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Otto Blumenthal (1876–1944) in retrospect

Paul Butzer[a,*], Lutz Volkmann[b]

[a] *Lehrstuhl A für Mathematik, RWTH Aachen University, 52056 Aachen, Germany*
[b] *Lehrstuhl II für Mathematik, RWTH Aachen University, 52056 Aachen, Germany*

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Abstract

This paper treats in detail the life and work of Otto Blumenthal, one of the most tragic figures of the 188 emigré mathematicians from Germany and the Nazi-occupied continent. Blumenthal, the first doctoral student of David Hilbert, was crucial in the publication and communication system of German mathematics between the two World Wars. There has been an unusual revival of interest in his mathematical work in the last three decades. Thus his work on orthogonal polynomials whose zeros are dense in intervals, called the *Blumenthal theorem* by T.S. Chihara (1972), lead to over two dozen recent papers in the field. The *Blumenthal–Nevai theorem*, with applications to scattering theory in physics, is one example. In modern work on Hilbert modular forms, increasingly being called *Hilbert–Blumenthal modular forms*, many recent papers even contain the word Blumenthal in their titles. This paper contains 212 references.

1. Introduction

This paper provides new biographical information on Otto Blumenthal, a mathematician who was crucial in the publication and communication system of German mathematics between the two World Wars. It discusses his contributions and career, giving new insights into the period of Nazi dictatorship and its consequences, particularly for the life of Blumenthal who, in spite of his early conversion to Protestantism, counted for Jewish according to the Nazi definition. Materials are also assembled to illuminate the paradox of the concentration camp Theresienstadt (Terezin).

During the last three decades, especially during the past decade, there has been an unusual revival of interest in the mathematical work of Otto Blumenthal. For example, Hilbert modular forms are now increasingly being called *Hilbert–Blumenthal modular forms*, recognizing that Blumenthal first placed them in a broad setting to connect three different fields. His work on

* Corresponding author.
E-mail addresses: butzer@rwth-aachen.de (P. Butzer), volkm@math2.rwth-aachen.de (L. Volkmann).

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orthogonal polynomials whose zeros are dense in intervals, first called the *Blumenthal theorem* on orthogonal polynomials by T.S. Chihara in his recollection of it in 1968 led to over two dozen papers by experts on orthogonal polynomials, and a *Blumenthal–Nevai theorem* with applications to scattering theory in physics. Blumenthal's extension of the basic theorem of Stokes–Helmholtz on vector analysis was essentially only improved upon in 1993; it has proved to be of importance in elasticity, fluid mechanics, electrodynamics and density functional theory. The *Picard–Hadamard–Borel–Blumenthal theorem* on entire functions paved the way for R. Nevanlinna's value distribution theory for meromorphic functions. Conjectures and problems of Blumenthal from 1907 on *Blumenthal's maximum curves* of analytic functions have only been recently solved or are still open.

Yet Blumenthal was also one of the most tragic figures of the 188 [1] refugee mathematicians from Germany and the Nazi-occupied continent, who suffered as a consequence of the political convolutions of that era. The paper presents the "case" made for his dismissal from his university position, followed by his removal from several professional journals. It elaborates on the ironies of the "enlightened" internment camp at Theresienstadt, where he remained intellectually active until his death. The contextual issues are documented in some detail since upcoming generations will be increasingly less familiar with the human and scientific implications of Nazi racism and repression. The inherent contradictions between science and totalitarianism are more than "historical" footnotes. They are part of the present and the future, and they are ultimately about values and human integrity.

A series of five papers [2] has documented the mathematicians from Germany and Austria who were forced to leave Germany, or taken to Nazi concentration camps and possibly perished there during 1933–1945. The first three of these articles were written by Maximilian Pinl [3] (1897–1978)

[1] The figure 188 is the number of mathematicians treated in the five papers of M. Pinl (see footnotes 3, 20) plus Peter Thullen (see footnote 11).

[2] This series of papers is said to belong to the first publications in Germany by a professional group dealing with the fate of its members during the Nazi era. According to Furtmüller [158], it is "a kind of pioneer work. ... Similar surveys could be suggested for other branches of science". The papers appeared in the "Jahresbericht der Deutschen Mathematiker-Vereinigung" (=DMV), the official journal of the German Mathematical Society. For later work based in part on these see, e.g. [122,186] also [198,199]. As to the history of the DMV see Gericke [66] and [179]. Research in Germany concerning the general intellectual immigration after 1933, the so-called *Exile-Research*, only began on a larger scale circa 1983; an exception was H. Pross [161]. In the USA, a parallel *immigration history* began in the early seventies. See the various papers in [45]. In 1972 work began on the monumental "Biographisches Handbuch der deutschsprachigen Emigration 1933–1945"; it was completed in 1983, containing circa 9000 short biographies, and was the work of joint American, Jewish and German research, see [175,176].

[3] The author of M. Pinl's obituary notice [112] did not include some important details of Pinl's own hardships in the period circa 1938–1955. Firstly, Pinl obtained his postdoctoral qualification (Habilitation) at the German University of Prague in 1936. After teaching relativity theory—the "new Physics"—as Dozent for a semester, he was taken into custody by the Gestapo for six months because he did not denounce his threatened friends; he was not Jewish. This was a few days after the German occupation of Czechoslovakia in May 1939. Since he was no longer allowed to teach at any German university, he ironically managed to go under cover in the Messerschmidt aircraft industry at Augsburg (1940/43) and then at the Luftfahrtforschungsanstalt (Aviation Research Institute) Braunschweig 1943/45 (see [89,15,80]) under Gustav Doetsch (see Remmert [169], as to Doetsch (1892–1977)). In December 1945 he was re-habilitated at the University of Cologne. Since he saw no career opportunities in the Western Zones of Germany and declined offers of professorships in the Soviet occupied Zone, he finally accepted an offer as Head of the Department of Mathematics at the University of Dacca (East Pakistan, now Bangladesh, see [35]), where he stayed from 1949–54, when his five-year appointment expired. Pinl, now 57 years old, had to return to Germany. Since the Cologne Faculty of Science is alleged to have turned down an appointment exceeding that of just a Diätendozent (lecturer) for Pinl—recall that he was an Anti-Nazi—, the Minister of Education of the State Nordrhein-Westfalen appointed him—against the motion of the whole Faculty—as KW-Professor, i.e., a professorship that expires upon retirement. Ms. Claudia Pinl (Cologne), the daughter of Max Pinl, kindly supplied the authors with information about her father.

[156], and the fourth, dealing with Austrian and Czech mathematicians, was co-authored by Pinl and Auguste Dick [4] (1910–1993) [157]. An English version of this series, carefully abridged and reorganized, and placed in a political context by Lux Furtmüller [158], appeared in the Year Book of the Leo Baeck Institute. [5]

These landmark papers document the scope of what became a brain drain set in motion by the National Socialist government that amounted to a self-amputation of mathematics and science in Germany. [6] Of those mathematicians who emigrated and survived, the greater part found refuge in the USA. [7] Some 28 fled to Britain, [8] and stayed there temporarily or permanently. [9] Many

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4 According to a written communication by Dr. Christa Binder (Vienna), there was no obituary essay for Frau Hofrat Dr. Dick, but her mathematical-historical *Nachlass* is located in the Archive of the Austrian Academy of Science. Articles and material concerning Blumenthal also appeared in the Neue Deutsche Biographie (Vol. 2, Berlin, 1955, p. 332), in E. Milkutat, Blumenthal, in: Historische Kommission bei der Bayerischen Akademie der Wissenschaften, in P. Arnsberg, Geschichte der Frankfurter Juden seit der Französischen Revolution, Band III. Biographisches Lexikon der Juden in den Bereichen: Wissenschaften, Kultur, Bildung, Öffentlichkeitsarbeit in Frankfurt am Main, Eduard Roetker Verlag, Frankfurt am Main, p. 48 f. as well as in the private papers of R. Courant, O. Veblen and Harlow Shapley ([186, p. 301]).

5 The noted Leo Baeck Institute, founded in London, 1954, is named after the rabbinical scholar Leo Baeck (1873–1956), author of “Das Wesen des Judentums” (Nathausen and Lamm, Berlin ^{1}1905, ^{6}1960). He was a leading figure of Judaism in Germany between the two world wars, and internationally after 1945. He was deported to Terezin in 1943 but survived; see A. Friedländer, Leo Baeck, Teacher of Theresienstadt, New York 1968; Hilberg [91, Vol. 2, p. 448].

6 It is estimated that between 1100 and 2500 professors of all categories at German universities and scientific academies were dismissed on racial or political grounds during the period 1933–1940. This is said to correspond to some 15–25% of the total professorial population in Germany. Furtmüller [158, p. 134] gives various sources with very different figures. Between 1932 and 1939 the total number of students attending universities in Germany declined from ca. 100,000 to ca. 40,000. In mathematics, the field with the most dramatic drop, the decline was from 4,245 mathematics students down to 306, just 7.2% of the pre-Nazi figure. This also reflects other policy changes. In this context see Rowe [178], Macrakis [127], also Kalkmann [102, p. 65]. When Hilbert was asked by Minister Rust (see footnote 28) in 1934 whether the mathematical institute at Göttingen suffered by the departure of the Jews and their friends, Hilbert’s reply was: “Jelitten? Dat hat nicht jelitten, Herr Minister. Dat jibt es doch janich mehr!” (Suffered? It hasn’t suffered, Mr Secretary; it simply doesn’t exist anymore.); see Fraenkel [57, p. 159] Concerning the exodus of intellectuals see [139,151,21,136,96].

7 As to the mathematicians (and other scientists) who emigrated to the USA see [158,186], Duren [46]. The articles dealing with refugee mathematicians in the USA, found in the last two works, in particular N. Reingold [168, pp. 175–200], S. Lefschetz (pp. 201–207), L. Bers [12, pp. 231–243], do not seem to be aware of the fundamental papers by M. Pinl, and their inventory is somewhat less than complete.

8 The self-help organization “Society for the Protection of Science and Learning”, founded by Fritz Demuth, Philipp Schwartz et al. in Switzerland in 1933, and moved to new headquarters in London in 1936, providing major assistance to emigrés under the patronage of Sir William Beveridge (see also [13]). They compiled a “List of Displaced German Scholars”, see [49]. The archive of this organization is now located in the Bodleian Library, Oxford; see Sherman [183].

9 For the dozen or so refugee mathematicians in Britain see the list in e.g. [158,186]. As Walter Hayman (London) kindly informed us, there does not seem to be any global paper dealing with the contributions of the emigré mathematicians to mathematics in Great Britain. For German-speaking exiles in general see Rider [174], Hirschfeld (Hartley) [92], Wallace [207], and the many individual articles contained therein. After June 1940, all nine German and Austrian refugee mathematicians then in Britain, spent many months in internment camps, and one was deported to Australia, see [158, p. 144], also [103,186, p. 104]. For background see F.L. Carsten in [92, pp. 138–154]. As a referee reports, the Austrian composer Hans Gál (1890–1987), who was appointed Director of the Conservatory in Mainz in 1929 (dismissed 1933 with all his works banned), fled in 1938. He settled in Edinburgh, was arrested on Whit Sunday, May 1940, along with other Edinburgh refugees, brought to a camp a Huyton (Liverpool), and a month later to Douglas, Isle of Man. He was released already late September 1940. At Huyton he composed his *Huyton Suite* (Op. 92) for flute and two violins (only instruments available at the camp). See [62]. The first named author’s father spent three weeks at Huyton before he was released.

found at least temporary asylum in a host of countries [10] from North and South [11] America to Turkey [12] and China. [13]

At the Aachen University of Technology (RWTH Aachen), 12 faculty members [14] were expelled, ten of whom managed to flee abroad and one who died in Theresienstadt. The twelfth, the

[10] These countries and their emigré mathematicians covered Denmark (including H. Busemann, W. Fenchel), France (including S. Bergmann, E.J. Gumbel, F. Pollaczek (1892–1981) [180], A. Weinstein), Switzerland (P. Bernays, A. Pringsheim, H. Samelson, A. Weinstein), Turkey (H. Geiringer, R. von Mises ⟨1933–1939⟩, W. Prager ⟨1934–1941⟩), Sweden (W. Feller), Palestine (R. Artzy, A. Cohn, M. Schiffer, I. Schur, W. Sternberg, O. Toeplitz), Norway (M. Dehn, E. Jacobsthal, W. Romberg), Canada (R. Brauer, P. Scherk, A. Weinstein), Australia (F. Behrend, H. Schwerdtfeger) South America (R. Frucht (Chile), P. Thullen (Ecuador)), or Russia (Cohn-Vossen, F. Noether).

