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The sensitivity of the calculation of ΔV to vehicle and impact parameters

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ABSTRACT

ΔV is frequently used to describe collision severity, and is often used by accident investigators to estimate speeds of vehicles prior to a collision, and by researchers looking for correlations between severity and outcome. This study identifies how ΔV varies over a wide range of input uncertainties allowing the direct comparison of different methods of input data collection in terms of their effect on uncertainty in the calculation of ΔV.

Software was developed to implement this sensitivity analysis and was validated against examples presented in the CRASH3 manual. The findings are therefore representative of, and relevant to, commercially available tools such as CRASH3 and AIDamage.

It is possible to measure the vehicle and collision parameters with sufficient accuracy to determine ΔV to a level of precision that is useful to predict occupant fatality. In many cases, ΔV is largely insensitive to the input parameter and category values or values determined from photographs may be used. A vehicle specific value of the stiffness parameter B should be used. Direct measurement of crush measurements and vehicle mass (including the best estimates of fluid loss) should be used. Similarly the mass of occupants and cargo should be measured directly rather than estimated from 50th centile values. Calculation of ΔV is sensitive to PDOF which should be measured with a precision of better than $\pm 6^\circ$.

1. Introduction

ΔV is frequently used to describe collision severity, and is often used by accident investigators to estimate speeds of vehicles prior to a collision, and by researchers looking for correlations between severity and outcome, however there is a lack of error analysis in its calculation. Current ΔV calculation requires specialist computer programs and several post impact vehicle measurements which must be taken from both vehicles. The purpose of this study is to identify suitable measurement protocols for a desired level of output accuracy. It considers the influence of each input parameter in the determination of ΔV to identify the specific accuracy needed for each input parameter. Therefore the suitability of different methods of data collection can be ascertained; for example, could photographs of the damaged vehicle be sufficient for the calculation, or do direct measurements need to be taken?

A correlation of ΔV with risk of fatality has led to the suggestion that on-site calculation of ΔV has the potential to influence emergency treatment, allowing medical staff to predict both severity and type of injury (Jokcsh, 1993, Nance et al., 2006); and to allow suitable preparation before the vehicle occupant arrives at hospital. For example Joksch (1993) described a simple relationship between ΔV and risk, r, of death as shown in Eq. (1).

$$r = \left(\frac{\Delta V}{61 \cdot 5}\right)^4 \tag{1}$$

Other more recent research has created more specific correlations, such as Winnicki and Eppinger (1998) who established relationships between probability of different AIS scoring chest injury and ΔV. This study suggests probability values for particular AIS ranks at different ranges of ΔV; for example, probability of sustaining an $\geq$AIS 5 injury was shown to be 0.001 for a 0–10 mph ΔV range; and 0.84 for a the 51+ mph ΔV range. Whilst their study also presents an equation for the risk of fatality, it does so only in terms of fatality as a consequence of chest injury; this study will use the older relationship by Jokcsh due to its applicability to all injury types and fatality causes. Statistical analysis has shown that ΔV is a strong predictor of injury as demonstrated by Gabauer and Gabler (2006). This study addresses the correlation with predicted injury by ΔV and actual injury as well as predicted injury by occupant impact velocity (OIV) and actual injury; it was shown that both injury prediction parameters correlated well with real life injury.

It is worth noting that the Joksch's formula does not differentiate between types, presence, or use of restraint systems and that

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whilst the present day risk is likely to be lower due to the increase in restraint system integrity and use, the pattern is likely to be similar. The value of ΔV can also allow medical staff to be vigilant of hidden injury, for example those at risk from thoracic aortic rupture, a condition often missed due to its lack of external signs. It was shown that occupants of crashed vehicles, with a ΔV of greater than 20 mph and interior intrusion greater than 15 cm, were at increased risk of aortic injury (Horton et al., 2000). Such on-site calculation does not typically occur as the current methods of ΔV calculation require computer access and timely measurements; however it may be that simple estimates of these measurements coupled with modern mobile computing devices could provide a reliable indicator of rupture risk.

This study explores both the options of using photographic evidence to estimate crash parameters such as crush measurements, and values based on the category of vehicle (rather than the vehicle itself) to estimate vehicle parameters such as vehicle stiffness. We propose a "target uncertainty in ΔV determination", based upon prediction of injury severity, and use this as a bench mark to judge different methods of crash and vehicle parameter determination.

A previous study of the accuracy of ΔV calculations suggested that there is an underestimation of approximately 10% and 5% in frontal and side collisions respectively (Lenard et al., 1998); but this was based on the assumption that there is high level of accuracy in the input parameters. The sensitivity of the ΔV output to the input parameters has not been wholly explored; Smith and Noga (1982) documented uncertainty in ΔV as a result of individual input parameter uncertainty. They found that the output uncertainty due to all potential input parameter uncertainty varied according to the orientation and speed of collision. They found that front–front and front–back high speed collisions exhibited the lowest level of ΔV uncertainty, 9–11% (these values are expressed as a fraction of the ΔV) and that low velocity side impacts exhibited the greatest (19.1–23%). The parameter seen to typically cause a maximum overall output uncertainty was the Principle Direction of Force (PDOF), contributing to up to 98% of the overall ΔV uncertainty; whereas the contribution from the mass and the crush values was significantly lower.

This study intends to take this work a stage further, by identifying how ΔV varies as a result of a range of input uncertainties. This allows direct comparison of methods of input data collection in terms of their effect on output uncertainty. This study will have larger input uncertainties for several parameters, compared to Smith and Noga (1982), as it will also be addressing non-direct measurements, therefore it cannot be presumed that the largest influence to ΔV uncertainty will be due to the PDOF.

