelastic system with a freeplay. The bilinear or hysteresis aeroelastic system was divided into several linear subdomains by switching points. So a mathematical technique based on the point transformation method was developed by Liu et al.[19,20] A perturbation-incremental method was developed to investigate the dynamic response of the two-DOF aeroelastic system with a freeplay structural nonlinearity[21] and hysteresis stiffness.[22] An expert system was developed by Popescu et al.[23] to predict the steady-state nonlinear behavior of a two-dimensional airfoil oscillating in pitch and plunge. Chen et al.[24,25] investigated the nonlinear flutter system of an airfoil using equivalent linearization menthod (ELM).

In the fields of nonlinear aeroelasticity, the usage of theoretic method is limited to specific conditions. If the number of DOF is higher than two, the theoretic scheme would be too complex to be applied. Furthermore, the initial conditions are difficult to be considered in the solution of the theoretic method. Numerical methods are widely applied for the investigation of nonlinear aeroelasticity in time domain. The classical Runge–Kutta integration is still popular for the solution of differential equation of nonlinear aeroelastic system.[26,27] From the numerical results, phrase plane, Poincaré map, bifurcation diagram, and Lyapunov exponents can be obtained to describe the aeroelastic behavior which will be discussed in the next section. Finite difference scheme is another numerical method that has been used in the solution of nonlinear aeroelastic problems. But there is no new result of nonlinear aeroelasticity obtained by using finite difference method. With the development of computers and commercial software, numerical approaches are becoming more and more convenient for the investigation of nonlinear aeroelasticity. But the numerical results should be verified through experimental studies.

Except for the flight test, wind tunnel experiments are the way that can validate the results from numerical or theoretic schemes. In the past few years at Duke University, Dowell and his colleagues have constructed a typical airfoil section aeroelastic experimental model with control surface freeplay. They have also designed and installed an experimental rotating slotted cylinder (RSC) gust generator in the Duke University low-speed wind tunnel, which was used to create a periodic or a linear frequency sweep gust excitation field.[28] Using these experimental facilities, a series of theoretical and experimental studies, such as flutter and LCOs, gust responses and alleviation,[29] has been completed. Overall, experimental investigation on nonlinear aeroelastic problems is much less than theoretical or numerical works. With the development of nonlinear aeroelasticity, experiments are necessary especially for engineering application.

2.3. Aeroelastic behavior

With structural nonlinearities, an aeroelastic system may exhibit a variety of phenomena, such as LCO and chaotic vibration. When the flow speed is smaller than the critical velocity, the system response is stable and the motion converges to an equilibrium point. And the point may be or not located at the zero point. Hopf bifurcation will arise at the critical flutter speed. An example is given in Fig. 2. In the bifurcation diagram, periodic or quasi-periodic motions will happen if there is more than one but exact number of points at a specific flow speed. If a very large number of points occur at one velocity, this suggests that the motion is probably chaotic. Further evidences to the existence of chaos are the phase trajectory and Poincaré section. For the chaotic motion, a classical phase trajectory is “two-well potential”. In the Poincaré section, an extremely large number of points indicate the motion is non-periodic. Furthermore, if there is some “structure” in the Poincaré section, the motion is not random but is most probably chaotic. A far more definitive method is the use of Lyapunov exponents, which give a measure of the rate of divergence or convergence of nearby orbits in phase space; a positive Lyapunov exponent indicates a chaotic system.

