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Nonlinear throughflow effects on thermally modulated porous medium

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Abstract Effect of vertical throughflow on Darcy convection has been investigated subject to time-periodic temperature modulation of the boundaries. The amplitudes of temperature modulation at the lower and upper surfaces are considered to be very small, and the disturbances are expanded in terms of power series of amplitude of convection. A weak nonlinear stability analysis has been performed for the stationary mode of convection, and heat transport in terms of the Nusselt number, which is governed by the non-autonomous Ginzburg–Landau equation, is calculated. The effect of vertical throughflow is found to be either to destabilize or stabilize the system for downward or upward throughflows in the case of impermeable boundary conditions. The effect of amplitude and frequency of modulation, Prandtl–Darcy number on heat transport has been analyzed and depicted graphically. Further, the study establishes that the heat transport can be controlled effectively by a mechanism that is external to the system.

1. Introduction

The buoyancy driven convection in fluid saturated porous media is of fundamental interest due to its practical applications such as geothermal energy utilization, enhanced recovery of petroleum reservoirs, insulation of reactor vessels, polymer engineering, ceramic processing and nuclear waste repositories, to mention a few. The enormous volume of work devoted to this field is well documented in the literature, Ingham and Pop [1], Nield and Bejan [2], Vafai [3]. Because of these applications, together with the fact that porous media occur in many natural situations, several studies have been undertaken to analyze the effects of different phenomena connected with such media. An excellent review of most of these studies has been reported in Nield and Bejan [4]. In the aforementioned applications, control of convective instability plays an important role. One of the effective mechanisms that control convective instability is that of maintaining a nonlinear temperature gradient. Recently, considering various convective flow models in porous medium [5–7], fluid layer [8–10] the phenomenon of heat or mass transfer investigated, where the concept of regulating either heat or mass

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transfer is missing. The temperature gradient can be achieved by time-dependent heating or cooling at the boundaries, the related problems have been investigated by Nield [11], Chhuon and Caltagirone [12], Rudraiah et al. [13], Rudraiah and Malashetty [14], Caltagirone [15], Bhatia and Bhadauria [16,17], Bhadauria [18–24], Bhadauria and Suthar [25], Bhadauria and Srivastava [26], Bhadauria et al. [27], Bhadauria and Kiran [28,29] and Kiran and Bhadauria [30].

However, several geophysical and technological applications involve non-isothermal flow of fluids through porous media, called throughflow (i.e., there is flow across the porous medium and the basic flows not quiescent). Such a flow alters the basic temperature profile from linear to nonlinear with layer height, which in turn affects the stability of the system significantly. The effect of throughflow on the onset of convection in a horizontal porous layer has been studied by Wooding [31], Sutton [32], Homsy and Sherwood [33], Jones and Persichetti [34]. Nield [35] and Shivakumara [36] showed that a small amount of throughflow can have a destabilizing effect, if the boundaries are of different types. Khalili and Shivakumara [37] have investigated the effect of throughflow and internal heat generation on the onset of convection in a porous medium. They have shown that throughflow destabilizes the system even if the boundaries are of the same type; a result which is not true in the absence of an internal heat source. The non-Darcian effects on convective instability in a porous medium with throughflow have been investigated in order to account for inertia and boundary effects by Shivakumara [38]. Shivakumara and Nanjundappa [39] investigated analytically, the effects of quadratic drag and vertical throughflow on double diffusive convection in a horizontal porous medium using the Forchheimer extended Darcy equation. It is found that, irrespective of the nature of boundaries, a small amount of throughflow in either of its direction destabilizes the system; a result which is in contrast to the single component system. Shivakumara and Sureshkumar [40] have studied convective instability in non-newtonian fluid saturated porous medium in the presence of vertical throughflow and found that throughflow has stabilizing or destabilizing effect depending on the boundaries and the directions of the flow. Brevdo [41] investigated three-dimensional absolute and convective instabilities at the onset of convection in a porous medium with inclined temperature gradient and vertical throughflow. Barletta et al. [42] analyzed the convective roll instabilities of vertical throughflow with viscous dissipation in a horizontal porous medium. The effects of hydrodynamic and thermal heterogeneity, horizontal throughflow on the onset of convection in a horizontal layer of a saturated porous have been investigated by Nield and Kuznetsov [43]. They found that the horizontal throughflow has no effect on the stability. When the permeability increases in the direction of the throughflow a small amount of throughflow may destabilize the transverse modes and so destabilize the layer as a whole. Reza and Gupta [44] investigated the effect of throughflow on the onset of convection in a horizontal layer of electrically conducting fluid, confined between two rigid permeable boundaries, and heated from below in the presence of a uniform vertical magnetic field. They found that magnetic field inhabits the onset of steady convection, and a positive throughflow is more stabilizing than negative throughflow. Patil and Rees [45], investigated the effects of local thermal nonequilibrium on the linear stability of the thermal boundary layer formed by a constant downward throughflow. They found that the basic temperature field is altered from the pure exponential form which arises when the phases are in LTE. They also found that, small values of either inter-phase heat transfer coefficient or the porosity-modified conductivity ratio cause the boundary layer to split into two distinct regions, an inner region, which arises because of the effect of the intrinsic suction velocity, and an outer region, which is due to the poor transfer of heat between the phases. Recently Nield and Kuznetsov [46], considering iso-flux and iso-temperature boundaries, investigated the effect on onset of convection in a layered porous medium with vertical throughflow and found that throughflow has a stabilizing effect whose magnitude may be increased or decreased by the heterogeneity.

