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CONTENTS

2.2. Risk and reliability theory, Part I

F. DeVylder, M. Goovaerts, Upper bounds for ruin probabilities in a new general risk model by the martingales methods.

M. Ruohonen, Applicability of the corrected diffusion approximation to the problem of ruin in risk processes.

2.3. Queuing theory, Part I

M.I. Reiman, A multi-class feedback queuing system in heavy traffic.

W.A. Rosenkrantz, The M/G/1 queue—a martingale approach.

S. Stidham Jr., T. Rolski, Continuous versions of the queueing formulas $L = \lambda W$ and $H = \lambda G$.

2.4. Stochastic analysis and martingales, Part I

M.A. Berger, A.D. Sloan, Signed measures on function space and higher order elliptic operators.

H. Korezlioglu, Two-parameter stochastic differential equations.

R.J. Vanderbei, Toward a stochastic calculus for several Markov processes.

S.J. Wolfe, A characterization of certain stochastic integrals.

2.5. Probability models

R. Durrett, A model of polymer degradation.

N. Hadidi, On the question of the power of incumbency.

H. Mendelson, Models of price creation.

2.6. Random walk and limit theorems, Part II

R.A. Doney, On the existence of the mean ladder height for random walks.

L. Saulis, On large deviations for a sum of random numbers of multidimensional random vectors.

2.7. Markov processes and diffusions, Part I

N.U. Ahmed, A class of generalized nonlinear functionals of Brownian motion.

H. Kaspi, Regenerative systems and Markov additive processes.

R. Sentis, Properties of the reflection operator associated with a transport Markov process and application.

2.8. Optimization, Part I

C. Cresswell, Optimization of control system modelling using stochastically disturbed test measurements.

H. Deppe, Existence of average cost optimal strategies in multichained semi-Markov decision problems.

2.9. Stochastic analysis and martingales, Part II

E. Bolthausen, Exact convergence rates for some martingale central limit theorems.

D. Green, Continuity of a class of random integrals.

J.H.B. Kemperman, J.C. Smit, On the moments of martingales with conditionally symmetric increments.

2.10. Inference, Part I

J. Brode, On the estimability of quasi-diffusions.

V.E. Beneš, I. Karatzas, Estimation and control for linear, partially observable systems with non-Gaussian initial distribution.

A.F. Karr, State estimation for Cox processes on general spaces.

2.11. Random walk and limit theorems, Part III

C.C Heyde, On the asymptotic behaviour of random walks on an anisotropic lattice.

D. Kannan, A random walk model of the Helminth Trichostrongylus Retortaeformis.

P. Ney, Dominating points and asymptotics of large deviations on R^d.

2.12. Risk and reliability theory, Part II

R.C. Gupta, On the determination of mixing density in reliability studies.

R. Maller, S.I. Resnick, Limiting behaviour of sums and the term of maximum modulus.

M. Shimura, A class of conditional limit theorems related to ruin problems.

2.13. Point processes and geometric probability

E. Arjas, Estimating dependent life lengths: a point process approach.

H. Berbee, On covering single points by randomly ordered intervals.

Y. Ito, Generalized functionals of a Poisson process.

M.S. Ali Khan, Least upper bound for a class of triangles.

J.P. Rasson, Geometric probability and entropy of some point processes.

2.14. Queueing theory, Part II

O.J. Boxma, A.G. Konheim, Exponential queueing networks with finite capacities.

J.W. Cohen, O.J. Boxma, The M/G/1 queue with alternating service formulated as a Riemann–Hilbert problem.

J.D. Griffiths, A traffic intersection model with non-Poisson arrivals on the major road.

J.H.A. DeSmit, On a system of Wiener–Hopf integral equations occurring in the theory of the multi-server queue.

2.15. Stochastic and dynamic systems, Part I

D.N. Assaf, B.Z. Levikson, Closure of phase type distributions under operations arising in reliability theory.

H. Crauel, Ergodic theory of linear stochastic systems.

Y. Yavin, A class of observers for nonlinear stochastic systems with jump process observations.

2.16. Games and information theory

B. Colin, G. Giroux, Une égalité sur la conservation de l'information.

P. Hammad, A. Taranco, Information géneralisée liée à certains processus stochastiques.

J.H.B. Kemperman, On the Shapley index of power for a weighted majority game.

M. Pachter, Y. Yavin, One pursuer and two evaders on the line—A stochastic pursuit–evasion differential game.

2.17. Optimization, Part II

J. Franke, Necessary conditions on optimal Markov controls for stochastic processes.

G. Mazziotto, J. Szpirglas, Optimal stopping with discontinuous partial observation.

F.A. Van der Duyn Schouten, A. Hordijk, On the existence of average optimal policies in Markov decision drift processes with general state and action spaces.

2.18. Markov processes and diffusions, Part II

R. Bass, A representation of additive functionals of d-dimensional Brownian motion.

E. Csáki, P. Révész, Strong invariance for local times.

J. Glover, Markov processes with identical last exit distributions.

2.19. Branching processes, Part I

L.G. Gorostiza, Convergence of supercritical branching random fields.

O. Nerman, On the convergence of supercritical general (C–M–J) branching processes.

O. Nerman, P. Jagers, The stable doubly infinite pedigree process of supercritical branching populations.

W.A. O'Waugh, Kin numbers in Galton–Watson populations.

2.20. Distribution theory

P. Franken, B. Lisek, On Wald's identity for dependent variables.

C.M. Goldie, Differences and quotients of record values.

R.L. Smith, Rates of convergence in extreme value theory.

2.21. Inference, Part II

W.J.R. Eplett, The role of Gaussian processes in the asymptotic theory of rank tests.

T. Lusin, I.W. Saunders, A Markov model for on-stream quality estimation.

I. Weissman, Confidence intervals for the threshold parameter.

2.22. Stochastic and dynamic systems, Part II

L. Arnold, V. Wihstutz, Stationary solutions of linear systems with additive noise.

J. Bulatović, Correlation function and measurable approximations of an arbitrary second order process.

J.M. Harrison, S.R. Pliska, A stochastic calculus model of continuous trading: Complete markets.

Y. Kifer, Direct and inverse problems for small random perturbations of dynamical systems.

2.23. Miscellaneous topics

P.J. Brockwell, S.I. Resnick, J.M. Gani, Birth, immigration and catastrophe processes.

A.T. Lawniczak, Gaussian measures on spaces of measurable functions.

W. Vervaat, Spaces of upper semicontinuous functions and their use in probability.

2.24. Inference, Part III

J. Šindelář, How to measure complexity in statistical decision making.

J.S. White, Maximum likelihood estimation for a diffusion process.

2.25. Branching processes, Part II

M. Adès, J.P. Dion, G. Labelle, K. Nanthi, Recurrence formulas in a simple branching process and the maximum likelihood estimation of the age.

T. Kuczek, The convergence of the age distribution for supercritical Bellman–Harris process and corollaries.

INTRODUCTION

The Tenth Conference on Stochastic Processes and their Applications was held at the Université de Montréal in Montréal, Canada, over the period 23–28 August 1981. The Conference was arranged under the auspices of the Committee for Conferences on Stochastic Processes of the ISI's Bernoulli Society for Mathematical Statistics and Probability. It was sponsored by the Natural Sciences and Engineering Research Council of Canada, Quebec Department of Education and the University of Montreal.

The Conference was attended by 232 scientists coming from the U.S.A. (73), Canada (54), France (15), the Netherlands (13), West Germany (11), Great Britain (9), Israel (8), Japan (6), Australia (5) and several other countries.

The scientific program consisted of 20 invited papers and 77 contributed papers. The following are the abstracts of the papers presented. For papers whose abstracts were not received, only the titles and the authors are listed, and they are marked with an asterisk.

1. INVITED PAPERS

Large Deviations in the Branching Random Walk

J.D. Biggins, *University of Sheffield, England*

By the branching random walk we mean the following process (or some variant of it). An initial ancestor is at the origin. He has children with positions given by a point process on R (or sometimes R^n). Each of these children has children in a similar way, their position being given by independent copies of the original point process centred at each parent's position. Subsequent generations are formed similarly. If μ is the intensity measure of the first generation point process and μ^n is its $n$fold convolution then μ^n is the intensity measure of the point process formed by the nth generation. When the process is supercritical results on the asymptotic properties of μ^n lead naturally to analogous results for the branching random walk. In particular 'large deviation' results for μ^n have (or should have) counterparts in the properties of the nth generation point process. These results will concern the density of nth generation people a long way from the 'centre' of the generation. A number of illustrations of this will be given.

Asymptotic Normality of Finite Fourier Transforms of Stationary Processes

David R. Brillinger, *University of California, Berkeley, CA*

Under regularity conditions, including stationarity and mixing, the Fourier transform of a segment of a random process may be shown to be asymptotically Gaussian. This work presents such a central limit theory for the case of random Schwartz–Bruhat distributions over a locally compact abelian group.

Such central limit results may be used to develop a variety of statistical methods of use in inference. This work discusses such methods for spectrum estimation, system identification and parametric estimation. The operation of tapering is seen to play an essential role.

Other results discussed include: almost sure bounds, Berry–Esseen bounds, large deviation results and Edgeworth expansions for finite Fourier transforms.

Erdös–Rényi Laws Versus Strong Invariance

Miklós Csörgö, *Carleton University, Canada*

The Erdös–Rényi (J. Anal. Math. 23 (1970) 103–111) law has played an important role in sorting out what invariance versus noninvariance is all about. It implies for example that if the i.i.d. rv $X_1, X_2, \ldots$ have mean zero, variance one and a moment generating function (m.g.f.) in a neighbourhood of zero, and if F is the distribution function of X_1 and W is a standard Wiener process such that $|S_n - W(n)| \overset{\text{a.s.}}{=} o(\log n)$, then $F = \Phi$, the unit normal distribution function (this is a special case of a theorem of Bártfai, Studia Sci. Math. Hungar. 1 (1966) 161–168). Here, and also in the sequel, $S_n := X_1 + \cdots + X_n$. We have also that, if the m.g.f. of X_1 does not exist in a neighbourhood of zero, then $S_n - W(n) \geq c \log n$ i.o. for any $c > 0$ and any standard Wiener process W (cf., e.g., Theorem 2.3.2 in Csörgö and Révész, Strong Approximations in Probability and Statistics, Academic Press, New York, 1981). Consequently, with $F \neq \Phi$, a best invariance principle can, at best, have the form $|S_n - W(n)| \overset{\text{a.s.}}{=} O(\log n)$, and that only if F has a m.g.f. (cf. Komlós, Major and Tusnády, Z. Wahrsch. Verw. Gebiete 32 (1975) 111–131 and 34 (1976) 33–58, where it is proved (one among many) that the latter form of strong invariance is indeed true for all those distributions which do have a m.g.f.). A further interplay of some new Erdös–Rényi laws and invariance principles will be also discussed on the basis of some recent developments in this area (cf. Révész, Carleton Math. Series No. 164 (1980) and Csörgö and Steinebach, Carleton Math. Series No. 166 (1980))

A Survey of Spatial Branching Processes

K. Fleischmann, *Academy of Sciences, GDR*

The purpose of this talk is to report on recent results in branching theory we obtained in our group in Berlin.

Each particle δ_x at position x in a complete separable metric space A generates a (random) cluster χ_x of particles situated in A. Different particles produce independent clusters. Intuitively, we get branching processes $(\Phi_t)_{t=0,1,\ldots}$ where the states are infinite particle systems. Our first purpose is to generalize known results from the critical spatially homogeneous case and substochastic case to this more general type of processes. This concerns limit theorems for (Φ_t) and structure assertions for cluster-invariant systems. Moreover, we are interested in ergodic properties of the equilibrium dynamics. In particular, we formulate conditions for past-tail-triviality of time stationary processes $(\Phi_t)_{-\infty<t<\infty}$.

Finally, in the spatially homogeneous case under a second moment assumption for χ_0 all extreme cluster-invariant systems are tail-trivial (in space). Analogously, measure-valued branching processes can be treated.

Counting Processes and Multiple Time Transformations

Thomas G. Kurtz, *Department of Mathematics, University of Wisconsin at Madison, Madison, WI 53706*

A family of counting processes $N = (N_1, N_2, \ldots)$ is usually characterized by specifying the jump intensities $\lambda_k(t, N)$, $k = 1, 2, \ldots$, that is nonnegative functions such that at time t, $\lambda_k(t, N)$ depends only on values of N at times at or before time t. The corresponding family of counting processes is to satisfy

$$\mathbf{P}\{N_k(t+\Delta t)-N_k(t)>0 \mid \mathfrak{F}_t\}=\lambda_k(t, N)\Delta t+o(\Delta t). \tag{1}$$

Given the jump intensities, these counting processes can be obtained as solutions of the system of equations,

$$N_k(t)=Y_k\left(\int_0^t \lambda_k(s, N)\,\mathrm{d}s\right), \quad k=1,2,\ldots, \tag{2}$$

where the Y_k are independent Poisson processes.

Under very general conditions it can be shown that the solution of (2) depends continuously on the λ_k.

Various scaling limit theorems can be obtained. If $\lambda_k^{(n)}(t, N)=n\lambda_k(t, n^{-1}N)$, then the solution $N^{(n)}$ of (2) satisfies

$$n^{-1}N_k^{(n)}(t)=n^{-1}Y_k\left(n\int_0^t \lambda_k(s, n^{-1}N^{(n)})\,\mathrm{d}s\right). \tag{3}$$

Under appropriate hypotheses the law of large numbers can be applied to $n^{-1}Y_k(nu)$ to show that $n^{-1}N^{(n)}$ converges to the solution of the deterministic system of integral equations

$$Z_k(t)=\int_0^t \lambda_k(s, Z)\,\mathrm{d}s. \tag{4}$$

A corresponding central limit theorem for

$$\sqrt{n}(n^{-1}N_k^{(n)}(t)-Z_k(t)) \tag{5}$$

can be obtained using the weak convergence of $n^{-1/2}(Y(nu)-nu)$ to Brownian motion.

Some Applications of Stochastic Approximation Schemes

D.L. McLeish, *University of Alberta, Canada*

Stochastic approximation has been applied in a number of statistical problems. We discuss a few of these applications. For maximum benefit from the use of stochastic approximation, two conditions should be satisfied:

(a) A reasonably complex numerical procedure needs to be run for each sample size (e.g., for estimating a parameter).

(b) Observations are taken sequentially and there is some advantage (e.g., for formulating a stopping rule) in keeping an updated version of the estimator.

Some examples in statistics, where considerable reduction in computation results from the use of stochastic approximation, are discussed.

