Physics Letters B 727 (2013) 136–140

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

U-spin breaking in CP asymmetries in B decays

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ARTICLE INFO

Article history:
Received 21 August 2013
Received in revised form 25 September 2013
Accepted 26 September 2013
Available online 17 October 2013
Editor: G.F. Giudice

ABSTRACT

U-spin symmetry predicts equal CP rate asymmetries with opposite signs in pairs of $\Delta S = 0$ and $\Delta S = 1$ B meson decays in which initial and final states are related by U-spin reflection. Of particular interest are six decay modes to final states with pairs of charged pions or kaons, including $B_s \to \pi^+K^-$ and $B_s \to K^+K^-$ for which asymmetries have been reported recently by the LHCb collaboration. After reviewing the current status of these predictions, highlighted by the precision of a relation between asymmetries in $B_s \to \pi^+K^-$ and $B^0 \to K^+\pi^-$, we perform a perturbative study of U-spin breaking corrections, searching for relations among asymmetries which hold to first order. No such relation is found in these six decays, in two-body decays involving a neutral kaon, and in three-body B^+ decays to charged pions and kaons.

1. Introduction

Charmless hadronic B decays provide valuable tests for the pattern of CP violation in the Cabibbo–Kobayashi–Maskawa (CKM) framework. Methods using isospin symmetry of strong interactions have been developed [1,2] and applied very successfully to a large amount of data [3,4] testing the CKM framework to a high level of precision. Flavor SU(3) relating B decay amplitudes [5,6] is much richer than isospin alone. However, it involves symmetry-breaking effects introducing *ab initio* unknown SU(3)-breaking parameters into the analyses [7]. These parameters have been studied using experimental data [8–13].

A particularly useful SU(2) subgroup of flavor SU(3) is U-spin [14], under which the quark pair (d, s) behaves like a doublet while the u quark and heavier quarks are singlets. The strangeness-conserving and strangeness-changing parts of the effective weak Hamiltonian responsible for B meson decays transform like d ("up") and s ("down") components of a U-spin doublet operator. This and unitarity of the CKM matrix [15], $\mathrm{Im}(V^*_{ub}V_{us}V_{cb}V^*_{cs}) = -\mathrm{Im}(V^*_{ub}V_{ud}V_{cb}V^*_{cd})$, have led to the following very simple and powerful general U-spin prediction phrased as a theorem [16]: *CP rate differences in pairs of decay processes in which both initial and final states are obtained from each other by a U-spin reflection, $U_r : d \leftrightarrow s$, are equal in magnitude and have opposite signs.* A dozen processes involving B meson decays to two pseudoscalars, divided into half a dozen U-spin pairs obeying this theorem, were listed in Ref. [16]. Other decays involving one or two vector mesons in the final state have been discussed in the framework of U-spin in Refs. [16] and [17]. Well-known examples, which have been studied extensively by experiments, are the pairs [16,18,19] ($B^0 \to K^+\pi^-, B_s \to \pi^+K^-$) and [20] ($B^0 \to \pi^+\pi^-, B_s \to K^+K^-$) and two pairs of three-body B^+ decays [21], ($B^+ \to \pi^+\pi^+\pi^-, B^+ \to K^+K^+K^-$), ($B^+ \to \pi^+K^+K^-, B^+ \to K^+\pi^+\pi^-$).

Symmetry-breaking corrections in U-spin relations between CP asymmetries have been discussed in [22] under various theoretical assumptions including factorization. A general analysis of U-spin breaking in decays including neutral vector mesons in the final state has been presented in [23], with specific applications to B decays into states involving a charm meson or a charmonium state.

Typical U-spin breaking corrections estimated by $\sim (m_s - m_d)/\Lambda_{\mathrm{QCD}}$ or $f_K/f_\pi - 1$ are of order 20–30%. They may be assumed to be treated perturbatively in hadronic matrix elements for B decays to energetic two-body final states. Ideally, one would seek cases in which first order U-spin breaking corrections vanish or cancel, leaving second order corrections which are expected to be small at a level of 5%. A quantity which has been shown recently to vanish in the U-spin symmetry limit and to first order in U-spin breaking, leaving only second order U-spin breaking corrections, is the D^0–$\bar{D}^0$ mixing amplitude [24]. This property had been shown previously to follow from a broader assumption of flavor SU(3) [25]. Another case of SU(3) symmetry, in which first order SU(3) breaking corrections have been shown to be further suppressed by small quantities, has been studied several years ago in a sum rule involving decay rates for $B \to K\pi$ and $B \to K\eta^{(\prime)}$ [26]. A U-spin behavior similar to the one exhibited by D^0–$\bar{D}^0$ mixing may apply to certain U-spin relations among CP asymmetries in B decays. The purpose of this Letter is to search for such relations.

2. $B^0 \to K^+\pi^-$, $B_s \to \pi^+K^-$ and U-spin related decays

In order to motivate our study we consider first the rather advanced experimental situation of the very early U-spin prediction [16,18] for the ratio of asymmetries $A_{CP}(B_s \to \pi^+K^-)/A_{CP}(B^0 \to$

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.09.056

Table 1
Branching fractions and direct CP asymmetries for B^0 and B_s decays to pairs involving a charged pion or kaon. Data are taken from Ref. [4] unless quoted otherwise.