From the above paragraph, it is observed that a huge amount of analysis on throughflow has been discussed on the onset of convection for various flow models. However, not much work has been done on throughflow considering nonlinear theory, which is essential to analyze the effect of heat transfer on the system. Further, to the best of authors' knowledge, not even a single study which considers linear/nonlinear thermal instability on throughflow under modulation is available in the literature. Therefore, in this paper, we intend to study, the effect of constant throughflow on Darcy convection, subjected to temperature modulation of the boundaries, by making a weak nonlinear stability analysis. The heat transport across the porous medium is quantified in terms of the Nusselt number, obtained by solving the non-autonomous Ginzburg–Landau equation.

2. Mathematical formulation

We consider an infinitely extended horizontal porous medium saturated by Newtonian fluid, confined between two free–free boundaries at $z = 0$ and $z = d$, and heated from below. The temperature of the boundaries varies periodically in a time-dependent manner. The temperature difference across the porous medium is kept at ΔT. We choose Cartesian frame of reference as, origin in the lower boundary and the z-axis in vertically upward direction. The schematic diagram is shown in Fig. 1, given below. It is assumed that the mechanical properties and thermal properties in x and y-directions are same. Further, Darcy law and the Oberbeck–Boussinesq approximation are considered. Under these assumptions, the equations which describe the system are given by the following:

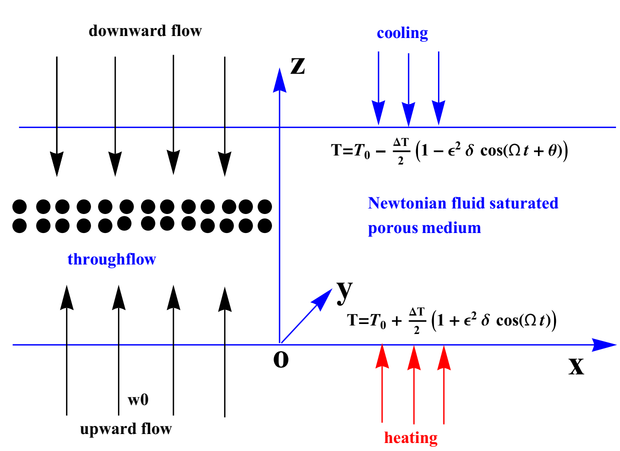

Figure 1 A sketch of the physical problem.

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{q}}{\partial t} = -\nabla p + \rho \vec{g} - \frac{\mu}{K}\vec{q}, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa_T \nabla^2 T, \tag{3}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0)], \tag{4}$$

where $\vec{q}$ is velocity (u, v, w), μ is a viscosity, K is permeability, κ_T is the thermal diffusivity, T is temperature, β_T is thermal expansion coefficient, γ is the ratio of heat capacities. For simplicity γ is taken to be unity in this paper. ρ is density, $\vec{g} = (0, 0, -g)$ is gravitational acceleration, T_0 is the temperature at which $\rho = \rho_0$ is reference density. The externally imposed thermal boundary conditions considered in this paper are as follows:

$$\begin{aligned} T &= T_0 + \frac{\Delta T}{2}[1 + \epsilon^2\delta \cos(\omega t)] \quad \text{at } z = 0 \\ &= T_0 - \frac{\Delta T}{2}[1 - \epsilon^2\delta \cos(\omega t + \theta)] \quad \text{at } z = d, \end{aligned} \tag{5}$$

where ϵ is the perturbation parameter, δ represents the amplitude of temperature modulation, ω is the modulation frequency and θ is the phase difference. The basic state is assumed to be quiescent and the quantities in this state are given by the following:

$$q_b = (0, 0, w_0), \quad \rho = \rho_b(z, t), \; p = p_b(z, t), \; T = T_b(z, t), \tag{6}$$

$$\frac{\partial p_b}{\partial z} = \frac{\mu}{K} w_0 - \rho_b g, \tag{7}$$

$$\frac{\partial T_b}{\partial t} + w_0 \frac{\partial T_b}{\partial z} = \kappa_T \frac{\partial^2 T_b}{\partial z^2}, \tag{8}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0)]. \tag{9}$$

The solution of Eq. (8) subject to the thermal boundary conditions Eq. (5), is given by the following:

$$T_b(z, t) = f(z) + \epsilon^2 \delta Re[f_1(z, t)]. \tag{10}$$

Here $f(z)$ is the steady part, while $f_1(z, t)$ is the oscillatory part of the basic temperature field, and will be defined later. The finite amplitude perturbations on the basic state are superposed in the form,

$$\vec{q} = \vec{q}_b + \vec{q}', \quad \rho = \rho_b + \rho', \; p = p_b + p', \; T = T_b + T'. \tag{11}$$

Since, we are considering only two dimensional flow model, therefore, introduce the stream function, ψ as $u' = \frac{\partial \psi}{\partial z}$, $w' = -\frac{\partial \psi}{\partial x}$. Using Eq. (11) in Eqs. (1)–(4), eliminating the pressure term, non-dimensionalizing the physical variables by $(x, y, z) = d(x^*, y^*, z^*)$, $t = \frac{d^2}{\kappa_T} t^*$, $\psi = \kappa_T \psi^*$, $T' = \Delta T\, T^*$, we obtain the following equations (after dropping the asterisk)

$$\frac{1}{Pr_D}\frac{\partial}{\partial t}(\nabla^2 \psi) = -\nabla^2 \psi - Ra\frac{\partial T}{\partial x}, \tag{12}$$

$$-\frac{\partial T_b}{\partial z}\frac{\partial \psi}{\partial x} - \left(\nabla^2 - Pe\frac{\partial}{\partial z}\right)T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi, T)}{\partial(x, z)}. \tag{13}$$