[11] A mathematician who found refuge in Ecuador and who has hardly been mentioned in the older emigration literature is Peter Thullen (1907–1996), a student of H. Behnke [10] in Münster. He was already well-known in 1934, thanks to his book "Theorie der Funktionen mehrerer komplexer Veränderlichen" (Springer, 1934), which he authored together with his teacher. Following a year in Rome as assistant to Prof. Severi, he left Germany in April 1935 for reasons of conscience due to his strong anti-Nazi convictions and to his connections with Catholic youth movements. Pinl III [156] omitted any reference to him (see, e.g. [187,169]). Thullen spent 16 years in exile in Ecuador, Colombia and Panama where, in addition to pursuing his mathematical interests as a university professor, he acquired expertise in actuarial mathematics. In 1952 he was offered an assignment to the International Labor Office's social security department in Geneva, Switzerland. Upon retirement in 1967 as director of that department, he taught at the Universities of Zurich and Fribourg, Switzerland, until 1970 and 1977, respectively. In his memoirs, written in 1988 and published in 2000 [200] he recalls the situation in Germany at the time of his emigration: "We hated the Nazis because we loved Germany" (translation by authors). After his return to Europe he adds: "We found a Federal Republic of Germany in which—different than we had hoped—democracy was imposed by the Allies, from outside. Apart from the condemned war criminals and those who had fled abroad, most had remained in their positions or returned to them, continuing in their professions, including Hitler's "blood judges" (*Blutrichter*) or the euthanasia 'doctors' ". These are harsh words but they reveal the spirit of the time. The authors would like to thank Dr. George Thullen (Genthod, Switzerland), political scientist and eldest son of Peter Thullen, for his insights into Nazi policies towards dissenters and the German post-war era. They are also grateful to Ms. Pinl for passing on to them the publications [200,187]. In support of Peter Thullen's thesis, certain scientists who had worked at the renowned Kaiser-Wilhelm-Gesellschaft, which passed over into the Max-Planck-Gesellschaft (MPG) in 1949, and who were involved in Nazi crimes were able to continue their careers unhindered after 1945. One such case is Otmar von Verschuer (1896–1969), who became Professor of Human Genetics at Münster University in 1951. In 1997 the MPG initiated a research project under its new President dealing with the matter. Its first major result is the 2-volume [105] edited by Prof. Doris Kaufmann (Bremen University), containing 33 papers devoted to "race-hygiene", genetics and virus research, psychiatry and neuropathology, but also with wartime research in chemistry, physics, mathematics and aeronautics. See also E. Klee [106,107], Frei [58] and the many volumes listed in the "Rheinischer Merkur", No. 25, 2001, Section "Wissenschaft und Praxis". The account [210] of Hiltgunt Zassenhaus, a sister of the mathematician Hans Zassenhaus (1912–1991) (see Plesken [159]), gives an additional picture of terror in Nazi times, with her regular, courageous visits from her home in Hamburg to various German prisons, helping Scandinavian internees. She writes: "Das Misstrauen war im Dritten Reich der zuverlässigste Wächter. Keine Dienststelle mochte aufzufallen." (Lack of trust was the most reliable watchman in the Third Reich. No office wished to be noticed.) (p. 147). Further, "There were those who had just one weapon: the good deed in silence ... They were just a few . . . On the opposite side were those who worked together with the evil. Also their numbers were small in comparison with the majority who claimed they did not want to see, as long as their own safety was not jeopardized".

[12] As to the emigration of scientists to Turkey—some 200 in all—see, e.g. the fine paper by R. Erichsen in [195, pp. 73–104]. See also [208,144].

[13] See e.g. Kranzler [113].

[14] The 12 faculty members made up almost 10% of the 123 professors teaching at Aachen in the Winter Semester 1932/33 ([102, pp. 120, 136]). As to the colleagues at Aachen under attack after 1933 one may consult the section "Vertriebene Professoren" in [77, pp. 181–274], a *Festschrift* published on the occasion of the 125th Anniversary of the RWTH in 1995 and edited by the mathematician Klaus Habetha, Rector at the time. He deserves credit for insisting that this explicit memorial be added to the Festschrift, noteworthy since this subject was—except for pp. 106–109—omitted in the earlier centennial Festschrift [111]. See also [48]. For useful background material on the forced emigration of intellectuals see K. Düwell [47], K. Ricking [173] and [24,22,185,195,11,134].

only person dismissed at the RWTH on other than racist grounds was Alfred Meusel (1896–1960), an economist who was regarded as a left-wing socialist or communist. Concerning the mathematical sciences, the applied mathematician Ludwig Hopf [15] (1884–1939), a student of Arnold Sommerfeld (1868–1951) and former assistant of Einstein, found a haven in Trinity College (Dublin) in 1939, after a protracted job-search. Theodore von Kármán [16] (1881–1963), Professor of Mechanics and Aerodynamics [17] in Aachen from 1913–1934, who had spent each winter semester at the California Institute of Technology, Pasadena since 1929, and the summer semester at Aachen, decided in 1933 to remain permanently at the Guggenheim Aeronautical Laboratory. His appointment at Aachen was terminated in 1934, although ironically in that same year he also had an offer to join the air ministry from its new director, Adolf Baeumker. After the war, he visited Aachen occasionally and eventually died there.

2. Otto Blumenthal: career and tragedy

2.1. The years before 1933

There are at present three obituary publications on Blumenthal, one [18] written in 1951 by his teacher and later colleague Arnold Sommerfeld, [19] together with Franz Krauss (1889–1982) [191],

[15] As to Hopf see Müller-Arends and Kalkmann [142]. Hopf was highly respected and admired by his students, as the parents of the first named author often recalled. His efficient, animated conduct of the regular problem sessions attached to the mathematical courses did indeed attract students.

[16] As to von Kármán, see Krause and Kalkmann [114], and concerning the surprising offer from Baeumker (1891–1976) see Hein [89, p. 72]. T. von Kármán even visited him in regard to the offer. However, von Kármán had already met Baeumker (a cousin of Chancellor Heinrich Brüning who fled to the USA in 1934) (see Hein [89, p. 25]). For Baeumker's central role in aviation research and industry during the Third Reich, and his close connections with Gustav Doetsch and Wilhelm Süss (1895–1958), see especially Remmert [169,170], also Ludwig [126]. Surprising or not, Baeumker worked with the U.S. Air Force 1946–1957, and became a US citizen.

[17] Aachen was famous in aerodynamical research at least as early as 1913 when Hans Reissner (1874–1967), Professor of Mechanics at Aachen 1906–1913, founded the Aerodynamisches Institut. Hugo Junkers (1859–1935), Professor of Thermodynamics at the RWTH 1897–1912, built the first all-metal airplane with self-supporting wings, the J 1, in 1915. For the history see [61] and for German aeronautical research 1914–1918, see Trischler [202] and [80,140].

[18] The paper [191] includes a useful autobiography of Blumenthal, a list (incomplete) of his publications transmitted by his son Dr. Ernest Blumenthal, as well as a good description of Blumenthal as researcher, teacher, and especially as a human being.

[19] Arnold (Johannes Wilhelm) Sommerfeld, a student of Hurwitz, Lindemann and Hilbert in Königsberg (East Prussia, now Kaliningrad), but who regarded Klein [26] as his real teacher—having been his assistant in Göttingen 1894–1897—was from 1897 to 1900 Professor for Mathematics in Clausthal (Germany), from 1900 to 1906 Professor of Mechanics in Aachen, from 1906 until 1938 Professor of Theoretical Physics in Munich, as successor to Boltzmann. Among his students are four Nobel Prize Laureates, namely Peter Debye from Aachen, and W. Heisenberg, W. Pauli and H.A. Bethe. Already in 1934 Sommerfeld, for whom the highest political virtue was the preservation of the German fatherland, wrote to Einstein "that the national feeling that I strongly felt, has stopped entirely after the misuse of the word "national" by those in authority. I would now have nothing against the disintegration of Germany as a power and its incorporation into a peaceful Europe". Thus he already had European Union thoughts in 1934. See Hermann [90], A. Sommerfeld [189,190], DSB, XII, pp. 525–532.

a former assistant, and eventually successor in office. The second is due to Heinrich Behnke [20] (1898–1979) in 1958 [9], and the third in 1995 [29]. Blumenthal is also treated in Pinl [156,158], as the first mathematician discussed, since the articles are alphabetically ordered according to universities.

Ludwig Otto Blumenthal was born in Frankfurt am Main (Germany) on July 20, 1876, the son of the physician Ernst Blumenthal and his spouse Eugenie, née Posen. After attending the Goethe Gymnasium in Frankfurt am Main, Otto Blumenthal first studied a semester of medicine (1894) at Göttingen (Germany), but then turned to mathematics and the sciences at Göttingen and Munich; he spent the summer semester at the latter. His most influential teachers were David Hilbert (1862–1943), Felix Klein (1849–1925) and Arnold Sommerfeld. According to the curriculum vitae attached to his doctoral thesis {1}, at Göttingen Blumenthal also heard the lectures of the mathematicians E. Ritter (1867–1895), A.M. Schoenflies (1853–1928), and the physicists W. Nernst (1864–1941), E. Riecke (1845–1915), W. Voigt (1850–1919), and H. Weber (1842–1913) as well as of the mineralogist T. Liebisch (1852–1922), the psycho-physicist G.E. Müller (1850–1934), and the chemist O. Wallach (1847–1931). At Munich he attended the lectures of the mathematicians C.L.F. Lindemann (1852–1939) and A. Pringsheim (1850–1941) as well as of the physicist A. Korn (1870–1945) and mechanist A. Foeppl (1854–1924).

Göttingen was then at its academic peak, and Blumenthal greatly admired the young *Privatdozent* Sommerfeld. He had heard his lectures on probability theory in the summer semester 1895, and on projective geometry the following year. To document his attachment to Sommerfeld, Blumenthal dedicated his doctoral dissertation (Dr. Phil.) "Über die Entwicklung einer willkürlichen Funktion nach den Nennern des Kettenbruches für $\int_{-\infty}^{0}(z-\xi)^{-1}\varphi(\xi)\,d\xi$" {1} (oral examination on May 25, 1898, with overall grade *summa cum laude*) to him. Behnke [9] notes that Sommerfeld wrote in his reminiscences that "Blumenthal was my favorite student", that he was extremely modest and unassuming, and that he had too little rather than too much self-confidence.

Nevertheless, Blumenthal is regarded as Hilbert's first doctoral student. Hilbert apparently became aware of Blumenthal's talents during a joint seminar conducted by Hilbert and Klein. In 1899 Blumenthal took the exams enabling him to teach mathematics, chemistry, and physics in secondary schools, and spent the winter of 1899/1900 in Paris, studying first and foremost

[20] H. Behnke, a senior mathematician at the University of Münster at the time, in fact suggested to the chief-editor (Butzer) of the "Jahresbericht der Deutschen Mathematiker-Vereinigung" in 1964/65 that Max Pinl (recall Notes 1,2) would be the ideal author for the series of articles. Max Pinl was pleased to accept this almost overwhelming task and within a comparatively short time assembled an impressive bibliographical corpus of information. It came to a total length of 185 pages, outlining the life and work of 188 individuals. Pinl knew many of the individuals personally or through his mathematical-biographical work with the "Jahrbuch über die Fortschritte der Mathematik" in Berlin circa 1926–1935. At the yearly meeting of the DMV, at Karlsruhe, in September 1967, the executive members of that society nonetheless suggested to the first author (Butzer), then chief-editor, that he cancel the project of remembering those members in the "Jahresbericht der DMV" persecuted under the Nazi regime. But the first author, who belonged to the presidency of the DMV in view of his office, refused to cancel the project, so that it was subsequently suggested that he relinquish his position or be voted out of office at the next yearly meeting, when his three-year appointment terminated. However, another past editorial board member, Walter Benz (Hamburg), was subsequently appointed chief-editor and he, together with the first author, now a co-editor, were able to see the project to completion over the years 1966–1976. They regularly met with Max Pinl in Cologne to discuss the delicate problems arising with the DMV; its president 1966–1969 was Karl Stein. It should also be emphasized that the Deutsche Forschungsgemeinschaft (DFG) and its referees had previously approved the project and continued to support publication financially. Thus it is inaccurate to state that "Pinl was commissioned by the Deutsche Mathematiker-Vereinigung to compile his report on the life and work of mathematicians persecuted or displaced in 1933–1945 . . .", as Furtmüller [158, p. 150], writes. On the history of the DFG during the Third Reich see N. Hammerstein [81].

with Émile Borel (1871–1956) and Camille Jordan (1838–1922). Already in 1901 he received the Habilitation in mathematics at Göttingen, with the thesis "Über Modulfunktionen von mehreren Veränderlichen" {3},{5}, and taught there from autumn 1901 to summer 1905 as *Privatdozent* (Emmy Noether (1882–1935) attended a course of his 1903/04). In between he substituted for a full professor at Marburg University. Blumenthal's reminiscences as a student and *Privatdozent* in Göttingen and at Marburg reveal the scientific atmosphere in two mathematical institutes at the time.[21] Sommerfeld, who was offered the chair of mechanics at the RWTH Aachen in 1900 as successor to August Ritter (1826–1899), pointed out Blumenthal to Adolf Wüllner (1835–1908), physicist and later rector at RWTH Aachen.