Self-similar Extremal Processes

G.L. O'Brien, *York University, Canada*
P. Torfs and W. Vervaat, *University of Nijmegen, The Netherlands*

Let $\{Y(t),\ t\in\mathbb{R}\}$ be a stationary process with values in $\bar{\mathbb{R}}=[-\infty,\infty]$. Let $M(t)=\sup\{Y(s): 0\leqslant s\leqslant t\}$. We call M an extremal process. M is said to be self-similar if, for some $H\in\mathbb{R}$, $M(at)$ has the same finite-dimensional distributions as $a^H M(t)$ for all $a>0$. Some properties of self-similar extremal processes are discussed and some examples are given. One objective is to learn about the possible limit distributions of sequences of partial maxima from stationary sequences.

Maximal Rule and Weak Solutions for Stochastic Differential Equations

J. Pellaumail, *INSA*, *France*

Let us consider the family $\mathcal{R}$ of the 'rules' (see [1, 2]) which are the 'weak solutions' (see [4]) of the stochastic differential equation $\mathrm{d}X=a(X)\,\mathrm{d}Z$ when a depends on all the past and is continuous for the uniform topology and when Z is a general semi-martingale. This family $\mathcal{R}$ is studied; namely, the following two results are stated:

(1) $\mathcal{R}$ is nonempty (cf. [3]).

(2) There exists an element R of $\mathcal{R}$ which is 'maximal' in $\mathcal{R}$, i.e., for every element R' of $\mathcal{R}$ dominated by R, one has $R'=R$ (new result).

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Recurrence of Diffusions

S. Ramasubramanian, *North-Eastern Hill University, India*

As asymptotic behaviour of a homogeneous diffusion is known to depend on whether the diffusion is transient, positive recurrent or null recurrent ([6, 7]), obtaining criteria for recurrence, transience, etc. is important. Criteria for recurrence, transience and positive recurrence for homogeneous diffusions with smooth coefficients were announced by Khas'minskii [6], and were independently proved by Friedman [5]. Extensions of such criteria to general nonhomogeneous diffusions (in the sense of Stroock and Varadhan), a dichotomy theorem for homogeneous diffusions, connections with exterior Dirichlet problems, a functional central limit theorem for positive recurrent homogeneous diffusions, etc. may be found in [1] or [4]. A nice exposition and survey may be found in [2, 3]. Recurrence of projections of diffusions is discussed in [8].

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On the Lengths of Increasing Runs

P. Révész, *Hungarian Academy of Sciences, Hungary*

Let $U_1, U_2, \ldots$ be a sequence of independent uniform $(0, 1)$ r.v.'s and let $R_1, R_2, \ldots$ be the lengths of increasing runs of $\{U_i\}$, i.e., $X_1 = R_1 = \inf\{i: U_{i+1} < U_i\}, \ldots, X_n = R_1 + R_2 + \cdots + R_n = \inf\{i: i > X_{n-1}, U_{i+1} < U_i\}$. The first theorem states that the sequence $(3/2)^{1/2}(X_n - 2n)$ can be approximated by a Wiener process in strong sense.

Let $\tau(n)$ be the largest integer for which $R_1 + R_2 + \cdots + R_{\tau(n)} \leq n$, $R_n^* = n - (R_1 + R_2 + \cdots + R_{\tau(n)})$ and $M_n = \max(R_1, R_2, \ldots, R_{\tau(n)}, R_n^*)$. Here, M_n is the length of the longest increasing block. A strong theorem is given to characterize the limit behaviour of M_n.

The limit distribution of the lengths of increasing runs is our third problem.

Equilibrium States for Potlatch Processes

F. Spitzer, *Cornell, New York, U.S.A.*

Let S be a countable set, and define configuration space as $X = (R^+)^S$. The object is to define a Markov process η_t with state space X as follows. (i) There is a given rate $\Pi(x, y) \geq 0$ for selecting an ordered pair $\{x, y\}$ of sites $x, y \in S$. (ii) When $\{x, y\}$ has been selected, η changes instantly to η' where $\eta'(z) = \eta(z)$ for all $z \neq x$ and

$\neq y$, while

$$\begin{pmatrix} \eta'(x) \\ \eta'(y) \end{pmatrix} = \begin{pmatrix} \alpha & 1-\beta \\ 1-\alpha & \beta \end{pmatrix} \begin{pmatrix} \eta(x) \\ \eta(y) \end{pmatrix}$$

where (α, β) is a random variable in $[0, 1]^2$. Of course i.i.d. versions of (α, β) are used on separate occasions.

The process η_t is called *regular potlatch* if

$$\sum_y \Pi(x, y) = \sum_y \Pi(y, x) < \infty \quad \text{and} \quad \mathbf{E}[\alpha(1-\alpha) + \beta(1-\beta)] > 0.$$

Theorem. *When $|S| < \infty$ and the regular potlatch process, η_t, has a unique equilibrium state μ, up to scalar multiple, then $\mathbf{E}_\mu[\eta(x)\eta(y)]$ is independent of x and y as long as $x \neq y$.*

This theorem suggests the conjecture that, for regular potlatch on an infinite set S, all pairs $\eta(x)$ and $\eta(y)$ with $x \neq y$ should be uncorrelated in equilibrium. This can indeed be proved in the special case when S is an abelian group with $\Pi(x, y) = \Pi(0, y-x)$.

Approximations for Point Processes and Networks of Queues

Ward Whitt, *Bell Laboratories, Holmdel, NJ 07733*

Complex systems such as computers and communication networks require complex models such as networks of queues, which in turn require simulation or approximation. Of the many approximation schemes (see [2]), the decomposition method appears particularly promising [3]. The first step is to decouple or decompose the model by replacing all the component flows (point processes) by independent stochastic processes. With this approximation, each queueing facility within the network can be analyzed in isolation, providing there is a method to treat feedback. The second step is to work with simple summary descriptions of each stochastic process involving two or three parameters. The idea is to have an elementary calculus for transforming the parameters to describe the basic operations of composition (superposition), decomposition (thinning), flow through a queue, overflow, etc. For this purpose, it is convenient to use renewal processes as the approximating point processes and the moments of the renewal interval as the basic parameters.

In this context, the question is: how should we approximate a point process by a renewal process? This can be done in two steps: first, properties of the point process are used to specify a few moments of the interval between renewals; then, if necessary, a convenient distribution can be fit to these moments. However, the renewal process or renewal-interval moments we get depend on the point process properties we use. In [4] two different methods are suggested for specifying the moments of the renewal interval. The stationary-interval method equates the moments of the renewal interval with the moments of the stationary interval in the point process to be approximated. The asymptotic method, in an attempt to account for the dependence among successive intervals, determines the moments of the renewal interval by matching the asymptotic behavior of the moments of the sums of successive intervals. These two procedures have been applied to approximate the superposition (merging) of point processes, and compared with the aid of computer simulation in the setting of a single-server queue with multiple renewal arrival processes. Both procedures have regions where they perform well, but also both have regions where they perform poorly. Much better than either procedure alone is a refined hybrid procedure [1]. For a large class of $\sum G_i/G/1$ queues, the average error in the mean queue length for the hybrid procedure was about 3 percent.

It is not difficult to apply the basic approximation methods and develop hybrids for the other operations arising in a general network of queues. Such approximation procedures are currently being developed and tested. They yield simple algorithms for the classical Jackson networks of queues (the Markov case) modified to have non-Poisson arrival processes, multiple servers at each facility and nonexponential service-time distributions.

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2. CONTRIBUTED PAPERS

2.1. Random Walk and Limit Theorems, Part I

Some Limit Theorems for Random Walks

A.K. Aleškevičienė, *Academy of Sciences of the Lithuanian SSR, SSSR*

Let $\xi_1, \xi_2, \ldots$ be a sequence of i.i.d. random variables with common distribution function $F(x)$. Define

$$\mathbf{E}\,\xi_1 = 0, \qquad D\xi_1 = 1, \qquad \beta_m = \mathbf{E}|\xi_1|^m, \qquad S_0 = 0, \qquad S_n = \sum_{l=1}^{n} \xi_l,$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(u^2)/2}\,du, \qquad g_1 < 0, \qquad g_2 > 0,$$

$$v_n(g_1, g_2) = \mathbf{P}\Big\{\min_{1 \leq k \leq n} S_k > g_1,\ \max_{1 \leq k \leq n} S_k < g_2\Big\},$$

$$v(g_1, g_2) = \mathbf{P}\Big\{\inf_{0 \leq t \leq 1} \xi(t) > g_1,\ \sup_{0 \leq t \leq 1} \xi(t) < g_2\Big\}$$

$$= \frac{1}{\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} \int_{g_1}^{g_2} [\exp\{-\tfrac{1}{2}(x + 2k(g_2 - g_1)^2\}$$

$$- \exp\{-\tfrac{1}{2}(x - 2k(g_2 - g_1))^2\}]\,dx$$

and $\xi(t)$ is the standard Wiener process. It is well known that $v_n(g_1\sqrt{n}, g_2\sqrt{n}) \Rightarrow v(g_1, g_2)$. The uniform estimates for the rate of convergence of the distribution laws $v_n(g_1\sqrt{n}, g_2\sqrt{n})$ to $v(g_1, g_2)$ can be found in the papers of Prokhorov, Skorohod, Nagayev and Sakhanenko, where the problem with two curved boundaries was considered, their best statement being as follows:

$$\sup_{g_1 < 0, g_2 > 0} |v_n(g\ \sqrt{n}, g_2\sqrt{n}) - v(g_1, g_2)| \leq L\beta_3/\sqrt{n}, \quad \beta_3 < \infty,$$

where L is the absolute constant.

In this talk we shall formulate several theorems on this subject.

In many applications of statistics, it is important to know the distribution functions of the range $R_n = \max_{0 \leq k \leq n} S_k - \min_{0 \leq k \leq n} S_k$ of the sequence of partial sums of i.i.d. random variables and of the time $T_r = \min\{k \geq 1, R_k \geq r\}$ at which the range first exceeds some given value [1].

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Bivariate Domains of Attraction for Stopped Random Walk

P. Greenwood and E. Perkins, *University of British Columbia, Canada*

Let S_n be a random walk, and let T be a stopping time for S_n. Consider the pair (T, S_T). If S_1 is in a domain of attraction and if T is in a domain of attraction, index $0<\alpha<1$, then is (T, S_T) in a bivariate domain of attraction? Some general results about this question and some examples are given.

Strong Approximations of Partial Sums of I.I.D. Random Variables in the Domain of Attraction of a Symmetric Stable Distribution

J. Mijnheer, *University of Leiden, The Netherlands*

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed (i.i.d.) symmetric random variables with common distribution function G, satisfying, for $x>0$, $G(-x)=1-G(x)=x^{-\alpha}L(x)$, $0<\alpha<2$, where L is a slowly varying function at infinity. Let $\{a_n\}$ satisfy $a_n^\alpha \sim nL(a_n)$ for $n\to\infty$. The distribution function of $a_n^{-1}(X_1+\cdots+X_n)$ converges weakly to the distribution function $F(\cdot,\alpha)$ of the symmetric stable distribution with characteristic exponent α. We prove the following theorem.

Theorem. *Let $\{X_n\}$ be defined as above. There exist sequences of positive real numbers τ_n and ϕ_n, $n\geq 1$, and a sequence of i.i.d. random variables Y_n such that $\tau_n Y_n$ has distribution function $F(\cdot,\alpha)$ and $|X_1+\cdots+X_n-(Y_1+\cdots+Y_n)|=o(\phi_n)$ a.s.*

We discuss the properties of τ_n and ϕ_n and give some examples.

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2.2. Risk and Reliability Theory, Part I

Upper Bounds for Ruin Probabilities in a New General Risk Model by the Martingales Methods

F. De Vylder and M. Goovaerts, *University of Louvain, Belgium*

We consider a portfolio of stochastically variable size in time, in an insurance business, which is composed of independent identical contracts, each of fixed duration. The contract number process is a general random point process defined by a sequence of intensities.

Essential differences with the classical actuarial risk model are the following:
- the generality of the involved stochastic processes;
- the discontinuous, more realistic, character of the premium income process.

The tribute to pay for this generality is that it is an almost impossible task to calculate exact ruin probabilities. But we can obtain bounds for the ruin probability in infinite or finite time intervals by replacing the surplus process by a minimizing surplus process and to the latter we apply the martingale argument used by Gerber [2] in the extended form employed by De Vylder [1].

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Applicability of the Corrected Diffusion Approximation to the Problem of Ruin in Risk Processes

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In a recent paper Siegmund presented a corrected diffusion approximation in first passage problems for random walks. It was based on the assumption that the random variables $X_1, X_2, \ldots$ are, for θ in some open interval containing 0, under P_θ, independent and have probability density function $\exp(\theta x - \psi(\theta))$ relative to some non-arithmetic measure F. In this note we consider the applicability of this result to the problem of ruin in risk processes. We show that the approximation is applicable only when all the claims are equal. This case corresponds to pure Poisson process. Our computer calculations indicate that the corrected diffusion approximation in this case is remarkably accurate when ruin in the infinite time period is considered. We have also compared the ruin in N claims given by the approximation to the exact probability of ruin in time N.

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2.3. *Queueing Theory, Part I*

A Multi-class Feedback Queueing System in Heavy Traffic

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We consider a single station multi-server queueing system with several customer classes. Each customer class has its own arrival process, with class membership determining the nature of service required by the customer. For a given class, the total service requirement of a customer is divided into a (possibly random) number of service quanta, where the distribution of each quantum may depend on the customer's class and the other quanta of that customer. Customers are served in a first come first serve manner. After receiving a quantum of service, a customer either feeds back to the end of the queue if he requires additional service, or leaves the system if his service requirement has been fulfilled.

We prove a heavy traffic limit theorem for the above system which states that as the traffic intensity approaches unity, properly normalized queue length and sojourn time processes converge weakly to (one-dimensional) reflected Brownian motion. The drift and variance of the limit process are given explicitly in terms of the parameters of the original arrival and service processes.

The M/G/1 Queue—A Martingale Approach

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Let $\eta(t)$ denote the virtual waiting time process for the M/G/1 queue with expected interarrival time b^{-1}, service time distribution $H(y)$ and $M(\alpha) = \int_0^\infty \exp(\alpha y)\,\mathrm{d}H(y)$. We denote the duration of the busy period by d and define the function $\lambda(\alpha) = \alpha + b(M(\alpha) - 1)$. We prove the following theorem: $X(t) = \exp[\lambda(\alpha)(t \wedge d) + \alpha\eta(t \wedge d)]$ is a martingale. From this we can derive in a new way the Laplace transform of d as well as other results on $\sup_{0 \le s \le t} \eta(s)$, too complicated to state here.

Let $F(t, x) = \mathbf{P}(\eta(t) \le x)$ and $F_0(x) = \mathbf{P}(\eta(0) \le x)$. It was shown by Takács that for each t and almost every x $F(t, x)$ satisfies the integro differential equation

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x} - bF + b\int_0^x H(x-y)\,\mathrm{d}F(t, y), \quad x \in R^+ = (0, \infty) \text{ with } F(0, x) = F_0(x). \tag{1}$$

Let $D = \{F; F$ is of bounded variation on R^+, F is absolutely continuous with derivative F' of bounded variation on R^+, F' is absolutely continuous$\}$.

