Assessment of thermal performance of semicircular fin under forced air convection: application to air-preheater

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Abstract

The performance of semi-circular fins subjected to forced convection has been studied analytically. The expression for its efficiency has been derived by solving a two-dimensional thermal energy balance equation and the computed efficiency has been compared with an equal volume circular fin. Under the operating conditions studied, the thermal efficiency of the semi-circular fin exceeded that of its circular counterpart. The effects of various parameters on the heat transfer coefficient and the exit air temperature have been determined for semi-circular finned air preheater. The heat transfer coefficient under forced air convection has been found to increase with an increment of the air velocity and reduction in the tube pitch, pipe size and fin spacing. The extent of air-preheats increases with an increase in pipe size and decrease in fin spacing, air velocity and tube pitch. Due to higher thermal performance of semi-circular fins compared to circular fins under identical conditions, improved energy efficiency can be achieved in air pre-heater.
Nomenclature

\[ B \]  thickness of semi-circular fin  \\
\[ CF \]  circular fin  \\
\[ c_p \]  specific heat capacity of air  \\
\[ d \]  pipe diameter  \\
\[ f \]  no. of semi-circular fins at a cross section  \\
\[ h \]  heat transfer coefficient of air  \\
\[ I \]  modified Bessel’s function of first kind  \\
\[ K \]  modified Bessel’s function of second kind  \\
\[ k_{air} \]  thermal conductivity of air  \\
\[ k_f \]  thermal conductivity of fin  \\
\[ m_{air} \]  mass flow rate of air  \\
\[ Pr \]  Prandtl Number  \\
\[ r \]  radius of semi-circular fin  \\
\[ Re \]  Reynold’s Number  \\
\[ R_1 \]  radius of pipe  \\
\[ R_2 \]  radius of circular fin  \\
\[ SF \]  semi-circular fin  \\
\[ S_n \]  tube pitch  \\
\[ T \]  temperature at any point on the fin  \\
\[ T_a \]  ambient air temperature  \\
\[ T_{a1} \]  inlet air temperature  \\
\[ T_{a2} \]  exit air temperature  \\
\[ T_b \]  base temperature of fin  \\
\[ u_{max} \]  maximum air velocity between two rows of tubes  \\
\[ u_\infty \]  approach air velocity  \\
\[ \nu_a \]  kinematic viscosity of air  \\
\[ \mu \]  viscosity of air

1. Introduction

Air is a primary component in furnaces and boilers. In each of these equipments, the ambient air needs to be heated up to very high temperatures. Preheating the incoming air largely improves the thermal efficiency of the system, thereby increasing the energy savings of the industry and results in lower operating costs. In fact, every 22°C rise in combustion air temperature increases the boiler efficiency by nearly 1%. Heat exchangers can be used to recover the heat from various processes to preheat the air. However, the heat transfer coefficient of air is low, and hence, fins or extended surfaces are used to enhance the heat transfer. It is a common industrial practice to utilize the heat of exhaust gases or flue gases and process steam to preheat ambient air. Improvisations in the design of existing heat exchangers can lead to better thermal efficiencies and cost savings rendering economical operations. A key feature in this aspect is improvisations in the design of the extended surfaces or fins.