The non-dimensionalizing parameters in the above equations are as follows : $Pe = \frac{w_0 d^2}{\kappa_T}$ is Péclet number, $Pr_D = \frac{\phi \nu d^2}{K \kappa_T}$ is Prandtl–Darcy number, $Ra = \frac{\beta_T g \Delta T d K}{\nu \kappa_T}$ is thermal Rayleigh number. Eq. (13) shows that the basic state solution influences the stability problem through the factor $\frac{\partial T_b}{\partial z}$, which is given by the following:

$$\frac{\partial T_b}{\partial z} = f'(z) + \epsilon^2 \delta Re[f_1'(z, t)], \tag{14}$$

where $f' = \frac{Pe e^{Pe z}}{1 - e^{Pe}}$, $f_1'(z, t) = [B(\theta_2)e^{\theta_2 z} + B(-\theta_2)e^{-\theta_2 z}]e^{-i\omega t}$, $B(\theta_2) = \frac{\theta_1 + \theta_2}{2}\frac{(e^{-i\phi} - e^{\theta_1 - \theta_2})}{e^{\theta_1}(e^{\theta_2} - e^{-\theta_2})}$, $\theta_1 = \frac{Pe}{2}$, $\theta_2 = \frac{\sqrt{Pe^2 + 4\lambda^2}}{2}$ and $\lambda = (1 - i)\sqrt{\frac{\omega}{2}}$. Assuming small variation of time, and re-scaling it as $\tau = \epsilon^2 t$, we study the stationary mode of convection of the system. We write the nonlinear system of Eqs. (12) and (13) in the matrix form as given below:

$$\begin{bmatrix} \nabla^2 & Ra\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\left(\nabla^2 - Pe\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi \\ T \end{bmatrix} = \begin{bmatrix} -\frac{\epsilon^2}{Pr_D}\frac{\partial}{\partial \tau}(\nabla^2 \psi) \\ -\epsilon^2 \frac{\partial T}{\partial \tau} + \frac{\partial(\psi, T)}{\partial(x, z)} + \epsilon^2 \delta f_1(z, \tau)\frac{\partial \psi}{\partial x} \end{bmatrix}. \tag{15}$$

To solve Eq. (15), we consider impermeable and isotherm boundary conditions as given below:

$$\begin{aligned} &\psi = 0 \text{ and } T = 0 \text{ at } z = 0, \\ &\psi = 0 \text{ and } T = 0 \text{ at } z = 1. \end{aligned} \tag{16}$$

3. Amplitude equation for stationary instability

We introduce the following asymptotic expansions in Eq. (15):

$$\begin{aligned} Ra &= R_0 + \epsilon^2 R_2 + \epsilon^4 R_4 + \ldots, \\ \psi &= \epsilon \psi_1 + \epsilon^2 \psi_2 + \epsilon^3 \psi_3 + \ldots, \\ T &= \epsilon T_1 + \epsilon^2 T_2 + \epsilon^3 T_3 + \ldots, \end{aligned} \tag{17}$$

where R_0 is the critical Rayleigh number at which the onset of convection takes place in the absence of temperature modulation. Now, we solve the system for different orders of ϵ.

At the lowest order, we have

$$\begin{bmatrix} \nabla^2 & R_0\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\left(\nabla^2 - Pe\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{18}$$

The solution of the lowest order system subjected to the boundary conditions, Eq. (16) is as follows:

$$\psi_1 = \mathbb{A}(\tau)\sin(k_c x)\sin(\pi z), \tag{19}$$

$$T_1 = -\frac{4k_c \pi^2}{\delta^2(4\pi^2 + Pe^2)}\mathbb{A}(\tau)\cos(k_c x)\sin(\pi z), \tag{20}$$

where $\delta^2 = k_c^2 + \pi^2$.

The critical Rayleigh number and the corresponding wave number for the onset of stationary convection are calculated, the expressions for Rayleigh number and wave number are given by

$$R_0 = \frac{\delta^4(4\pi^2 + Pe^2)}{4\pi^2 k_c^2}, \tag{21}$$

$$k_c = \pi. \tag{22}$$

At the second order, we have:

$$\begin{bmatrix} \nabla^2 & R_0\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\left(\nabla^2 - Pe\frac{\partial}{\partial z}\right) \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \end{bmatrix}, \tag{23}$$

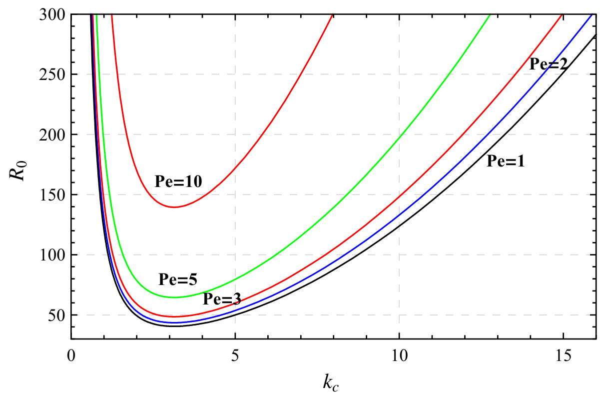

Figure 2 Effect of Péclet number on critical Rayleigh number R_0.