Theorem. *If $F_0 \in D$, then there exists a unique solution to* (1) *satisfying the conditions* (*i*) $t \to F(t, x)$ *is strongly continuous with respect to the total variation norm* $\|F\|$; (*ii*) $F(t, x) \in D\,\forall t \ge 0$ *so* (1) *holds for all* $(t, x) \in R^+ \times R^+$, *and* (*iii*) $\|F(t, x)\|$ *is bounded.*

Both these theorems are consequences of the functional analytic and martingale methods of the author's paper "Some martingales associated with queueing and storage processes" of which preprints are available.

Continuous Versions of the Queueing Formulas, $L = \lambda W$ and $H = \lambda G$

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T. Rolski, *Wrocław University, Poland*

The queueing formula $L = \lambda W$ and its generalization $H = \lambda G$ hold as deterministic relations between sample averages under very general conditions [1, 2, 3]. Here L and λ are the limiting time-average number of discrete units (customers) in the system and number of arrivals, respectively, and W is the limiting customer-average time spent in the system. In the relation $H = \lambda G$, H and G can be interpreted, respectively, as the limiting average cost per unit time and per customer, where each customer n incurs cost at rate $f_n(t)$ at time t. In this paper, we consider systems where input is continuous rather than in discrete units. Such a situation may arise in diffusion approximations or queues, in reservoir models, and other storage processes. Cumulative input is measured by a non-decreasing right-continuous process, $y(t)$, $t \geqslant 0$. Associated with particle s arriving at time $y^{-1}(s)$ is a function $f(s, t)$, which describes its behaviour after entering the system. We prove $L = \lambda W$ and $H = \lambda G$ under conditions that are natural generalizations of those needed in the discrete case. In particular, we need a condition to ensure that $g(s) := \int_0^\infty f(s, t)\,dt$, the total cost associated with particle s, does not grow too rapidly (in a Cesaro sense). We give examples.

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2.4. Stochastic Analysis and Martingales, Part I

Signed Measures on Function Space and Higher Order Elliptic Operators

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A random variable X on a signed probability space is said to be k-normal $\mathrm{N}_k(\mu, \sigma)$ (k is even) if it has density $f_X(x) = (1/\sigma)p_k((x-\mu)/\sigma)$ where $p_k(\xi) = (1/\pi)\int_0^\infty \cos\lambda\ \mathrm{e}^{-(\lambda^k/k!)}\,\mathrm{d}\lambda$. It is said to be $\chi^m_{n,k}$ if it has the form $\sum_{i=1}^n X_i^m$ where the X_i are i.i.d. $\mathrm{N}_k(0, 1)$. Using p_k for transition density one can define a signed measure on paths $\omega : [0, \infty) \to R$, analogous to Wiener measure. The stochastic process $\omega(t)$, analogous to Brownian motion, is a k-normal Markov process which can be used to set up a general stochastic integral. Limit properties for $\chi^m_{n,k}$ reveal that the stochastic calculus so generated is of order k, and the generalized k-diffusions generate elliptic operators of order k. The classification of these operators involves some interesting algebraic results. As a particular example, the operator $\sum_{|\alpha|=k} a_\alpha(x)\partial^\alpha$ can be generated from a k-diffusion only if $\sum_{|\alpha|=k} a_\alpha(x)\xi^\alpha = \sum_i (\langle \lambda_i(x), \xi\rangle)^k$.

Two-parameter Stochastic Differential Equations

H. Korezlioglu, *Ecole Nationale Supérieure des Télécommunications*

R_{st} represents the rectangle $\{(u,v)\in \boldsymbol{R}_+^2 : u\le s, v\le t\}$, s, u, a the horizontal coordinates and t, v, b the vertical coordinates of $\boldsymbol{R}_+^2$. B is a Brownian sheet on $\boldsymbol{R}_+^2$, M is defined by $M_{st} = \int_{R_{st}} G_{uv}\, \mathrm{d}B_{uv}$ where G is a nonrandom continuous function, and J is defined by $J_{st} = \int_{R_{st}^2} 1(a\le u, b\le v) G_{ub} G_{av}\, \mathrm{d}B_{ub}\, \mathrm{d}B_{av}$. The following stochastic differential equation is considered:

$$X_{st} = \tilde{x}(s) + x(t) - x_0 + \int_{R_{st}} \theta(u, v, X_{uv})\, \mathrm{d}u\, \mathrm{d}v + \int_{R_{st}} g(u, v, X_{uv}) M(\mathrm{d}u, \mathrm{d}v)$$
$$+ \int_{R_{st}} \varphi(u, v, X_{uv}) M(u, \mathrm{d}v)\, \mathrm{d}u + \int_{R_{st}} \tilde{\varphi}(u, v, X_{uv}) M(\mathrm{d}u, v)\, \mathrm{d}v$$
$$+ \int_{R_{st}} \psi(u, v, X_{uv}) J(\mathrm{d}u, \mathrm{d}v)$$

where $\tilde{x}$, x, θ, g, φ, $\tilde{\varphi}$, ψ are nonrandom continuous functions and $x_0 = \tilde{x}(0) = x(0)$.

Under Lipschitz and linear grown conditions on the coefficients, it can be shown that this equation has a unique solution. The solution is Markovian in the sense of [1] when $\varphi = \tilde{\varphi} = \psi = 0$. There are cases in which this last condition does not hold and the solution is still Markovian. In fact, if X also satisfies a diffusion equation in s driven by $(M_{st},\ s\in \boldsymbol{R}_+)$ and another in t driven by $(M_{st}, t\in \boldsymbol{R}_+)$, conditions can be expressed for X to be Markovian in the sense of [2]. The Markov property of [2] implies that of [1] when it is defined with respect to the natural filtration of X.

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Toward a Stochastic Calculus for Several Markov Processes

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In this paper we investigate a class of harmonic functions associated with a pair $x_t = (x_{t^1}^1, x_{t^2}^2)$ of strong Markov processes. In the case where both processes are

Brownian motions, a smooth function f is harmonic if

$$\Delta_{x^1}\Delta_{x^2}f(x^1, x^2) = 0.$$

For these harmonic functions we investigate a certain boundary value problem which is analogous to the Dirichlet problem associated with a single process. One basic tool for this study is a generalization of Dynkin's formula which can be thought of as a kind of stochastic Green's formula. Another important tool is the use of Markov processes $\tilde{x}^i_{t^i}$ obtained from $x^i_{t^i}$ by certain random time changes. We call such a process a stochastic wave since it propagates deterministically through a certain family of sets; however, its position on a given set is random.

A Characterization of Certain Stochastic Integrals

S.J. Wolfe, *University of Delaware, U.S.A.*

A full probability measure μ defined on R^k is said to be a Lévy probability measure if there exist sequences $\{A_n\}$, $\{a_n\}$ and $\{X_n\}$ of respectively linear operators, vectors, and independent random variables such that the probability measures of $A_n(X_1+\cdots+X_n)-a_n$ converge weakly to μ and in addition the random variables $\{A_nX_j: j=1, 2, \ldots, n, n=1, 2, \ldots\}$ form an infinitesimal system.

In this paper the following theorem will be proved: A random variable X defined on R^k has a full Lévy probability measure iff there exists a centered stochastic process $\{B(t): 0 \leq t < \infty\}$ with independent and homogeneous increments and a linear operator A, all of whose eigenvalues have negative real part, such that the stochastic integral $\int_0^\infty e^{At}\, dB(t)$ has the same probability measure as X. Other related results will also be presented. These theorems are obtained by using theorems of Urbanik [1] and the author [2]. They generalize one-dimensional theorems of the author [3].

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2.5. Probability Models

A Model of Polymer Degradation

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Consider a system in which an object of size h lives an exponential amount of time with mean $h^{-\alpha}$, $\alpha \geq 0$, and then splits into two pieces of size hV and $h(1-V)$

where V has distribution F. This model with $\alpha = \frac{2}{3}$ has been used as a model of polymer degradation. (The rationale for $\frac{2}{3}$ is splitting rate $\approx$ surface rate $\approx$ (volume)$^{2/3} \approx$ (polymer length)$^{2/3}$.) In my talk I will state some results that Mike Brennan and I have obtained for l_t, the length of a randomly chosen interval.

When $\alpha = 0$ if we consider the logarithms of the sizes as particles in $(-\infty, \infty)$ we get a branching random walk so it follows from known results that, for suitable a, b,

$$(\log l_t - at)/bt^{1/2} \to \text{a normal distribution.}$$

When $\alpha > 0$ the behaviour changes,

$$t^{1/\alpha} l_t \to \text{a limit } G_\alpha \quad \text{(which depends on } V\text{)}.$$

If V is uniform on $(0, 1)$, G_α has a density $A_\alpha \exp(-(B_\alpha x)^\alpha)$. This result is easy to prove when $\alpha = 1$ (two lines) and is left as a pre-talk exercise for the reader.

On the Question of the Power of Incumbency

N. Hadidi, *University of Wisconsin, U.S.A.*

What do politics and probability have in common? Markov chains among other things! Consider $s \geqslant 1$ political offices for each of which a member of $m \geqslant 1$ parties may be a candidate. Each member may serve r consecutive terms, $1 \leqslant r \leqslant k$. For each term $n \geqslant 1$, let X_n depict the characteristic of the successful candidate. $X_n = i$, $1 \leqslant i \leqslant km$, implies a member of party $[i/k]+1$ is serving his/her $(i-[i/k]k)$th term. $[\omega]$ denotes the greatest integer less than ω. In view of the quality of incumbency it is not unreasonable to model X_n as a Markov chain. "Markov properties are clearly inherent in certain political processes" [1]. To determine when a significant change in the behaviour of the electorate has occurred the underlying variability of the sequence X_n should be quantitatively studied—'politimetrics' [2]. As an illustration the data from Wisconsin legislature are analyzed. Assuming homogeneity the maximum likelihood estimates of transition probabilities are computed by district and as a whole. These are in turn used to suggest indicators of trends.

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Models of Price Creation

H. Mendelson, *University of Rochester, U.S.A.*

This paper is concerned with the application of stochastic models to study the behavior of prices in securities markets. We derive the behavior of the market, given the following:

(i) a stochastic demand process, which governs the arrival of buy orders at different prices to the market;

(ii) a stochastic supply process, depicting the arrival of sell orders; and

(iii) a market mechanism, which transforms these orders into transactions.

Specifically, we are concerned with the distributions of the market price, the quantity traded and the market surplus, and with the effects of the market mechanisms and the specification of the demand and supply processes on market behavior. We consider a model where prices are set by a market-maker; a continuous-price model where prices are determined by periodic market-clearing; and a discrete-price model with periodic market-clearing. Potential research in this application area will also be indicated.

2.6. Random Walk and Limit Theorems, Part II

On the Existence of the Mean Ladder Height for Random Walks

R.A. Doney, *University of Manchester and University of British Columbia*

If X denotes a typical step in a random walk and Z_+, Z_- denote the magnitudes of the first ascending and descending ladder heights, it is known that when $\mathbf{E}(X)=0$ and $\mathrm{Var}(X)=\infty$ we have $\mathbf{E}(Z_+)\mathbf{E}(Z_-)=\infty$. Thus either both of Z_+ and Z_- have infinite mean, or one has finite mean and the question considered in this paper is whether we can distinguish the cases by looking at the distribution of X. We derive an integral involving $\mathbf{P}\{X \geqslant x\}$ and $\mathbf{P}\{X \leqslant -x\}$ which is finite whenever $\mathbf{E}(Z_-)<\infty$, and in the special case that $\mathbf{P}\{X \geqslant x\}$ is regularly varying we show that this integral is finite iff $\mathbf{E}(Z_-)<\infty$. This last result allows us to improve a result in [1] on the exact asymptotic behaviour of the distribution of the first ladder epoch when X belongs to the domain of attraction of a completely asymmetric stable law of index α, $1<\alpha<2$.

Reference

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On Large Deviations for a Sum of Random Number of Multidimensional Random Vectors

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Let $X_1, X_2, \ldots$ be independent random vectors (r.v.) in Euclidean space R^s with mean zero, unit covariation matrix and common distribution function $F(A)$. Let $(\cdot, \cdot)$ denote the scalar product in R^s, $\|x\| = (x, x)^{1/2}$ and $|x| = |x_1| + |x_2| + \cdots + x_s|$. The standard Gaussian distribution in R^s is denoted by Φ. Let the random variables X_i satisfy the condition: $\exists$ a constant $K > 0$ such that (B) $|\mathbf{E}(X_i, z)^k| \leq \frac{1}{2}(k!)(K|z|)^{k-2}\|z\|^2$, $k = 2, 3, \ldots$. Further let ω be an independent from X_i one-dimensional integer random variable with the mean $\mathbf{E}\omega = \alpha = \alpha(t)$ depending on some parameter t and its cumulant of the kth order $\Gamma_k\{\omega\}$ satisfying the condition: there exist a number K_2 and a non-negative d such that (S) $|\Gamma_k\{\omega\}| \leq k! K_2^{k-1} \alpha^{1+(k-1)d}$, $k = 2, 3, \ldots$. Under some rather general restrictions on the set $A = B^{(1)} B^{(2)}$ where $B^{(i)}$, $i = 1, 2$, are the convex sets, the theorems dealing with large deviations in the Cramer zone are proved for the random vector $Z_\omega = \sum_{i=1}^{\omega} X_i/\sqrt{\alpha}$ with the distribution function $F_\omega(A)$. Namely, $F_\omega(A) = \Phi(A) \exp\{\lambda(x^0)\}(1 + \theta \max\{\rho, \ln \Delta_\alpha\}/\Delta_\alpha)$. Here $\lambda(x)$ is the multidimensional Cramer series; $\rho = \|x^0\|$, $1 \leq \rho \leq c_0 \Delta_\alpha$, $0 < c_0 < 1$, $|\theta| \leq C(s, c_0)$, $\Delta_\alpha = [\alpha(\max 4(1 + k^2), 3k_2\alpha^d\})^{-1}]^{1/2}$, $x^0 \in b\, dyA$. Analogous statements are received in a case when the random vectors are summarized with weights.

2.7. *Markov Processes and Diffusions, Part I*

A Class of Generalized Nonlinear Functionals of Brownian Motion

N.U. Ahmed, *University of Ottawa, Canada*

Most of the fundamental results on the theory of nonlinear functionals of Brownian motion are related to the functional analysis of the L_2 space over the Brownian motion space or the space of generalized White noise. In this paper, inspired by the recent work of Hida [1, 2] and the author [3], we introduce new classes of generalized functionals of White noise which have integral representation with kernels either belonging to any L_p space $(1 \leq p \leq \infty)$ or any Sobolev space $H^{s,p}$, $s \in R$, $p \in [1, \infty]$. Our results are not covered by the recent result of Hida [1, 2] and also our method of construction differs from that of Hida. In our approach we view the generalized functionals as elements of a vector space which is the dual of another vector space of (very!) regular functions of White noise. We then obtain the generalized functions as the limit in an appropriate topology of a sequence of regular functions of White noise. We also construct a locally convex topology on the space of generalized functionals.