Rectangular and annular fins are most commonly used in heat exchangers [1]. The analysis of semi-circular fins (SF) was first conceived by Chakraborty and Sirkar [2]. They made a comparison between circular fins (CF) and SF circumscribing circular pipes. They considered the case of natural convection and concluded that when the temperature difference between the ambient and the base of the fin is not too large, and the numbers of SF at a
cross-section < 7, SF show better efficiency than circular fins for large pipes. Khaled [3] modelled and analyzed a system of “hairy fins”. These refer to a system of primary rectangular fins with numerous slender rods, that can be conceived of as secondary fins. It was demonstrated that this system can be more effective in terms of heat transfer as compared to a rectangular fin under specific conditions. The effect of size of the rods, their thermal conductivity and the convection coefficient on the heat transfer rate was studied. The author claimed that the increase in heat transfer that is obtained in this system is significant and can be incorporated in thermal systems. Kundu and Das [4] analyzed elliptic fins with a constant base temperature and heat transfer to the surroundings by convection. They found that an elliptic fin competes well with an eccentric annular fin; and under certain circumstances performs better than eccentric annular fins. Heat transfer from finned heat exchangers subjected to forced convection has been studied by Chen and Hsu [5]. Vertical annular circular fins with a non uniform heat transfer coefficient were considered in this study. The authors calculated the heat transfer coefficient and efficiency for a number of operating conditions and their results show that the heat transfer coefficient increases with the air velocity and the fin spacing. Sapkal et al. [6] carried out an optimization of air preheater design and evaluated the effect of various parameters for an inline tube arrangement. The performance of the system was analyzed with the help of Computational Fluid Dynamics (CFD). Yodrak et al. [7] investigated a heat pipe air preheater system using a combination of theoretical modelling and experimental observations. It was reported that the heat transfer rates in heat pipe air preheater systems increased on increasing the hot gas temperature and tube diameter. Experimental studies on heat transfer rates from an array of pin fins subjected to forced air convection was conducted by Tahat et al. [8]. They determined the optimal spacing of the fins and found the dependence of heat transfer on the Reynold’s No., Nusselt No. and the spacing between fins. Kobus and Oshio [9] theoretically modelled a vertical pin fin system subjected to both free and forced convection. The model predicted the effect of various parameters on the thermal resistance of the heat sink. Experimental validation of the model was also reported.

To the authors’ knowledge, there has been no previous work on the application of SF in air preheaters. The present study analyzes the thermal performance of SF circumscribing circular pipes and their applications in air preheaters. First, fins at one cross-section of one tube have been considered. A mathematical model has been proposed for evaluating the performance of SF, subjected to forced convection. Under such conditions, the heat transfer coefficient is a function of the Reynolds number and Prandtl number. The efficiency of a semicircular fin has been calculated and compared to that of a circular fin, keeping the same material volume for fabrication of the fins. The effect of various geometrical and physical factors – pipe size, air velocity, thermal conductivity of fin, number of SF and base temperature on the thermal efficiency have been analyzed. The SFs have been subsequently implemented in the design of air-preheaters with an inline arrangement of tubes. The influence of various parameters like air velocity, thermal conductivity of the fin material, pipe size, fin spacing and tube pitch have been studied. In the present study, in order to investigate the effect of thermal conductivity on fin efficiency, two alloys – one of aluminium and copper [10], and the other of magnesium, zinc and manganese [11] have been considered. Both these alloys have high thermal conductivities, and changing the compositions of the alloys result in different thermal conductivities.

2. Mathematical model

2.1. Fins at one cross-section on a single pipe

A steady state analysis has been carried out on semi-circular fins circumscribing a circular pipe at a single cross-section. The two-dimensional thermal energy balance is based on the following assumptions:
(i) The temperature across a fin is a function of \( x \) and \( y \), i.e., the temperature distribution is two-dimensional. The temperature gradient along the \( z \) direction is considered to be negligible as the thickness of the fin is very small.

(ii) No heat is lost from the tip of the fin, i.e., the tip is considered to be insulated. In addition, heat loss from the lateral surface of the fin is neglected as thin fins are considered.
(iii) The thermal conductivity of the fin material is constant. The value has been taken as the thermal conductivity of the material at the temperature corresponding to the base temperature of the fin.

(iv) There is no thermal contact resistance between the wall of the pipe and the fin.

(v) Heat is lost to the surroundings only by means of convection. Any form of heat loss by radiation is neglected.

(vi) Curvature effects at the base of the fin are neglected. The base is considered to be straight. This assumption is fairly accurate as the dimension of the fin is much smaller compared to the pipe size.

(vii) The heat transfer coefficient of the ambient air is a function of the air velocity, and hence the Reynolds number. It is also a function of the Prandtl number.