In October 1905 Blumenthal followed Lothar Heffter (1862–1962), who had spent the 1904/1905 year at Aachen, as successor of the number theorist Hans von Mangoldt.[22] It was the first chair in Mathematics at Aachen, one which Blumenthal held until 1933. Aachen had three chairs[23] in mathematics and one in mechanics at the time, and only one regular research assistantship since 1911. The parallel chair in mathematics was occupied in succession by P. Furtwängler ⟨1907–1910⟩, M.W. Kutta ⟨1910–1912⟩, G. Hamel ⟨1912–1919⟩, E. Trefftz ⟨1919–1922⟩ and L. Hopf ⟨1923–1934⟩, the chair in geometry by E. Kötter ⟨1897–1922⟩, H. Brandt ⟨1921–1930⟩ and H. Graf ⟨1931–1932⟩, and in mechanics by H.J. Reissner ⟨1906–1913⟩ and T. von Kármán ⟨1913–1934⟩.

Of interest are the names of the members of the Department of Mathematics and Mechanics who received their Habilitation during Blumenthal's time. They are A.A. Timpe (1882–1959) in the year 1910, L. Hopf in 1914, E. Trefftz in 1917, Otto Foeppl (1885–1963) in 1920, K.O. Friedrichs (1901–1982) in 1928, F.N. Scheubel (1899-) in 1930, H. Graf (1897–1985) in 1931 and R.L.M. Iglisch (1903–1987) in 1931. All of these received positions at institutes of technology or universities in Germany, Austria or USA.

In 1908 Blumenthal married Mali Ebstein, the daughter of Prof. Wilhelm Ebstein (of Göttingen University) and his wife Elfriede, née Nicolaier. They had two children, a son Ernest (1914–1974) and a daughter Margrete (1911–1980).

During 1914–1917 Blumenthal took part in World War I, two years of which he spent as head of a military weather station.[24] In 1918 he worked in the Siemens–Schuckert aircraft construction firm, returning to the Aachen Institute in 1919. His paper {24} on double-decker airplanes is based on his work at this aircraft firm.[25]

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[21] Concerning these documentary recollections of his teachers and colleagues at these two universities see W. Lorey, [125, pp. 351–57, 371, 383].

[22] For von Mangoldt (1854–1925) and his contribution to the proof of the prime number theorem, carried out independently by de La Vallée Poussin and J. Hadamard in 1896, see Mawhin [133], Landau [116].

[23] For the mathematicians holding chairs at the RWTH Aachen see [5].

[24] In Blumenthal's own biography [129, p. 376], it is mentioned that he was in charge of several meteorological stations, including a zeppelin port in Hannover.

[25] The "cradle" of German aviation research was at Göttingen, where Felix Klein from 1890 on insisted upon linking mathematical-scientific research with technological-practical training, building on a division of work between the classical universities and the institutes of technology (Technische Hochschulen). He attracted Ludwig Prandtl (1875–1953) to Göttingen, and blazed a trail from hydrodynamics to aerodynamics. The joint seminars held together with Carl Runge on aerodynamical problems also made this possible. Prandtl's collaborators included Georg Fuhrmann, Otto Foeppl (Dissertation and Habilitation in Aachen 1911 respect. 1920), Carl Wieselsberger (Professor of Applied Mathematics and Fluid Mechanics at Aachen—1930–1941) and Albert Betz. He had many students, including von Kármán. See e.g. Ludwig [126], Trischler [202].

Photo from the collection of Eva Wohl (daughter of Otto Toeplitz). Courtesy of the Department of Mathematics, Technion.

At Aachen he held several offices: department head (1907/08 and 1919/20), elected senator (1914/15 and 1921), dean (1927/28), and senate representative for the Faculty of Science (1928/29). In 1922 Blumenthal founded the *Ausseninstitut*, to represent the humanities and social sciences at the RWTH with respect to the hard-core engineering curriculum; he served as its chair until 1927. In 1921 Blumenthal had been considered for a professorship at Frankfurt University. [26]

[26] In a letter of W. Blaschke to L. Bieberbach of January 27, 1921, concerning the problem of Bieberbach's successor at Frankfurt when he would leave for Berlin, Blaschke thought Pólya far better than Blumenthal, and that the latter's "fame" stemmed primarily from von Kármán's desire to get rid of him at Aachen. E. Hecke, in his letter to Bieberbach, also of January 27, 1921, shared Blaschke's view: "Kármán recommends him very warmly, that is already not a good sign". Possibly there was a rivalry between von Kármán and Blumenthal. Max Dehn, whom Hecke recommended above all, became Bieberbach's successor in Frankfurt. See Segal [182, pp. 341–345], and the literature cited there, also for Hecke's letter.

2.2. From dismissal to Theresienstadt

Shortly after the Nazis took over, the Aachen student association, the AStA,[27] denounced Blumenthal as an alleged communist in a letter of March 18, 1933, signed by Josef Hermann and Karl Bauer, to Bernhard Rust (1883–1945),[28] who was the minister of Science, Culture and Education. On April 27, 1933, Blumenthal and the economist Alfred Meusel were taken into "protective custody" (*Schutzhaft*) by the Gestapo[29] and, on May 10, Rust relieved him of his duties as a university instructor.

The Rector of the RWTH, Paul Roentgen (1881–1968), as well as the Pro-Rector, Felix Rötscher (1873–1944), did their best to help both faculty members, but without success;[30] Bernhard Rust did not even admit him for a personal discussion. Neither the Faculty for General Science nor Prof. Erich Trefftz (1888–1937) in Dresden, a former student and colleague of Blumenthal at Aachen, were able to influence[31] the decision of the ministry. Hermann Starke (1874–1960), Professor of Physics at Aachen and Dean of the Faculty at the time, himself a victim of Nazi attacks, reported[32] that a few students had also denounced Blumenthal with unsupported claims.

The investigation against the suspended professors was carried out by the chief government administrator (*Regierungspräsident*) at Aachen in his capacity as commissione[33] of the RWTH.

[27] The denunciations by the AStA of the RWTH Aachen of March 18, 1933, to the ministry are located in the Hochschularchiv Aachen (university archive; cited below as HAAc) under No. 508. The full letter is also reproduced in Lepper [120, Vol. II, pp. 1117–1119]. Blumenthal's assistant Dipl.-Ing. Hellmann is mentioned with the same accusation, likewise Prof. Meusel. In a further denunciation of April 10, 1933, the AStA, now signed by Karl Baum and Franz August Becker, refer to Professors Fuchs and Maedge for their communistic views, and to Professors von Kármán, Ruer, Salmang, Levy and Dr. Strauss for the fact that they are Jewish members of the university (but not politically active). The letter of April 10 is also to be found in HAAc, No. 508 as well as in Lepper [120, p. 1138]. The 2-volume monumental work of Dr. Herbert Lepper, former Director of the City Archives, gives an excellent account together with an amazing number of documents of the history of the Jewish Synagogue Congregation from 1801 to its destruction by the Nazis in 1942. These documents are complemented by the excellent, parallel study of the Technische Hochschule Aachen during the Third Reich by Kalkmann [102]; see also [153].

[28] B. Rust was a teacher who already began his Nazi political career in 1924 as *Gauleiter* of Hannover and was minister from 1933 until his suicide; see Pedersen [154]. The theoretical protagonists of the Nazi educational system were Professors Ernst Krieck (1882–1947) and Alfred Baeumler. Both are said to have failed in their aims; see Tenorth [22, p. 251], and [181]. Schwabe [181] treats the basic political outlook of German academia. The majority are said to have sympathized with the conservative right-wingers, radicalized since World War I. For a more discerning opinion, as a function of the time, see M.G. Steinert [22, pp. 474–487], also [108].

[29] Prof. Krauss visited Blumenthal during the *Schutzhaft* fortnight together with his friend Robert Sauer (1898–1970), Professor of Applied Mathematics and Descriptive Geometry in Aachen from 1932–1944; see Geller [65, p. 174], for further details. The latter, well-documented book treats in detail not only history of physics at Aachen during the chaos of World War II but also deals with some of its mathematicians. Hermann Starke, himself denounced by Nazi assistants at the RWTH already March 1933 (a teacher of the first named author's father), also visited Blumenthal while in prison.

[30] A letter of the Rector to the ministry of April 27, 1933, and the letter of the ministry of May 10, 1933, to Blumenthal are also located in HAAc. Professors Hopf, Fuchs, Meusel, Mautner, Levy and Drs. Strauss and Pick were relieved of their duties in a telegram to the Rector by Rust on April 29, 1933. See again Lepper [120, p. 1141]. The Rector passed on the message to Dr. Strauss on April 29 (perhaps also to the other members). See also Kalkmann [102, p. 121].

[31] Whether the undated letter of the Faculty was ever sent is not known. Trefftz's letter to the Rector is dated May 27, 1933.

[32] The report of Starke is dated June 16, 1933, again HAAc 1201 a; [102, p. 234].

[33] The commissioner's report, dated July 18, 1933, is located in the Hauptstaatsarchiv (main state archive) at Düsseldorf (cited as HStA) under Regierung Aachen, 20065.

Although his report emphasized that Blumenthal's political assertions did not exalt Russia and that he was at most an "idealist", the ministry nevertheless decided on the dismissal.

Together with E.J. Gumbel (1891–1966) [75,99] in Heidelberg, Blumenthal was one of the early victims of the National Socialist intervention among mathematicians [55, p. 36]. On the basis of Paragraph 4 of the earliest racist and ideological statutes, euphemistically called the "Gesetz zur Wiederherstellung des Berufsbeamtentums" (Professional Civil Service Restitution Act) of April 7, 1933, [34] the minister suspended Blumenthal from the civil service on September 22, 1933; his salary was discontinued at the end of December. This happened despite the fact that Blumenthal's parents and he himself had converted from the Jewish to the Protestant faith in 1895, and he was active in the Protestant community of Aachen for many years. Conversion to another religion did not matter according to Nazi laws.

Sommerfeld and Krauss [191] have written that Blumenthal was suspended from office "not on racial grounds but because of his membership" in "international and pacifist organizations". According to the official statements of the ministry [35] Blumenthal lost his professorship on account of his memberships in the "Gesellschaft der Freunde des neuen Russlands" (Society of Friends of the New Russia) as well as the "Deutsche Liga für Menschenrechte" (German League for Human Rights). Blumenthal had taken part in the All-Russian Mathematical Congress in Kharkov in 1930, and again traveled to Russia to give lectures there in 1931. He was specifically accused of giving two lectures in Aachen about his impressions of the Russian educational system. But Blumenthal had already been in Russia in 1900, long before the Bolshevik revolution. During 1933–35 he gave several mathematical lectures in Delft, Leyden, Utrecht, Zürich, Brussels and Sofia. In the last city in 1935 he offered a month-long course in Russian on integral equations, as well as a lecture on "The life and scientific work of David Hilbert" {44}.

In the questionnaire stipulated by the laws of April 7, 1933, Blumenthal had to give explicit information on his "racial" background. He identified his parents as well as his four grandparents as having been members of the Jewish congregation in Frankfurt. For whatever reason, he was prohibited from professional activity in Germany after 1935. Although he had managed to find refuge for his children [36] in England—his daughter received her doctorate at Cologne in 1934 and left in 1936 for England, his son studied from 1933 on at Manchester University—Blumenthal and his wife remained in Aachen until July 13, 1939, when they found refuge together with kindred academic emigrés in a hostel (House Zuilenveld) near Utrecht. In fact in March 1939 he accepted an offer supported by David van Dantzig (1900–1959) to do private teaching at the Delft Institute of Technology. [37]

[34] For this Act see the *Reichsgesetzblatt* 34, 1933, p. 175 and for an excerpt concerning Paragraph 3, the minister suspended, see Lepper [120, p. 1137]. In fact, Blumenthal should have been doubly exempt, as a frontline fighter of World War I, decorated with the Iron Cross, and as a prewar civil servant. See [158, p. 157].

[35] A photocopy of the dismissal document is to be found in Gerstengarbe [69]. It is actually the only one for 614 university lecturers treated there (with attached photocopy).