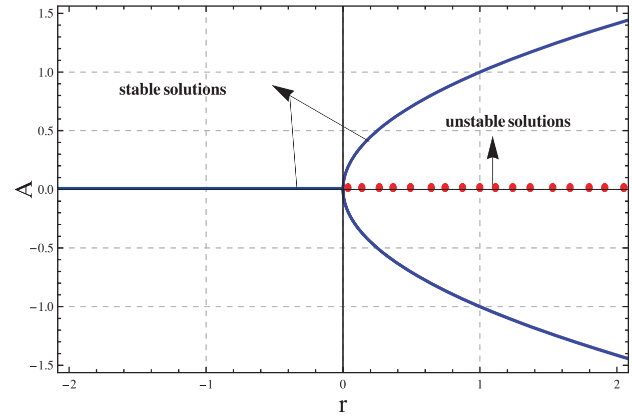

Figure 3 Supercritical pitch fork bifurcation diagram (OPM).

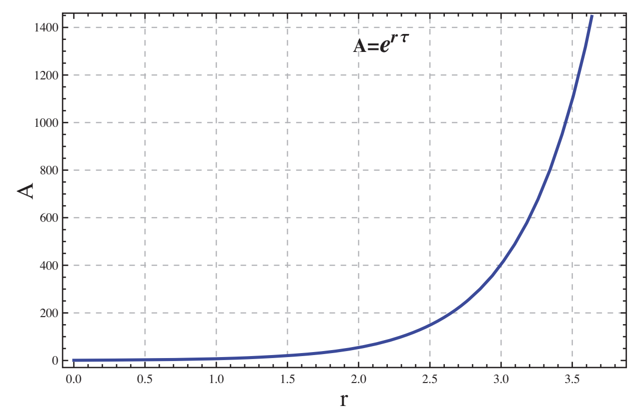

Figure 4 Supercritical pitch fork bifurcation diagram (LBMO).

where

$$R_{21} = 0, \tag{24}$$

$$R_{22} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial T_1}{\partial x}. \tag{25}$$

The second order solutions subjected to the boundary conditions, Eq. (16) is obtained as follows:

$$\psi_2 = 0, \tag{26}$$

$$T_2 = -\frac{2k_c^2\pi^3}{\delta^2(4\pi^2 + Pe^2)^2}\mathbb{A}^2(\tau)\sin(2\pi z) + \frac{Pe k_c^2\pi^2}{\delta^2(4\pi^2 + Pe^2)^2}\mathbb{A}^2(\tau)\cos(2\pi z). \tag{27}$$

The horizontally averaged Nusselt number Nu, for the stationary mode of convection, is given by

$$Nu(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}\left(\frac{\partial T_2}{\partial z}\right)dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}\left(\frac{\partial T_b}{\partial z}\right)dx\right]_{z=0}} = 1 + \frac{4\pi^4 k_c^2(e^{Pe} - 1)}{\delta^2 Pe(4\pi^2 + Pe^2)^2}\mathbb{A}^2(\tau). \tag{28}$$

The above results, Eqs. (21) and (22) are obtained by Lapwood [47], Siddheshwar et al. [48,49], Bhadauria et al. [28] for $Pe = 0$ and isotropic porous medium.

At the third order, we have

$$\begin{bmatrix} \nabla^2 & R_0\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\left(\nabla^2 - Pe\frac{\partial}{\partial z}\right)\end{bmatrix}\begin{bmatrix}\psi_3 \\ T_3\end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32}\end{bmatrix}, \tag{29}$$

where

$$R_{31} = -\frac{1}{Pr_D}\frac{\partial}{\partial \tau}(\nabla^2\psi_1) - R_0\frac{\partial T_2}{\partial x} - R_2\frac{\partial T_1}{\partial x}, \tag{30}$$

$$R_{32} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_2}{\partial z} + \delta f_1(z,\tau)\frac{\partial \psi_1}{\partial x} - \frac{\partial T_1}{\partial \tau}. \tag{31}$$

Substituting ψ_1, T_1 and T_2 into Eqs. (30) and (31), we can easily obtain the expressions for R_{31} and R_{32}. Now, applying the solvability condition for the existence of third order solution, we get the Ginzburg–Landau equation for the stationary mode of convection, with time-periodic coefficients, in the form

$$A_1\mathbb{A}'(\tau) - A_2\mathbb{A}(\tau) + A_3\mathbb{A}(\tau)^3 = 0, \tag{32}$$

where $A_1 = \frac{\delta^2}{Pr_D} + \frac{4R_0\pi^2k_c^2}{\delta^4(4\pi^2+Pe^2)}$, $A_2 = \frac{4R_2\pi^2k_c^2}{\delta^4(4\pi^2+Pe^2)} - \frac{2R_0k_c^2}{\delta^2}\delta I_1$, $A_3 = \frac{2R_0\pi^4k_c^4}{\delta^4(4\pi^2+Pe^2)^2}$ and