Because of the sensitivity to disturbance and the uncertainty of chaotic motion, it is challenging to fully understand the phenomenon that appears in a nonlinear aeroelastic system. Chaotic motion in a nonlinear aeroelastic system with various structural nonlinearities has been studied by many researchers. A two-DOF airfoil with a cubic nonlinearity in pitch and with quasi-static aerodynamics that depends only on the instantaneous pitch angle was investigated by Zhao and Yang.[30] They found that the chaos from the phase plane would occur at velocities in excess of the velocity required for static divergence. For small structural preloads with bilinear (or freeplay) nonlinearity, narrow regions of chaotic motion were obtained by Price et al.[31] Unsteady aerodynamics based on Wagner’s function was adopted. Furthermore, the existence of chaos was confirmed for the cubic nonlinearity via the positive Lyapunov exponents. Using a mathematical approach, chaotic motion was found for the two-DOF aeroelastic system with freeplay or hysteresis nonlinearities in pitch by Liu et al.[14,16] For the same aeroelastic model, when the nonzero circulatory moment in pitch is adopted, the local chaos was detected by Li and Xiang[32], and frequent switching between local motions then leads to global chaos.

As chaotic motion was obtained when using the cubic nonlinearity, Price et al.[31] pointed out that the cause of the chaos is not the discontinuous nature of the bilinear nonlinearity. The research work of Li et al.[13] has investigated the cause of the chaotic motion in nonlinear aeroelastic systems. From the analysis of the nonlinear restoring moment in pitch, the “energy flat” in the nonlinear stiffness is determined to be a necessary condition for chaotic motion. Furthermore, the effect of the elastic axis center position, the airfoil/air mass ratio, and the structural preload on the chaotic motion of the system was investigated using bifurcation diagrams in pitch. When the elastic axis shifts from the mid-chord to the leading edge, the chaotic motion will disappear. The mass ratio has little effect on the occurrence of chaos. The aeroelastic system will not show a chaotic response if a structural preload is included.

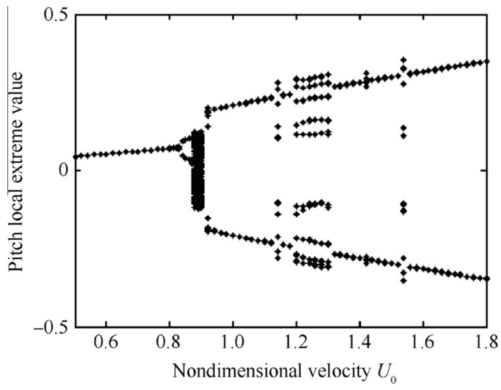


Fig. 2 A typical bifurcation diagram of pitch response.[13]

2.4. Aeroelastic control

For the two-dimensional wing section with structural nonlinearity, research effort has been made to develop control strategies to suppress flutter. In the early stage, the classical linear full-state feedback control law was derived for a wing section with nonlinear stiffness to stabilize the nonlinear system in some circumstances. Partial feedback linearization methodology was also applied to the design of nonlinear controllers for nonlinear aeroelastic system. In order to derive a globally stabilizing controller, a full feedback linearization controller based on two control surfaces was designed.[33] The state-dependent Riccati equation (SDRE) method was developed for nonlinear control problems, and used to design suboptimal control laws of nonlinear aeroelastic systems considering both quasi-steady[34,35] and unsteady aerodynamics.[36] Based on the SDRE control law, the effect of freeplay and time delay in the control surface of a closed-loop system was investigated by Li et al.[37] A global robust control law for an aeroelastic model of uncertainty was derived considering output feedback in Ref.[38] Time delay feedback was successfully used by Ramesh and Narayanan[39] to control the chaotic motions in a two-dimensional airfoil.