[36] According to a letter of November 18, 1933, to von Kármán, Blumenthal's daughter Margrete studied English literature at Cologne, and hoped she would be allowed to do her doctorate, being "non-Aryan". See [186, p. 72]. As to the admission of non-Aryan students to academic exams, see the decree of 1934, reprinted in Lepper [120, p. 1169].

[37] Thanks are due to V. Felsch who located these facts in the Blumenthal diary which is in the possession of the Blumenthal descendants in England.

Two nights before their departure they (again) visited the Krauss family “wo es wieder herzlich ist” (where it is again heartwarming), as Blumenthal wrote in his diary; Krauss had been Blumenthal’s assistant for nine years, his colleague and successor in April 1934. It hardly ever happened that a non-party member was appointed as chair-holder at Aachen; he was one of just four such cases. In fact, Franz Krauss was never a member of the NSDAP, nor the SA or the SS (see [102, p. 64]).

Surprisingly the Blumenthals received British visas to visit their children in England on August 20, 1939. In view of the tense political situation their daughter Margrete (perhaps uncomprehendingly) urged her parents to return to the Netherlands promptly. And they did leave on August 26, just five days before the outbreak of war. [38] What a tragic mistake considering the Nazis had already occupied Austria, and Bohemia-Moravia in March 1939.

Paul Ewald (1888–1985) among others had tried to find a university position [39] for Blumenthal in England and in a letter [40] of January 13, 1938, to Erich Hecke (1887–1947), Blumenthal asked him to try to help him with respect to the USA. In the spring of 1939, J. Hadamard found for Blumenthal a position as chairman of a mathematical institute in Rosario, Argentina. Blumenthal was most happy to accept the offer but it finally went up in smoke. [41]

Earlier, C. Caratheodory (1873–1950), who was Carl-Schurz Visiting Professor at Madison, Wisconsin, had tried to convince S. Lefschetz, President of the American Mathematical Society at the time, to invite Blumenthal for a US lecture tour on mathematical-historical topics. Lefschetz’s reply of December 7, 1936, was somewhat coarse and sarcastic: “I am only the President of the American Mathematical Society (until the end of this year) and not its Führer! . . . There exists a specific Committee of the American Mathematical Society in charge of the visiting lectureships”; see [186, p. 209].

Theodor von Kármán, according to a letter to Blumenthal of April 29, 1936, also intervened unsuccessfully ([186, p. 262]). In a letter to von Kármán of November 18, 1933, Blumenthal expressed his emigration wish in very modest terms: “Ich wage nicht, an eine dauernde Auslandsstelle zu denken: das ist ein schöner Traum. Aber vielleicht findet sich die Möglichkeit zu Vorträgen oder Semesterkursen. Kannst Du mir zu dergleichen verhelfen? SOS.” (I dare not think of a permanent position in a foreign country: that is just a beautiful dream. But there is perhaps the possibility of giving lectures or a one-semester course. Can you help me get something like that?) In his final letter to von Kármán, dated January 10, 1940, from Delft, Blumenthal wrote that he was allowed to cross the border (on July 13, 1939) with his furniture but without any funds or valuables, and that von Kármán had helped obtain an “affidavit” of support for him; but affidavits would allow him to enter the USA only after ten(!) years ([186, p. 120]).

[38] Recall the foregoing footnote.

[39] Taken from C. Reid [166, p. 212].

[40] According to written communication from Rotraut Stanik, Hamburg.

[41] This information is taken from Blumenthal’s diary, dated from April 5 to April 21, 1939. Dr. Felsch kindly copied the explicit entries for us; he intends to make the diary available to the public. Beppo Levi received the position.

The basic problem facing Jewish emigrés was that they were not admitted to England or the USA for permanent residence without firm job offers. [42] Here one must distinguish between immigration and emigration. [43] Lord Templewood, Chamberlain's Home Secretary and author of some ten books, mentioned that temporary visas were issued to refugees in order to circumvent (the tight) immigration laws. Thus temporary visas without the right to work were much easier to obtain.

Blumenthal was 62 in 1938 and many other refugee mathematicians were also seeking jobs outside of Germany. After the Nazi occupation of the Netherlands (1940), Blumenthal and his spouse [44] had to endure the sufferings of other Jewish German citizens. On April 22, 1943, they were taken to the notorious internment camp at Vught, and 18 days later to Westerbork; there his wife died on May 21, 1943, after having gone through a degrading experience. Yet in a letter to a Dutch friend Blumenthal wrote "I thank God that he gave me a bearable life, but also that he relieved my wife so early. For her that was intolerable which I accepted quietly. Looking back at the past year, it was painful but quiet" (see [191]). In the summer of 1943 he had heard that his sister had been taken to Theresienstadt. Now, after the death of his wife, he requested to be transferred there, too, and this was granted. [45]

According to more recent documents, [46] Blumenthal was transferred (under Transport No. 64) from camp Westerbork, Netherlands, on January 18, 1944 at 10:42 am, to join 870 other so-called

[42] The German emigrées faced intimidating problems both from the German side and the potential host country. On the one hand they had to obtain an exit visa, pay the (often exorbitant) *Reichsfluchtsteuer* (capital flight tax), and abandon property and means (apart from 10 marks, although in at least some cases furniture was somehow transferred across the border); on the other hand, they had to meet the restrictive immigration policies of the country of refuge. For example, the US immigration act of 1924 admitted at most 25,000 Germans per year. There were certain "preferential" exceptions for university personal with affidavits of support. Britain after November 1938 admitted 55,000 refugees, more than any other European country. As to all host countries, the current estimate is a total of close to half a million German-speaking refugees. See e.g. Strauss [195], Rider [174], Pross [161], Siegmund-Schultze [186, pp. 83–124]. A referee gives the example of the Austrian dentist Desider Furst and his family who crossed illegally near Aachen into Belgium at Christmas-time 1938, lived there for two months, evading police. On March 1, 1939, he was admitted to England as one of forty dentists; see [60]. The fate of Hans Schneider (1927–) and his parents is similar. They entered Czechoslovakia illegally from Austria in June 1938, and were refugees without resources, status or prospects. Whereas his father was admitted to Britain also as dentist, and settled 1939 in Edinburgh, Hans reached Edinburgh via Warsaw, a flight from there to Amsterdam, and a Quaker school in the Netherlands for refugee children (see `www.math.wisc.edu/~hans/pers_ hist.txt`). Hans' wife Miriam Schneider emigrated to Scotland via the "Kindertransport"; for this transport, which brought ca. 10,000 children to Britain when war broke out, see Göpfert [72]. Already on April 28, 1933, 11 people were caught trying to cross the border at Aachen without exit permits; see Lepper [120, p. 1142]. Whereas the emigration of Jewish Germans was encouraged, if only because their possible capital could be appropriated, the emigration of non-Jewish Germans was strictly illegal. See e.g. E.B. Bukey's book [23] on the popular sentiment in Austria during the Nazi era 1938/45. The first named author with his parents and brother, as Catholic dissenters, left Germany secretly in 1937, via Aachen to England. See [25].

[43] It was a referee who brought up this basic distinction, in particular the reference to the book of Viscount Samuel John Gurney Hoare [197].

[44] It is curious that the name Mali Blumenthal (née Ebstein) is not found in the volume "In Memoriam" of 1995 [95] containing some 111,500 names.

[45] Facts found in the Blumenthal diary.

[46] Concerning the "Richtlinien" (guidelines) of May 15, 1942, for carrying out the "evacuation" of Jews to the "old people's ghetto" Theresienstadt (Terezin), see Lepper [120, pp. 1339–1341]. There are various informative volumes treating Terezin. See e.g. Adler [4,3], Lederer [119], Makarova-Makarov and Kuperman [129], Starke [194] and Troller [204]. Norbert Troller (1896–1981), an artist and architect from Brünn (Brno), the uncle of George Stefan Troller (whose films paralleled his experiences during his escape), presents a very vivid description of life in Terezin during his stay there between March 1942 and September 1944, when he was transported to Auschwitz. See his memoirs in [204]; they include a useful and informative introduction, notes, and biographical information by Joel Shatzky, the editor. The book contains many of the over 300 drawings and watercolors Troller made while in Terezin. See also Hilberg [91].

bevorzugte ("privileged") Jews. He was on transport No. XXIV/2 which arrived in the elite concentration camp Theresienstadt (part of it officially called the "Polizeigefängnis Theresienstadt" (Terezin), Czech Republic, 60 km north of Prague) two days later. These "privileged" inmates had participated in World War I and had received decorations such as the Iron Cross, first or second class, with casualty insignia; there was in fact a list with nine distinguishing marks. There also were so-called "Prominente", to which Blumenthal did not belong. Prisoners tried to remain in Westerbork as long as possible, in order to avoid deportation to the East with its disastrous reputation. Westerbork was regarded as "the transitory ghetto of the Netherlands" which "maintained a blooming cultural and religious life"; the deportees from Holland later made up "an ideologically heterogeneous yet active and influential group in Terezin" (see [129, p. 104]).

"In Terezin ghetto, some of the most brilliant intellectual elites of Czech, German, Austrian, Dutch and Danish Jewries were incarcerated in the period between 1941 and 1945" ([129, p. 10]). At the end of the war, only 132 of the 870 "privileged" inmates survived. It is said that some ten percent of the inmates were Christians. [47] Terezin has been called the "University over the Abyss" [129, p.11]: 489 lecturers gave 2309 lectures during 1942-44. Topics covered included literature (with 361 lectures), personalities (360), history (310), Jews and Judaism (432), philosophy (164), society (161), science (121), art and culture (223). [48] The library at Terezin included a central library, a children's library, a central medical library, a technical library, a young Jewish library, and the readers' circle. It was said that "This library, it appears, will become in time the greatest Jewish library in Europe and the world" (see [129, p. 30]). According to Troller [204, p. XXIV], the activities "permitted" in the ghetto included concerts, opera performances, and lectures: "The older ghetto inhabitants found consolation and uplift in the cultural activities, which helped them to forget temporarily their dreary present. For the young people, it provided a source of strength, mental discipline, and courage". (We report these almost farcical views without further comment.)

Blumenthal also gave lectures at Terezin. These included Accident law (on March 3, 1944); Errors as a source of knowledge in mathematics (March 4, 1944); Mathematics and experimentation (February 16, 1944); Functions and curves (June 17 and June 26, 1944); Conformal representations (June 20, 1944); The world of derivatives and integrals (July 1, 1944); Circle relationships and stereometric projections (July 10, 1944); and Integral theorems of the theory of functions (August 8, 15, and 22, 1944). These lectures were interrupted from March till May due to heart and lung problems, pneumonia and dysentery. [49]

"Created as a publicity bluff, a propaganda decoy, the privileged Jewish settlement of Terezin was touted to the world as a "humane solution to the Jewish question"!" In fact, KZ Terezin, [50]

[47] Heads of the Protestant community were Dr. Jur. Arthur Goldschmidt (1873–1947), Dr. Otto Stargardt (1874–), and Domine Max Enker. Heads of the Catholic congregation were Dr. Jur. Hans W. Hirschberg (1893–), Lieutenant General Field Marshal Hans J.G.F.H. von Friedländer (1882–1944). Dr. Anna Auředničková (1873–), the author of "Three years in Terezin" (1945), translator of some 70 books of Czech authors into German, wrote "It was a paradox: the 'Jews' in the ghetto enjoyed freedoms that the 'Christians' in 'freedom' did not" [129, p. 116]. For a daily account (of 986pp.) of life, organization, lectures in Terezin 1942 – 44—not a typical recollection of the past—see the unusual volume by Philipp Manes (born 1875 in Elberfeld, murdered 1942 in Auschwitz); see Manes [130, p. 347].

[48] The Terezin lecturers included Dr. Leo Baeck and many other prominent internees. See Makarova et al. [129].

[49] As to the lectures, recall Makarova et al. [129]. The information on Blumenthal's health is taken from various letters in the possession of the Blumenthal descendants.

[50] It is said that "although the victims were never murdered outright, they were placed into such conditions that their survival over any long period of time... was highly unlikely". See Troller [204, p. XXIII]. The Nazis even invited the Red Cross to inspect Terezin. So that it would not appear to be overcrowded, the Nazis deported some 5,000 of the inmates to the East shortly before the inspection of the ghetto by the Danish Red Cross on June 23, 1944.

"considered to be a model ghetto by the Nazi hierarchy", was a transit hub; those meant to be exterminated were generally sent on to Auschwitz–Birkenau, a few to Dachau.