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Regenerative Systems and Markov Additive Processes

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We consider a regenerative system (X, M) [1, 2] where M is the regeneration set and X is the mark process. It is shown that when stopping times whose graphs are contained in $\bar{M} - M$ are totally inaccessible, it is possible to construct a Markov additive process $(\Omega, \mathscr{H}, \mathscr{H}_t, (X_t, \tau_t), \theta_t, P^x)$ whose range is (X, M). This result was previously known [1] for regeneration sets that are perfect. We allow M to have isolated points and limits of such points. Applications of this result to excursions of Hunt processes from Borel sets will be discussed.

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Properties of the Reflection Operator Associated with a Transport Markov Process and Application

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Let (X_t, V_t) be a Markov process on $R \times V$ (V is a compact set of R^n, symmetric with respect to 0) whose infinitesimal generator is

$$f = f(\eta, v) \to v \frac{\partial f}{\partial \eta} + Q_\eta f \tag{1}$$

where v is the first coordinate of v, and Q satisfies $Q_\eta g(v) = \int \sigma_1(\eta, v, w) g(w)\, \mathrm{d}w - \sigma(\eta, v) g(v)$, $\sigma(\eta, v) = \int \sigma_1(\eta, v, w)\, \mathrm{d}w$. σ_1 is strictly positive and $\sigma_1(\cdot\,. v, w)$ is periodic with period 1. We first analyze the behaviour of the 2nd component (V_t) of the process when the first (X_t) goes to infinity. Let R be the operator from $L^\infty(V_-)$ to $L^\infty(V_+)$ ($V_+ = \{v \mid v \geq 0\}$ and $V_- = -V_+$) defined by $Rf(v) = \mathbf{E}_{0,v} f(V_\tau)$; τ is the first time where $X_t \notin R^+$. We show that R satisfies a discrete Riccati equation. In the case where the structure is homogeneous we show that R satisfies another Riccati equation (which is of the classical aspect) and that there exist $\delta_0 > 0$ such that

$$Rf \geqslant \delta_0 \int_{V_-} f(v)|v|\,\mathrm{d}v \quad \forall f \geqslant 0. \tag{2}$$

Application. We consider a domain which is the juxtaposition of two strips $x_1 \in [0, I]$ and $x_1 \in [I, J]$. Let Q^{g} be an operator defined as in (1) and Q^{d} an operator of the same type but without dependence in η. We have $f \to (Q(x)f) = Q^{\mathrm{g}}_{x/\varepsilon} f$ if $x < I$ and $Q^{\mathrm{d}} f$ if $x > I$. Consider ($\alpha \geqslant 0$)

$$-\frac{1}{\varepsilon} \sum_i v_i \frac{\partial u^\varepsilon}{\partial x_i} + \frac{1}{\varepsilon^2} Q(x) u^\varepsilon - \alpha u^\varepsilon = 0.$$

Then, if we denote R^{g} and R^{d} the operators of reflection associated with Q^{g} and Q^{d}, we show with (2) that the Freedholm alternative is true for the operator $R^{\mathrm{g}} R^{\mathrm{d}} - I$. So we can introduce boundary layers at the interface $x = I$ and find the limit of u^ε, when ε goes to 0.

2.8. Optimization, Part I

Optimization of Control System Modelling Using Stochastically Disturbed Test Measurements

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The paper examines the robustness of several optimization techniques which establish low order system models using test data embedded with stochastic noise patterns. Applications of the techniques in different control systems will be used as case studies. In one of the studies the system models produced were used to simulate the dynamics of a complex non-linear control system to be determined.

With the increase in the availability of microprocessors and microcomputers in the control field test data is being collected and processed more easily and quickly. The paper will, using the case studies, report on the application of microprocessors in data acquisition and their suitability for providing on-line system modelling facilities.

Existence of Average Cost Optimal Strategies in Multichained Semi-Markov Decision Problems

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We consider a semi-Markov decision model with a denumerable state space I and compact metric action sets. For the transition law we assume the usual continuity properties [1] with slight sharpenings where needed. The expected one step costs $\rho_i(\cdot)$ are bounded below and lower semi-continuous (lsc) w.r.t. the actions. For each stationary strategy there exists a finite subset K of the set of states j with finite mean recurrence time μ_{jj} such that the probability for reaching K having started in i, f_{ik}, equals 1. Further the expected absolute costs incurred until the first entry into K are finite for every starting state. Our criteria are the (expected) average costs defined as the upper limit of the expected costs up to time t divided by t, when t tends to infinity. We allow arbitrary strategies for which the nth action may depend on the whole history up to the nth decision time point.

We show four theorems which generalize all results in the literature concerning the existence of average optimal strategies. Especially, neither unchainedness nor communicatingness need be assumed.

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2.9. *Stochastic Analysis and Martingales, Part II*

Exact Convergence Rates for Some Martingale Central Limit Theorems

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Let $X_{n,i}$, $i \leqslant n$, be a square integrable martingale difference array, i.e., $\mathbf{E}(X_{n,i} \mid \mathcal{F}_{n,i-1}) = 0$ a.s. where $\mathcal{F}_{n,k}$ is the σ-algebra generated by $X_{n,1}, \ldots, X_{n,k}$. For the sake of simplicity, it is assumed that $\mathbf{E}(X_{n,i}^2) = 1$ for all i and n. Let $S_{n,k} = \sum_{i=1}^k X_{n,i}$, $\sigma_{n,k}^2 = \mathbf{E}(X_{n,k}^2 \mid \mathcal{F}_{n,k-1})$, $V_n^2 = (\sum_{k=1}^n \sigma_{nk}^2)/n$, $\delta(n) = \sup_t |\mathbb{P}(S_{nn}/\sqrt{n} \leqslant t) - \Phi(t)|$, where Φ is the standard normal distribution function. Several results on the behaviour of $\delta(n)$ are obtained:

(1) If $\sup_{i,n} \|X_{n,i}\|_3 < \infty$, $\sigma_{n,i}^2 = 1$ a.e. for all i, n, then $\delta(n) = O(n^{-1/4})$.

(2) If $\sup_{i,n} \|X_{n,i}\|_\infty < \infty$, then $\delta(n) = O(n^{-1/2} \log n + \|V_n^2 - 1\|_1^{1/3}$.

(3) If $\sup_{i,n} \|\mathbf{E}(|X_{n,i}|^3 \mid \mathcal{F}_{n,i-1})\|_\infty < \infty$, $\sup_n \|\sigma_{n,i}^2 - 1\|_1 = O(i^{-1/2})$, then $\delta(n) = O(n^{-1/2} \log n)$.

(4) If $\sup_{i,n} \|\mathbf{E}(X_{n,i}^4 \mid \mathcal{F}_{n,i-1})\|_\infty < \infty$, $\sup_n \|\sigma_{n,i}^2 - 1\|_2 = O(i^{-1/2})$, $\sup_n \|\mathbf{E}(X_{n,i}^3 \mid \mathcal{F}_{n,i-1}) - \mathbf{E}(X_{n,i}^3)\|_2 = O(i^{-1/4})$, then $\delta(n) = O(n^{1/2})$. The rates are all best possible under the stated conditions.

Continuity of a Class of Random Integrals

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Let $(S, \mathcal{S}, m)$ be a finite measure space, p satisfy $1 < p \leqslant 2$ and M be an independently scattered random measure on $\mathcal{S}$ such that, for each $A \in \mathcal{S}$, $M(A)$ is symmetric with $\mathbf{E}M(A)=0$ and $\mathbf{E}|M(A)|^p=M(A)$. Let $\{g(t,\cdot): t\in[0,1]^d\}$ be a family of functions in $L^p(S,\mathcal{S},m)$. Define

$$X(t)=M(g(t,\cdot))=\int_S g(t,\lambda)\,\mathrm{d}M(\lambda),\quad t\in[0,1]^d.$$

By approximating $\{X(t): t\in[0,1]^d\}$ by a sequence of subgaussian processes, it is shown, using methods similar to those of Marcus [3] and Fernique [1], that $\{X(t): t\in[0,1]^d\}$ has a version with continuous sample functions if $\int_0 \underline{f}^{1/p}(u)\,\mathrm{d}u/(u(\log 1/u)^{1/2})<\infty$, where $f(u)=\sup_{|t-s|\leqslant u}\int_S|g(t,\lambda)-g(s,\lambda)|^p\,\mathrm{d}m(\lambda)$ and $\underline{f}(u)$ is the modification of Hahn and Klass [2], $\underline{f}(u)=\inf_{y\geqslant 1} y^p f(u/y)$.

References

[1] X. Fernique, Continuité et théorème central limite pour les transformées de Fourier des mésures aléatoires du second ordre, Z. Wahrsch. Verw. Gebiete 42 (1978) 57–66.

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On the Moments of Martingales with Conditionally Symmetric I crements

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J.C. Smit, *University of Nijmegen, The Netherlands*

We will say that the sequence of random variables $S_1, S_2, \ldots$ has conditionally symmetric increments if the following assumption holds:

(1) $\mathbf{P}(X_{k+1}\leqslant x\,|\,S_k)=\mathbf{P}(X_{k+1}\geqslant -x\,|\,S_k)$ a.s. for all $x\in R$, $k\geqslant 1$, where $X_{k+1}=S_{k+1}-S_k$ if $k\geqslant 1$, $X_1=S_1$.

We will show that under this assumption the following inequalities hold for all $n\geqslant 1$ (and any $p>2$):

$$(2)\qquad \sum_{k=1}^{n}\mathbf{E}|X_k|^p\leqslant \mathbf{E}|S_n|^p\leqslant C(p)n^{p/2-1}\sum_{k=1}^{n}\mathbf{E}|X_k|^p.$$

The best possible choice for $C(p)$ will be shown to be $C(p)=(p-1)^{p/2}$. If $1<p<2$, (2) holds with the same expression for $C(p)$ but the inequality signs reversed.

Note. Although (1) does not imply that $S_1, S_2, \ldots$ is a martingale, the constant in (2) cannot be improved if we restrict our attention to the class of martingales which satisfy (1). This justifies the term martingale in the title.

2.10. Inference, Part I

On the Estimability of Quasi-diffusions

J. Brode, *I.S.C.A.E., Maroc*

Feller (1954) defined diffusions to include processes whose infinitesimal generators could be the kernel of a stable distribution with characteristic exponent $1<\alpha<2$. Given that these processes are not square integrable, what remains of the estimability of the standard diffusion? It will be shown that an unique Feller semi-group of solutions exists for the backward equation. With further topological assumptions, it will be shown that a best approximation exists as well as a measure for the infinite dimensional case.

Estimation and Control for Linear, Partially Observable Systems with Non-Gaussian Initial Distribution

V.E. Beneš, *Bell Laboratories, N.J., U.S.A.*
I. Karatzas, *Columbia University, U.S.A.*

The nonlinear filtering problem of estimating the state of a linear stochastic system from noisy observations is solved for a broad class of probability distributions on the initial state. It is shown that the conditional density of the present state, given past observations, is a mixture of Gaussian distributions, and is parametrically determined by two sets of sufficient statistics which satisfy stochastic differential equations; this result leads to a generalization of the Kalman–Bucy filter to a structure with a conditional mean vector, and additional sufficient statistics that obey nonlinear stochastic equations and determine a generalized (random) Kalman gain. The theory is used to solve explicitly a control problem with quadratic running and terminal costs, and bounded controls.

State Estimation for Cox Processes on General Spaces

A.F. Karr, *The Johns Hopkins University, U.S.A.*

Let N be an observable Cox process on a locally compact space E directed by an unobservable random measure M. Techniques are presented for estimation of M, using the observations of N to calculate conditional expectations of the form $\mathrm{E}[M]|\mathcal{F}_A]$, where $\mathcal{F}_A$ is the σ-algebra generated by the restriction of N to A. We introduce a random measure whose distribution depends on N_A, from which we obtain both exact estimates and a recursive method for updating them as further observations become available. Application is made to the specific cases of estimation of an unknown, random scalar multiplier of a known measure, of a symmetrically distributed directing measure M and of a Markov directed Cox process on R_+. By means of a Poisson cluster representation, the results are extended to treat the situation where N is conditionally additive and infinitely divisible given M.

2.11. *Random Walk and Limit Theorems, Part III*

On the Asymptotic Behaviour of Random Walks on an Anisotropic Lattice

C.C. Heyde, *CSIRO, Australia*

This talk will deal with a random walk on a two-dimensional lattice with homogeneous rows and inhomogeneous columns which could serve as a model for the study of some transport phenomena. Subject to an asymptotic density condition on the columns both the horizontal and vertical motions of the walk are asymptotically like that of rescaled Brownian motion. Various consequences of this are derived including central limit, iterated logarithms, and mean square displacement results for the components of the walk.

A Random Walk Model of the Helminth Trichostrongylus Retortaeformis

D. Kannan, *University of Georgia, U.S.A.*

We consider a one-dimensional random walk model of the Helminth *Trichostrongylus Retortaeformis* which is parasitic on sheep and rabbit. The larvae that hatched from the eggs in the excreta of hosts move through grass pasture. After a random length of time, each larva climbs up a blade of grass and stays there until ingested by a grazing animal. The proposed random walk model is as follows: the larva moves with a uniform speed v and at the end of each unit of time Δt it either changes its direction with probability $p\Delta t$, gets absorbed with probability $q\Delta t$, or continues its motion in the same direction with probability $1-(p+q)\Delta t$. (Note that each state acts as a quasi-absorbing state.) The associated motion is governed by the hyperbolic equation $v^{-2}f_{tt}(t,x)+2v^{-2}(p+q)f_t(t,x)=f_{xx}(t,x)+v^{-2}(2p+q)(a(x)-f(t,x))$ with $f(0,x)=a(x)$ and $f_t(0,x)=0$. A diffusion approximation is obtained by speeding up the random walk such that $v, p, q\to\infty$ with $2(p+q)/v^2\to b$, and $q^2/v^2\to c$; this results into the quasi-linear equation $f_t(t,x)=b^{-1}f_{xx}(t,x)+(c-a)(a(x)-f(t,x))$. Our purpose is to study these two equations using Markov Process Methods.

Dominating Points and Asymptotics of Large Derivations on R^d

Peter Ney, *University of Wisconsin, Madison*

Let μ be a probability measure on R^d with generating function $\varphi(\cdot)$, $S=$ convex hull of support of μ, $D(\varphi)=\{\alpha\in R^d: \varphi(\alpha)<\infty\}$. For $\alpha\in D$, define the Cramer transform of μ by $\mu_\alpha(dx) = [\varphi(\alpha)]^{-1} e^{\alpha \cdot x}\mu(dx)$, and let X^α be a r.v. with p.m. μ_α. If D is open and $v \in \mathrm{In}(S)$, then grad $\varphi(\alpha) = v\varphi(\alpha)$ has a unique solution $\alpha = \alpha_v \in \mathrm{In}(D)$, and α_v is a homeomorphism on $\mathrm{In}(S)$.