The arrangement of the fins around the pipe has been shown in a previous paper [2].

Figure 1 shows the control volume over which we make a thermal energy balance. Such a treatment gives us the partial differential equation

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{2h}{k_f B} (T - T_a) = 0
\]  

(1)

The heat transfer coefficient is related to the Reynolds’s No. And Prandtl No. by the relation

\[
\frac{hd}{k_{air}} = C \text{Re}^\alpha \text{Pr}^{\beta/2}
\]

(2)
The values of the constants $C$ and $n$ in Eq. 2 are obtained from [12].

The solution of the partial differential equation (1) is assumed to be a polynomial of the form

$$T - T_a = a_1x^3 + a_2x^2 + c_1xy + c_2x + e_1x^2y^2 + e_2$$

subject to the boundary conditions

(i) $x = 0, -r \leq y \leq r, T = T_b$
(ii) $x = 0, y = r, \frac{dT}{dx} = 0$
(iii) $x = 0, y = r, \frac{dT}{dy} = 0$
(iv) $x = r, y = 0, \frac{dT}{dx} = 0$
(v) $x = r, y = 0, \frac{dT}{dy} = 0$

Applying the above boundary conditions on Eq. 5, we get the temperature profile as

$$\frac{T - T_a}{T_b - T_a} = 1 + \frac{m^2x^3}{3r} - \frac{m^2x^2}{2} - \frac{m^2}{2} \left( \frac{1}{r^2} + \frac{m^2}{6} \right) x^2y^2$$

where

$$m = \sqrt{\frac{2hr^2}{k_fB}}$$

Once the temperature profile has been obtained, the expression for the efficiency can be determined. The thermal efficiency of a fin is defined as the ratio of the actual amount of heat transferred by the fin to the amount of heat that would have been transferred by the same fin, if the temperature throughout the fin were equal to its base temperature.

The efficiency is calculated as:
For the temperature profile obtained, the expression for the efficiency is

\[ \eta_{\text{semi}} = 1 - 0.0044m^2 - 0.0347m^4 \]  

For the same conditions, the efficiency for a circular fin is given by [13]

\[ \eta_{\text{circular}} = \frac{2R_1}{m_c(2R_2^2 - R_1^2)} \left( I_0(m_c R_1) K_1(m_c R_2) - K_0(m_c R_2) I_1(m_c R_1) \right) + \frac{I_1(m_c R_2) K_0(m_c R_1)}{I_0(m_c R_1) K_1(m_c R_2)} \]  

\[ m_c = \sqrt{\frac{2h}{k_f B}} \]  

The volume of the circular fin is equal to the volume of the semi-circular fin. For this constraint to be valid, the following equation is to be satisfied [2].

\[ R_2^2 = R_1^2 + \frac{fr^2}{2} \]  

\[ r = R_1 \sin \left( \frac{180}{f} \right) \]  

2.2. Stack of tubes used in air preheater systems

After finding out the thermal performance of SF at a single cross-section, circumscribing a single pipe, the application and performance of such fins in air preheater systems is evaluated. Air preheater systems consist of a stack of finned tubes, over which air flows. The hot fluid which heats up the air usually flows through the tube side. Flue gases and process steam are commonly used heating fluids. In our study, we have taken condensing steam passing through the tube side as the heating fluid; and have considered inline arrangement of the tubes. It is considered that the air flows in a cross flow pattern over the tubes.

For the system of a stack of tubes, the equation relating the \( h_d \), \( Re \) and \( Pr \) is given by [12], [14]

\[ \frac{h_d}{k_{air}} = C \left( Re_{\text{max}} \right)^n \left( Pr \right)^{0.36} \]
\[ \text{Re}_{\text{max}} = \frac{u_{\text{max}} d}{v_a} \]  
\[ u_{\text{max}} = u_\infty \left( \frac{S_n}{S_n - d - 2r} \right) \]  

The values of \( C \) and \( n \) are taken from [12], [14].