According to Monsignor Tomáš Fedorovič,[51] Blumenthal died in Terezin on November 12, 1944, the cause of his death not being officially known. Krauss and Sommerfeld [191] add that he died of pneumonia[52] in the camp hospital after three days of unconsciousness. Emil Jilovsky, a Czech engineer, wrote to a relative of Blumenthal in 1945 that the head doctor of the general hospital of the city of Terezin took appropriate care of Blumenthal, whose legs and arms were swollen. The autopsy stated that he had old-age tuberculosis and water in the brain; thus he apparently died of natural causes.[53] But the deaths of perhaps 85% of the 870 "privileged" inmates within two or three years makes clear that life at Terezin was very harsh, presumably in terms of nutrition, hygiene, clothing and warmth. A far more critical examination of the mortality statistics in relation to listed proximal causes of death, such as pneumonia, is called for.[54]

Blumenthal's sister, Anna Storm, had died in Theresienstadt on June 13, 1943, some six months before her brother's arrival; one can imagine how disappointed Blumenthal must have been not to have met her there. But Blumenthal had an almost superhuman spirit of endurance according to his colleague Franz Krauss [191]: "During the times of increasing persecution, shortly before his emigration to the Netherlands, in his and in my presence, friends, who were also being persecuted, came up with embittered accusations about the Nazi henchmen. Then Blumenthal called out in his shocked, beseeching voice: 'No, we can't even hate our enemies'." Later, when Dutch friends offered to hide him in their quarters, he refused with the comment that "he does not wish to bring his friends into danger".

2.3. Professional services and their termination

Blumenthal's professional services also are illuminating, both as to his personality and the arbitrariness of his exclusion from his professional community. He was the long-term managing editor of the "Mathematische Annalen", the most prestigious mathematics journal of that time; it was an assignment he carried out with painstaking care, from 1906 to 1938, for 32 years.[55] In fact, Blumenthal bore the burden of the management of the Math. Annalen, even though it was nominally in the hands of Hilbert and Klein, the latter having been editor since 1902.

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[51] The authors are very grateful to Monsignor Fedorovič, Památnik Terezin, Principova alej 304, CZ — 41155 Terezin for supplying this information. He also made the authors aware of the books Adler [3, pp. 30–35], [129, pp. 376–377]. The authors also thank Miroslav Kárný, Tererova /b, CZ - 14900 Praha 4 [104]. His "Institut Theresienstädter Initiative" gave out the "Theresienstädter Gedenkbuch. Die Opfer der Judentransporte aus Deutschland nach Theresienstadt 1942–1945" (2000), containing the names and files of 42,124 German Jews deported there. Blumenthal's name does not appear, perhaps because he had converted to the Protestant faith in 1895. See also Bondy [16].

[52] Disease played a very significant part in increasing the mortality rate in Theresienstadt. In all, a total of 18,000 of the inmates died of infectious diseases, while between 1942 and 1944 over 6,000 succumbed to pneumonia and other respiratory infections. See Adler [4, p. 528], Lederer [119, p. 264], Troller [204, p. 166].

[53] Taken from a report by Jilovsky to the lawyer Jaroslav Jiny, Prague, of October 1945, sent on to Ernest Blumenthal.

[54] Recall also the pre-preceding footnote.

[55] The so-called *"Grundlagenstreit"* of the Mathematische Annalen of 1928–1930, a notorious conflict which ensued when Hilbert sought to oust L.E.J. Brouwer from the editorial board, has been carefully chronicled by Dirk van Dalen [38]. This fight, involving Hilbert, Brouwer, the neutral Einstein, the would-be mediator Blumenthal, as well as Courant, H. Bohr, Bieberbach and Ferdinand Springer, the publisher of the Annalen, carried broader political implications; they set the stage for events that followed after the Nazi takeover. See also [55, p. 54].

He was also appointed editor of the "Jahresberichte der Deutschen Mathematiker-Vereinigung" (DMV) in 1924, a function he also carried out conscientiously until 1933. He had been a member of the DMV since 1900. The editor of the "Jahresberichte" automatically is one of three executive members of the DMV, raising the question of why Blumenthal should have resigned in June 1933. Fischer, Hirzebruch, Scharlau and Törnig [55, p. 52], write that this decision was connected with his being taken into custody and his suspension in Aachen. Specifically, Ludwig Bieberbach (1886–1982) in a letter of June 26 in that year wrote to Helmut Hasse (1898–1979) that he had discussed the necessity of such a resignation with Blumenthal. Both Hasse, the treasurer of the DMV, and Bieberbach, secretary of the DMV, together with Blumenthal, constituted the editorial board of the "Jahresberichte". One may assume that Bieberbach, the advocate of a "Deutsche Mathematik" and the de facto sole editor of the "Jahresberichte", suggested to Blumenthal that he resign. [56]

In regard to Blumenthal's editorship of the "Mathematische Annalen" it is relevant that on November 15, 1938, Rust's ministry [57] issued the so-called "Akademieerlass" (academy decree) according to which the German scientific academies were required to modify their statutes. The "Führerprinzip" was ordained, and regular members had to be German citizens according to German civil law; "German Jews" were to be asked to resign. This decree clearly applied to scientific societies such as the DMV. On January 2, 1939, Blumenthal was excluded from the DMV. [58] This also applied to Alfred Brauer, Dehn, Hamburger, Hellinger, Rosenthal, Schur and Toeplitz. This decree presumably spelled the end of Blumenthal's editorship of the "Mathematische Annalen".

Springer, the publisher of the "Mathematische Annalen", had agreed to pay Blumenthal a pension [59] when he was forced from the *Annalen* board because Hecke, Behnke and van der Waerden had insisted on it as a condition of their continuing to put out the journal. After Blumenthal left Germany this pension was paid, at Blumenthal's request, to his sister.

This decree probably also applied to Blumenthal's work for the "Zeitschrift für Angewandte Mathematik und Mechanik" (ZAMM). From 1934–1938 he wrote the English and French abstracts for the articles appearing in this journal. Probably made possible due to the efforts of Erich Trefftz, this again highlights Blumenthal's dedication to service. His unusual linguistic skills—including eight languages—underscore his broad worldview. He spoke, read and wrote fluently in French, English and Russian, knew some Italian, Dutch and Bulgarian and could read Latin and Greek.

On April 18, 1997, a commemorative plaque honoring the contributions of Blumenthal was unveiled in the main building of the RWTH, in a central location near that documenting the found-

[56] Also relevant is that the 1933 business meeting of the DMV was held jointly with the *Mathematischer Reichsverband*, an organization to defend professional interests founded 1921, on September 20 in Würzburg [97]. According to the minutes of the *"Mitgliederversammlung"* (meeting of the membership) [138], the chairman of the *Reichsverband*, Georg Hamel (1877–1954), Professor of Mechanics at Berlin, introduced and carried the motion that the "*Führerprinzip*" be adopted by the *Reichsverband*. That meant election of a council of "Aryan" collaborators, basically by the "Führer". Bieberbach was named to both this council and as liaison to the DMV. Richard Baldus (1885–1945), chairman of the DMV, presided over the joint meeting, which illustrates the authoritarian and racist tenor of the times, even though the DMV in 1935 under the influence of Knopp and Hasse backed away from this principle. For this material see especially "Jahresbericht der DMV", 43 (1934), pp. 80–82, also [55], Mehrtens [135,137].

[57] As to the "*Akademieerlass*" and its consequences, see e.g. [55, p. 70].

[58] According to a citation from the Blumenthal diary of January 2, 1939: "The DMV crossed me off their lists (president 1924, member of executive board till 1933)".

[59] See Segal [182, pp. 239/40], and the letters between Behnke, Hecke and van der Waerden listed there. In volume 120 of the *Annalen* for 1947–1949 (volume 119 covered 1943–1944) Blumenthal appears with Klein, Hilbert and Hecke as distinguished previous editors. The acting editors of this volume at the time were Behnke, Heinz Hopf, Franz Rellich, Richard Courant, Kurt Reidemeister and van der Waerden.

ing of the RWTH in 1870. The initiative for the belated recognition came from the mathematics group.

3. Aspects of Blumenthal's research

The mathematical work of Otto Blumenthal has received attention in two obituary notices [191,9], but in German. We will consider his main fields of research, focusing on aspects that have become of particular interest during the past three decades.

3.1. Analytical and physical-technological work

Blumenthal was primarily an analyst, above all a complex function theoretician, in the spirit of the classical Göttingen tradition, that also anticipated applications. This tradition was not only defined by Klein and Hilbert, but also by Hermann Minkowski (1864–1909) and Carl Runge (1856–1927). His dissertation {1} under Sommerfeld and Hilbert on Stieltjes continued fraction expansions, referred to above, was already a generalization of the expansion of a function in terms of surface spherical harmonics, a topic to which he later devoted some 10 papers. Thus in {16} he examined an approximate representation for such harmonics for increasing indices—work applied by K. Federhofer [53]—, and in {26} he studied the equilibrium problem for a membrane clamped between two elastic rods using series expansions and eigenvalue determinations, and in {45} the Vianello procedure in determining the static stability of an important beam problem under longitudinal forces. The expansion of functions such as in {1} later became of greater importance in communication theory in the hands of Wilhelm Cauer [60] (1900–1945). Further, a set of his papers {2, 17, 19, 20, 24, 38} is concerned with physical-technological problems. In some of these he acknowledges his Aachen colleagues T. von Kármán, L. Hopf and E. Trefftz for suggestions. In Vol. 3 of his well-known lectures (in 6 Vols.) on technical mechanics—which went into at least 10 editions—August Foeppl [56, p. 113], presents Blumenthal's work on tension trajectories in beams.

The applied mathematical and technological problems that interested Blumenthal at Aachen are also indicated by the doctoral theses he directed. Since Aachen was primarily an engineering school at the time, his students and assistants came from this group; this must be taken into account.

The dissertation "Querstabilität und Seitensteuerung von Flugmaschinen" (Crossstability and side steering of airplanes) by Karl Gehlen [61] (1883–1933) (Printed by R. Oldenbourg, Munich in 1913, 31pp.) of February 1912, which had Prof. Dr. Ing. Reissner as First Referee and Advisor and Blumenthal as Second, solved an open problem on the longitudinal instability of airplanes, a topic which was of basic importance at that time. Recall that two papers of Blumenthal himself {17, 24} are concerned with tensions and stress in airplane wings.

The topic of the dissertation "Über Spannungen in ungleichmässig erwärmten Platten mit Ausblick auf massive Brücken" (On tensions in non-uniformly heated plates with a view to massive bridges) by Ernst Münter (Printed by Emil Ebering, Berlin in 1922, 56pp.) of January 1919 was given to him by Prof. Dr. Ing. Pohl of Berlin's Institute of Technology. The candidate

[60] As to Cauer and his work see the *Festschrift* [132]. Concerning communication theory, in particular modern sampling theory of signal analysis, see [28] and the literature cited there.

[61] Karl (Maria Hubert) Gehlen played such an important role in the German and Dutch aviation industry that he is even mentioned in the "Neue Deutsche Biographie, Duncker and Humboldt, Berlin, 1964, vol. 6, p. 134".

tried to solve the problem using Ritz's method, but without success. Blumenthal, who became Münter's advisor, suggested a different but effective approach.

The dissertation "Über die gewendelte Schale unter besonderer Berücksichtigung der unendlich langen, aussen eingespannten, innen freien Wendelschale" (On the helical shell with special consideration of the infinitely long clamped outside and inside free helical shell) by Fritz Wingerter (Printed by Universitätsverlag, Robert Noske, Borna-Leipzig in 1931, 51pp.) of July 1928 was a topic of Blumenthal's lecture {38} at the DMV meeting in Bad Kissingen, September 1927.

The dissertation [62] "Potentialströmungen in Ventilen" (On potential flows in valves) by Bruno Eck (1899-) of 1924 (Typewritten, 64pp.) has connections to Blumenthal's work on surface spherical harmonics and turbulence {19}.