2. Methods

2.1. Target level of uncertainty

To make appropriate judgements about the utility of different measurement protocols it is important to have an *a priori* concept of a target level of the precision in the value of ΔV calculated. Clearly, this level of uncertainty depends on the context in which the measurement of ΔV is to be used. To allow the reader to apply their own criteria, we will display the variation of ΔV over the full range of input parameters. However, for the purposes of this paper, we have chosen a target uncertainty of ΔV that corresponds to 90% confidence interval for risk of fatality, according to Josck's relationship (Eq. (1)). This condition requires the ΔV value to be accurate to 2.6%, the calculation of this is shown in Appendix 2. The study performed by Winnicki and Eppinger predicted different AIS probabilities for different 10 mph ranges of ΔV. The difference in $\geq$AIS 5 probabilities between adjacent high ΔV groups was large; for example, the probability of suffering an $\geq$AIS 5 injury from a ΔV of 41–50 mph was shown to be 0.472, whereas for 51+ mph it was shown to be 0.842. We have therefore chosen to use a 2.6% variation in ΔV as our acceptable limit, this is a narrower range to give more helpful results in terms of fatality risk.

2.2. The C-ΔV program

To perform this study we needed to run the calculations, used in the industry standard accident analysis software, a very large number of times over a range of values for each of the parameters studied. We therefore developed our own program (referred to within this paper as C-ΔV). The calculations used are well established, and their derivation is given in Appendix 1. The C-ΔV program was written in C, due to its versatility across computing platforms. It was written so that both input and output parameters could have either imperial or SI units, to facilitate its use on all sources of collision data.

2.2.1. Program validation

The program was validated with examples from the CRASH3 user manual. Table 1 shows the examples used to validate the C-ΔV program. All the examples were replicated accurately. Example 2 gave the largest discrepancy, of 1%, between the two pieces of software. However, this difference was very small and can be attributed to rounding differences since the two programs store values to different levels of precision. The correspondence between C-ΔV and CRASH3 was excellent for Example 1, which is the scenario studied in depth in this paper.

2.3. Inputs for the uncertainty analysis

The examples used in the CRASH3 manual are considered to be "typical" scenarios; therefore one of these cases was used as the studies baseline. We chose Example 1, which is a low speed side impact with both an offset and an angled PDOF, because this type of impact has been shown to have the highest uncertainty in ΔV calculation (Smith and Noga, 1982). The results presented are for vehicle 2.

There will be a level of uncertainty in all measured parameters; for this sensitivity analysis we needed to establish what these ranges should be. In many cases, we identified two levels of measurement precision; more precise direct vehicle specific measurements and less precise values from vehicle category data or photographic measurement. The values chosen and their justification are given below.

2.3.1. Vehicle parameters

The majority of commercial programs contain an internal database in which vehicles are categorised as belonging to one of 5–10 different categories. Analysis can be performed using category values rather than values that specifically correspond to the actual vehicle. Using these values in place of vehicle specific values has the potential to introduce a large amount of uncertainty. Therefore, we considered the possible magnitude of such discrepancies as part our investigation.

The low precision range we chose for size parameters was determined by the 9% maximum variation in vehicle width and length found within categories. More precise values can be obtained using databases, such as the National Highway Traffic Safety Administration's (NHTSA) Light Vehicle Inertial Parameter Database (NHTSA, in press). This reports vehicular length and width is given to the nearest inch, resulting in an expected uncertainty of $\pm$0.5in, or $\pm$0.0127 when converted to SI units; X_f and Y_s (being half these values) will therefore be studied over the range $\pm$0.00635 m.

Table 1
Results of program validation against CRASH3 manual examples.

Test and vehicle number			Energy			ΔV		
			CRASH3 (lbin)	C-ΔV (lbin)	% Diff (%)	CRASH3 (mph)	C-ΔV (mph)	% Diff (%)
1	T-Bone collision between vehicle 1 (front) and vehicle 2 (right hand side)	V 1	765,510	764,734	0	16.7	16.7	0
		V 2	361,205	361,536	0	21.6	21.6	0
2	Head on collision between vehicle 1 into an immovable barrier	V 1	979,421	980,099	0	33.9	34.3	-1
		–	–	–	–	–	–	–
3	Rear collision between vehicle 1 (front) and vehicle 2 (rear)	V 1	444,318	443,584	0	18.3	18.4	0
		V 2	574,902	575,998	0	23.7	23.7	0
4	Head on collision between vehicle 1 and vehicle 2	V 1	1,005,694	1,007,367	0	25.8	25.9	0
		V 2	1,031,724	1,034,138	0	33.3	33.3	0

Table 2
Vehicle input parameters.

Parameter	Units	Expected minimum value		Actual value	Expected maximum value	
		C	VS		VS	*C*
Y_s	(m)	0.777	0.847	0.853	0.860	0.930
K^2	(m^2)	1.675	1.900	1.904	1.908	2.132
A	(N/m)	17,163	22,066	24,518	26,970	31,874
B	(N/m^2)	323,448	415,861	462,068	508,275	600,688

C: Using category data as the input source. VS: Using vehicle specific data as the input source.

A variance of up to 12% was found for the value of radius of gyration (K^2) within a given vehicle category, therefore this will form the 'low precision' range. If using vehicle specific values from online databases (Carfolio.com 2011), the value of *I* is given in *ft*·*lb* s^2 with the values rounded to the nearest whole number, *I* relates to *K* by the following equation (where *m* is the mass of the vehicle):

$$K^2 = \frac{I}{m} \quad (2)$$

This gives a precision of K^2 of $\pm 6\,\text{in}^2$ ($3.871 \times 10^{-3}\,\text{m}^2$); and consequently the program shall be run through this range when looking at expected output ΔV uncertainty from this method of input.

Again, the limiting factor in stiffness parameters is the range of values within a vehicle category; up to 30% for A and B stiffness parameters. Vehicle specific database values (Accident reconstruction software, 2010) are given to three decimal places. This implies that the input range for this situation should be 24517.995–24517.0005 N/m and 462067.9995–462068.0005 N/m^2 for A and B respectively. However the stiffness value is still calculated using the assumption that b_0 (velocity at which there is no residual crush) is always 5 mph. Previous research has suggested that the accuracy of such calculation is ±10% (Smith and Noga, 1982). Therefore we used this range to determine the likely input uncertainty when using vehicle specific values.