The heat of the hot fluid will be transferred to the ambient air through the exposed fin surface, as well as that part of the pipe which is unfinned. For the general case, where there are \( m_1 \) rows and \( m_2 \) columns of tubes in the stack, and \( N \) number of fins per meter of the pipe, the total amount of heat that is lost by the hot fluid is given by

\[ q = m_1 m_2 h(\eta_{\text{semi}} (2\pi r^2 f) N + \pi d (1 - NB))(T_b - T_a) \]  

Assuming that there are no heat losses in the system, the heat lost by the hot fluid will be taken up by the ambient air, the heat transfer can be written as

\[ q = m_{\text{air}} c_p (T_{a2} - T_{a1}) \]  

Equations (17) and (18) can be solved to yield the value of the exit air temperature. Its expression comes out to be

\[ T_{a2} = \frac{AT_b + (m_{\text{air}} c_p - 0.5 A) T_{a1}}{m_{\text{air}} c_p + 0.5 A} \]

\[ A = m_1 m_2 h(\eta_{\text{semi}} (2\pi r^2 f) N + \pi d (1 - NB)) \]

Fig. 2 : Schematic of the inline finned tube arrangement in the air preheater
3. Results and Discussion

In the present study, Schedule 40 pipes with nominal diameters ranging from 25 mm to 90 mm have been considered [6], [7]. The ambient air temperature is taken as 25°C.

A comparative study for SF and CF has been presented in this section. For a given pipe size, the volume of fin material used for both SF and CF is same. The radii of the SF and CF for the range of pipe size considered have been presented in Table 1. The two cases which were modelled, i.e., fins at one cross-section of the pipe, and fins on a stack of tubes are now elucidated separately.

3.1 Fins at a single cross-section

Here, the influence of a number of geometrical and physical parameters on the fin efficiency has been studied.

3.1.1 Effect of number of semi-circular fins

Table 1 indicates that radii of both SF and CF decrease as the number of SF at a cross-section is increased keeping the volume of fin material unchanged. The effect of variation in number of semicircular fin on thermal efficiency is depicted in Fig. 3(a). As the number of SF at a particular cross-section increases from 4 to 10 (keeping all other parameters constant); evidently, the fin efficiency is found to increase for both circular and semicircular geometries. This can be ascribed to the fact that, increasing the number of SF at particular cross-section results in a smaller radius of each fin (Table 1); which means that the conductive resistance offered by the fin decreases; accordingly, the temperature drop across the fin gets reduced. In other words, the temperature within the fin remains closer to the base temperature; and thus, enhancing the fin efficiency. Notably, efficiency of the SF is more than its circular counterpart, because in every case, the radius of each SF is less than the equivalent CF.

<table>
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<tr>
<th>Nominal pipe size (m)</th>
<th>Pipe radius, $R_1$ (m)</th>
<th>$r$ (m)</th>
<th>$R_2$ (m)</th>
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<td>0.0359</td>
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</tr>
</tbody>
</table>

3.1.2 Effect of pipe size

Fig. 3(b) shows that the efficiency of both SF and CF decreases with the increase in pipe size for any given number of SF. When the pipe size is increased, keeping the number of SF at a particular cross-section constant, the radius of each SF and the equivalent CF both increase (Table 1). Thus, the conductive resistance increases, leading
Fig. 3: Influence of factors on thermal efficiency - A comparison between semi-circular and circular fins

(a) Effect of no. of semi circular fins

(b) Effect of pipe size

(c) Effect of air velocity

(d) Effect of base temperature

(e) Effect of thermal conductivity
to a decrease in the efficiency. Since the radius of a semi-circular fin is smaller than the circular fin, its efficiency is more.