$$I_1 = \int_0^1 f_1(z,\tau)\sin^2(\pi z)\,dz.$$

The Ginzburg–Landau equation given by Eq. (32) is a Bernoulli equation and obtaining its analytical solution is difficult due to its non-autonomous nature. So, it has been solved numerically using the in-built function NDSolve of Mathematic 8 subjected to the initial condition $\mathbb{A}(0) = a_0$, where a_0 is the chosen initial amplitude of convection. In our calculations we use $R_2 = R_0$, to keep the parameters to minimum. For unmodulated case, the analytical solution of the above Eq. (32) takes the form:

$$\mathbb{A}(\tau) = \frac{1}{\sqrt{\left(\frac{A_3}{A_2} + C_1\mathrm{Exp}\left[-\frac{2A_2}{A_1}\tau\right]\right)}}, \tag{33}$$

where A_1, A_3 are same as in Eq. (32), $A_2 = \frac{4R_2\pi^2k_c^2}{\delta^4(4\pi^2+Pe^2)}$ and C_1, which appears in Eq. (33), is an integration constant, can be found by using suitable initial condition. We have calculated the mean value of Nusselt number $(\overline{Nu})$ for better understanding the effect of temperature modulation on heat transport, a representative time interval that allows a clear comprehension of the modulation is chosen. The interval $(0, 2\pi)$ seemed an appropriate interval to calculate $(\overline{Nu})$. The time-averaged Nusselt number $(\overline{Nu})$ is defined as

$$\overline{Nu} = \frac{1}{2\pi}\int_0^{2\pi} Nu\, d\tau. \quad (34)$$

Since the amplitude $\mathbb{A}(s)$ is obtained numerically and hence $(\overline{Nu})$ is also obtained numerically. The factor I_1, determines whether the modulation amplifies or diminishes the amplitude of convection. A discussion of the results now follows culminating in a listing of conclusions.

4. Results and discussion

In this paper, we study the combined effect of temperature modulation and vertical throughflow on Bénard–Darcy convection in a porous medium. A weakly nonlinear stability analysis has been performed to investigate the effect of temperature modulation and vertical throughflow on heat transport. The effect of temperature modulation on Bénard–Darcy system has been assumed to be of second order $O(\epsilon^2)$. This means we consider only small amplitude temperature modulation. Such an assumption helps us in obtaining the amplitude equation of convection in simple and elegant manner, and is much easier to obtain than in the case of the Lorenz model. The purpose of weak nonlinear theory is to study heat transfer, which linear study could not support. External regulation of convection is important in the study of convection in porous media. The objective of this article was to consider such as candidates, temperature modulation and vertical throughflow for either enhancing or inhabiting convective heat transfer as is required by a real application. The temperature modulation has been considered in the following three cases:

1. In-phase modulation (IPM) $(\theta = 0)$.
2. Out-phase modulation (OPM) $(\theta = \pi)$.
3. Only Lower boundary modulated (LBMO) $(\theta = -i\infty)$, which means that the modulation effect will not be considered in upper boundary but only in lower boundary.

Since the porous medium is assumed to be closely packed, the Darcy model is considered in governing equation. Vadasz [50], pointed that there are some modern porous medium applications, such as mushy layer in solidification of binary alloys and fractured porous medium, where the value of Pr_D may be considered to be unity order, therefore the time-derivative term in the present study has been retained. Further, this is the reason that the values of Pr_D have been kept around one in our calculations. The values of δ are consider very small, between 0 and 0.1, since we are studying the effect of small amplitude modulation on the heat transport. Also, since the effect of modulation on the onset of convection as well as on the heat transport is maximum at low frequencies, therefore the modulation of temperature is assumed to be of low frequency. A small amount of throughflow in a particular direction can either destabilize or stabilize the system, therefore, the values of Pe are taken around one.

It can be noticed that the critical Rayleigh–Darcy number is an even function of Pe and as Pe increases, R_0 also increases, thus onset of convection is delayed due to throughflow as shown in Fig. 2. The reason for this according to Reza and Gupta [44] is that as we increase throughflow velocity, a temperature boundary layer forms at the one of the plates, this decreases the effective thickness of the stratified layer of fluid, while the temperature difference across the layer remains constant, thus R_0 would increase with Pe. However, due to nonlinear effects we obtain the results opposite in heat transfer. Using linear stability analysis, Reza and Gupta [44] found that upward flow stabilizes more than downward flow for two rigid plates.