The curb mass (the vehicle mass with all the fluids necessary for correct function), can typically be found using databases. Uncertainties are likely to arise from incorrect assumption of fuel level, occupant mass and cargo. Although fuel is included in the curb mass, a full tank is assumed, and the difference between this and an empty tank can substantial. The average fuel tank of a vehicle holds 40 kg of fuel, giving a potential uncertainty of a 40 kg over estimate. In post collision analyses where there is no information about the mass of the occupants, each occupant is typically estimated as a 50th centile male at 75 kg. However the typical range of male mass is between 55 kg and 95 kg (5th–95th centile) (Pheasant, 2003) resulting in a potential uncertainty of ±20 kg. As 5 people may be in the vehicle, this could amount to a maximum input uncertainty of ±100 kg. Finally the cargo is often not included in estimations of vehicle mass; this could be as much as 40 kg. With five 5th centile male occupants and an empty petrol tank the mass of the vehicle may be overestimated by 140 kg; with five 95th centile male occupants and cargo, the mass may be underestimated by 180 kg. This was the range over which we ran our analysis. This is clearly an approximation because in the case of an unbelted occupant the occupant's mass will not be directly coupled to the vehicle and its inclusion in the overall mass for the ΔV calculation is a simplification. Furthermore, even with belt restrained occupants, the occupant will not move as one with the vehicle, lowering the accuracy of the ΔV calculation. Addressing occupant movement separately is beyond the scope of this study, however the implications should be acknowledged.

Ideally the mass vehicle and cargo should be measured directly. Portable weighpads are generally used to determine the vehicle mass following a collision, such mats tend to have an accuracy of

Table 3
Collision input parameters.

Parameter	Units	Expected minimum value		Actual value	Expected maximum value	
		P	DM		DM	*P*
Weight	(kg)	980	1098	1160	1222	1300
PDOF	(°)	46	46	50	80	80
Crush	(m)	−0.0508	−0.0127	0	+0.0127	+0.0508
Crush Length	(m)	2.3622	2.4003	2.413	2.4257	2.4638
Offset	(m)	−0.5017	−0.4445	−0.4255	−0.4064	−0.3493

P: Using photographic evidence as the data input source. DM: Using direct measurement as the data input source.

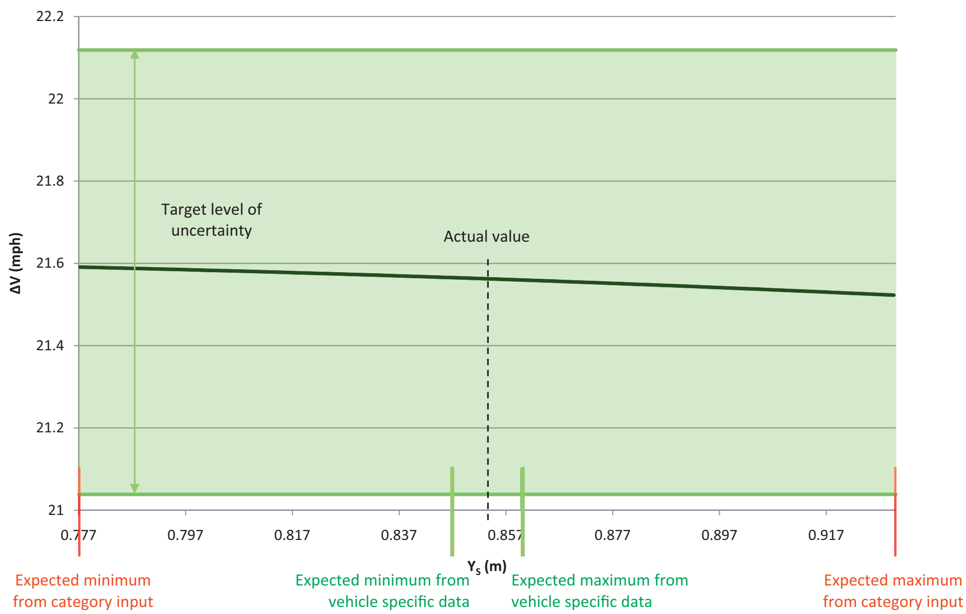

Fig. 1. Sensitivity of ΔV to Y_s.

±1% (Hawkley International, 2007). There is, however, the possibility of an immeasurable amount of fluid spillage during some collisions. Substantial fluid loss is possible from the fuel, oil and coolant reservoirs. A typical mid to large car, as used in our example, with a 4–6 cylinder engine will hold approximately 60 l of fuel, 5 l of water in the cooling systems and 4 l of oil. The mass of this much fluid is 40 kg, 5 kg and 5 kg respectively. This gives a potential of up to 50 kg potential fluid loss.

2.3.2. Collision parameters

As discussed, the use of photographs to estimate input crash parameters will be tested. The ideal case of direct vehicle measurement will also be addressed to compare the two levels of error, giving an idea as to which method should be used for each parameter.

The PDOF is estimated according to the visible crush pattern and knowledge of the scenario. For their sensitivity analysis, Smith and Noga (1982) investigated the difference between the PDOF estimation of a lay person and the value given by professional investigators for 35 vehicles, they found the estimates to differ by up to 30° (one clock face value). The example we evaluate has a PDOF of 50°, therefore the analysis can only be run between 46° and 80° (given the limitation of the requirement of a direct rather than glancing collision, Fig. 11). There is no direct measurement for this parameter without direct observation of the crash.