3.1.1. *Orthogonal polynomials: The Blumenthal–Nevai theorem*

Let us consider one specific theorem from Blumenthal's dissertation {1}, pp. 16–20 which plays a fundamental role in the study of orthogonal polynomials whose zeros are dense in intervals. The most characteristic feature of orthogonal polynomials (other than orthogonality itself) is the fact that every sequence of orthogonal polynomials satisfies a three-term recurrence of a very special form. Written for monic polynomials $\{P_n(x)\}$, this takes the form

$$P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x), \quad n \in \mathbf{N},$$
$$P_{-1}(x) = 0, \quad P_0(x) = 1, \quad c_n \text{ real}, \quad \lambda_{n+1} > 0.$$

According to a result of J. Favard [52] [32, p. 21] (1935), these conditions are necessary and sufficient for $\{P_n(x)\}$ to be an orthogonal sequence with respect to a distribution $d\mu(x)$, namely

$$\int_{-\infty}^{\infty} P_n(x)P_m(x)\, d\mu(x) = k_n \delta_{n,m} \quad (k_n > 0)$$

$\mu(x)$ being increasing, bounded. Now $P_n(x)$ has n real, distinct zeros, denoted by $x_{n,i}$, with $x_{n,1} < x_{n,2} < \cdots < x_{n,n}$, and from the well-known separation theorem, there follows $x_{n+1,i} < x_{n,i} < x_{n+1,i+1}$ for $i = 1, 2, \ldots, n$. Thus $\xi_i = \lim_{n\to\infty} x_{n,i}$ and $\eta_j = \lim_{n\to\infty} x_{n,n-j+1}$ exist (or are $\pm\infty$), with $-\infty \leqslant \xi_1 \leqslant \xi_2 \leqslant \cdots \leqslant \eta_2 \leqslant \eta_1 \leqslant \infty$. In particular, $[\xi_1, \eta_1]$ is the smallest interval containing all of the zeros, $x_{n,i}$, thus the true interval of orthogonality according to J.A. Shohat, also referred to as the "spectral interval". Since $\xi_i \leqslant \xi_{i+1} \leqslant \eta_{j+1} \leqslant \eta_j$, let us set $\sigma = \lim_{i\to\infty} \xi_i$, $\tau = \lim_{j\to\infty} \eta_j$, and $X = \{x_{n,i} : 1 \leqslant i \leqslant n, \quad n = 1, 2, \ldots\}$. Then σ and τ are respectively the smallest and largest limit points of the derived set X', as well as of the spectrum of μ provided the Hamburger moment problem associated with the given sequence is a determined one. Now Blumenthal {1} proved the following result: If $\lim_{n\to\infty} c_n = c$ and $\lim_{n\to\infty} \lambda_n = \lambda > 0$, c and λ being finite, then

$$\sigma = c - 2\sqrt{\lambda}, \quad \tau = c + 2\sqrt{\lambda},$$

and the set X of the zeros of $P_n(x)$ is dense in $[\sigma, \tau]$.

The proof, which is not trivial—it uses a theorem of H. Poincaré on difference equations (see [160]), a fact which Blumenthal especially emphasizes, as well as the Stieltjes-Vitali theorem on uniformly bounded sequences of analytic functions—is sketched in Chihara [32, p. 121]. In

[62] The list of the four doctoral students associated with Blumenthal can also be found in "The Mathematics Genealogy Project" (`http://www.genealogy.ams.org`). However, Eck's dissertation could not be found in Aachen. He regarded himself as a student of T. von Kármán. He was the author of very popular books: His "Technische Strömungslehre" (Springer, 1941) had its ninth edition in two volumes in 1988.

fact, this author seems to have been the first to call the result Blumenthal's theorem on orthogonal polynomials. Geronimus in his appendix to the Russian translation of Szegö's book [196] mentions it ([68, pp. 69, 94]), referring explicitly to Shohat (1934), Akhiezer-Krein (1938) and himself (1957), but not Blumenthal. Szegö refers to Blumenthal's dissertation in general {1}; also Naiman [143] worked in the matter. Máté, Nevai and Van Assche [131] and others have shown that the Blumenthal theorem can be proven from a theorem of Weyl about perturbations of the spectra of self-adjoint operators. Chihara himself obtained an analogue of Blumenthal's theorem for an unbounded case, [63] $c = \infty$, but σ finite with X being dense in $[\sigma, \infty)$, as well as a mild generalization. Ready applications in the area are to the Legendre polynomials, the Chebyshev polynomials of second kind, Jacobi polynomials, polynomials studied by F. Pollaczek [64] (and J. Meixner and W. Hahn in a related context), those by W.A. Al-Salam and L. Carlitz, Askey and Ismail [8], etc. An interesting result for polynomials on analytic Jordan arcs which satisfy a three-term recurrence relation and is connected to a converse of Blumenthal's theorem, was found by Duren [46]. For the operator-theoretic approach see [70,71].

Papers containing extensions or applications of the Blumenthal theorem and which appeared in the past dozen years include Dehn [40], Ifantis and Siafarikas [93], López, et al. [123], Nevai and Zhang [148], Nevai [145], Siafarikas [184], Zhang [211], Lasser and Obermaier [118], Nevai and Totik [147]. In this respect the fine survey papers by Chihara [33,34] are of special interest. There many further results and papers are discussed following up the Blumenthal ideas. Notable are results by P. Nevai and his collaborators. Indeed, the density of the zeros would suggest (but not imply) that the interval $[\sigma, \tau]$ belongs to the spectrum (= support of $d\mu(x)$). The fact that this is, indeed, true follows from an argument of Nevai. It is the Blumenthal–Nevai theorem [65] [148] (Theorem 4.2.14). This is one of their many results. An extension of their results due to Geronimo and Case [67] has significance for applications to scattering theory in physics. In "Asymptotic representation for the Blumenthal–Nevai orthogonal polynomials in the essential spectrum", Spigler and Vianello [192] obtain qualitative information on the asymptotic behavior of the so-called Blumenthal–Nevai class of orthogonal polynomials.

The article "Orthogonal matrix polynomials: zeros and Blumenthal's theorem" by Duran and Lopez–Rodriguez [44] is concerned with an investigation of orthogonal matrix polynomials P_n from their matrix three-term recurrence relation $tP_n(t) = D_{n+1}P_{n+1}(t) + E_n P_n(t) + D_n^* P_{n-1}(t)$, where D_n and E_n are $N \times N$ matrices. The authors deal with Blumenthal's theorem for matrix polynomials, assuming that the recurrence coefficients E_n and D_n converge to limits E and D, the limits being finite Toeplitz matrices. It is shown that the support of the measure μ is an

[63] Blumenthal further asserted that the limit points of the zeros of the polynomials outside the interval $[\sigma, \tau]$ are at most finite. That this assertion is not true was shown by [31]; a concrete counterexample to this effect, connected with the Pollaczek polynomials, was found recently [94].

[64] Felix Pollaczek, born 1892 in Vienna, belonged to the Austrian refugees. He escaped 1938 with his wife on one of last airplanes leaving Prague for Zurich, went to Paris and in November, 1942, when southern France became occupied, he fled from Lyon to the French Alps, and again in July, 1943 to an even more remote mountain spot where the mayor, priest and inhabitants concealed them for a year. At our Charlemagne conference of 1995 our colleague Friedrich Schreiber (died 2000) lectured on Pollaczek; see the article [180]. Mrs. Schreiber kindly told us that Pollaczek's wife, with whom she is still in contact, lives in Paris (in her high nineties). The polynomials due to Pollaczek, Meixner and Hahn are part of the Askey scheme of polynomials; see Tom Kornwinder: `http://www.science.uva.nl/~thk/art/informal.html`.

[65] A referee observed that Paul Nevai [145, pp. 20–24], proved more than just the Blumenthal density theorem. In fact, with Blumenthal's conditions, for every point x in the support of the measure and for every subsequence $\{n_k\}$, there is a sequence of zeros of p_{n_k}, say $\{x_k\}$, that converges to x. In addition, the interval $[\sigma, \tau]$ is in the support of the measure, and the support of the measure has no accumulation points outside this interval; [146].

interval and possibly two sequences of numbers outside this interval accumulating at the end points. If the matrices are not Toeplitz, the problem seems to be open. Zygmunt [212] extended the matrix version of Blumenthal's theorem [66] —a keyword of this article. Let us add the recent paper "Blumenthal's theorem for Laurent orthogonal polynomials" by Ranga and Van Assche [164].

3.1.2. Extensions of Helmholtz's fundamental decomposition theorem

The solutions of many problems of applied mathematics and mathematical physics depend delicately on the applicability of the classical Stokes—Helmholtz decomposition theorem (of 1849 and 1858), called the fundamental theorem in vector analysis by Sommerfeld [189, p. 147] (see also [19]). It states that either every arbitrarily given 3-dimensional vector function $\mathbf{u}(\mathbf{x})$ (subject to some condition of differentiability—letters in boldface designate vectors) can be decomposed into a curl-free vector plus another divergence-free vector (the weak version), or that it can be decomposed into the gradient of a scalar function $\theta(\mathbf{x})$ plus the curl of another vector function $\mathbf{b}(\mathbf{x})$, i.e.,

$$\mathbf{u}(\mathbf{x}) = \nabla\theta + \nabla \times \mathbf{b}.$$

The conventional (simple) proof requires $\mathbf{u}(\mathbf{x})$ or $\nabla \cdot \mathbf{u}$ to be of order

$$\mathbf{u}(\mathbf{x}) = O(|\mathbf{x}|^{-2-\delta}), \quad |\mathbf{x}| \to \infty, \quad \delta > 0 \tag{*}$$

if the region $D \subset \mathbf{R}^3$ under consideration is infinite. Since this order condition is artificial and unduly restrictive from the viewpoint of applications, various authors have attempted to relax such restrictions. Now the proof of Helmholtz's theorem reduces to the existence of a solution to the vector Poisson equation $\nabla^2\mathbf{v}(\mathbf{x}) = \mathbf{u}(\mathbf{x}), \quad \mathbf{x} \in D$, ∇^2 being the Laplacian, namely the Newtonian potential integral

$$\mathbf{v}(\mathbf{x}) = \frac{1}{4\pi}\int_D \frac{\mathbf{u}(\mathbf{y})\, dv}{|\mathbf{x}-\mathbf{y}|}. \tag{**}$$

However, this volume integral may fail to exist if D is unbounded; it however does if the order condition $(*)$ holds; it is actually this part of the proof which requires the restrictive condition. In order to weaken this condition, Blumenthal {9} was the first to employ the Green's function $(1/|\mathbf{y}-\mathbf{x}| - 1/|\mathbf{y}|)$ instead of $1/|\mathbf{y}-\mathbf{x}|$, in which case $\mathbf{u}(\mathbf{x})$ needs only be of order $O(|\mathbf{x}|^{-1-\delta})$ at infinity. Ton Tran-Cong [201] in 1993 refined the Blumenthal approach, extending the process to higher-order terms, showing that this order condition may be replaced by the weaker $\mathbf{u}(\mathbf{x}) = O(|\mathbf{x}|^{\nu})$ as $|\mathbf{x}| \to \infty$ for any fixed ν. Here D can be simply or multiply connected, and this work also extends Helmholtz's theorem into an N-dimensional version, using the calculus of differential forms. Already in 1962 Gurtin [76] had considered a milder weakening of restriction $(*)$. The volume integral may also fail to exist if $\mathbf{u}(\mathbf{x})$ has a singularity at the point $\mathbf{x} = \mathbf{a}$ lying in D; such points are a common and important idealization in applications. Formula $(**)$ clearly holds if $\mathbf{u}(\mathbf{x}) = O(|\mathbf{x}-\mathbf{a}|^{-3-\delta})$ as $|\mathbf{x}| \to \mathbf{a}$, for some $\delta > 0$. Now Gregory [74] in 1996 even showed that all restrictions on the rate of growth of $\mathbf{u}(\mathbf{x})$ as $|\mathbf{x}| \to \infty$, or as singular points are approached, are unnecessary, apart from the fact that the field $\mathbf{u}(\mathbf{x})$ now has to belong to $C^2(D)$ rather than $C^1(D)$.

[66] A referee emphasized that Rakhmanov's theorem [162] should be mentioned in conjunction with the Blumenthal theorem. The main players included P. Nevai, A. Máté, V. Totik, G. López Lagomasino, M. Bello Hernández, A. Duran, S.V. Khrushchev, S.A. Denisov. The referee referred to Barry Simon's recent book [188] in the context of the unit circle.

Consequently all representation theorems dependent upon Helmholtz's theorem are also freed from the sharp restrictions. The proof is based on representing the required **v** as an infinite series, the existence and convergence being the essential difficulty.

All in all, Blumenthal's result, apart from that of Gurtin of 1962, remained until 1993 the least restrictive of the directly derived versions of the Stokes–Helmholtz theorem.

For examples of their use and the significance of the above discussed restriction in elasticity and fluid mechanics see Tran-Cong [201] and the literature cited there. For the use of Blumenthal's theorem in modern density functional theory see Capelle and Gross [30].

Blumenthal's method of accelerating the convergence of the solution of Poisson's equation is similar to the technique now used in analysis to prove Carleman's inequalities see (e.g. [101]). Concerning Blumenthal's result in connection with vacuum electron acceleration, the Lawson–Woodward theorem, etc., see e.g. [203], and in regard to anisotropic media see e.g. [39].