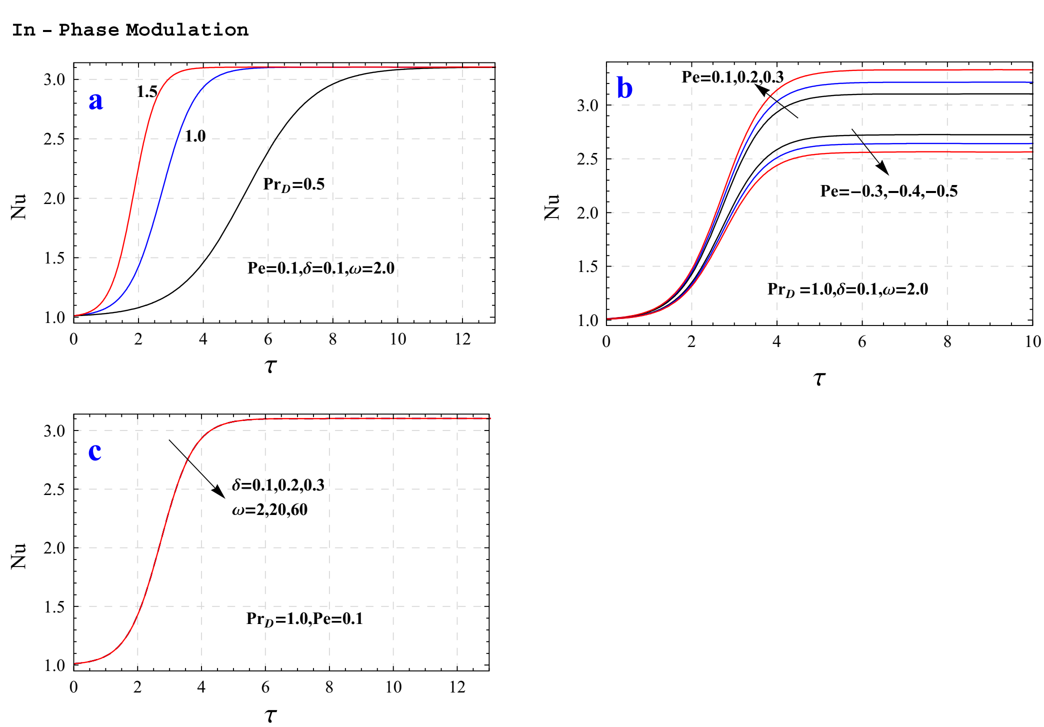

Figure 5 Effect of (a) Pr_D (b) Pe (c) δ, ω on Nu with respect to time τ.

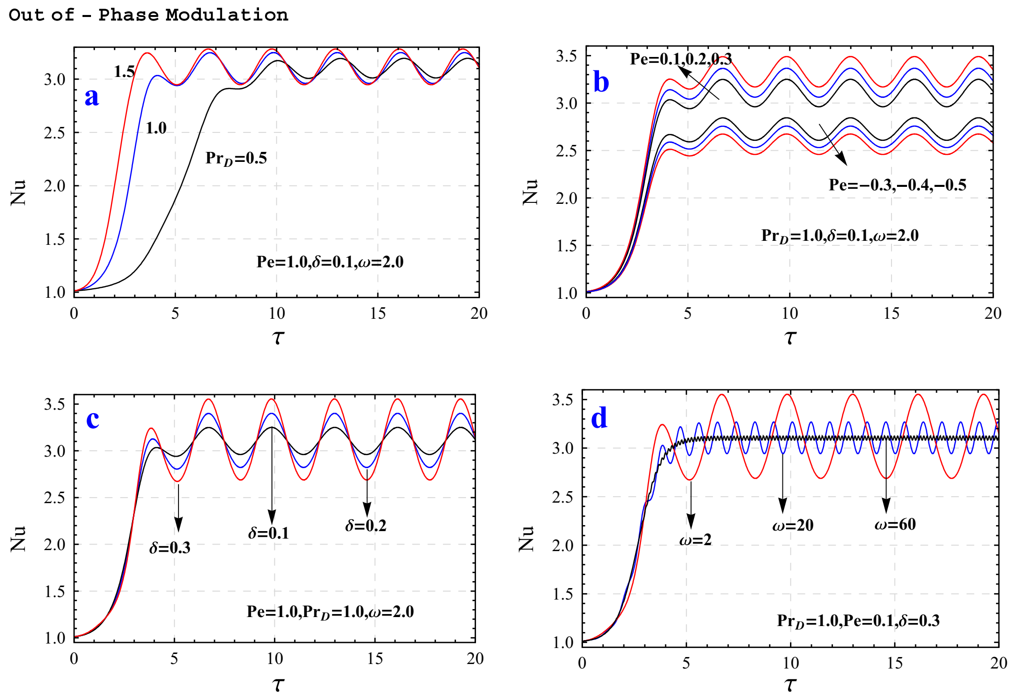

Figure 6 Effect of (a) Pr_D (b) Pe (c) δ (d) ω on Nu with respect to time τ.

One can observe from Eq. (32) that the coefficients $A_1,\ A_2 > 0$ and A_2 is a function of time. The solution gives as $\tau \to -\infty$, $\mathbb{A} \to 0$ is unstable solution, and a new stable solution develops, $r = \pm\sqrt{\frac{A_2}{A_3}}$ as $\tau \to \infty$, whatever be the value of $\mathbb{A}_0$. This is known a supercritical pitch fork bifurcation. For the range of τ as $(-\infty, \infty)$ the coefficient A_2 takes +values, the origin has become unstable. Two new stable fixed points appear on either side of the origin as shown in Figs. 3 and 4. If we assume that A is very small in Eq. (32), then the equation may be approximated by $A_\tau = rA$ whose solutions are $A = e^{r*\tau}$. Thus, for very small amplitude initial disturbances the flow grows in strength is exponentially. Also R_2 represents the deviation of Ra away from critical Rayleigh number, these disturbances eventually attained by nonlinear effects given in Eq. (33).

The numerical results for Nu, obtained from the expression in Eq. (28) by solving the amplitude Eq. (32) are presented in the (Figs. 5–8). It is clear to see the expression in Eq. (28) in conjunction with Eq. (32) that $Nu(\tau)$ is a function of system parameters. The effect of each type of modulation on heat transport is shown in (Figs. 5–8), wherein the plots of Nusselt number Nu versus τ are presented. It is found from the figures that the value of Nu starts with one and remains constant for quite some time, thus showing the conduction state initially. Then the value of Nu increases with time, thus showing the convection state and finally becomes constant on further increasing τ, thus achieving the steady state.