Use of photographs can introduce errors when estimating crush measurements. Certain elements of the photograph with known dimensions can be used as a scale, however photograph perspective and quality will impede the process and reduce the accuracy. Research has shown that the error of measuring length using 3D photogrammetry is ±1 in (Fenton et al., 1999), but as only a small amount of photos may available of the studied collisions, 3D photogrammetry is not necessarily possible, we have therefore increased the likely error from using a singular photograph to ±2

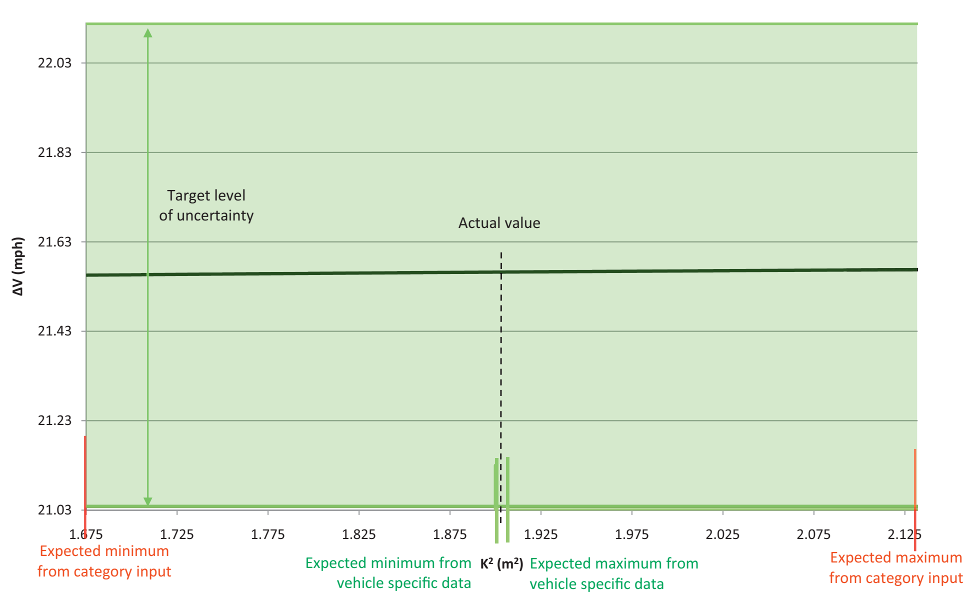

Fig. 2. Sensitivity of ΔV to radius of gyration.

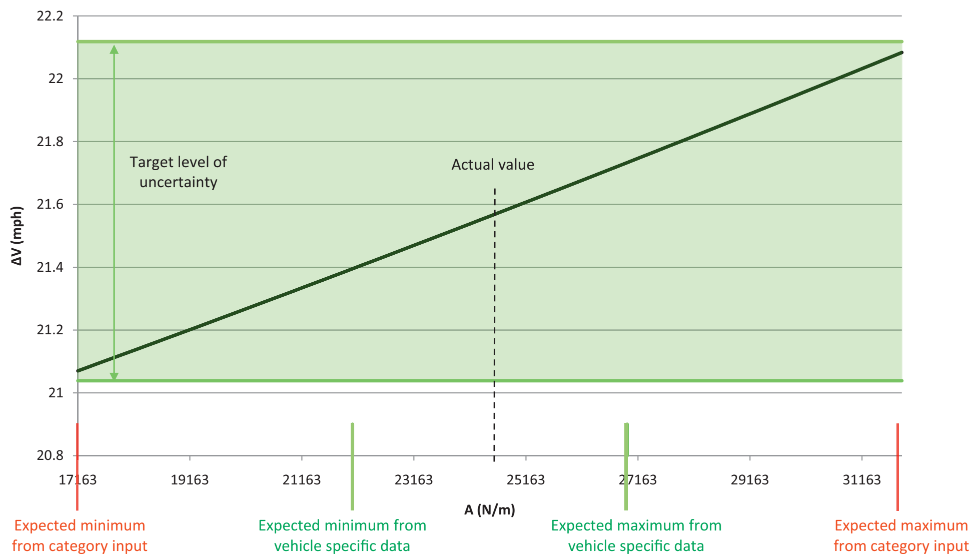

Fig. 3. Sensitivity of ΔV to stiffness parameter A.

in or ±0.0508 m; therefore an expected uncertainty of ±0.0508 m was used for both the crush measurements and length of the crush to account for such effects. When directly measured, crush values are typically rounded to the nearest inch, giving an expected uncertainty of ±0.5 in (0.0127 m) (NHTSA 1981).

The offset of the crush is the distance between the centre of the crush and the centre of the vehicle, it is found by measuring both the length of the crush area, and the location of the crush with respect to the vehicle edge. It therefore has two opportunities for uncertainty. To allow for this we used 1.5 times the uncertainties determined for crush parameters.

2.3.3. Summary of inputs

Tables 2 and 3 show the inputs that were used as the expected input uncertainty range for each example, note that Y_s is being used as X_f is more relevant for a side impact and compared to a frontal impact.

3. Results

The results for each parameter have a number of common features. They are plotted over the full 'less precise' range to indicate the worst case variation in ΔV that might result. Where there is a more precise method of determining an input parameter this is also indicated on the x-axis. Similarly, the actual example value is indicated. The ±2.6% target range for ΔV is indicated as a shaded corridor that is projected onto the x-axis so that the required precision in each parameter can be directly visualised.

3.1. Vehicle parameters

The results show that uncertainties in individual vehicle parameters do not influence uncertainty in ΔV uniformly. The Y_s (Fig. 1) and K^2 (Fig. 2) were found to have very minor effects on ΔV. Even large uncertainties, due to variation within a catergory, only