3.2. Function theory

Presumably during his research stay in Paris 1899/1900 Blumenthal was confronted with problems in the theory of entire functions which was developed especially in the hands of Borel [18], Pierre Boutroux (1880–1922), Jacques Hadamard (1865–1963), Ernst Lindelöf (1870–1946), Henri Poincaré (1854–1912), Karl Weierstrass (1815–1897) and Anders Wiman (1865–1959).

3.2.1. The three-circle theorem and extensions

Suppose that $f(z)$ is an analytic or holomorphic function of the complex variable z in the open disc $|z| < R$. Then, as usual,

$$M(r) = M(r, f) = \max_{|z|=r} |f(z)| \quad (0 \leqslant r < R)$$

is called the *maximum modulus of* f. A function which is analytic in the whole complex plane $\mathbf{C}$ is said to be *entire*. The maximum modulus $M(r)$ is of fundamental importance in complex analysis, in particular, for the investigations of entire functions, the classical theorems of the theory of entire functions being for the most part expressible in the form of inequalities involving $M(r)$.

It is easy to see that the maximum modulus $M(r, f)$ is continuous, and the maximum principle shows that it is a steadily increasing function in r, if f is not constant. In 1907 Blumenthal {10} even proved that $\log M(r)$ is a convex function of $\log r$. That means:

Let f be an analytic function in the closed annulus $r_1 \leqslant |z| \leqslant r_2$. Then, for each r with $r_1 < r < r_2$,

$$\log M(r) \leqslant \frac{\log r_2 - \log r}{\log r_2 - \log r_1} \log M(r_1) + \frac{\log r - \log r_1}{\log r_2 - \log r_1} \log M(r_2).$$

In connection with this inequality, Hardy (1877–1947) [84] wrote: "This theorem was discovered independently by Blumenthal {10}, Faber [51] and Hadamard [79]. The first statement of the theorem was due to Hadamard and the first proof to Blumenthal." This inequality, nowadays known as Hadamard's three circle theorem, is a special case of the two constants theorem, discovered independently by the brothers Frithjof Nevanlinna and Rolf Nevanlinna [149] in 1922 and Ostrowski [152] in 1923. The three-circle theorem can be generalized in various directions, in particular, for other metrics and for harmonic and subharmonic functions. It seems to be the first example of the concept of logarithmic convexity. Not only this concept but also the theorem itself enabled Thorin (1912-) in 1948 to establish the so-called Riesz-Thorin convexity theorem concerning the interpolation of linear operators in Banach spaces. For details see e.g. Butzer [27] and the extensive literature cited there.

3.2.2. Blumenthal's maximum curves

Blumenthal {11}, however, established much deeper results than Hadamard's three-circle theorem.

If $f(z)$ is different from the function cz^n, then $|f(z)| = G(r,\theta)$ with $z = re^{i\theta}$ attains its maximum $M(r, f)$ for at most a finite number of values of θ. If on each circle we mark the point corresponding to each such value of θ, we obtain a number of curves, which Blumenthal {10}, {11} called the *maximum curves* of the function $f(z)$. In {11} Blumenthal investigated and discussed the maximum curves very precisely. He proved, for example, that in every closed disc the maximum curves are differentiable at all except possibly a finite number of points. Furthermore, he showed that $M(r)$ is an analytic function of r, except at an isolated number of points $r_1 < r_2 < \cdots < r_n < \cdots$, so that $M(r)$ is represented by distinct analytic functions in the intervals $r_n \geqslant r \geqslant r_{n+1}$. Blumenthal {11} and subsequently Hardy [83] have constructed examples, showing that the maximum curves can actually have discontinuities in the complex plane. By adapting an example of Hardy [83], Tyler [205] has even shown recently that for $r > 4$, the entire function $g(z) = e^{e^{z^2}+2z\sin^2 z}$ takes its maximum modulus only on the real axis, and that $M(r, g) = e^{e^{r^2}+2r\sin^2 r}$ at every point of the positive real axis with isolated points appearing on the negative real axis when $\sin^2 r = 0$. Unfortunately, Tyler makes no mention of Blumenthal {10},{11}, although Hardy [83] had clearly emphasized this significant contribution.

The results of Blumenthal concerning the analytic nature of $M(r)$ were later used (see for example the book of Valiron (1884–1955) [206, pp. 22–27, 64–67 and 125–132]) to establish the existence of proximate orders of entire functions in order to use Phragmén-Lindelöf theorems for the proof of Wiman's [209] classical 1905 theorem, which states that $\limsup_{r\to\infty} m(r, g) = \infty$ for each entire function g of order less than 1/2, where $m(r, g)$ is the minimum modulus of g on the cycle $|z| = r$ (the definition of the order of an entire function is given below). The entire function $h(z) = \cos\sqrt{z}$ of order 1/2 shows that this theorem of Wiman is best possible, since $|\cos\sqrt{r}| \leqslant 1$, and therefore $m(r, h) \leqslant 1$.

In {10} Blumenthal remarks that the above mentioned properties by no means suffice to characterize the function $M(r)$ as the maximum modulus of an analytic or entire function. Furthermore, in {11} Blumenthal raised the following problems and questions (see also Bieberbach [14, p. 127]):

Problem 1 (*Blumenthal 1907*). Determine necessary and sufficient conditions for a real valued function $M(r)$ to be the maximum modulus of an analytic or entire function.

Problem 2 (*Blumenthal 1907*). Let $f(z)$ be analytic in the disc $r = |z| < R$ such that $f(0) = 0$. If ε and η are complex numbers with $|\varepsilon| = |\eta| = 1$, then the analytic functions $f(z)$, $\varepsilon f(\eta z)$, and $\varepsilon\bar{f}(\eta\bar{z})$ have the same maximum modulus $M(r, f)$. Do there exist further analytic or entire functions with the maximum modulus $M(r, f)$?

Conjecture 3 (*Blumenthal 1907*). *There exists a cubic polynomial P with the property that the maximum curves of P have discontinuities.*

In connection with Problem 1 of Blumenthal, in 1951 Hayman [85] points out: "The fact that $M(r)$ need not be given by just one analytic function makes the problem of its characterization for instance for the class of entire functions $f(z)$ very difficult. We shall solve here a simpler problem, namely the local characterization of $M(r)$ near $r = 0$." Following the ideas of Blumenthal {11}, Hayman [85] proved, for example, that if $M(r) = 1 + a_k r^k + \cdots$ with $a_k \neq 0$, then for $k = 1, 2$ there exists a unique function $f(z)$ with $M(r, f) = M(r)$, attaining its maximum modulus for small r on an assigned regular arc through the origin. That this result is not true for $k \geqslant 3$ is also demonstrated. At the end of his article [85], Hayman also presents some related problems.

For the special case of entire functions, Problem 2 can also be found 60 years after Blumenthal in Hayman [87] as Problem 2.15. This time Hayman [87] does not give any hint to Blumenthal {10} {11} nor to Bieberbach [14]. But Hayman has given in [87] the following interesting examples. The functions $(1 - z)e^z$ and $e^{\frac{z^2}{2}}$ have the same value of $M(r)$ for $0 < r < 2$, as do e^{z-z^2} and $e^{z^2+\frac{1}{8}}$ for $r \geqslant 1/4$.

Remark. The second example of Hayman leads us to the following infinite family of such examples. If $a, b > 0$ are real numbers, then let

$$f(z) = e^{az-bz^2} \quad \text{and} \quad g(z) = e^{z^2+\frac{a^2}{8b}}.$$

Now it is readily seen that $M(r, f) = M(r, g)$ for $r \geqslant a/4b$. Thus, we conclude that for every $\varepsilon > 0$, there exist two different entire functions f and g such that $M(r, f) = M(r, g)$ for $r \geqslant \varepsilon$.

In 1986 Jassim and London [100] solved Conjecture 3. They proved that the maximum curves of the polynomial $z^3 + az^2 - z + b$ must have a discontinuity if $b > 0$ and $ab > 1 + 2b$.

Altogether, the reader can appreciate that the theory of maximum curves, introduced by Blumenthal {10},{11} in 1907, has remained an important area of research. Nonetheless there has been a tendency to overlook the pioneering role of Blumenthal. For related questions and solutions, see also [87], Problems 2.16–2.18, [36], Problems 2.16–2.18, and [6], Problems 2.16, 2.18 and 2.49–2.51.

3.2.3. Entire functions

In 1879 Picard (1856–1941) [155] had established his celebrated theorem that an entire function g must assume all values in the complex plane with at most one exception. The well-known entire function e^z, which never assumes the value zero, shows that the theorem of Picard is best possible. The question naturally arose whether anything further could be said about the roots of the equation $g(z) = a$ for different complex values of a.

In the special case that g is a polynomial of degree $n \geqslant 1$, by the fundamental theorem of algebra (first proved by Carl Friedrich Gauß (1777–1855) in 1799), g has n zeros if each zero is counted according to its multiplicity. Since $g(z) - a$ also is a polynomial of degree n, it follows that g takes on every complex value a precisely n times. With the aid of its zeros $a_1, a_2, \ldots, a_n$, the function can be represented in the form

$$g(z) = c(z - a_1)(z - a_2) \ldots (z - a_n),$$

and hence we have the asymptotic equation

$$\lim_{r\to\infty} \frac{M(r, g)}{r^n} = |c|.$$

This shows that the degree n also tells one exactly how rapidly the polynomial g grows as $r \to \infty$ and vice versa. Consequently, there is a significant symmetry between the rate of growth of the polynomial g and the number of all its a-points, also called its *value distribution*.

The complete results on polynomials served as a model for the value distribution theory of entire functions in general. More than a century ago, Borel succeeded in combining and improving results of Picard, Poincaré and Hadamard in such a way that a value distribution theory began to take shape. More precisely, if g is an entire function, then

$$\rho = \rho(g) = \limsup_{r\to\infty} \frac{\log\log M(r, g)}{\log r}$$

is called the *order* of g. Its formal definition was given in 1897 by Borel [17] who realized its central role in the theory of entire functions. For example, the orders of the entire functions $\cos\sqrt{z}$, $\sin z$, e^{z^n} and e^{e^z} are $\frac{1}{2}, 1, n$ and ∞, respectively. Furthermore, let $n(r, a, g)$ denote the number of a-points of an entire function g in the disc $|z| \leqslant r$, i.e. the number of roots of the equation $g(z) = a$, each root being counted according to its multiplicity. The *exponent of convergence*, or *order* $\rho(a, g)$ *of the a-points*, is defined by

$$\rho(a, g) = \rho(a) = \limsup_{r\to\infty} \frac{\log n(r, a, g)}{\log r}.$$

In 1893 Hadamard [78] showed that $\rho(a, g) \leqslant \rho(g)$ for every a. Of course, in Hadamard's inequality $\rho(a, g) < \rho(g)$ is possible, since $n(r, a)$ can even be zero for every r. On the other hand, Picard's theorem says that $n(r, a)$ can vanish identically for one value a only. In 1897 Borel [17] notably generalized Picard's theorem in the case of finite order $\rho(g) < \infty$, by proving $\rho(a, g) = \rho(g)$ for all a with at most one exception and that such an exception can occur only if $\rho(g)$ is a positive integer.

Some years later, Lindelöf [121], Boutroux [20] and Maillet [128] refined and extended this result of Borel for entire functions of finite iterated order, i.e., those that are entire functions g with the property that there exists a positive integer m such that

$$\limsup_{r\to\infty} \frac{\log_{m+1} M(r, g)}{\log r}$$

is finite, where $\log_{m+1} M(r, g) = \log(\log_m M(r, g))$. However, already Borel [17] knew examples of entire functions that have no finite iterated order. Following Borel's [17] indications, in 1907, Blumenthal {10},{12} avoided such growth restrictions by introducing his complicated but useful concept of so called *comparison functions* ("Vergleichsfunktionen"). The major contributions of Blumenthal {10},{12} finally led to a complete generalization of Picard's theorem in the sense of Borel. We present this result in the following simplified form.

Theorem 1 (*Picard–Hadamard–Borel–Blumenthal*). *If g is a non-constant entire function, then $\rho(a, g) = \rho(g)$, with the possible exception of one single value a.*

Consequently, not only polynomials but all non-constant entire functions exhibit a remarkable symmetry in the distribution of their values and, up to one value, the number of a-points is determined by the rate of growth of the function.

In connection with Theorem 1, Blumenthal {10}, pointed out that his methods would also lead to a corresponding theorem for (meromorphic) algebroid functions; this essentially extends some results of the 1905 thesis by Rémoundos [171] on algebroid functions of finite order. However, the proper key for a general value distribution theory was found by Rolf Nevanlinna (1895–1980) in the 1920's. He succeeded in creating a far-reaching value distribution theory for meromorphic functions, in such a way that it contained as a special case the theory of entire functions in an improved form. In 1925 the essentials of what is now called *Nevanlinna Theory* were there to be read in a pure and elegant form [150]. The central results in Nevanlinna's pioneering treatise [150] are the *First* and *Second Main Theorem* of value distribution theory; they lead quite easily to the theorem of Picard–Hadamard–Borel–Blumenthal (see also the textbooks of Hayman [86] and Jank and Volkmann [98]).