Definition. v is a *dominating point* of $B \subset R^d$ if $v \in \partial B$ and $B \subset v + \{x \in R^d : x \cdot \alpha_v \geq 0\}$.

It is easy to show that for any $v \in \mathrm{In}(S)$ $\mu^{*n}(nB) = \rho^n(v) \int_{n(B-v)} e^{-x \cdot \alpha_v} \nu^{*n}(dx)$, where $\nu(\cdot)$ is the p.m. of $x^{\alpha_v} - v$ with $\int x \, d\nu = 0$, and $\rho(v) = e^{-v \cdot \alpha_v}\varphi(\alpha_v)$. If v is a dominating point for B, then this representation, together with standard classical central limit estimates, can be used to obtain asymptotic estimates of $\mu^{*n}(nB)$ which are sharper than presently known.

For example, if B is convex with $\mathrm{In}(B) \neq \emptyset$, and $\int x \, d\mu \notin B$, then there exist c_1, c_2, $\delta > 0$ such that $\rho^n n^{-d/2}[c_1 + O(n^{-\delta})] \leq \mu^{*n}(nB) \leq \rho^n n^{-1/2}[c_2 + O(n^{-\delta})]$. Under stronger assumptions on B one obtains estimates like $\mu^{*n}(nB) = \rho^n n^{-\gamma}[c + O(n^{-\delta})]$ for suitable γ. Sharper asymptotic expansions are also available.

A fixed point argument is used to prove the following main result.

Theorem. *If B is convex with* $\mathrm{In}(B) \neq \emptyset$, $\int x \, d\mu \notin B$, *and $D(\varphi)$ is an open set, then there exists a unique dominating point of B.*

2.12. Risk and Reliability Theory, Part II

On the Determination of Mixing Density in Reliability Studies

R.C. Gupta, *University of Maine, U.S.A.*

In reliability studies the three quantities (1) the survival function, (2) the failure rate and (3) the mean residual life function are all equivalent in the sense that given one of them, the other two can be determined.

In this paper we have considered the class of exponential type distributions and studied its mixture. This class of distribution contains the two parameter exponential and three parameter Weibull distributions as special cases. Given any one of the above mentioned three quantities of the mixture a method is developed for determining the mixing density. Several examples are provided as illustrations. Some well-known results follow trivially.

Limiting Behaviour of Sums and the Term of Maximum Modulus

R. Maller, *CSIRO, Western Australia*
S.I. Resnick, *Colorado State University, U.S.A.*

Suppose $\{X_n, n \geq 1\}$ are i.i.d. rv's with common continuous distribution function F. Set $S_n = X_1 + \cdots + X_n$, $M_n = \bigvee_{i=1}^n X_i$ and let $X_n^{(1)}$ be the term of maximum modulus, i.e., the X_i among $X_1, \ldots, X_n$ for which $|X_i|$ is largest. The influence of the extreme terms on the sample sum is studied by examining the behaviour of $S_n/X_n^{(1)}$ and S_n/M_n. The main results center about conditions for these quantities to converge to 1 in probability and almost surely. Related results deal with ratios of order statistics and ratios of record values of $\{X_n\}$. A novel feature of our approach is to study the behaviour of $\{S_n\}$ between successive record values of $\{X_n\}$.

A Class of Conditional Limit Theorems Related to Ruin Problems

M. Shimura, *University of Tsukuba*

Let $S_0 = 0$ and $S_n = \sum_{k=1}^{n} X_k$, $(n \geq 1)$, where $\{X_k\}$ is a sequence of i.i.d. random variables with mean 0 and variance σ^2. For $x \geq 0$ set $N_-(x) = \min\{n : x + S_n < 0\}$, $M(x) = \max\{x + S_n : 0 \leq n < N_-(x)\}$ and $A(x;\kappa) = \sum_{0 \leq n < N_-(x)} (x + S_n)^\kappa$, κ a positive constant. In this paper we consider conditional limit theorems concerning $N_-(x)$, $M(x)$ and $A(x;\kappa)$. For example we obtain the following result.

Let $\{x_n\}$ be a sequence of non-negative numbers such that $n^{-1/2}x_n \to 0$ as $n \to \infty$, and let ρ be a positive number. Then we have, for each ξ,

$$\mathbf{P}(M(x_n) \leq \sigma n^{1/2}\xi \,|\, N_-(x_n) > n\rho) \to p_{1,\rho}(\xi)$$

and

$$\mathbf{P}(N_-(x_n) \leq n\xi \,|\, M(x_n) > \sigma n^{1/2}\rho) \to p_{2,\rho}(\xi)$$

as $n \to \infty$. Here, $p_{i,\rho}(\cdot)$ $(i = 1, 2)$ are the continuous distribution functions of the functionals of some specified excursions of a reflecting Brownian motion starting at 0. By the result we obtain the asymptotic formulas for the tails of the distribution functions of $M(x)$ and $A(x;\kappa)$.

2.13. Point Processes and Geometric Probability

Estimating Dependent Life Lengths: A Point Process Approach

E. Arjas, *University of Oulu, Finland*

We consider a series system of n components, denoting by $\boldsymbol{T} = (T_i)_{1 \leq i \leq n}$ the vector of the component lifetimes and respectively by $\tau = \min_{1 \leq i \leq n} T_i$ and $\xi = \{1 \leq i \leq n;\, T_i = \tau\}$ the corresponding system life length and the failure pattern. If I is a subset of $\{1, 2, \ldots, n\}$, let $\tau_I = \min_{i \in I} T_i$ and $\xi_I = \{i \in I : T_i = \tau_I\}$. In various applications (reliability, biomedical, etc.) it is of interest to ask: under what conditions, and how, can inference concerning (τ, ξ) be used for (τ_I, ξ_I)? Langberg, Proschan and Quinzi [1] have recently considered this question in the interesting case where the lifetimes T_i $(1 \leq i \leq n)$ are not a priori assumed to be independent.

We interpret their result in the general framework of marked point processes and the associated hazard (=compensator) processes, adding complementary remarks.

Reference

[1] N. Langberg, F. Proschan and A.J. Quinzi, Estimating dependent life lengths, with applications to the theory of competing risks, Ann. Probab. 9 (1981) 157–167.

On Covering Single Points by Randomly Ordered Intervals

H. Berbee, *Free University at Amsterdam, The Netherlands*

Say that an interval I precedes an interval J if $\sup I \leqslant \inf J$. Let $a_1 \geqslant a_2 \geqslant \cdots > 0$ be a descending sequence of numbers with total sum 1. We construct disjoint random intervals I_i in $(0, 1)$ with length a_i, $i \geqslant 1$, such that each order is equally probable. To this end let Y_i, $i \geqslant 1$, be independent random variables, uniformly distributed on $(0, 1)$. Write $i \propto j$ if $Y_i < Y_j$. The random intervals $I_i := (\sum_{j \propto i} a_j, a_i + \sum_{j \propto i} a_j)$, $i \geqslant 1$, satisfy our requirements. Their union is an open set in $(0, 1)$ and the interval I_i precedes I_j if $i \propto j$.

Theorem. $P(r \in \bigcup I_i) = 1$, $0 < r < 1$.

This result proves a long open conjecture in game theory concerning regularity of certain games (see [1]). Consider the process $X_t := \sum a_i 1_{\{Y_i \leqslant t\}}$, $0 \leqslant t \leqslant 1$. The theorem will follow if it is shown that X hits r with probability 0. For Levy processes a similar result is known and was conjectured by Chung. The idea of the proof is to consider X locally at t as a Levy process with a random Levy measure. Then the method introduced by Carleson to solve Chung's problem can be used to prove our theorem.

Reference

[1] L.S. Shapley, in: Recent advances in game theory, Princeton Univ. Conf. (1962) 113–118.

Generalized Functionals of a Poisson Process

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This is essentially an application on Hida calculus [1] to Poisson functionals. With the defining measure μ on S' of a Poisson noise, the space (L^2) of square-integrable functionals on (S', μ) is defined and it enjoys a direct sum decomposition

(Wiener–Ito decomposition): $(L^2) = \bigoplus C_n$, where C_n is a space spanned by Charlier polynomials of degree n in $\langle x, \eta_1\rangle, \ldots, \langle x, \eta_n\rangle$, $\eta_j \in S$. By means of the Hida diagram, (L^2) can be extended to $(L^2)^-$, where the renormalization of nonlinear functionals such as $\dot{P}(t)^n$, $\exp(\dot{P}(t))$, live and operations such as differentiation with respect to $\dot{P}(t)$, multiplication by $\dot{P}(t)$, and so on can be rigorously defined. The renormalization of $g \in (L^2)$ is defined by $g(P(t)) := U^{-1}g(e^{i\xi(t)})$, with the U-transforms $(Ug)(\xi) = C(\xi)^{-1}\int g(x)\, e^{i\langle x,\xi\rangle}\, d\mu(x) = C(\xi)^{-1}(Tg)(\xi)$ [2]. Once a polynomial or an exponential function in $\langle x, \eta_1\rangle, \ldots, \langle x, \eta_n\rangle$ is renormalized, it attains its limit in $(L^2)^-$ as η_i goes to δ_{t_j}, $j = 1, \ldots, n$. The differential operator and the adjoint operator are defined by

$$\frac{\partial}{\partial \dot{P}(t)} = U^{-1}\frac{\delta}{\delta\, e^{i\xi(t)}}U, \qquad \left(\frac{\partial}{\partial \dot{P}(t)}\right)^* g, h = g, \frac{\partial}{\partial \dot{P}(t)} h$$

respectively. These operators give the following formulae, where attention must be paid to the domains:

$$\frac{\partial}{\partial \dot{P}(t)} g(\dot{P}) = g(\dot{P} + \delta_t) - g(\dot{P}), \qquad \int g(\dot{P})\, dP(t) = \int \left(\left(\frac{\partial}{\partial \dot{P}(t)}\right)^* + 1\right) g(\dot{P})\, dt,$$

$$\dot{P}(t) \cdot g(\dot{P}) = \left(\left(\frac{\partial}{\partial \dot{P}(t)}\right)^* + 1\right)\left(\frac{\partial}{\partial \dot{P}(t)} + 1\right) g(\dot{P}).$$

Here $\dot{P}(t)\cdot$ is a multiplicative operator.

References

[1] T. Hida, Carleton Math. Lecture Notes No. 13, 2nd ed. (1978).
[2] T. Hida and N. Ikeda, Proc. 5th Berkeley Symp. Math. Stat. Probab. 2 (1967) 117–143.

Least Upper Bound for a Class of Triangles

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We are considering the class $\{\frac{1}{2}xH(x);\ x \geq 0\}$ and $\{\frac{1}{2}x^2H(x);\ x \geq 0\}$ of triangles where $H(x) = \mathbf{P}(X \geq x)$, X being a non-negative random variable. In the first class x is the base and $H(x)$ is the perpendicular distance and in the second x is the base and $xH(x)$ is the perpendicular distance. It is known that if $\mathbf{E}(X)$ exists then an upper bound of the first class is $\frac{1}{2}\mathbf{E}(X)$ and an upper bound of the second class is $\frac{1}{2}\mathbf{E}(X^2)$ provided it exists. We obtained in this paper the least upper bound of $\frac{1}{2}xH(x);\ x \geq 0\}$ when $\mathbf{E}(X)$ exists as well as when $\mathbf{E}(X)$ does not exist. In the latter it is possible when the limit $\lim_{x\to\infty} x^2 f(x)$ exists, f being the density function of X. Similarly we are finding a least upper bound for $\{\frac{1}{2}x^2H(x);\ x \geq 0\}$ when $\mathbf{E}(X^2)$ exists and when it does not. This approach has been extended to the class $\{-\frac{1}{2}xF(x);\ x \leq 0\}$ of triangles where $-x$ is the base and $F(x)$ is the perpendicular distance with $F(x) = \mathbf{P}(X \leq x)$. Finally a least upper bound for the class $\{\mathbf{P}(|X| \geq \lambda);\ \lambda > 0\}$ of probabilities has been obtained without the assumption of the existence of $\mathbf{E}(X^2)$. The case when $\mathbf{E}(|X|)$ does not exist has also been discussed. Some areas of application of these results are mentioned at the end of the paper.

Geometric Probability and Entropy of Some Point Processes

J.P. Rasson, *Facultés Universitaires de Namur, Belgium*

Davidson [1] proves the theorem: "Amongst the line-processes satisfying some hypotheses, the mixed Poisson process, Q_M, has the largest specific intersection rate inside a given circle in the plane". We try to find the connections between this property P, and the more recent theory of entropy of point processes [2]. The very nature of P suggests us to look for the connections with one component of the entropy, which we call the locational entropy of second order, H_{L_2}. But, although H_{L_2} is maximal for Q_M, as well there is no direct connection with P. We establish a new property P_2 of Q_M, regarding also the intersection rate of the lines, and we explain why P_2 is more natural than P and implies P. We prove that P_2 implies the fact that H_{L_2} is maximal for Q_M.

We make some comments on the way Davidson limited his class of processes which seems to be arbitrary in some sense; if we do not use his hypothesis, P and P_2 are also true for processes other than Q_M.

We propose some generalizations of P_2 and study the event "The triangle formed by three lines cutting a given circle contains the centre of this circle". We prove an optimality property of Q_M which implies the optimality of H_{L_3}, the locational entropy of third order. We generalize P_3 to P_n and show the connection with H_{L_n}.

References

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2.14. Queueing Theory, Part II

Exponential Queueing Networks with Finite Capacities

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Consider a network of service stations $Q_0, \ldots, Q_M$. Requests arrive at the centers according to independent Poisson processes; they travel through the network demanding independent negative exponentially distributed amounts of service at those centers which they visit, and finally depart from the network. Some of the service stations have finite waiting rooms. When the capacity at Q_i is reached, service at all nodes which are currently processing a request destined next for Q_i is instantaneously interrupted. This blocking phenomenon makes an exact analysis intractable and a numerical solution computationally infeasible for most exponential systems.

We consider the possibility of extending an approximation procedure [1] to networks with feedback and show that it can be easily obtained for the two-stage queueing system with feedback and finite intermediate waiting room studied in [2]. We also point out how the approximation procedure can be used to derive accurate estimates for throughput rates in exponential queueing networks.

References

[1] O.J. Boxma and A.G. Konheim, Approximate analysis of exponential queueing systems with blocking, Acta Inform. 15 (1981) 19–66.

[2] A.G. Konheim and M. Reiser, A queueing model with finite waiting room and blocking, J. Assoc. Comput. Mach. 23 (1976) 328–341.