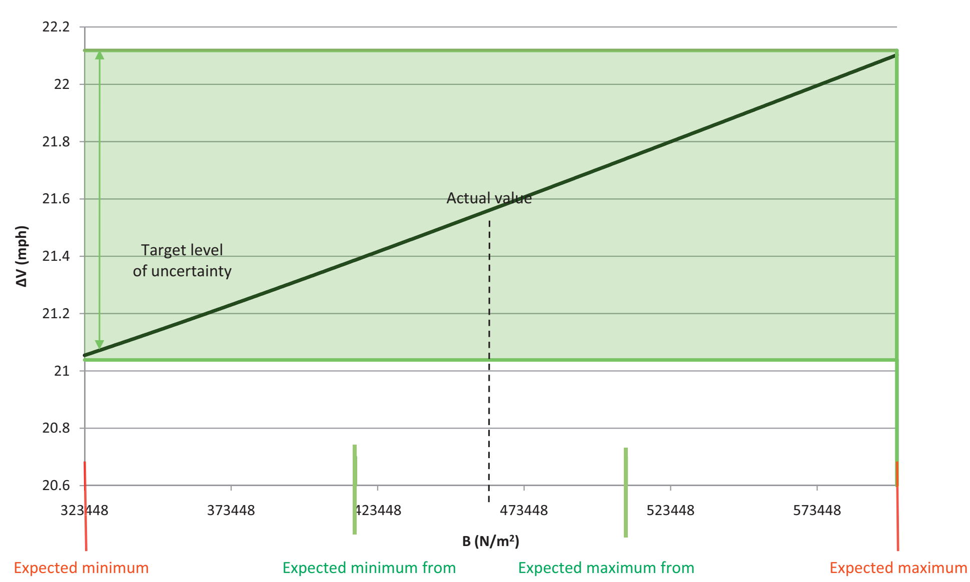

Fig. 4. Sensitivity of ΔV to stiffness parameter B.

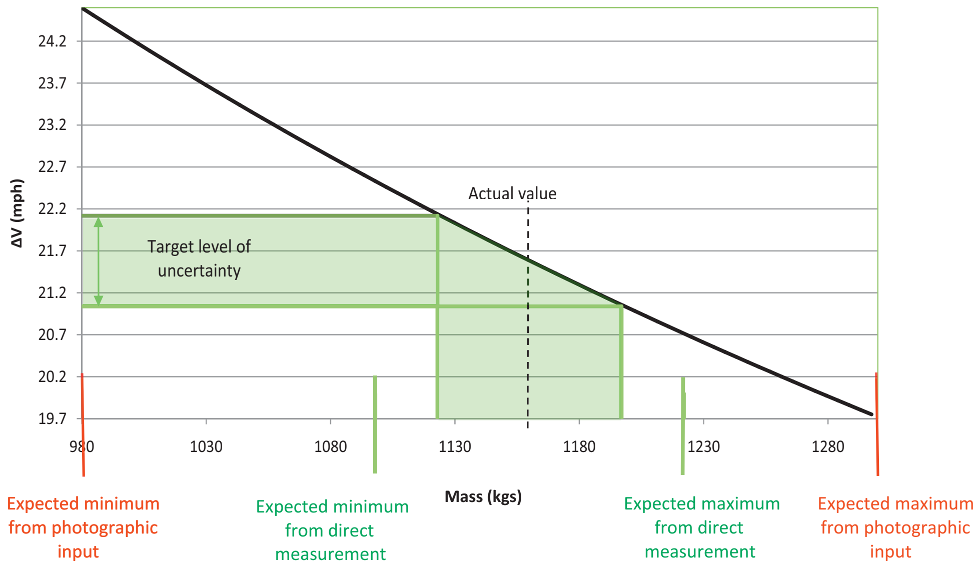

Fig. 5. Sensitivity of ΔV to vehicle mass.

changed the value of ΔV by 0.18% and 0.02% respectively. The stiffness parameter A (Fig. 3) had a greater influence on ΔV, but even the uncertainty due to using the category value did not take the value of ΔV beyond the target range.

The stiffness parameter B (Fig. 4) was shown to have a greater influence on the output, however the uncertainty arising from both use of the category value and the vehicle specific data changed the value of ΔV by 2.5% and 0.83% respectively keeping the ΔV within the target range of uncertainty for both input methods.

Either method of measuring the mass of the vehicle (Fig. 5) resulted in the ΔV moving out of the target uncertainty range; even direct measurement resulted in a ΔV uncertainty of up to 4.5%. For the value of ΔV to remain in the target range, the uncertainty in the mass of the vehicle and payload must be no more than ±3% (approximately 40 kg in this case).

3.2. Collision parameters

The uncertainty introduced into measurements of crush length (Fig. 8) and offset (Fig. 9) by photographic estimation had minor effects; changing the value of ΔV by 0.34% and 0.2% respectively and therefore keeping it within the target uncertainty range.

The uncertainty associated with estimating the crush measurement values (Fig. 7) from photographic evidence resulted in a 7.7% uncertainty in the value of ΔV, moving it significantly outside the target uncertainty range. Whereas, the uncertainty associated with direct measurement only changed the value of ΔV by 1.9% keeping it within range.

There was only one range of uncertainties for the input value of PDOF and the range was not centrally distributed. The results (Fig. 6) showed that if the input uncertainty was less than 6° the output ΔV remained in the target range.

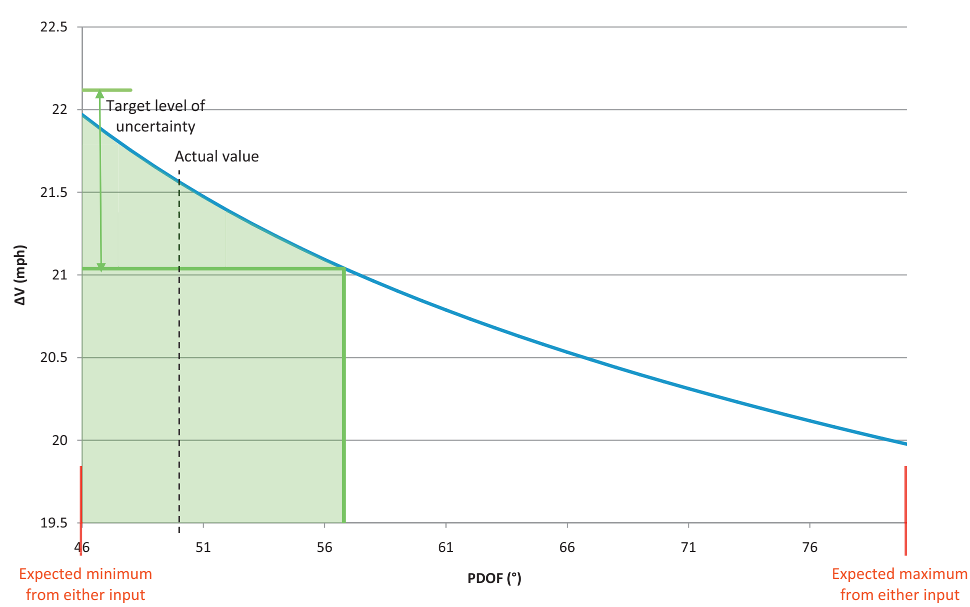

Fig. 6. Sensitivity of ΔV to principle direction of force.