When the uncertainty in the structural nonlinearity was taken into account, an adaptive control method would be used to depress the aeroelastic flutter. The impact of uncertainty on aeroelastic response prediction has received substantial attention in the literature. General sources of uncertainty that complicate airframe design and testing were briefly described by Pettit.[40] Pettit and Beran[41] investigated the effects of uncertainty on airfoil LCO by using of Monte Carlo simulation (MCS). Parametric uncertainty was modeled in the third- and fifth-order stiffness coefficients of the pitch spring. Different computational methodologies, such as Wiener–Haar, Cyclic and B-spline projection methods have been developed to quantify the uncertain response of an airfoil aeroelastic system in limit-cycle oscillation, subject to parametric variability.[42] Uncertainties are specified in the cubic coefficient of the torsional spring and in the initial pitch angle of the airfoil. When the uncertainty was considered in the flutter suppression, adaptive controllers based on partial or full feedback linearization were derived. In Ref.[43], experimental results were presented to exam the adaptive controller derived theoretically. A series of adaptive controllers was derived for flutter suppression by Singh and Brenner[44] and unstructured uncertainties were also taken into account.[45] In order to improve the performance of the adaptive controller, both leading-edge (LE) and trailing-edge (TE) control surfaces were used in the design of multiple-input multiple-output control strategies in Refs.[46–48] Recently, an output feedback and an adaptive decoupled fuzzy sliding-mode control laws have been implemented for suppressing flutter and reducing the vibrational level in subcritical flight speed range.[49,50] Based on the tensor-product model transformation and the parallel distributed compensation, a control law for prototypical aeroelastic wing section was designed and presented in Ref.[51] An ultrasonic motor was also used for the flutter control of the aeroelastic system with a nonlinear stiffness in pitch.[52] Structured model reference (SMR) adaptive control method has been developed for a special type of structure, and used for flutter suppression of an aeroelastic system.[53]

Damping uncertainty in airframe structure and control system is inevitable and may have significant effect on the aeroelastic behavior.[54] It is very difficult to establish an accurate damping model, and much of experimental data is normally needed. Recently, the investigation in Ref.[55] focused on deriving an adaptive controller for flutter suppression of a nonlinear aeroelastic system with damping uncertainty. Two examples of the wing sections with single trailing-edge or leading- and trailing-edge were taken. Adaptive controllers based on partial feedback linearization and structured-model reference were designed. The numerical simulation results show that the damping uncertainty has a positive effect on the control effectiveness. The closed-loop system considering damping uncertainty has quicker response to control and greater critical flutter velocity.

3. Nonlinear aeroelasticity of high-aspect-ratio wings

As shown in Fig. 3, high-aspect-ratio flexible wings exhibits large deformation under aerodynamic loads, and geometric nonlinearity has to be taken into account in structural modeling. For the aeroelastic analysis of high-aspect-ratio wings, linear or nonlinear unsteady aerodynamics needs to be considered. Presently, nonlinear beam model and two-dimensional strip theory have been extensively favored in identifying the critical nonlinear aeroelastic phenomena of flexible structure. This section gives a brief introduction on the nonlinear structural modeling and the unsteady airfoil aerodynamic modeling.

3.1. Structural modeling

With the application of stronger, lighter weight and more flexible synthetic materials, high-aspect-ratio wings may suffer from large deformation, which makes geometrically nonlinear analysis of structure become essential. Generally, geometric nonlinearity can be categorized into three types[56]: large displacement/large rotation/small strain, large displacement/small rotation/small strain and large displacement/large rotation/large strain, among which the "large displacement/large rotation/small strain" geometric nonlinear behavior is commonly seen in HALE aircraft and helicopter blades. On hypothesis of small strain, the structure model of high-aspect-ratio flexible wing is usually represented as a beam considering geometric nonlinearity, and two types of fundamental structural models are widely used, the nonlinear moving beam and the geometrically exact intrinsic beam.

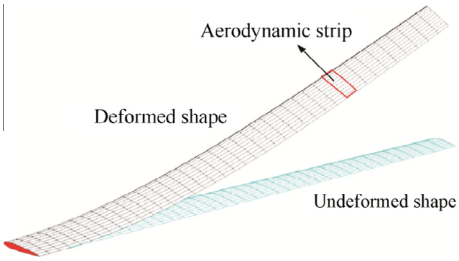


Fig. 3 Deformation of high-aspect-ratio flexible wing.