The M/G/1 Queue with Alternating Service Formulated as a Riemann–Hilbert Problem

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The single server queue with two Poissonian arrival streams is considered; the server handles alternatingly a customer of each queue if the queues are not empty. Customers of the same arrival stream have the same service time distribution $B_1(\cdot)$ and $B_2(\cdot)$, respectively. Only the stationary situation is considered, and herefore it is shown that the determination of the joint queue length distribution at a departure epoch can be formulated as a Riemann–Hilbert boundary problem. This problem can be completely solved for general $B_1(\cdot)$ and $B_2(\cdot)$, and the numerical evaluation of all queueing quantities of interest can be effectuated.

The aim of the study is to show that the formulation of queueing problems, which require the solution of functional equations for the determination of joint distributions, as Riemann–Hilbert boundary problems is an extremely powerful approach in analyzing such queueing problems.

A Traffic Intersection Model with Non-Poisson Arrivals on the Major Road

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This paper is concerned with the estimation of vehicle queue lengths in the minor road of a non signalized major–minor road intersection (T-junction). Several authors, e.g., Tanner [1], have considered such delays when vehicle headways on both roads are negative exponentially distributed, but little work has been published concerning major road traffic with an arbitrary distribution for gaps. One explanation for this is that only in the exponential case the process is regenerative, and considerable mathematical difficulties arise with other distributions. The present paper deals with a model of an intersection where traffic on the major road consists of a stream of vehicles forming alternately gaps of constant length and blocks of arbitrary lengths. A form of imbedded Markov Chain analysis is used, and an explicit expression is derived for the vehicle mean queue length in the minor road. Numerical results are also presented.

The fixed cycle traffic-light situation is derived as a special case, and comparisons are made with the results of other authors.

Reference

[1] J.C. Tanner, A theoretical analysis of delays at an uncontrolled intersection, Biometrika 49 (1962) 163–170.

On a System of Wiener–Hopf Integral Equations Occurring in the Theory of the Multi-Server Queue

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The analysis of the queue $GI/H_m/s$, i.e., the multi-server queue with renewal input and hyper-exponential service times, gives rise to a system of integral equations which is of Wiener–Hopf type. In this paper we obtain the solution of this system. The solution shows that the stationary waiting time distribution is a mixture of $\binom{m+s-1}{s}$ exponentials and a concentration at 0, as already suggested by Pollaczek [1].

Reference

[1] F. Pollaczek. Théorie Analytique des Problèmes Stochastiques Relatifs á un Groupe de Lignes Téléphoniques avec Dispositif d'Attente (Gauthier-Villars, Paris, 1961).

2.15. Stochastic and Dynamic Systems, Part I

Closure of Phase Type Distributions under Operations Arising in Reliability Theory

D.N. Assaf and B.Z. Levikson, *University of Haifa, Israel*

Consider three basic operations arising in reliability theory: finite mixtures, finite convolutions, formation of coherent systems of independent components. In this paper it is shown that (i) the class of all phase type distributions is closed under all three operations, and (ii) the class of all phase type distributions having a representation in which the matrix is upper triangular is the smallest class which is closed under all three operations and contains all exponential distributions.

The paper includes a section of preliminaries in which the relevant material regarding phase type distributions and reliability theory is summarized.

Ergodic Theory of Linear Stochastic Systems

H. Crauel, *University of Bremen, Federal Republic of Germany*

We consider the system $\dot{x}(t)=A(t,\omega)x(t)$, where A is a stationary, ergodic $(n\times n)$-matrix-valued process. Using Osedelec's multiplicative ergodic theorem we prove the existence of a fundamental matrix $\Psi(t,\omega)$, allowing a Floquet-type decomposition $\Psi(t)=S(t)\cdot\exp(\Lambda t+o(t))$, where S is a (partially) stationary $R^{n\times n}$-valued process and Λ is a constant matrix, determined by the Lyapunov-numbers of the system.

Finally we show that a linear system $\dot{x}(t)=Ax(t)$ can be stabilized by adding a mean zero stationary process F to A iff tr $A<0$. In this case F can be taken skew symmetric and depending on a one-dimensional diffusion ξ only: $F(t,\omega)=F(\xi(t))$.

A Class of Observers for Nonlinear Stochastic Systems with Jump Process Observations

Y. Yavin, *CSIR*, *South Africa*

Nonlinear filtering problems are discussed, involving a process $\{X_t, t\geq 0\}$ determined by the stochastic differential equation $\mathrm{d}x=f(x)\,\mathrm{d}t+\sigma(x)\,\mathrm{d}W$, $t>0$, $x\in R^m$, with the observation process $\{Y_t, t\geq 0\}$ given by $\mathrm{d}y=c_0\,\mathrm{d}N$, $t>0$, $c_0>0$, $y\in R^p$, where W is a R^m-valued Wiener process. Two cases are dealt with: (1) On a probability space (Ω, F, F_t, P), $N_t=\int_0^t\lambda(X_s)\,\mathrm{d}s$, $t\geq 0$, is an R^p-valued (F_t, P) martingale, where $F_t=\sigma(X_s, N_s, s\leq t)$; $F=\bigvee_{t\geq 0}F_t$, and $\lambda: R^m\to R^p$ is a given function; (2) $N_t=\int_0^t c(X_s,u)\nu(\mathrm{d}s,\mathrm{d}u)$, $t\geq 0$, where ν is a Poisson random measure on $[0,\infty)\times R^m$, and $c: R^m\times R^m\to R^p$ is a given function. An observer is required of the form: $\mathrm{d}z=f(z)\,\mathrm{d}t+G(y,z)(\mathrm{d}y-c_0\lambda(z)\,\mathrm{d}t)$, $t>0$, $z\in R^m$, in case (1); and $\mathrm{d}z=f(z)\,\mathrm{d}t+G(y,z)(\mathrm{d}y-\int_{R^m}c(z,u)\Pi(\mathrm{d}u)\,\mathrm{d}t)$, $t>0$, $z\in R^m$ in case (2), where $E\nu(t,A)=t\Pi(A)$. In both cases the 'gain matrix' G is to be determined. Let D be the unit cube in R^{2m+p} and let τ_i where i labels the two cases be the first time that $(X_t, Y_t, Z_t)\notin D$ given that $(X_0, Y_0, Z_0)\in D$ (Z_t is the state of the observer). The approach we adopted is to choose a matrix $G_*^{(i)}$, of a bang-bang type, in such a manner that $E_{x,y,z}\Lambda\{t: 0\leq t<\tau_i, |X_t-Z_t|\leq\varepsilon\}$ is maximized on a class of admissible gain matrices. Here Λ is the Lebesgue measure on R and $\varepsilon>0$ is given. For each $i=1,2$ sufficient conditions on the maximizing gain matrices are derived. These conditions can be applied off-line and can be used in the design of the observers. Several examples are solved numerically.

2.16. Games and Information Theory

Une Egalite sur la Conservation de l'Information

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Partant du théorème suivant [1]: Soient $\xi_1, \xi_2, \ldots, \xi_n$ n variables aléatoires indépendantes définies sur le même espace probabilité $(\Omega, F, \mathbb{P})$ et prenant un nombre fini de valeurs et soit ζ une autre variable aléatoire arbitraire définie sur le même espace et prenant également un nombre fini de valeurs. On a alors: $\sum_{k=1}^n I(\xi_k, \zeta) \leq I((\xi_1, \xi_2, \ldots, \xi_n), \zeta)$. Nous avons essayé de généraliser cette notion d'inégalité entre informations mutuelles en considérant des variables aléatoires quelconques sur lesquelles nous avons tenté de traduire cette relation plutôt sous la forme d'une égalité informationnelle. Posant: μ: loi de $(X_1, X_2, \ldots, X_n)$, $\tilde{\mu}$: loi de $(X_1, X_2, \ldots, X_n, Y)$, μ_k: loi de X_k, ν: loi de Y, ν_k: loi de (X_k, Y),

$$\hat{\mu}(A_1 \times A_2 \times \cdots \times A_n \times B) = \int_B \prod_{k=1}^n \frac{\partial \tilde{\nu}_k}{\partial \nu}(y, A_k)\,(\mathrm{d}y)$$

ou

$$\frac{\partial \tilde{\nu}_k}{\partial \nu}(y, A_k) = \int_{A_k} \frac{\mathrm{d}\tilde{\nu}_k}{\mathrm{d}\mu_k \otimes \nu}(x_k, y)\mu_k\,(\mathrm{d}x_k).$$

Nous avons (en supposant $\tilde{\mu} \ll \bigotimes_{k=1}^n \mu_k \otimes \nu$ et moyennant deux lemmes portant essentiellement sur l'absolue continuité des mesures $\hat{\mu}$ et $\tilde{\mu}$ par rapport respectivement a $\bigotimes_{k=1}^n \mu_k \otimes \nu$ et $\hat{\mu}$): $D(\mu, \bigotimes_{k=1}^n \mu_k) + D(\mu, \mu \otimes \nu) = \sum_{k=1}^n D(\tilde{\nu}_k, \mu_k \otimes \nu) + D(\tilde{\mu}, \hat{\mu})$ qui ont l'égalité informationnelle cherchée.

Référence

[1] I. Csiszar, Ann. Probab. 3 (1975) 146–158.

Information Géneralisée Liée à Certains Processus Stochastiques

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On s'intéresse ici au régime de non-équilibre statistique de certains types de processus stochastiques (p.s.) tels que les processus de diffusion, files d'attente sans approximation, etc. l'évolution dans le temps de ces processus, markoviens et gaussiens, est très bien décrite au moyen de la notion d'information simple ou généralisée. Lorsque ces p.s. sont soit homogènes dans le temps, soit linéaires, l'information joue très bien le rôle de 'guide' du p.s. et sa valeur extrémale reflète bien l'état d'équilibre stable du processus. Ce rôle s'applique au cas de certains systèmes de files d'attente (M/M1; GI/G/1, réseaux . . .) sous l'approximation de la diffusion. Pour ces systèmes, on s'intéresse la plupart du temps à leur régime permanent (équilibre) pour des raisons bien connues. L'intérêt de l'information est qu'elle permet de mieux cerner le régime transitoire (non équilibre) de ces processus. A titre d'application, on étudie le cas d'une file d'attente GI/G/1 à capacité limitée et barrières absorbantes. On s'efforce de détecter quelques points 'critiques' de son évolution au moyen de l'information généralisée (D, ordre α). Enfin une dernière partie est consacrée à l'étude des p.s. markoviens en général: on y établit une condition nécessaire et suffisante pour que l'information attachée à un p.s. non-homogène et non additif décroisse constamment au cours du temps; cette condition est liée à la nature de la distribution d'équilibre du processus.

On the Shapley Index of Power for a Weighted Majority Game

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The weighted majority game is characterized by a finite set $N = \{1, 2, \ldots, n\}$ of players with corresponding weights $w_1, \ldots, w_n$. A subset S of N is a winning coalition if $w(S) = \sum_{j \in S} w_j$ exceeds a preassigned quota c. Fix $i \in N$ and let $N_i = N/\{i\}$. A swing for i is defined as a non-winning subset of N_i which becomes a winning coalition by adding the player i. The Shapley index of power of i is defined as the probability ϕ_i that a random subset S of N_i is a swing for i, the probability $P(S)$ of S depending only on its size $|S|$, in such a way that $P(S = m) = 1/n$ $(m = 0, 1, \ldots, n-1)$.

A very practical application concerns the Electoral College of the United States which has $n = 51$ players and $c = 270$. In such a situation, the exact calculation of ϕ_i is hard. In the present paper we present an asymptotic expansion for ϕ_i when n is large. The proof makes an essential use of a local limit theorem due to Petrov [1]. Truncating this expansion, one obtains a sequence of easily calculated approximations $\phi_i^{(k)}$. In the above illustration, $\phi_i^{(5)}$ is accurate in 5 or 6 significant digits.

We also consider the case of a fixed number of majority players in the presence of an 'ocean' of minority players. As one application, our formulae allows us to determine rather precisely when it is favorable for two minority players to pool their resources.

Reference

[1] V.V. Petrov, Sums of independent random variables (Springer, New York, 1975).

One Pursuer and Two Evaders on the Line–A Stochastic Pursuit–Evasion Differential Game

M. Pachter and Y. Yavin, *CSIR, South Africa*

A differential game of pursuit and evasion on the real line is discussed with one pursuer and two evaders, the motion of the players being affected by noise. The game of degree is considered where the pursuer strives to maximize the probability of his winning the game, i.e. of capturing at least one of the evaders, the probability function being given as a solution to a certain partial differential equation. The degenerate situation here arises in a natural way and it is possible to present a quite detailed analysis of this case.

2.17. Optimization, Part II

Necessary Conditions on Optimal Markov Controls for Stochastic Processes

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We illustrate the applicability of locally convex optimization techniques, in the formulation of Dubovitskij and Milyutin to the control of stochastic processes with an example: Let $\{X_t; 0 \leq t \leq T\}$ be the solution of

$$\mathrm{d}X_t = a(X_t, u(t, X_t))\,\mathrm{d}t + b(X_t, u(t, X_t))\,\mathrm{d}W(t), \quad X_0 = x_0. \tag{1}$$

$W(t)$ is a standard Wiener process. $u(t, x)$ can be chosen from a closed, convex subset K with nonempty interior in the space of measurable functions on $[0, T] \times \mathbb{R}$ which are continuously differentiable w.r.t. x and for which $u(t, x)$ and $(\partial/\partial x)u(t, x)$ are bounded. We consider a loss functional of Bolza type: $L(u) = E\{R(X_t) + \int_0^T S(X_t, u(t, X_t))\,\mathrm{d}t = \min!$ We assume that R, S, a, b are continuously differentiable and their derivatives are bounded on sets $\{(x, u): |u| \leq r\}$, $r > 0$. Using theorems of Gihman and Skorokhod about the continuity and differentiability of solutions of stochastic differential equations depending on a parameter we derive the following.

Theorem. *Let u^0 be a local minimum for L in the set K of admissible controls. $\{X_t^0, 0 \leq t \leq T\}$ denotes the solution of* (1) *for $u = u^0$. For functions f of x and u we write $f_x^0(t, x) := (\mathrm{d}/\mathrm{d}x)\{f(x, u^0(t, x))\}$. Then for all $u \in K$ we have*

$$L_0'(u - u^0) = E\int_0^T \{u(t, X_t^0) - u^0(t, X_t^0)\}\{\alpha(t, X_t^0)\,\mathrm{d}t + \beta(t, X_t^0)\,\mathrm{d}W(t)\} \geq 0.$$

Here $L_0'(u)$ is the directional derivative of the functional L at u^0 in direction u.

$$\alpha(t,x)=\zeta(t,x)\left\{\frac{\partial}{\partial u}a(x,u^0(t,x))-b_x^0(t,x)\frac{\partial}{\partial u}b(x,u^0(t,x))\right\}$$

$$+\frac{\partial}{\partial u}S(x,u^0(t,x)),$$

$$\beta(t,x)=\zeta(t,x)\frac{\partial}{\partial u}b(x,u^0(t,x)),$$

$$\zeta(t,x)=E\left\{R'(X_T^0)Y_T/Y_t+\int_t^T S_x^0(s,X_s^0)Y_s/Y_t\,\mathrm{d}s|X_t^0=x\right\},$$

$\{Y_t, 0\leq t\leq T\}$ *is the solution of the linear homogeneous equation* $\mathrm{d}Y_t=a_x^0(t,X_t^0)Y_t\,\mathrm{d}t+b_x^0(t,X_t^0)\cdot Y_t\,\mathrm{d}W(t)$, $Y_0=1$.