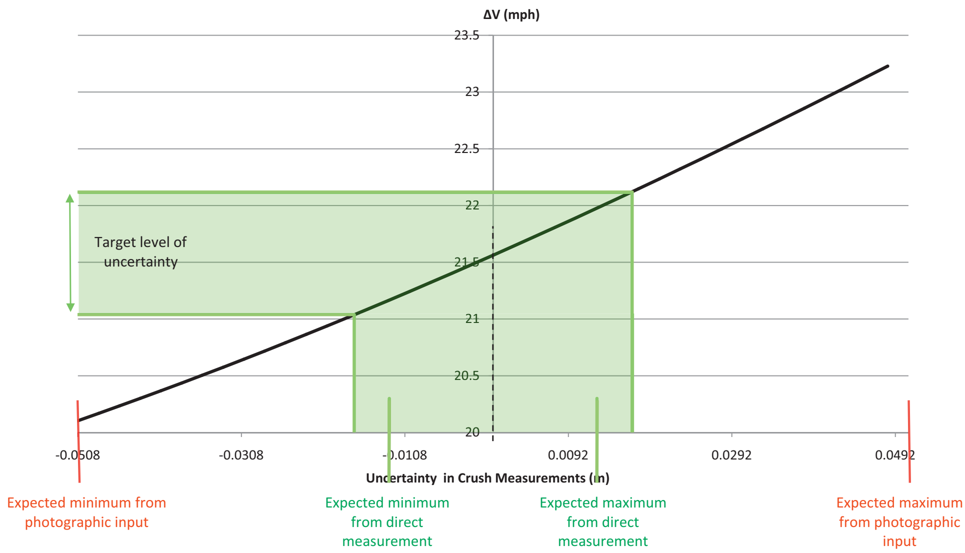

Fig. 7. Sensitivity of ΔV to crush measurements.

4. Discussion

The software developed to implement this sensitivity analysis was validated against examples presented in the CRASH3 manual. The findings are therefore representative of, and relevant to, commercially available tools such as CRASH3 and AIDamage.

Categorical values can be used for the input values of Y_s, K^2 and A to give the output value of ΔV within the target error range; however the input value of B should be found for the specific vehicle, and the category value should not be used, in order to give the output value of ΔV within the target range.

Photographic estimation is suitable for the input values of crush length and offset to give the output value of ΔV within the target error range; whereas the input value of crush measurement should be directly measured, as the use of photographic estimation for this input value has shown to give an unacceptable error in the value of ΔV.

Neither method of recording the value of mass provided a ΔV within the required accuracy range. Despite measuring the mass of the vehicle and cargo directly using weigh pads, the unknown amount of fluid loss results in unacceptable uncertainties in the output ΔV. Future research should be directed at providing a more accurate estimation of the amount of fluid lost; there are several methods that could be used; for example, in the case of no obvious penetration to any of the fluid reservoirs it is unlikely that there will have been any significant loss; in which case the input uncertainty is significantly reduced. A second method could be direct observation; if the collision has occurred on a road any fluid lost will be evident and therefore would allow the scene investigator to make an estimation of the amount lost. Furthermore, protocol could be established stating that if there is obvious reservoir punctures, all fluid reservoirs except fuel should be assumed to be full at the time of the collision as this should be the case for roadworthy vehicles. Methods of determining the amount of fuel prior to the collision could then be set-up to give a more definitive measure of the fluids within the vehicle at the time of collision. This would reduce the uncertainty to ±12 kg for the measured input method; giving an output value of ΔV in the target error range. Finally, as

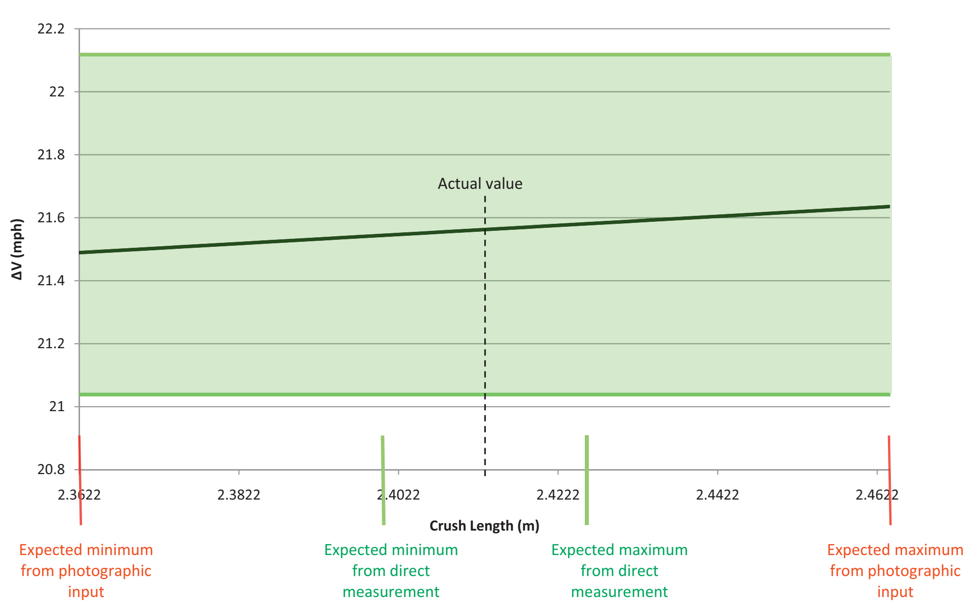

Fig. 8. Sensitivity of ΔV to crush length.