For **IPM,** the results are presented in Fig. 5a–d. From Fig. 5a, we observe that Nu increases with Prandtl–Darcy number, the effect is clearly visible at small values of Pr_D. On further increasing the time the effect of increasing Pr_D on heat transport diminishes. These results are earlier obtained by Bhadauria et al. [27], Bhadauria and Kiran [28,29] without throughflow. The effect of Pe on heat transfer is given in Fig. 5b, for the cases of downward and upward throughflows.

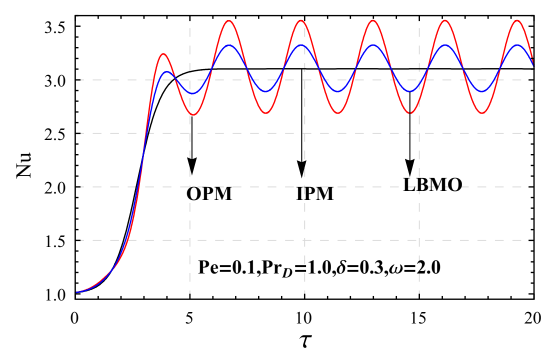

Figure 7 Comparison of three types temperature modulations.

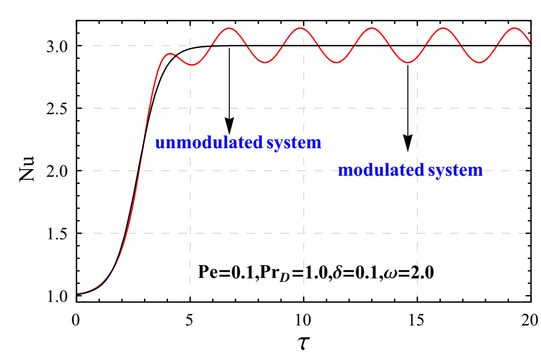

Figure 8 Comparison between modulated and unmodulated cases.

It is found that upward throughflow ($Pe > 0$) has destabilizing effect, whereas downward throughflow ($Pe < 0$) has stabilizing effect. The same results were also obtained by Nield [35] in the case of a fluid layer with small amount of throughflows.