Optimal Stopp'ng with Discontinuous Partial Observation

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A separation theorem is shown between filtering and optimal stopping for a partially observable system. The signal X is a Feller process. Its observation Y can be decomposed in the sum of a continuous and a discontinuous part. $Y=Y^c+Y^d\cdot Y^c$ has the classical form $Y^c=\int_0^t h(X_s)\,\mathrm{d}s+W_t$ where $\int_0^\infty h^2(X_s)\,\mathrm{d}s<\infty$ and W is a Brownian motion independent of X. Y^d is a point process with intensity $\int_0^t H(X_s)\,\mathrm{d}s$ such that $\int_0^\infty H^2(X_s)\,\mathrm{d}s<\infty$. Let G be the filtration generated by Y. We look after a G-stopping time T^X maximizing $\{E(J_T)$: T G-stopping time$\}$, with

$$J_t=\int_0^t \mathrm{e}^{-\alpha s}f(X_s)\,\mathrm{d}s+g(X_t),$$

f and g being bounded continuous functions.

A solution T^X of this problem, which is a Markovian function of the unnormalized conditional law of X knowing Y, is found.

On the Existence of Average Optimal Policies in Markov Decision Drift Processes with General State and Action Spaces

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A. Hordijk, *University of Leiden, The Netherlands*

Recently Hordijk and van der Duyn Schouten [1] introduced the concept of Markov decision drift process. These processes are generalizations of Markov decision processes with continuous time parameter. The special feature of a Markov decision process is the possibility to describe infinitesimal as well as impulsive control simultaneously.

In this paper we investigate the existence of average optimal policies for a Markov decision drift process. Sufficient conditions are derived which guarantee that a 'limit point' of a sequence of α-discounted optimal policies with α approaching zero is an average optimal policy.

When the α-discounted optimal policies generate regenerative stochastic processes an alternative set of sufficient conditions is obtained. This set of conditions has the advantage that it is easy to verify in several applications. The results are also applicable to Markov decision processes with discrete or continuous time parameter and to semi-Markov decision processes and in this sense they weaken the well-known conditions for the existence of an average optimal policy for Markov decision processes with finite or compact action spaces.

Reference

[1] A. Hordijk and F.A. van der Duyn Schouten, Markov decision drift processes; discretization and weak convergence II, to appear.

2.18. Markov Processes and Diffusions, Part II

A Representation of Additive Functionals of d-Dimensional Brownian Motion
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A well-known result of Ito and McKean [1] says that any additive functional A_t of l-dimensional Brownian motion can be written as $A_t = \int L_t^y \mu\,(\mathrm{d}y)$ for some measure μ, where L_t^y is local time at y. An analogue of this result is proved for additive functionals of a d-dimensional Brownian motion W_t. If '$\cdot$' denotes inner product, let $L(t, s, v)$ be local time of $W_t \cdot v$, where v is a unit vector. Let B be the collection of unit vectors. If A_t^μ is the additive functional corresponding to a finite meaure that has finite potential, then

$$A_t^\mu = \lim_{a\to\infty} \int\int_B \int_{-\infty}^{\infty} H_a(s - y \cdot v) L(t, s, v)\,\mathrm{d}s\,\mathrm{d}v\mu\,(\mathrm{d}y) \qquad (1)$$

where H_a is a known deterministic function depending only on the parameter a and the dimension d. If the right-hand side of (1) is denoted A_t^a and μ satisfies a very slight regularity condition, the limit is in the sense $\sup_{t\leq u} |A_t - A_t^a| \to 0$ a.s. as $a \to \infty$. In general, convergence will be in probability, while if μ has a sufficiently smooth density, (1) can be rewritten to eliminate any limit whatsoever. As a by-product, we get conditions for a.s. joint continuity for a family of additive functionals A_t^a indexed by a parameter.

Reference

[1] K. Ito and H.P. McKean Jr., Diffusions and their sample paths (Springer, New York, 1965).

Strong Invariance for Local Times

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Let $X_1, X_2, \ldots$ be a sequence of i.i.d. integer valued random variables, $S_i = X_1 + \cdots + X_i$, with $\mathbf{E}\, X_i = 0$, $\mathbf{E}\, X_i^2 = \delta^2 < \infty$. The second author [1] has shown that if $\mathbf{P}(X_i = +1) = \mathbf{P}(X_i = -1) = \frac{1}{2}$, then on a suitable probability space one can redefine the sequence $\{X_i\}$ together with a Wiener process such that $\sup_{x \in Z} |L(x, n) - N(x, n)| \ll n^{1/4+\varepsilon}$ a.s. for all $\varepsilon > 0$, where $L(\cdot, \cdot)$ is the local time of the Wiener process and $N(x, n)$ is the number of visits of the sequence $\{S_i\}$ to the point x in the time interval $(0, n]$. In the present paper we extend this result to a more general distribution, under certain moment conditions. We note that the rate of convergence can not be improved to $n^{1/4}$.

Reference

[1] P. Révész, Local time and invariance, to appear.

Markov Processes with Identical Last Exit Distributions

J. Glover, *University of Rochester, U.S.A.*

Let X and Y be two transient Hunt processes. If X and Y enjoy the same last exit distributions from compact sets, then Y is equivalent to a time-change of X by the inverse of a strictly increasing continuous additive functional. This result can also be interpreted (with natural auxiliary hypotheses) as a statement in potential theory involving equilibrium measures.

2.19. Branching Processes, Part I

Convergence of Supercritical Branching Random Fields

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Let N_t, $t \geq 0$, be a random field of particles on $\mathbb{R}^d$ defined as follows: N_0 is a homogeneous Poisson field, each initial particle generates a supercritical age dependent branching process where the particles perform Brownian motions (or other processes), and $N_t(A)$ is the number of particles in the Borel set $A \subset \mathbb{R}^d$ at time t. Under an appropriate high-density space-time scaling and centering, the process converges weakly to a generalized Gaussian random field X_t, $t \geq 0$, which satisfies the stochastic evolution equation

$$\frac{\partial X}{\partial t} = \tfrac{1}{2}\,\Delta X + \dot{W},$$

with initial condition X_0 = standard Gaussian white noise on $\mathbb{R}^d$, the space-time white noise $\dot{W}$ having covariance

$$\operatorname{cov}(\dot{W}(x, t),\ \dot{W}(y, s)) = \delta_2(t, s)\delta_d(x - y).$$

A. Martin-Löf studied the no branching case; D. Dawson, R. Holley and D. Stroock, the Markovian critical case; and K. Fleischman, a supercritical Galton–Watson case.

On the Convergence of Supercritical General (C–M–J) Branching Processes

O. Nerman, *University of Göteborg, Sweden*

Convergence in probability of Malthus normed supercritical general branching processes (i.e. Crump–Mode–Jagers branching processes) counted with a general characteristic are established, provided the latter satisfy mild regularity conditions. If the Laplace transform of the reproduction point process evaluated in the Malthusian parameter has a finite 'x log x-moment', convergence in probability of the empirical age distribution and more generally of the ratio of two differently counted versions of the process also follows.

Malthus normed processes are also shown to converge a.s., provided the tail of the reproduction point process and the characteristic both satisfy mild regularity conditions. If in addition the x log x-moment above is finite, a.s. convergence of ratios follows.

Further, a finite expectation of the Laplace-transform of the reproduction point process evaluated in any point smaller than the Malthusian parameter is shown to imply a.s. convergence of ratios even if the x log x-moment above equals infinity.

The Stable Doubly Infinite Pedigree Process of Supercritical Branching Populations

O. Nerman and P. Jagers, *University of Göteborg, Sweden*

An individual is sampled randomly from a supercritical general branching population and the pedigree process, which centers around this 'ego'-individual, is studied. The process describes not only lineage backwards and forwards, but also the lives of all individuals involved. Under mild conditions and in several senses, the process is shown to stabilize, as time passes. The limit is a doubly infinite population process, which generalizers the stable age distribution of branching processes and demography. It displays a nice independence structure, and can easily be constructed from the original branching law. The results are applied to certain kin-number problems, the process of ego's ancestors' births, and to the FLM-curve of cell kinetics.

Kin Numbers in Galton–Watson Populations

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The problem of 'kin numbers' in a population concerns the number of relatives of various degrees of affinity of a randomly chosen member of the population. If we call this individual 'Ego', then the observations are the size of Ego's sibling group, the number of her aunts (assuming a one-sex female-line study), her daughters, and so on.

The problem has traditionally been treated in the demographic literature by means of the life table and maternity function. The results are limited to expectations, and the methods assume large samples. This is not satisfactory for the small sizes of family groups. For these groups stochastic fluctuations and joint distributions must be studied, and this is done in the present work by supposing that Ego belongs to a population developing according to a Galton–Watson branching process.

Basic results for this problem which are shortly to appear in *Advances in Applied Probability* will be outlined. This work concerns (small) family groups in a large population. Current work with A. Joffe concerning the problem when the population itself is small (as in an isolated tribe) will also be discussed.

2.20. Distribution Theory

On Wald's Identity for Dependent Variables

P. Franken and B. Lisek, *Humboldt University, GDR*

Let P be the distribution of a stationary real-valued random sequence $\phi=(X_i)_{i=0}^{\infty}$, and $\tau(\phi)$ a stopping time with $E_p\tau(\phi)<\infty$. Then there exists a distribution Q of a stationary sequence $\psi=(X_i, Y_i)_{i=0}^{\infty}$, $Y_i\in\{0, 1\}$ with the properties (i) $Q(Y_0=1)>0$, $Q((X_i)_{i=0}^{\infty}\in(\cdot))=P(\phi\in(\cdot))$, (ii) $E_Q(\sum_{i=0}^{\tau(\phi)-1} X_i \mid Y_0=1)$ $=E_Q(\tau(\phi)\mid Y_0=1)E_P X_0$. (ii) is a generalization of Wald's identity. The proof of this result is based on the following approach to solve recursive stochastic equations of the type $Z_{i+1}=f(X_i, Z_i)$ (cf. [1]). Assume that there is a system $\{A(i, \varphi): i\in\Gamma^+,$ $\varphi\in\mathbb{R}^+\}$ of subsets of the range of values of Z_n with the property

$$P(f(X_i, A(i, \phi))\subseteq A(i+1, \phi) \text{ for all } i\in\Gamma^+, 0<\#A(0, \phi)<\infty)=1.$$

Then, under some regularity conditions, there is a stationary sequence $(Z_i)_{i=0}^{\infty}$ for which the given equation almost surely holds with respect to the common distribution of $(\phi, (Z_i)_{i=0}^{\infty})$.

Reference

[1] B. Lisek, A method for solving a class of recursive stochastic equations, Z. Wahrsch. Verw. Gebiete, to appear.

Differences and Quotients of Record Values

C.M. Goldie, *University of Sussex, England*

Let $R_1, R_2, \ldots,$ be the sequence of upper record values of i.i.d. observations whose distribution function F is continuous and satisfies $F(0^+) = 0$, $F(x) < 1$ for all $x < \infty$. Characterizations are given for convergence in distribution to non-degenerate and degenerate limits, and for almost sure convergence to 0 or ∞ of the differences $R_{n+1} - R_n$, and for uniform a.s. convergence of the empirical distribution function $G_n(x) := n^{-1} \sum_{k=1}^{n} I[R_{k+1} - R_k \leq x]$, $x \in \mathbb{R}$. By a simple transformation equivalent characterizations are obtained for convergence of quótients R_{n+1}/R_n. In particular, R_{n+1}/R_n converges in distribution to an exponential distribution when, but not only when, the tail $1 - F(x)$ is regularly varying, thus answering a conjecture of Smid and Stam in the negative. Indeed, convergence in distribution to degenerate limits is equivalent to, and to exponential limits is implied by, regular variation if $1 - F(\cdot)$ *off* some set of positive reals that has, in a certain sense, zero asymptotic density. Methods used include a Tauberian theorem of Bingham and some summability lemmas. Distributional convergence of the process $\{R_{n+1} - R_n, R_{n+2} - R_n, \ldots\}$ considered as the epoch process of a point process, is characterized in a similar way. The limit process has to be a Cox process satisfying a form of stationarity. Despite this, the class of possible limit processes is very large, including for example all Poisson processes, necessarily of constant rate, and all mixed Poisson processes. Hence the class of limit distributions of $R_{n+1} - R_n$ includes all mixtures of exponentials.

Rates of Convergence in Extreme Value Theory

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It is well known that, under certain conditions, the renormalized maxima of a sequence of independent, identically distributed random variables converge weakly to one of the three extreme value limit distributions. However, the convergence is sometimes very slow, as was first pointed out by Fisher and Tippett in their study of normal extremes. We develop a general theory applicable when the limit distribution is either $\exp(-x^{-\alpha})$ or $\exp(-(-x)^{\alpha})$, $\alpha > 0$. This, therefore, excludes the specific problem of normal extremes, for which the limit distribution is $\exp(-e^{-x})$, but that problem has been dealt with in detail by Hall and others. The theory developed here allows the calculation of the rate of convergence under very general conditions. Under more restrictive conditions it is possible to obtain an expression for the principal error term, analogous to a one-term Edgeworth expansion. Among the applications the theory casts a new light on some uses of extreme value theory in the reliability of multi-component systems.

2.21. Inference, Part II

The Role of Gaussian Processes in the Asymptotic Theory of Rank Tests

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Two new classes of limiting Gaussian processes are considered here. The first of these is obtained from the Kolmogorov–Smirnov test statistic when the distributions of the sample are discontinuous. The second is the following: suppose $\boldsymbol{R} = (R_1, \ldots, R_N)$ denotes the vector of ranks corresponding to the sample $X_1, \ldots, X_N$ possessing an exchangeable joint density. The standard linear rank statistic is given by $S_N(\boldsymbol{R}) = \sum_{i=1}^N c_i a_N(R_i)$. Assume $a_N(i)$ to be periodic with period N. For $0 \leq t \leq 1$ define $a_N(i, t) = a_N(i - [tN])$ and hence define the $[0, 1]$-process $S_N(\boldsymbol{R}, t) = \sum_{i=1}^N c_i a_N(R_i, t)$. The limiting distribution of $S_N(t)$ is obtained under two assumptions: (i) $\exists \phi(u) \in L_2([0, 1])$ for which $\|a_N(1+[uN]) - \phi(u)\| \to 0$ as $N \to \infty$, (ii) if $c_\mu = (c_{\mu 1}, \ldots, c_{N(\mu)})$ define a sequence of vectors, then as $N(\mu) \to \infty$, $\sum (c_{\mu i} - c_\mu)^2 / \max(c_{\mu i} - c_\mu)^2 \to \infty$. Then

$$(\textstyle\sum_i (c_{\mu i} - c_\mu)^2)^{-1/2} (S_{N(\mu)}(t) - c_\mu \sum_{i=1}^{N(\mu)} a_{N(\mu)}(i)) \xrightarrow{d} \xi(t)$$

where $\xi(t)$ is a Gaussian process with the properties that $E(\xi(t)) = 0$, $\mathrm{cov}(\xi(s), \xi(t)) = \int_0^1 \phi(u-s)\phi(u-t)\,\mathrm{d}u$ and $P\{\xi(t) \in C[0, 1]\} = 1$ (continuous sample paths).