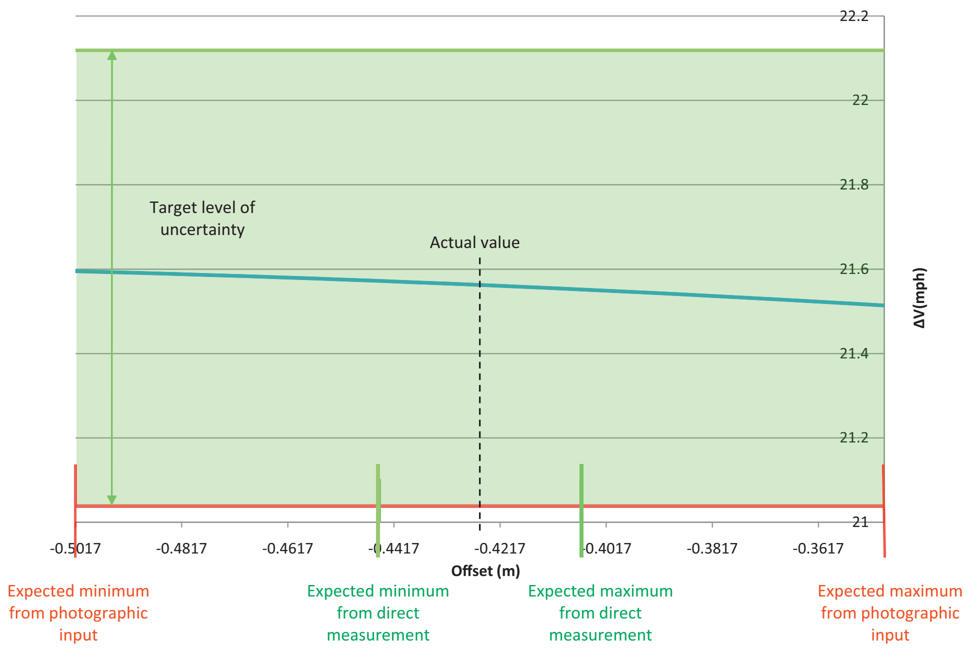

Fig. 9. Sensitivity of ΔV to crush offset.

noted in the methods section, the occupant and the vehicle may not be directly coupled; separation of the occupant's mass from the ΔV equation and inclusion with improved mathematical formula could increase the accuracy and lower the ΔV sensitivity to the vehicle mass parameter.

Estimation of PDOF was also found to introduce significant errors in the determination of ΔV. Our results indicate that PDOF should be quantified with a precision of better than $\pm 6^\circ$.

The estimated input uncertainty of 30° for the PDOF is perhaps outdated; it is likely that current collision investigators are more able to determine the PDOF precisely than in the 1980s, as discussed by Smith and Noga (1982); unfortunately there is no more recent documentation Determination of the PDOF values is now aided by collision simulation, a function offered by both standalone programs(Neptune et al., 1992) and the CRASH3 program itself, adding to the likelihood that the actual input uncertainty is lower. It is therefore suggested that future work is directed at determining the up to date uncertainty in this parameter input. The PDOF lies through the damage centroid of each vehicle at the time of collision. Therefore immediate reconstruction of the point of impact would lower the input uncertainty of this value; however the likelihood of this reconstruction being possible before the occupant reaches hospital is low, especially if specialist extraction from the vehicle is required. Methods of refining the PDOF are present by Neades (2011) that include PIM calculations (Planar Impact Mechanics),

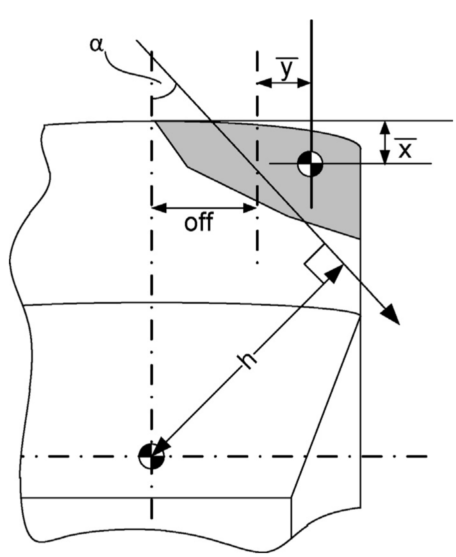

Fig. 10. Definition of terms.

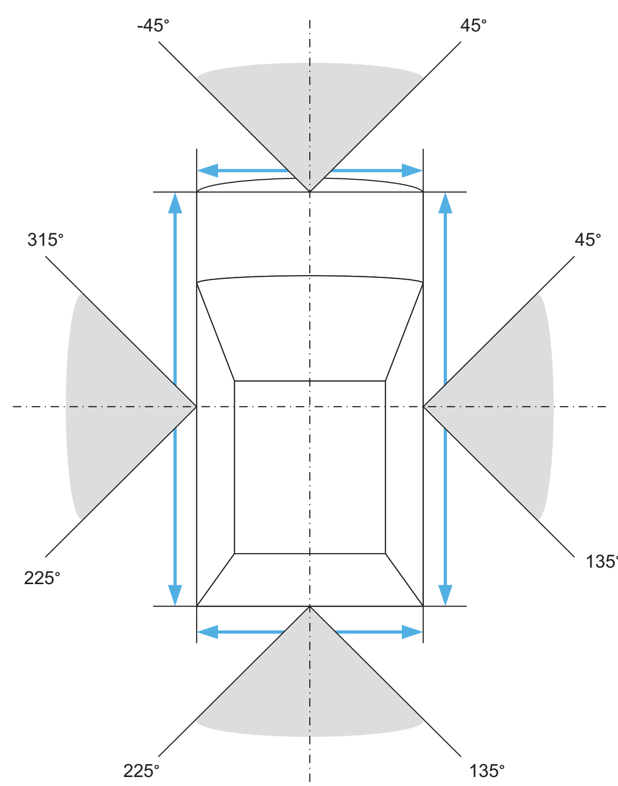

Fig. 11. Limitations to permissible impact directions and points for the analysis.

however an impact plane needs to be defined, which requires either specialist knowledge or reconstruction.