3.1.3 Effect of air velocity

Fig. 3(c) shows the effect of increasing the air velocity from 1.5 ms\(^{-1}\) to 3 ms\(^{-1}\) on the fin efficiency. The thermal efficiency of both SF and CF is found to decrease on increasing the air velocity. Increasing the velocity of incoming air increases the heat transfer coefficient of air. This means that more heat will be lost from the fin surface. In other words, the fin will become cooler as the air velocity is increased. Thus, the temperature inside the fin no longer remains closer to the base temperature. Hence, the efficiency reduces.

3.1.4 Effect of base temperature

Fig. 3(d) shows the influence of base temperature on the efficiency. The efficiency marginally increases when base temperature is increased from 150\(^{\circ}\)C to 200\(^{\circ}\)C. Changing the base temperature changes the thermal conductivity of the fin. The fin material chosen (an alloy of aluminium and copper) \[10\] has a very small value of thermal temperature coefficient. So, the thermal conductivity changes marginally with temperature. Hence, no significant change in efficiency is observed when the base temperature is changed.

3.1.5 Effect of thermal conductivity of fin

Fig. 3(e) shows the effect of increasing the thermal conductivity of the fin material on its efficiency. As is evident from the figure, the thermal efficiency increases when the thermal conductivity of the fin material increases. The thermal conductivity of the fin changes when the material of the fin changes. In this study, an alloy of aluminium and copper has been taken as the material of construction of the fin. The effect of thermal conductivity has been studied by altering the composition of each element in the alloy \[10\]. When a material of high thermal conductivity is used, the conductive resistance of the fin decreases. Thus the temperature inside the fin is closer to the base temperature. Hence, the fin efficiency increases.

3.2 Stack of tubes

For the air preheater system, tubes with 4 columns and 8 rows have been analyzed. In case of stacks having less than 20 columns of tubes, the value of \(h\) is multiplied by a factor. The values of the factor for different cases are given by \[12\] and \[14\].

The influence of a number of geometrical and physical parameters on the heat transfer coefficient and the exit air temperature has been studied.

3.2.1 Effect of fin spacing

Fig. 4(a) depicts the effect of fin spacing on heat transfer coefficient. The coefficient decreases very slowly as the fin spacing is increased. If fins are placed closer to one another, the local velocity of air between the fins will increase marginally due to the lower area available for its flow. However, since the spacing is not changed by a drastic amount, the increase in heat transfer coefficient is not significant.

The effect of fin spacing on the exit air temperature has been shown in Fig. 5(a). Clearly, the exit temperature decreases as the distance between two fins increase, provided all other parameters are constant. This may be attributed to the fact that the number of fins over a unit length increases as the fin spacing decreases. As a result, the area available for heat transfer increases. This leads to increased values of the exit air temperature. The figure shows that as the fin spacing is decreased from 10 mm to 4 mm, the exit air temperature increases from 36.85\(^{\circ}\)C to 45.31\(^{\circ}\)C for SF; and from 28.95\(^{\circ}\)C to 31.91\(^{\circ}\)C.
Fig. 4: Influence of factors on heat transfer coefficient (Wm⁻²K⁻¹) – A comparison between SF and CF
Fig 5.: Influence of factors on exit air temperature (°C) – A comparison between SF and CF
3.2.2 Effect of tube pitch

The effect of tube pitch on the heat transfer coefficient is exhibited in Fig. 4(b). Increasing the tube pitch has a negative impact on the transfer coefficient. A smaller tube pitch would increase the maximum velocity of air flowing between the tube stacks [as evident from equation (16)]; thus, augmenting the heat transfer coefficient appreciably.

In Fig. 5(b), the effect of tube pitch on the exit air temperature has been depicted. As the tube pitch is increased from 0.2 m to 0.45 m, the exit temperature decreases from 101.84°C to 59.24°C for SF and from 48.81°C to 34.76°C for CF. A lower value of tube pitch means that tubes are placed in close proximity of one another. This implies that there will be more number of tubes over a unit cross-section. Thus, the available heat transfer area increases. Also, since the heat transfer coefficient increases on reducing the tube pitch, the amount of heat transfer from the fins gets enhanced; hence, rendering higher exit air temperature.