The nonlinear moving beam model has the feature of describing the beam deformed position with displacements or Euler angles. The initial form was presented by Hodges and Dowell[57] for rotor blades using Hamilton's principle and Newtonian method in 1974. Their model coupled bending and torsion deflection with second-order nonlinear terms and used displacements to describe the beam deformed position. Then, some researchers improved the nonlinear beam model by involving more nonlinear terms so as to describe deformation more accurately. However, the results show that selection of higher-order nonlinear terms could lead to contradiction between the computational complexity and displacement accuracy. Later Minguet and Dugundji[58,59] used the Euler angles to describe the beam deformed position and derived equilibrium and compatibility equations without any limitation on magnitude of displacements. But the Euler angles somewhat make the aerodynamic loads calculation a bit complex. The nonlinear moving beam model has great influence on structural modeling of high-aspect-ratio flexible wing, even though it was originally developed for rotor aeroelastic analysis with some imperfection. Jaworski and Dowell[60] conducted the comparison of theoretical structural models with experiment for high-aspect-ratio wing. Liu and Xiang[61,62] studied the nonlinear flutter characteristics of high-aspect-ratio flexible wing. Pan et al.[63] investigated the flight loads analysis and optimization.

The geometrically exact intrinsic beam model was mainly completed by Hodges and his co-workers.[64–66] The model[64] provides a geometrically exact, fully intrinsic dynamic formulation for moving beam including initial curvature and twist, shear deformation, rotary inertia, and general anisotropy. The governing equations are characterized by partial differential equations of motion and kinematical formulas with unknowns only including stress resultants, generalized strains and generalized velocities, and appear to be unique for the absence of displacement and rotation variables. Ref.[65] presents a literature review of the fully intrinsic concept. According to geometrically exact intrinsic beam theory, the modeling of large deflection, moving, anisotropy 3-D beam can be decoupled into 1-D beam kinematic analysis and 2-D cross-sectional analysis, and more details can be found in Ref.[66] The main advantage of the theory is that the governing equations are geometrically exact and written in intrinsic form without being augmented with some forms of angular displacement variables, and with the degree of nonlinearity no higher than two, which simplifies derivation of aeroelastic equations and reduces computational cost. It has already been applied to structural modeling for high-aspect-ratio flexible wing aeroelastic analysis by some researchers and would have a wide application prospect in structural modeling. Chang and Hodges[67] presented parametric studies on ground vibration test of HALE aircraft. Zhang and Xiang[68,69] studied the nonlinear aeroelastic response of flexible wing and the parametric effect when subjected to a lateral follower force.

3.2. Aerodynamic modeling

The key for a successful aeroelasticity analysis is the calculation of the aerodynamic loads based on the motion/deformation of the aerodynamic body. For high-aspect-ratio flexible wing, aerodynamic loads calculation can be formulated based on different types of aerodynamic theory, such as strip theory, unsteady vortex-lattice method (UVLM), indicial response theory and Euler/Navier–Stokes CFD aerodynamic modeling techniques, while the strip theory has been widely applied for its higher computational efficiency.[70] Aerodynamic loads in strip theory are described by airfoil aerodynamic model, which can be divided into two major catalogues: frequency domain analytical models and time domain numerical models.

Frequency domain analytical models mainly include Greenberg' s aerodynamic model, Loewy's aerodynamic model, Theodorsen aerodynamic model and so on. In frequency domain analytical models, the motion of airfoil and airflow are assumed to be harmonic functions of time, and the equation of wing coupling with nonlinear structure and aerodynamic model can be formulated in the frequency domain. The Frequency domain analytical models can easily deduce the aeroelastic problems into an eigenvalue problem, and the flutter boundaries and stability analysis can be determined by eigenvalues analysis. It is equivalent in aeroelastic stability analysis between the treatment of time and frequency domains. However, frequency domain analytical models, strictly speaking, are only effective when parameters are close to flutter boundaries. It is inaccurate for transient aeroelastic analysis while the motion is far away from flutter boundaries.