Often the time available to investigate the scene and take measurements is small. Information regarding the collision is needed by the medical staff as soon as the occupant begins treatment and roads need to be reopened as soon as possible. This study has shown that the only measurements that need to be taken from the damaged vehicle are the crush measurements, PDOF and the mass. Once the vehicle make and model has been identified a vehicle specific stiffness parameter *B* value can be looked up, vehicle category standard values can be used for the remaining parameters.

5. Conclusion

It is possible to measure the vehicle and collision parameters with sufficient accuracy to determine ΔV to a level of precision that is useful to predict occupant fatality. In many cases, ΔV is largely insensitive to the input parameter and category values or values determined from photographs may be used. A vehicle specific value of the stiffness parameter B should be used. Direct measurement of crush measurements and vehicle mass (including the best estimates of fluid loss) should be used. Similarly the mass of occupants and cargo should be measured directly rather than estimated from 50th centile values. Calculation of ΔV is sensitive to PDOF which should be measured with a precision of better than $\pm 6^{\circ}$.

Appendix A. Appendix 1

Theory used to calculate ΔV.

A.1. Nomenclature

- r risk of injury
- M_i mass of vehicle i
- X_i displacement of impacted surface of vehicle i
- K_I stiffness of impacted surface
- X_i acceleration of vehicle i
- ΔV_i delta-V of vehicle i
- E_A energy of deformation of both vehicles
- K^2 radius of gyration
- γ_I proportion of vehicle i's mass acting in the collision
- $h, \bar{y}, \bar{x}, \alpha$ Shown in Fig. 10
- L length of crush
- A stiffness (crush resistance) at which no residual crush occurs
- B rate of stiffness change
- C crush
- G force that can be applied to produce no residual crush

A.1.1. Theory development

Emori (1986) applied Hooke's law to a collision scenario by using springs to represent the colliding vehicles; this was then combined with Newton's second law of motion, to give Eqs. (3) and (4) for a two vehicle collision:

$$M_1\ddot{X}_1 = -\left(\frac{K_1K_2}{K_1+K_2}\right)(x_1-x_2) \tag{3}$$

$$M_2\ddot{X}_2 = -\left(\frac{K_1K_2}{K_1+K_2}\right)(x_1-x_2) \tag{4}$$

McHenry (Nhtsa, 1981) simplified this by looking at the displacement of the two vehicles relative to each other allowing the combination of Eqs. (3) and (4). He then used standard equations for conservation of momentum and kinetic energy could be combined to eliminate any remaining displacement terms (assuming that the two vehicles reach a common velocity). A term, γ, was introduced to represent the proportion of each vehicles mass acting in the collision because the full mass of the vehicle was not involved in (Fig. 10) non-central collisions.

$$\Delta V_1 = \sqrt{\frac{2\gamma_1 E_A}{M_1\left(1+\frac{\gamma_1 M_1}{\gamma_2 M_2}\right)}} \tag{5}$$

where

$$\gamma = \frac{k^2}{k^2+h^2} \tag{6}$$

and

$$h = (\bar{y}+\text{off})\cos\alpha + (X_f+\bar{x})\sin\alpha \tag{7}$$

The method of determining E_A was outlined by Campbell (1974); the method assumes that the residual crush (in or m) is proportional to the stiffness (resistance to force - lb/in or N/m) to give a linear function that when integrated will give the energy dissipated in the formation of the crush (lbin or Nm) as shown in Eq. (8):

$$E_A = \int_o^L \left(AC + \frac{BC^2}{2} + G\right) dL \tag{8}$$

As the coefficients A, B and G are unknown functions of L, this integral must be solved using approximation methods such as the trapezium rule.

The stiffness coefficients A and B are stored within the ΔV programs, and are established *via* previous crash testing. The crush at two different velocities is measured and a linear relationship between the two is created; the combination of this function and Eq. (8) allows the values of A and B to be found.

A.1.1.1. Theory limitation. The theory does not work for "glance off" type of collisions, any crash that occurs between −45 and 45° is considered a frontal collision, a collision of −50° to the front of the vehicle is out of the scope of the program as this is considered a "glance off" collision. Fig. 11 shows the PDOF's that are within the scope of the program.

Appendix B. Appendix 2

Calculating the acceptable error in ΔV.

Looking at an error of −10% in the risk as stated. The erroneous risk is termed r_2 and the consequent ΔV is termed as $\Delta V_2 \cdot r_1$ and ΔV_1 are the original values.

$$r_2 = r_1 - 0.1r_1$$

$$r_2 = r_1(1-0.1)$$

$$\Delta V_1 = \sqrt[4]{61.5^4 r_1}$$

$$\Delta V_2 = \sqrt[4]{61.5^4 r_2}$$

$$\Delta V_2 = \sqrt[4]{61.5^4 r_1(1-0.1)}$$

$$\Delta V_{\text{error}} = \frac{\Delta V_2 - \Delta V_1}{\Delta V_1}$$

$$\Delta V_{\text{error}} = \frac{\Delta V_2}{\Delta V_1} - 1$$

$$\Delta V_{\text{error}} = \sqrt[4]{\frac{61.5^4 r_1(1-0.1)}{\sqrt[4]{61.5^4 r_1}}} - 1$$

$$\Delta V_{\text{error}} = \sqrt[4]{(1-0.1)-1}$$

$$\Delta V_{\text{error}} = -0.026$$

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