The reason for investigating $S_N(t)$ is that $\nu_N = \sup\{S_N(t)\}$ possesses a remarkable statistical property. It turns out that linear tests are inadmissible under Bahadur efficiency and the reason is that the test based on ν_N always dominates S_N in efficiency. The approximation of functionals of these Gaussian processes is discussed further. In particular a method of least squares approximation is recommended.

A Markov Model for On-Stream Quality Estimation

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A Markov model for the flow rate and quality of material (e.g. iron ore) passing along a conveyer belt is constructed. The model is used to compare the performance of different sampling schemes used in assessing the overall quality of material.

Confidence Intervals for the Threshold Parameter
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Let $Z_1, Z_2, \ldots$ be independent exponential random variables, each with mean 1. Let $\alpha > 0$, $\beta > 0$ and μ be fixed numbers and define the stochastic process $\xi = \{\xi_i : i = 1, 2, \ldots\}$ by $\xi_i = \mu + \beta(Z_1 + \cdots + Z_i)^{1/\alpha}$. Based on the observation of $\xi_1, \ldots, \xi_k$, we present two methods for constructing confidence intervals for the threshold parameter μ when the scale (β) and the shape (α) are unknown. A similar problem was discussed by de Haan at the Ninth Conference on Stochastic Processes and their Applications (Evanston, August, 1979).

The process ξ is in fact the limiting process (as $n \to \infty$) for the lower extremes of a sample of size n from a distribution which is regularly varying at its threshold μ (e.g. Weibull, beta, gamma). Thus the results are applicable in life-testing situations, where the sample size is large but only few failures have been observed.

2.22. Stochastic and Dynamic Systems, Part II

Stationary Solutions of Linear Systems with Additive Noise
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Let $z(t)$, $t \in \mathbb{R}$, be a strictly stationary process with values in $\mathbb{R}^d$ which is continuous in probability and has locally integrable trajectories. We aim at clarifying as completely as possible existence and uniqueness of stationary solutions of $\dot{x} = Ax + z$, $x(0) = x_0$, A a $d \times d$-matrix, without imposing any further conditions on z (e.g. existence of moments). A solution $x(t; x_0) = \exp(tA) \times (x_0 + Z(t))$, $Z(t) = \int_0^t \exp(-sA) z(s)\, ds$, is called weak, if it is possibly defined on another probability space. It is called strong if it is defined on the original probability space. A strong solution is called z-shift solution if $x(t; x_0) = \theta_t x_0$, x_0 z-measurable and θ_t the group of shifts associated with z. The main results are as follows.

Existence: for given A and z there exists a weak stationary solution iff there is a z-shift stationary solution of the original probability space. We give necessary and sufficient criteria (depending on the structure of A) in terms of $Z(t)$. Uniqueness: weak solutions are unique iff all $\operatorname{Re} \lambda_i(A) \neq 0$. Strong solutions are unique if all $\operatorname{Re} \lambda_i(A) \neq 0$, in case some $\lambda_i(A) = 0$ they are never unique. z-shift solutions are unique iff $H(A, z) = 0$, $H(A, z)$ being the set of z-shift solutions of the homogeneous (deterministic!) equation $\dot{x} = Ax$. Sufficient conditions for uniqueness are formulated in terms of ergodic properties of the noise z.

Correlation Function and Measurable Approximations of an Arbitrary Second Order Process

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The oscillation functions of second order processes and of its correlation function are defined, and connections between properties of these functions and of the process are considered. One process is called ε-approximation of another process if they are equal to each other (with probability one) everywhere except maybe on some set whose Lebesgue measure is not bigger than ε. The problem of finding conditions for the existence of a measurable ε-approximation of an arbitrary second order process is interesting and important in many applications. It is known [2] that a second order process is measurable if and only if its correlation function is measurable and its linear space is separable. In this paper sufficient conditions, in correlation functions terms, for the linear space of a process to be separable are given, as well as sufficient conditions for the existence of a measurable ε-approximation of a given second order process.

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A Stochastic Calculus Model of Continuous Trading: Complete Markets

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A paper by the same authors in the 1981 volume of *Stochastic Processes and Their Applications* (11 (3) (1981) 215–260) presented a general model, based on martingales and stochastic integrals, for the economic problem of investing in a portfolio of securities. In particular, and using the terminology developed therein, that paper stated that every integrable contingent claim is attainable (i.e., the model is complete) if and only if every martingale can be represented as a stochastic integral with respect to the discounted price process. This paper provides a detailed proof of that result as well as the following: The model is complete if and only if there exists a unique martingale measure.

Direct and Inverse Problems for Small Random Perturbations of Dynamical Systems

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Let S^t be *a* continuous in t group of homeomorphisms (the flow) of a complete metric locally compact space X with t being a continuous parameter $-\infty < t < \infty$

or a discrete one $t=\ldots,-2,-1,0,1,2,\ldots$. Consider a family of time-homogeneous Markov processes x_t^ε in X with continuous trajectories, where ε is a positive parameter. Let P_t^ε be the transition operator of x_t^ε and suppose that all x_t^ε satisfy the Feller property. The family x_t^ε is called a small random perturbation of the dynamical system S^t if for any continuous f and all $t>0$, $\sup_{x\in X}|P_t^\varepsilon f(x)-f(S^t x)|\to 0$ as $\varepsilon\to 0$. We distinguish two general problems about small random perturbations of dynamical systems. The first one is the direct problem, i.e., the study of asymptotic behaviour as $\varepsilon\to 0$ of parameters of the process x_t^ε provided some information about the dynamical system S^t and the type of perturbations are known. The second one is the inverse problem, i.e. the study of the dynamical system S^t when the asymptotic behaviour as $\varepsilon\to 0$ of some probabilistic parameters of the processes x_t^ε is known. Most papers on this matter deal with the direct problem. In this paper we consider the examples of the inverse problem which has applications to the elliptic singular perturbation problem in the theory of partial differential equations.

2.23. Miscellaneous Topics

Birth, Immigration and Catastrophe Processes

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J.M. Gani, *University of Kentucky*

There has recently been a great deal of interest in stochastic models for the growth of populations which are subject to catastrophe due to either deaths or large-scale emigrations [1, 2, 3]. Emphasis has been placed on semistochastic models (i.e. models with deterministic growth of population size interrupted by downward jumps occurring at random times). These semistochastic models do, however, exhibit some curious features; for example the expected time to extinction for the model of [3] approaches infinity as the initial population size goes to zero. In this paper we analyze linear birth–immigration–catastrophe processes, analogous to the standard linear birth–immigration–death process but with sample-paths whose downward jumps are specified by a catastrophe size distribution. Several specific forms for the catastrophe size distribution are examined (binomial, uniform and truncated geometric) for which it is possible to derive both the stationary distribution and the Laplace transform of the time for the population size to reach zero.

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Gaussian Measures on Spaces of Measurable Functions

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Gaussian measures on some non-Banach spaces of measurable functions are investigated. The main result is that every Gaussian measure on a separable Orlicz space L_Φ is an extension of the canonical cylindrical Gaussian distribution on the RKHS.

As an application we obtain some results concerning the support of Gaussian measures on L_Φ as well as the law of iterated logarithms for sequences of i.i.d. Gaussian distributed random elements with values in L_Φ.

Spaces of Upper Semicontinuous Functions and their Use in Probability

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Let E be locally compact with countable base. We consider functions $f: E \to \bar{R} := R \cup \{-\infty, \infty\}$ and set $f^\vee(A) := \bigvee_{t\in A} f(t)$ for $A \subset E$. A function f is called upper semicontinuous (usc) if $f(t) = \bigwedge\{f^\vee(G);\text{ open } G \ni t\}$ for all $t \in E$. Let US(E) be the set of all usc f. Endowed with the so called hypergraph topology can be characterized by $f_n \to f$ in US(E)$\Leftrightarrow \liminf{}^\vee f_n(G) \geqslant f^\vee(G)$ for open $G \subset E \Leftrightarrow \limsup f_n^\vee(K) \leqslant f^\vee(K)$ for compact $K \subset E$. The following considerations exhibit the usefulness of US(E) in probability theory. (1) Let $\mathscr{I}$ be the collection of bounded open intervals in R. Call the family of $\bar{R}$-valued rv's $(M(I))_{I\in\mathscr{I}}$ an extremal process if $M(\bigcup_{k=1}^\infty I_k) = \bigvee_{k=1}^\infty M(I_k)$ with probability 1 whenever $\bigcup_{k=1}^\infty I_k \in \mathscr{I}$. Then there is a US($R$)-valued rv X such that $(M(I))_{I\in\mathscr{I}}$ and $(X^\vee(I))_{I\in\mathscr{I}}$ have the same finite-dimensional distributions. (2) Call $[f, g]$ a semicontinuous pair if $-f, g \in$ US(E), $f \leqslant g$, and let SC(E) be the set of all such $[f, g]$. Then SC(E) is closed in $(-US(E)) \times US(E)$, so compact and metrizable. The set $C(E)$ of continuous R-valued functions on E becomes a subset of SC(E) by identifying $h \in C(E)$ with $[h, h] \in$ SC(E). The relative topology is the usual one of local uniform convergence. So SC(E) is a metrizable compactification of $C(E)$. This is the context for a new proof of Donsker's theorem without verification of tightness. (3) The space $\mathscr{F}(E)$ of closed subsets of E can be identified with the closed subspace of US(E) consisting of indicator functions. Therefore, random usc functions generalize random closed sets.

2.24. Inference, Part III

How to Measure Complexity in Statistical Decision Making

Jan Šindelář, *CSAV, Czechoslovakia*

There are two main directions in the field of complexity. First, the complexity of computation (the space and time complexity is considered). Second, the complexity in Kolmogoroff, or similarly, sense (the length of the 'shortest' method solving the problem is considered). The complexity is then characterized by a pair of naturals or a single natural, respectively.

The above mentioned methods were applied to measuring the complexity of statistical decision making based on Bayesian decision functions. But the obtained expressions were so complicated that their dependence on the parameters of the decision problem could not be followed.

The author is proposing new methods of measuring complexity characterizing it by a real vector whose dimension is given by the problem. Each component of this 'complexity' vector is a simple function of the parameters of the problem. Thus the dependence of complexity on parameters of the problem can be studied.

One of these methods was successfully applied to measuring the decrease of complexity of decision making when statistically weakly dependent observations are considered independent.

Maximum Likelihood Estimation for a Diffusion Process

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Let $X(t)$ be defined by the diffusion equation

$$\mathrm{d}X(t) = \theta X(t)\,\mathrm{d}t + \mathrm{d}W(t), \quad 0 \leqslant t \leqslant T, \quad X(0) = 0 \tag{1}$$

where θ is an unknown parameter and $W(t)$ is the standard Wiener process. The likelihood ratio for testing the hypothesis H_0: $\theta = 0$, $\mathrm{d}X(t) = \mathrm{d}W(t)$ against the alternative H_1: $\theta \neq 0$, $\mathrm{d}X(t) = X(t) + \mathrm{d}W(t)$ is, $L(\theta) = \exp(\theta \int X(t)\,\mathrm{d}X(t) - (\theta^2/2 \int X^2(t)\,\mathrm{d}t)$. The maximum likelihood estimate for θ is also the least squares estimate $\hat{\theta} = \int X(t)\,\mathrm{d}X^*(t) / \int X^2(t)\,\mathrm{d}t$. The moments of $\hat{\theta} - \theta$ are obtained from the moment generating function

$$M(u, v) = E\left(\exp\left(u\left(\int X\,\mathrm{d}X - \theta \int X^2\,\mathrm{d}t\right) - v \int X^2\,\mathrm{d}t\right)\right)$$

as $E(\hat{\theta} - \theta)^k = \int (\partial^k M(0, v)/\partial^k u) v^{k-1}\,\mathrm{d}v/(k-1)!$. This extends results of Shenton and Johnson [3], who obtained these moments for $\theta = 0$. The discrete analogue of (1) is obtained by partitioning the interval $[0, T]$ into N equal parts of length $d = T/N$; let $Y_i = X(id)$, then Y_i satisfies the auto-regressive equation $Y_i = \alpha Y_{i-1} + u_i$ where $\alpha = \exp(d\theta)$ and the u's and $\mathrm{NID}(0, \sigma^2)$. The relationship between the estimate $\hat{\alpha} = (\sum y_i y_{i-1}) / \sum y_{i-1}^2$ and $\hat{\theta}$ is examined and various limiting procedures are considered. These limits are essentially those obtained earlier by White [4] and tend to shed some light on some comments of Feigin [2].

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2.25. Branching Processes, Part II

Recurrence Formulas in a Simple Branching Process and the Maximum Likelihood Estimation of the Age

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Pursuing some recent work by Hwang and Hwang [2] and Adès, Dion and Labelle [1], this paper is concerned with the estimation of the age of a Galton–Watson process $\{X_n; n \geqslant 0, X_0 = 1\}$. When no closed form expression is known for $g_n(s)$, then the nth iterate of the offspring probability generating function $g(s)$ is given; when $(1+c+ds)g'(s) = a + bg(s)$, where a, b, c, d are constants, a recurrence formula for $P(X_n = K)$, $K = 1, 2, \ldots$, which can be computed recursively by using only $P(X_n = 0) = s_n = g(s_{n-1})$, $n = 1, 2, \ldots$ and $s_0 = 0$, is given. This covers the most important discrete distributions. A special case of particular interest, the negative binomial offspring distribution, is considered, and the maximum likelihood estimate of the age of the Galton–Watson process is obtained numerically and a comparison with Stigler's estimator [3] is made.

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The Convergence of the Age Distribution for Supercritical Bellman–Harris Processes and Corollaries

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Consider a Bellman–Harris process with lifelength distribution $G(\cdot)$, and offspring distribution $\{p_j\}$. Under gradually weakening of conditions on $G(\cdot)$ and/or $\{p_j\}$ (see [1–4]), it was shown that the empiric age distribution converged almost surely on the set of non-extinction of the process. Recently it has been shown that this convergence result holds under the sole assumption that $1 < \sum jp_j < \infty$ (see [5]). This result, along with some interesting corollaries, will be presented.

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