Time domain numerical models can obtain airfoil aerodynamic loads in arbitrary motion, and are widely used in nonlinear aeroelastic analysis. Currently, the most commonly used time domain numerical models include unsteady wake models and dynamic stall models. In unsteady wake models, the unsteady aerodynamic effects are considered as a consequence of the time-history of the induced velocity from the vorticity contained in the shed wake, coupled with the induced velocity contributed by the circulation contained in the trailed wake. There are two general approaches to model the induced effects produced by this cycloidal wake: dynamic inflow models and vortex wake models. At present, the most popular model of dynamic inflow theory is that of Pitt and Peters.[71] Ref.[72] conducts a brief literature review of its development. The dynamic inflow models have attractive mathematical forms and relatively high numerical efficiency, which will always be appealing for high-aspect-ratio flexible wing aeroelastic analysis.[73,74] However, the concept of the apparent mass applied to the time constants seems certainly not to be a rigorous analogy. The vortex wake models use "vortex methods" to represent the strengths (circulation) and spatial locations of the vertical elements and mainly contain the prescribed vortex method and the free vortex method. The disadvantage of the vortex wake models is the relatively higher computational cost.

Dynamic stall will occur on any airfoil or lifting surface when the effective angle of attack is above its normal static stall angle. This phenomenon has been extensively studied using oscillating 2-D airfoils in wing tunnel experiments. Based on experimental data, several dynamic stall models have been provided using parsimonious, semi-empirical formulas. A few of them are currently used for aeroelastic analysis, such as ONEAR model and Leishman-Beddoes model. The ONERA dynamic stall model was first developed by Tran and Petot.[75] They described the unsteady airfoil behavior in both attached flow and separated flow of a pitching airfoil using a set of nonlinear differential equations. Then some suggested modifications were added and a later common used version of ONERA model was documented by Petot.[76] The coefficients in these differential equations of the ONERA model are determined by parameter identification using experimental measurements on oscillating airfoils. At present, ONERA aerodynamic model is widely applied to aeroelastic analysis by many researchers such as Zhao and Hu[77] and Zhang and Xiang.[68,69] However the ONERA models usually lack rigor and generality and need a significant number of empirical coefficients. Later Troung[78] presented ONERA-BH model using a Vander Pol Duffing type nonlinear equation to represent the separated flow conditions. The model requires slightly fewer coefficients, and can give good description of the phenomenon of vortex exfoliation. The Leishman-Beddoes (L-B) model was first presented by Beddoes, and then developed by Leishman et al. Ref.[79] presented a brief review of its development. The L-B model includes static model, unsteady attached flow model, trailing edge separated flow model and vortex flow model, and has the ability of representing the unsteady lift, pitching moment and drag characteristics of an airfoil undergoing dynamic stall. An important feature of this model is that rigorous representations of compressibility effects are included, which are essential for helicopter applications. Although the model has also been developed as a set of differential equations and has fewer coefficients, it shows significant disagreement with the experimental data at low Mach number and has yet seldom been seen in high-aspect-ratio flexible wing aeroelastic analysis.

The strip theory coupled with two-dimensional airfoil aerodynamic model has been widely used for aerodynamic loads calculation as that it easily allows for corrections, including semi-empirical stall models, and steady viscous drag. However, it shows disadvantage in evaluation of spanwise variations, which may be critical for high-aspect-ratio wings when large deflections occur. Therefore, the approach of nonlinear beam model coupled with strip theory could cause relevant 3-D flow physics to be neglected, such as the accurate prediction of wing-tip effects, and the aerodynamic interference between wakes and lifting surfaces. Currently, Euler/Navier–Stokes equations and CFD techniques have still been used to simulate the 3-D flow of flexible aircraft, but incur computational expenses. Some researchers point out that the UVLM, which provides a medium-fidelity tool for aerodynamic calculation, may be unveiled as an outstanding one in future aeronautical research.[80,81]

4. Nonlinear aeroelasticity of aircraft

Nonlinear aeroelastic phenomena are commonly seen and becoming increasingly important in the aeroelastic analysis of the full aircraft, especially for HALE aircraft and fight aircraft. So in this section, some considerable research efforts on the nonlinear aeroelasticity of HALE and fight aircraft are presented.