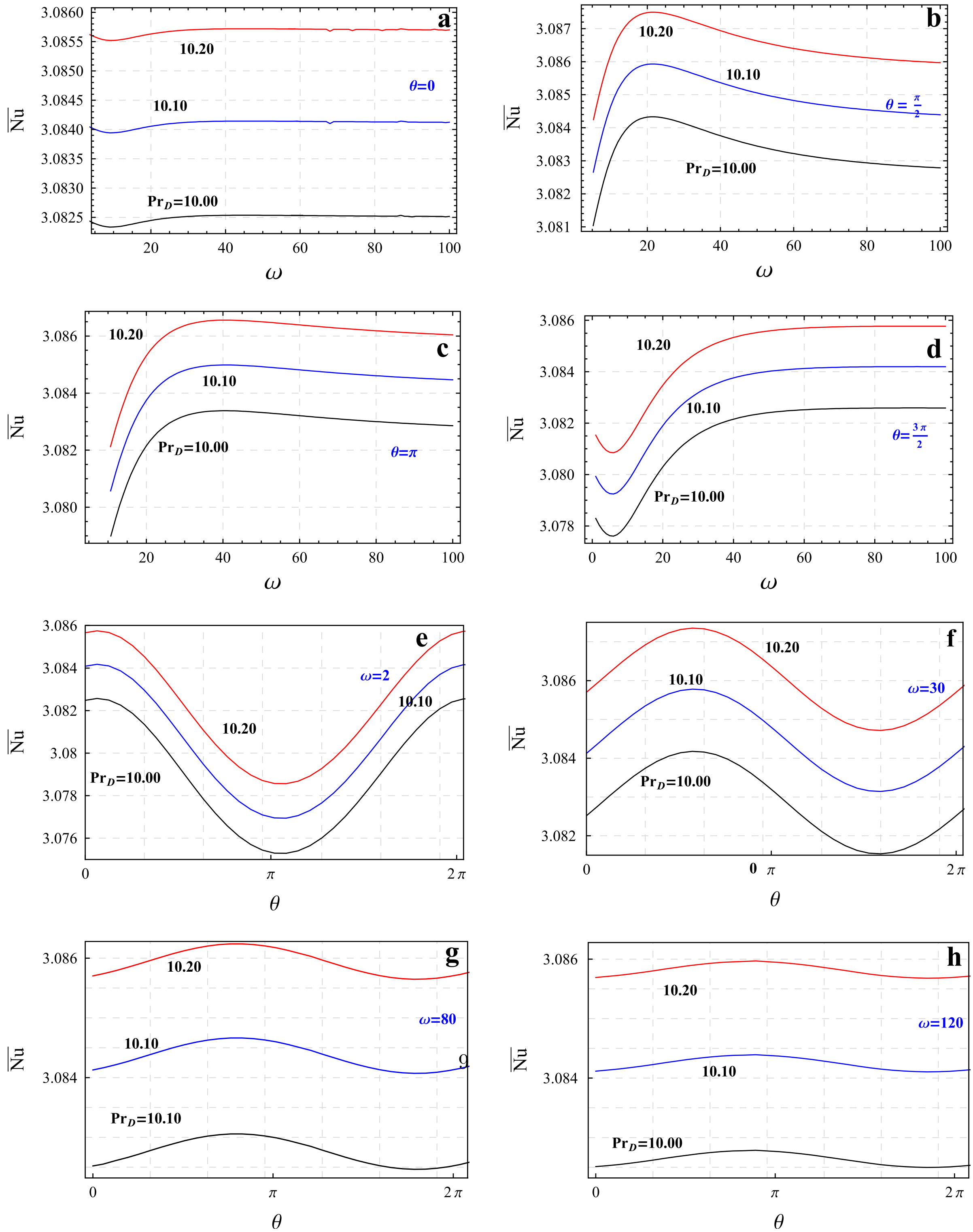

Figure 9 Effect of phase angle (θ) and frequency (ω) of modulation on mean Nusselt number $\overline{Nu}$.

The present results are compatible with the results obtained by Shivakumara and Sureshkumar [40] and Suma et al. [51]. According to Shivakumara and Sureshkumar [40], the destabilization effect may be due to the distortion of steady-state basic temperature distribution from linear to nonlinear by the throughflow. A measure of this is given by the basic state temperature and this can be interpreted as a rate of energy transfer into the disturbance by interaction of the perturbation convective motion with basic temperature gradient. The maximum temperature occurs at a place where the perturbed vertical velocity is high, and this leads to an increase in energy supply for destabilization. Further, the amplitude δ and the frequency modulation ω both have negligible effects on heat transport in this case given in Fig. 5c.

In Fig. 6a–d, we have depicted the variation of Nu with time τ for out of phase modulation. It is found that Nu starts with one, increases with increasing time τ, and then becomes oscillatory. However, on further increasing the time, we observe from Fig. 6a and b that the effects of Pr_D, Pe on heat transport are found to be similar to those of **IPM**. Further, we found in Fig. 6c that the effect of amplitude of modulation is to increase the magnitude of Nu, thus increasing the heat transport and advancing the convection. We note the following in respect of the influence of amplitude on heat transport.

$$Nu_{\delta=0.1} < Nu_{\delta=0.2} < Nu_{\delta=0.3}$$

Also, from Fig. 6d, we observe that an increase in the frequency of modulation decreases the magnitude of Nu, and so the effect of frequency of modulation on heat transport diminishes. At high frequency, the effect of temperature modulation on thermal instability disappears altogether. This result agree with the linear stability results of Venezian [52], where the correction in the critical value of Rayleigh number due to temperature modulation becomes almost zero at high frequencies. The effect of amplitude and frequency of modulation is of similar effect under rotation speed modulation [53] which is originating idea from temperature modulation. The results in the case of lower boundary temperature modulation only were also obtained, but found to be qualitatively very similar to those obtain in **OPM** case, therefore not presented here. Further, the magnitude of Nu in lower boundary temperature modulation only is found to be less than that in the case of **OPM**. In Fig. 7, a comparison of results of in phase modulation, out of phase modulation and when only lower boundary temperature is modulated, is presented. It is found that the magnitude of Nu for **LBMO** is greater than that obtained in case of **IPM**, but less than that of **OPM** as shown below:

$$Nu_{\mathbf{IPM}} < Nu_{\mathbf{LBMO}} < Nu_{\mathbf{OPM}}.$$

In Fig. 8, we presented the unmodulated result of Eq. (33) and compared it with the present results of modulated case. It is found that the unmodulated results are very similar to the results obtained for **IPM**, which also confirms that in-phase modulation does not affect heat transport in the system.

We also have presented our results (according to Siddheshwar et al. [49]) on mean Nusselt number ($\overline{Nu}$), which depends on both the phase difference θ and frequency ω of modulation than only on the choice of the small amplitude modulation. Fig. 9a–d shows the effect of phase angle θ on ($\overline{Nu}$) and Fig. 9e–h shows effect of ω on ($\overline{Nu}$). From the figures it is evident that for a given frequency of modulation there is a certain range of θ in which ($\overline{Nu}$) increases with increasing θ and another range in which ($\overline{Nu}$) decreases. Thus, one can conclude that, the suitable combination or choices of ω and θ can be used to regulate heat transfer depending on the demands on heat transport in an application situation. Heat transfer can be controlled (enhanced or reduced) with the external mechanism of temperature modulation effectively. We also can observe our results in Fig. 9 are the results which are similar to Siddheshwar et al. [49] for the Newtonian fluid case. It is clear that for temperature modulation the boundary temperatures should not be in in-phase modulation (synchronized), where the effect of modulation is negligible on heat transport.

5. Conclusions

We have analyzed the effect of temperature modulation and vertical throughflow on Bénard–Darcy convection by performing a weakly nonlinear stability analysis resulting in the real Ginzburg–Landau amplitude equation. The following conclusions are made:

1. The effect of in-phase modulation is negligible on heat transport, while it is oscillatory in nature for **OPM** and **LBMO**.
2. The effect of δ and ω is also found to be negligible on heat transport when the boundaries temperature is modulated in phase.
3. Effect of Pr_D is to enhance the heat transport for all three types of modulations at lower values of time and same at large values of time.
4. The effect of throughflow (Pe) enhances heat transport for upward direction, diminishes for downward direction of all three types of modulations.
5. The parameters θ and ω show significant effect on $\overline{Nu}$ given in Fig. 9.
6. Supercritical pitch fork bifurcation exits for Eq. (32).

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