3.2.3 Effect of air velocity

The heat transfer coefficient increases with increase of air velocity. It increases from 50.45 Wm$^{-2}$K$^{-1}$ to 78.08 Wm$^{-2}$K$^{-1}$ corresponding to an enhancement in air velocity from 1.5 ms$^{-1}$ to 3 ms$^{-1}$. This result corroborates well with the findings of Chen and Hsu [5].

A plot of exit air temperature against air velocity is provided in Fig. 5(c). The air velocity is increased from 1.5 ms$^{-1}$ to 3 ms$^{-1}$. The exit air temperature for the SF decreases from 47.02°C to 42.27°C; and from 31.93°C to 30.29°C for the CF. The reason for such an observation is that the contact time of hot and cold stream decreases when the velocity of incoming air increases. In other words, in a given time, the mass of air to be heated increases when the air velocity increases. On the other hand, the heat transfer coefficient increases when the air velocity increases. This tends to increase the heat transfer rate. However, the decrease in the time of contact between the two streams overshadows the effect of increased heat transfer coefficient.

3.2.4 Effect of pipe size

The heat transfer coefficient is found to decrease as the nominal diameter of the pipe increases. This trend is displayed in Fig. 4(d). The coefficient falls from 55.79 Wm$^{-2}$K$^{-1}$ to 47.84 Wm$^{-2}$K$^{-1}$ as the pipe size increases from 25mm to 90mm.

Fig. 5(d) demonstrates how the exit air temperature changes on changing the size of the pipe. When the nominal diameter of the pipe is increased from 0.25 m to 0.9 m, the exit temperature of the air increases from 41.83°C to 121.89°C for SF. When CF is used, the exit temperature increases from 30.94°C to 53.45°C. The radius of both SF and CF increase when the pipe size is increased (Table 1). This increases the surface area to a large extent, and hence, increases the exit air temperature. Notably, although the thermal efficiency and heat transfer coefficient decrease on increasing the pipe size, the effect of increased surface area is large enough to increase the overall rate of heat transfer. In every case, the surface area for the SF is much larger, almost twice than that for the CF. Hence, the exit air temperature is always more for SF.

3.2.5 Effect of thermal conductivity

Although the heat transfer coefficient is independent of the thermal conductivity of the fin material. But this parameter has an effect on the exit air temperature as shown in Fig. 5(e). The thermal conductivity values have been taken from [10] and [11]. The exit air temperature for both the fins has been computed to increase with an increase in material thermal conductivity. In practice, the thermal conductivity of the material makes an appreciable difference to the exit temperature. A higher value of this property gives a higher value of the exit temperature. For the SF, the exit air temperature increases from 102.34°C to 103.73°C as the thermal conductivity increases from
89.22 Wm⁻¹K⁻¹ to 127.394 Wm⁻¹K⁻¹.

4. Conclusion

In this study, the semi-circular fin has been analyzed analytically. The thermal efficiency for such a fin has been evaluated; and the effect of some thermo-physical properties on the efficiency has been studied. The application of such fins in air preheaters has been proposed. For such an application, the effect of some geometrical and physical properties on the exit air temperature has been assessed.

The efficiency of the semi-circular fin is found to be greater than the circular fin, under the conditions studied. As such, the thermal efficiency of the fin increases when the number of semi-circular fins at a cross-section increase and the thermal conductivity of the fin material increases. The efficiency increases marginally on increasing the base temperature. The thermal performance decreases on increasing the pipe size and the air velocity.

The use of semi-circular fins in air preheater always leads to a higher preheat value of the air, when both fins operate under the same conditions. This is because the surface area of a semi-circular fin is almost twice than that of a circular fin, when the same volume of fin material is used. Use of higher pipe sizes, lower tube pitch and lower fin spacing increases the exit temperature of air, thus increasing the thermal performance of the system. Depending on the geometrical and physical conditions used, the energy savings that can be achieved employing semi-circular fins is around 149.35 – 1081.20 kW.

5. References