4.1. HALE aircraft

For the HALE aircraft with flexible wing, as shown in Fig. 4, the frequency of elastic mode is quite low, which appears near to the frequency of flight mode, and the coupling effect of aeroelastic behavior and flight dynamics occurs known as body-freedom flutter. The previous researches showed that the phugoid mode is mildly unstable, and the structural dynamics and the rigid-body characteristics are strongly

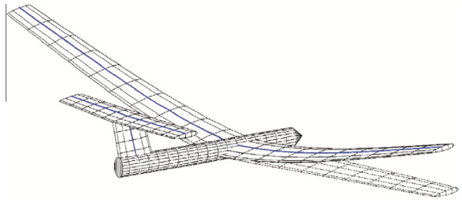


Fig. 4 Sketch of T-tail flexible aircraft deformation.

coupled due to the flexible nature of the wing. In order to understand the coupling effect of nonlinear aeroelastic and flight dynamics of the very flexible aircraft, several simulation tools and analysis method have been developed such as ASWING, UM/NAST, NATASHA and NANSI.

ASWING was initially presented by Drela[82] as an integrated simulation model for flexible aircraft of preliminary aerodynamic, structural, and control-law design. Its structural model is Minguet's nonlinear beam model, and aerodynamic is calculated by vortex lattice method considering Prandtl-Glauert correction for compressibility effect. ASWING aims to conduct rapid modeling and flight simulation of flexible aircraft including nonlinear static, dynamic response with gust fields and eigenmode analysis. Love et al.[83] used ASWING for aero-structural analysis of a sweep flying wing SensorCraft so as to better understand the effect of passive means on body freedom flutter. González et al.[84] modeled the Unmanned Airplane for Ecological Conservation as a flexible-body using the ASWING code and compared it with results from an analytical empirical method and potential flow codes, aiming to evaluate the aerodynamic and static stability of the aircraft. However, the ASWING is limited in capability for nonlinear aeroelastic analysis of joined-wing configurations aircraft.

UM/NAST (The University of Michigan's Nonlinear Aeroelastic Simulation Toolbox) was presented by Cesnik and Brown[85,86] to study the coupling effects of nonlinear aeroelastic and flight dynamic. They used a computationally effective strain-based structural formulation and the finite-state unsteady subsonic aerodynamic model. The code serves as a plant representation for HALE aircraft control design. It focuses on a reduced number of states to represent the complex nonlinear problem. Then some improvements were suggested by Shearer el al.[87–89] and Su and Cesnik.[74] The UM/NAST is capable of nonlinear aeroelastic modeling, integral wing actuation for generating maneuver loads, flutter boundary enhancement, gust load alleviation, trajectory control and overall nonlinear vehicle optimization of unconventional configurations. However, the code has not been validated by any flight experimental data. A scaled test HALE aircraft called X-HALE[90,91] is in the process of development for the purpose of collecting flight test data and subsequently uncovering the strengths and weaknesses of UM/NAST. The UM/NAST is capable of analyzing the nonlinear aeroelastic behavior of both conventional and unconventional configurations aircraft such as flying-wing configuration, and joined-wing configuration, etc.

NATASHA, a computer program named Nonlinear Aeroelastic Trim and Stability of HALE Aircraft, was firstly presented by Patil and Hodges[92] based on geometrically-exact fully intrinsic beam theory and 2-D the finite state induced-flow model of Peters. The initial program could provide trim, payload distribution, stability analysis and dynamic response for conventional and flying-wings configurations with dynamic stall effect unconsidered. Then Patil et al.[93,94] applied the methodology in the nonlinear gust response in frequency domain and time domain of highly flexible aircraft. Chang and Hodges[67,73] conducted a simulation of ground vibration test (GVT) environment and presented an analysis and parametric study of the flight dynamics of flexible aircraft. Mardanpour et al.[95] studied the effect of engine placement on aeroelastic trim and stability of flying wing aircraft. Moreover, the capability of NATASHA was updated by Sotoudeh et al.[96–100] and applied to statically indeterminate configuration such as joined-wing using incremental discretization method. The results from NATASHA were validated with a range of results from the well-known solutions of beam stability and vibration problems, published experiment data from scaled wind tunnel tests and results from rotorcraft comprehensive analysis system (RCAS). Although the capabilities and validation of NATASHA is still under development, the NATASHA's results are hoped to be used as benchmarks for their own codes.