To the topic of Section 3.2 there also belong {22}, {25}, {30}, {33}, {39} and {40}.

3.3. Hilbert–Blumenthal modular forms

We now turn to Blumenthal's Habilitation thesis {3},{5}. In Göttingen of the 19th century a central role was played by three directions of research: The theory of Abelian functions developed by Bernhard Riemann (1826–1866) and Weierstrass, already mentioned, the theory of automorphic functions of one variable as founded by Klein and Poincaré and, thirdly, Hilbert's theory of algebraic number fields. His dissertation builds, according to his own remarks, on notes by Hilbert from the years 1893–94 on the action of the modular group Γ_K of a totally real field K of degree n over $\mathbf{Q}$ on the product H^n of n complex upper half planes. It gave a sketchy description of general properties such as properly discontinuous action and fundamental domain but it contained precise information on the construction of modular functions by means of the theta functions. Blumenthal manages to connect these three fields in question. In fact, G. van der Geer writes the following in the first lines of his book [64]: "In a paper in Mathematische Annalen {3, 5} Blumenthal did the first pioneering work in a program outlined by Hilbert with the aim of creating a theory of modular functions of several variables that should be just as important in number theory and geometry as the theory of modular functions of one variable was at the beginning of this century."

Concerning this, Freitag [59] writes: "The Hilbert modular group $\Gamma_K = SL(2, \mathbf{o})$ is the group of all 2×2 matrices M of determinant 1 with coefficients in the ring $\mathbf{o}$ of integers of a totally real number field $K \supset \mathbf{Q}$. This group and the corresponding spaces and functions—the Hilbert modular varieties and Hilbert modular forms—have been the subject of many investigations starting with the Blumenthal papers {3},{5}."

In his own introduction on the subject Garrett [63] adds that three fundamental ideas and methods that arose during the period 1904–1961 must be attributed notably to Blumenthal, Hecke, Siegel, Maass, Klostermann and H. Petersson.

A further motivation can be deduced from efforts to approach Hilbert's 12th problem (explained by him in his lecture at the Int. Congress at Paris in 1900), namely the description of all possible abelian extensions of arbitrary number fields. It was hoped to find functions of several variables that for arbitrary algebraic number fields had a similar meaning as does the exponential function for the rational numbers or Klein's module function j for imaginary-quadratic number fields.

Beginning with Γ_K, by a linear fractional transformation of all matrices conjugate to M, one obtains a discontinuous group of biholomorphic automorphisms of H^n. Blumenthal first constructed an exact fundamental domain for Γ_K. He showed that the Hilbert modular forms, or the Hilbert–Blumenthal modular forms as they have often been called since circa 1978 (see e.g. [165,109,163,73,193,42]), determine an algebraic function field in n indeterminates. In his subsequent paper {6} Blumenthal studied to what extent Hilbert modular forms can be represented in terms of the zero values of theta functions in n variables.

Following the function theoretic investigations of Blumenthal, Hecke, another student of Hilbert, began to develop an arithmetic theory of Hilbert modular forms, work continued especially by Shimura, Klingen and Siegel. For this the reader may consult Freitag [59] and Garrett [63].

Finally, let us mention two newer results. In their article "Algebraische Zyklen auf Hilbert-Blumenthal-Flächen", Harder, Langlands and Rapoport [82] consider two famous conjectures of Tate of 1965, the first asserting and yielding more precisely that $Z(E) = H^2(X \times \overline{\mathbf{Q}}, \mathbf{Q}_l(1))^{\mathrm{Gal}(\overline{\mathbf{Q}}/E)}$ and the second that the dimension of $Z(E)$ is the order of the pole $s = 2$ of the Hasse–Weil zeta-function of X:

$$L^2(s, X_E) = \prod \det(1 - \mathbf{N}\mathfrak{p}^{-s}\Phi_{\mathfrak{p}}^{-1}|\tilde{H}^2(X \times \overline{\mathbf{Q}}, \mathbf{Q}_l))^{-1}.$$

In the foregoing, the Shimura variety associated to the reductive algebraic $\mathbf{Q}$-group $G = R_{K/\mathbf{Q}}GL(2)$ and to a congruence subgroup $\mathscr{K} \subset G(\mathbf{A}_f)$ is denoted by X (which is assumed to be smooth), K being a real quadratic number field. This is a quasiprojective algebraic surface over $\mathbf{Q}$, and the Hilbert–Blumenthal surfaces are the connected components of $X(\mathbf{C})$ of the form $\Gamma \setminus H \times H$, and $\Gamma \subset SL(2, \mathfrak{o}_K)$ a congruence subgroup. E is a finite extension of $\mathbf{Q}$, and the subspace $Z(E) \subset H^2(X \times \overline{\mathbf{Q}}, \mathbf{Q}_l(1))$ is assumed to be spanned by the algebraic cycles. The above product runs over all places $\mathfrak{p}$ of E, and the group $\tilde{H}^2$ is the image of the restriction map $H^2(\tilde{X} \times \overline{\mathbf{Q}}, \mathbf{Q}_l) \to H^2(X \times \overline{\mathbf{Q}}, \mathbf{Q}_l)$ for any smooth compactification $\tilde{X}$ of X; Φ is the Frobenius morphism acting on H^2.

The authors verify this conjecture for the Hilbert–Blumenthal surfaces X and any abelian extension E of $\mathbf{Q}$. In fact, in this case $Z(E)$ is the space $D(E)$ spanned by the Hirzebruch–Zagier cycles and some universal Chern classes.

A year later, Klingenberg [109,110] showed that the proof could be modified so that the result also holds for the nonabelian Tate cycles. Kumar Murty and Ramakrishnan [115] settled the nonabelian case by a quite different method, the Tate cycles being matched with Hodge cycles, this being done via period relations.

Achter and Cunningham [2] in their paper "Isogeny classes of Hilbert–Blumenthal abelian varieties over finite fields" give an explicit formula to measure the size of the isogeny class of a Hilbert–Blumenthal abelian variety over a finite field. Using work of Arthur–Clozel [7] (1989) and the affine Bruhat decomposition they evaluate all the relevant orbital integrals, thereby finding the isogeny class. They begin by recollecting constructions concerning Hilbert–Blumenthal moduli spaces. See also Ellenberg [50].

3.4. Hilbert's biography; conclusion

Blumenthal {44} is also the author of David Hilbert's *Lebensgeschichte*. It indirectly reveals his personal loyalty and broadly based competence. Being Hilbert's first student, [67] he felt deeply

[67] Of Hilbert's 73 doctoral students (with 11,347 descendants) the best may have been Hermann Weyl and Erich Hecke. Weyl received Hilbert's chair in 1930.

indebted to him his whole life and wanted to assist him and work in the interests of the fast growing Göttingen school. He did his best to produce a complete, objective and high standard account of Hilbert's life and work, based on all the details he himself knew or could gather about Hilbert. It again shows Blumenthal as a dedicated and meticulous author, who had command over the many subfields of the areas of mathematics in which Hilbert himself worked. Interestingly, this biographical account of one of Germany's most prestigious mathematicians was published in 1935 after Blumenthal had already been dismissed from office. It was cited thrice in the past ten years alone (see [41,43,37]).

Blumenthal wrote a foreword to a rather important collection of papers dealing with a variety of aspects of the relativity principle, written by H.A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, with comments by A. Sommerfeld. For the sixth edition of this collection see [124].

As to the breadth of his interests let us finally add that Mrs. Gertrud Magnus, the widow of a German pharmacologist whom the Blumenthals met in Utrecht, wrote in 1945 to Magrete Blumenthal [68] that she thought "very highly of him [Blumenthal]; he was always my 'Lexicon' and his knowledge was sharp and exhaustive, he never failed ... . He always discussed in depth with me the material treated in the bible course led by Pastor Bruno Benfey (1891–1962), and could explain everything so fundamentally". In a bible course for 12 Protestant Jews Blumenthal read the texts in the Greek bible, which he always had lying next to the Lutheran version.

In summary, Otto Blumenthal was an outstanding mathematician who contributed extremely generously of his time and energy in the interest of German mathematics, [69] in every way that he was allowed, despite the terrible injustice that he was made to endure. There is a small measure of poetic justice in that his research work has experienced an unusual world-wide revival during the past few decades. He maintained stable working relationships or loyal friendships across a lifetime, just as his professional and institutional loyalties were undiminished by the rejections he experienced with such stoic resignation and bravery. He was a pacifist in the most fundamental sense of the term, and his cultural and linguistic proclivities extended across Europe, building bridges in the interest of international collaboration. It is particularly tragic that such a man could not find a refuge in time of need.

Publications by Otto Blumenthal

{1} Über die Entwicklung einer willkürlichen Funktion nach den Nennern des Kettenbruches für $\int_{-\infty}^{0}(z-\xi)^{-1}\varphi(\xi)\,d\xi$. Dissertation, Dietrich Universitäts-Buchdruckerei, Göttingen 1898, 57pp.

{2} Die Bewegung der Ionen beim Zeeman'schen Phänomen, Zeitschr. f. Mathematik und Physik 45 (1900) 119–136.

{3} Über Modulfunktionen von mehreren Veränderlichen (Erste Hälfte). Math. Ann. 56 (1903) 509–548.

{4} Zum Eliminationsproblem bei analytischen Funktionen mehrerer Veränderlicher, Math. Ann. 57 (1903) 356–368.

[68] Letter in the possession of the Blumenthal descendants.

[69] Blumenthal was often asked to present celebratory lectures. Thus when a memorial tablet was added to the family home of Felix Klein (1849–1925) in Düsseldorf on October 12, 1926, it was Blumenthal who gave the festive lecture in the presence of the Klein family and numerous mathematicians from the Rhineland. When Hilbert turned 70 on January 23, 1932, it was Blumenthal who, together with H. Weyl, L. Bieberbach and H. Hasse, honored him in Göttingen, leading a student torch-light procession in the name of the DMV.

{5} Über Modulfunktionen von mehreren Veränderlichen (Zweite Hälfte). Math. Ann. 58 (1904) 497–527.
{6} Über Thetafunktionen und Modulfunktionen mehrerer Veränderlicher. Jahresber. Deutsch. Math.-Verein. 13 (1904) 120–132.
{7} Bemerkungen zur Theorie der automorphen Funktionen, Gött. Nachr. (1904) 92–97.
{8} Abelsche Funktionen und Modulfunktionen mehrerer Veränderlicher. Verh. Naturf. Ges. Cassel 2 (1904) 21.
{9} Über die Zerlegung unendlicher Vektorfelder, Math. Ann 61 (1905) 235–250.
{10} Über ganze transzendente Funktionen. Jahresber. Deutsch. Math.-Verein. 16 (1907) 97–109.
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{12} Principes de la théorie des fonctions entières d'ordre infini, Paris: Gauthier–Villars 1910, 150pp.
{13} H. Minkowski, Fortschritte der Mathematischen Wissenschaften in Monographien. Zwei Abhandlungen über die Grundgleichung der Elektrodynamik. Mit einem Einführungswort von Otto Blumenthal, Ed. Otto Blumenthal, B.G. Teubner, Leipzig, gr. 8 (1910) p. 82.
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{15} Bemerkungen über die Singularitäten analytischer Funktionen mehrerer Veränderlicher, "Festschrift Heinrich Weber zu seinem siebzigsten Geburtstag am 5. März 1912", Leipzig, B.G. Teubner 1912, 11–22. (Reprint: Chelsea Publishing Co., New York, 1971. viii+500pp.)
{16} Über asymptotische Integration linearer Differentialgleichungen, mit Anwendung auf eine asymptotische Theorie der Kugelfunktionen. Arch. der Math. u. Physik (3) 19 (1912) 136–174.
{17} Über die Druckverteilung längs Joukowskischer Tragflächen, Zeitschr. f. Flugtechnik und Motorluftschiffahrt 4 (1913) 125–130.
{18} Einfache Beispiele ungleichmässig konvergenter Reihen, Annales da Academia Polytechnica do Porto 8 (1913) 68–73.
{19} Zum Turbulenzproblem, Sitzungsberichte d. Bayerischen Akademie der Wissenschaften (1913) 563–595.
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{31} Bemerkung zu der Arbeit des Herrn Popoff "Über die Gewinnung summierbarer Potenzreihen aus summierbaren Fourier-Reihen", Math. Ann. 89 (1923) 126–129.
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