NANSI is a computational aeroelastic tool presented by Wang et al.[101] for Nonlinear-Aerodynamics/Nonlinear-Structure Interaction. The initial methodology was integrated via tightly coupling a geometrically exact nonlinear intrinsic beam model and the generalized unsteady nonlinear vortex-lattice aerodynamic model, and had the capability of nonlinear time-domain aeroelastic simulation. Then it was enhanced to include the capability of handing effect of gust and stall flow.[102,103] However, the NANAI is still limited in its capabilities and seems a bit inconvenient for analyzing in frequency-domain.

Additionally, some other simulation frames have been developed to analyze the coupled behavior of flight dynamics and aeroelasticity. Zhao and Ren[104] applied multi-body dynamic approach in flexible aircraft structural modeling and studied the nonlinear aeroelastic characteristic coupled with ONERA aerodynamic model. Zhang and Xiang[105] completed a rigid-flexible coupling simulation frame, considering geometrical nonlinearities, dynamic stall, material anisotropy and rigid body motion with elastic motion of fuselage neglected, via fully intrinsic beam theory and Extended-ONERA aerodynamic model.

Recently, considerable research efforts have been made to know more about the nonlinear aeroelastic phenomena of HALE aircraft. However, the problem of nonlinear aeroelasticity coupled with nonlinear flight dynamics is still not completely followed. Simulation tools have been presented to improve predictions of HALE aircraft response, stability, and overall performance. However, these codes have still been under development and none of them have been completed validated with real flight test data from a HALE aircraft. Mostly, they are validated in a piecemeal fashion against beam models such as a simple cantilevered beam model and wind tunnel data. This is because currently there is scarcely any aircraft flight data available for validation. Consequently, these nonlinear aeroelastic solvers are mainly applied to conceptual design and analysis of HALE aircraft. Among these tools, flexible beam theory and strip theory are the main approaches for aeroelastic modeling for the reason of rapid modeling and solving, which would be employed in future research.

4.2. Fight aircraft

Aeroelastic systems of the fight aircraft are inherently nonlinear due to both aerodynamic and structural nonlinearity. Nonlinear aeroelasticity in structural aspects is mainly caused by control surface freeplay, and store-induced LCO. Due to the complexity of aircraft structure, it is almost impossible to develop a theoretical technique suitable for the nonlinear aeroelastic analysis of full aircraft. Computational aeroelasticity including CFD and CSD, also called fluid–structure interaction (FSI) techniques, is an effective way to solve the aeroelastic problems of full aircraft. Ref.[106] presented a survey of the state of computational aeroelasticity and a discussion of its success and continuing challenges. A virtual flight test (VFT) technique based on FSI utilizing physics-based modeling and simulation was developed to provide the capability of predicting aeroelastic phenomena on complex full-aircraft models. And it was used to analyze the influence of F-16 store on LCO.[107] Structural nonlinearities at the wing-fold hinge represented by a bilinear spring, and the outboard leading-edge flap hinge represented by a freeplay were investigated. For the bilinear nonlinearity, limit-cycle oscillations can occur with considerable increase in flutter speed above that for nominal hinge stiffness. For the freeplay nonlinearity, it was shown that limit-cycle oscillations are possible within a small range before the critical flutter speed. A medium-fidelity transonic small-disturbance aerodynamic theory was used to investigate the LCO of F-16.[108] Higher-order spectral analysis was performed to identify nonlinear aeroelastic phenomena that are associated with LCO encountered in the F-16 flight test.[109] The results show that nonlinearities associated with LCO are most observable at the forward locations on the wing-tip and underwing launchers. In the vertical direction, the nonlinearity is cubic and leads to the generation of a third harmonic component. In the lateral direction, the nonlinearity is quadratic and leads to the generation of the second and higher-order harmonics. Methods that can identify the onset of nonlinear aeroelastic phenomena, such as LCO have been developed for the flight flutter test data of F/A-18 by Silva and Dunn.[110] Standard correlation and power spectral density techniques, experimentally-identified impulse responses and higher-order spectral techniques are successfully applied to the data.

For the fight aircraft with stealth, there is no based on the classical flutter solution and an empirical buzz criterion with the Den Hartog equivalent spring equation, flight test techniques for max-freeplay flutter testing were developed for F-22.[111] Then successful flight flutter tests program were flown to meet the prime objectives of flutter testing with maximum freeplay on aileron, rudder, and horizontal while the control surface's hinge moment was at or near zero. The maximum freeplay test results, in spite of the earlier wind tunnel test results, indicated that the design for freeplay for both flutter and buzz may be conservative, and that some relief in freeplay limits may be possible. So flight test is necessary in the design of control surface freeplay. Although there is not many published literature as the military secrecy, experimental test and verification on nonlinear aeroelasticity are necessary for the high performance fight vehicles.

5. Conclusions and outlook

In this paper, an extensive review on recent advancement in nonlinear aeroelasticity of two dimensional airfoil, high-aspect-ratio wing, HALE aircraft and fight aircraft is presented. Two-dimensional airfoil model is convenient to detect the nonlinear aeroelastic behavior and its mechanism. Besides the three classical nonlinear models, time delay has been considered in the closed-loop nonlinear aeroelastic problem. Rational polynomial has been verified to be more accurate to describe the nonlinear relationship from experimental test. LCO and chaos are the two typical kinds of behavior of nonlinear aeroelastic system. The route to chaos and the cause of the chaotic motion are concluded. Nonlinear research techniques, such as numerical, theoretical, and wind tunnel test, and various aeroelastic flutter control methods are discussed in detail.

Notable progress has been made in structural and aerodynamic modeling. Experimental and theoretical investigations have been made for well understanding of the nonlinear aeroelastic behavior. For high-aspect-ratio flexible wing, geometrically exact intrinsic theory shows the advantage in structural modeling and the time domain numerical airfoil aerodynamic model seems more applicable in describing aerodynamic loads. However, none of the current models can be regarded as the last word on describing the nonlinear aeroelastic phenomena, and it is likely that three-dimensional effects of both structure and aerodynamic will attract more attention. Nevertheless accurate and higher computational efficient approaches have been required for nonlinear aeroelastic analysis. Several simulation codes have been developed to understand the behavior of nonlinear aeroelasticity coupled with nonlinear flight dynamics for HALE aircraft. However, there is currently no aircraft flight data available for validation. Flight experiments should be carried out in future research. For the fight aircraft, control surface freeplay and store-induced LCO are the two nonlinear factors that have to be taken into account. Ground and flight tests are necessary to eliminate the risk from nonlinear aeroelasticity.

With the development of adaptive wing, such as folded wing, variable camber wing, new nonlinear aeroelastic problems have appeared.[112] As the large deformation of morphing wing in trailing edge, geometrically nonlinear relation would occur in both span wise and chord wise. Furthermore, nonlinear stress–strain relation is likely to be another obstacle for the modeling of aeroelastic morphing wing. The freeplay nonlinearity caused by morphing mechanism was investigated by Li et al. with traditional aeroelastic model.[113] But further investigation has to be done for the geometrically nonlinear model and CFD aerodynamics. With the development of new materials, it has become possible not to avoid LCO, but try to utilize it. Some potential applications of nonlinear aeroelasticity are extracted, such as energy harvesting based on piezoelectric element and LCO-based flapping design, detailed information of which may be found in Ref.[113–117]

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