Decay mode	Branching fraction (10^{-6})	Direct CP asymmetry
$B^0 \to K^+\pi^-$	$19.57^{+0.53}_{-0.52}$	-0.082 ± 0.006
$B_s \to \pi^+K^-$	5.4 ± 0.6	0.26 ± 0.04
$B^0 \to \pi^+\pi^-$	5.10 ± 0.19	0.31 ± 0.05[a]
$B_s \to K^+K^-$	24.5 ± 1.8	$-0.14 \pm 0.11 \pm 0.03$ [27]
$B^0 \to K^+K^-$	0.12 ± 0.05	–
$B_s \to \pi^+\pi^-$	0.73 ± 0.14	–

[a] World-averaged value includes $A_{CP} = 0.38 \pm 0.15 \pm 0.02$ from Ref. [27].

$K^+\pi^-$). Denoting CP rate differences by $\Delta\Gamma(B \to f) \equiv \Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)$, the U-spin theorem quoted in the introduction predicts

$$\Delta\Gamma\big(B_s \to \pi^+K^-\big) = -\Delta\Gamma\big(B^0 \to K^+\pi^-\big), \tag{1}$$

or

$$\frac{A_{CP}(B_s \to \pi^+K^-)}{A_{CP}(B^0 \to K^+\pi^-)} = -\frac{\tau(B_s)}{\tau(B^0)}\frac{\mathcal{B}(B^0 \to K^+\pi^-)}{\mathcal{B}(B_s \to \pi^+K^-)}. \tag{2}$$

That is, the ratio of B_s and B^0 decay asymmetries is predicted to be negative and equal in magnitude to the inverse ratio of corresponding decay rates.

Branching fractions and direct CP asymmetries, taken from Refs. [4] and [27], are given in Table 1 for all B and B_s decays to pairs involving a charged pion or kaon. (We use the standard convention, $A_{CP}(B \to f) \equiv [\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)]/[\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)]$.) Several of these measurements have been recently improved substantially by the LHCb collaboration [28,29]. For the ratio of B_s and B^0 lifetimes we will take the value [4] $\tau(B_s)/\tau(B^0) = 0.998 \pm 0.009$.

Using the values in Table 1 we calculate

$$-\frac{\tau(B_s)}{\tau(B^0)}\frac{\mathcal{B}(B^0 \to K^+\pi^-)}{\mathcal{B}(B_s \to \pi^+K^-)} = -3.62 \pm 0.41 \tag{3}$$

for the one side of (2) and

$$\frac{A_{CP}(B_s \to \pi^+K^-)}{A_{CP}(B^0 \to K^+\pi^-)} = -3.17 \pm 0.54 \tag{4}$$

for the other. The ratio of asymmetries is negative and larger than one around 3–4, consistent with the ratio of decay rates (3). Turning the argument around, one might have used the CP asymmetry in $B^0 \to K^+\pi^-$ to predict $A_{CP}(B_s \to \pi^+K^-) = 0.30 \pm 0.04$, in good agreement with the value in Table 1 which has been obtained by averaging very recent measurements by the LHCb and CDF collaborations [29,30].

The current precision of the U-spin prediction (1) may be measured by the deviation of $-\Delta(B_s \to K^-\pi^+)/\Delta(B^0 \to K^+\pi^-)$ from one:

$$\begin{aligned} 1 &+ \frac{\Delta(B_s \to \pi^+K^-)}{\Delta(B^0 \to K^+\pi^-)} \\ &= 1 + \frac{\mathcal{B}(B_s \to \pi^+K^-)A_{CP}(B_s \to \pi^+K^-)\tau(B^0)}{\mathcal{B}(B^0 \to K^+\pi^-)A_{CP}(B^0 \to K^+\pi^-)\tau(B_s)} \\ &= 0.12 \pm 0.18. \end{aligned} \tag{5}$$

U-spin breaking in decay amplitudes is enhanced by a factor four in (5), originating in the ratio of two differences of squared amplitudes for processes and their charge-conjugates. [See Eq. (19) in Section 3.] We see that the U-spin asymmetry relation (1) is obeyed quite well. Current experimental errors, dominated by measurements of $\mathcal{B}(B_s \to \pi^+K^-)$ and $A_{CP}(B_s \to \pi^+K^-)$, allow for its violation by about 20–30% including this factor of four. A resulting stringent constraint on suitably normalized U-spin breaking corrections in decay amplitudes, at most of order several percent, will be given in Eq. (20).

The apparent success of this prediction may be accounted for by small U-spin breaking corrections such as occurring in an approximation based on naive factorization [22]. A question which we will address in the next section is whether first order U-spin breaking corrections in (1) are further suppressed or vanish in a general perturbative analysis. Another possibility would be to combine the rate asymmetries in (1) with asymmetries in other U-spin related processes, to be discussed now, such that the combined asymmetry vanishes at first order U-spin breaking.

The other two U-spin pairs in Table 1 ($B^0 \to \pi^+\pi^-$, $B_s \to K^+K^-$), ($B^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$), which require flavor-tagging and time-dependence, involve considerably larger experimental errors. The U-spin relation

$$\Delta\Gamma\big(B_s \to K^+K^-\big) = -\Delta\Gamma\big(B^0 \to \pi^+\pi^-\big) \tag{6}$$

predicts a small negative asymmetry in $B_s \to K^+K^-$,

$$\begin{aligned} &A_{CP}\big(B_s \to K^+K^-\big) \\ &\quad = -\frac{\tau(B_s)}{\tau(B^0)}\frac{\mathcal{B}(B^0 \to \pi^+\pi^-)}{\mathcal{B}(B_s \to K^+K^-)}A_{CP}\big(B^0 \to \pi^+\pi^-\big) \\ &\quad = -0.064 \pm 0.012. \end{aligned} \tag{7}$$

A very recent measurement reported by the LHCb collaboration [27], $A_{CP}(B_s \to K^+K^-) = -0.14 \pm 0.11 \pm 0.03$, is in agreement with this prediction but is also consistent with zero due to a large statistical error. It would be interesting to watch the change in central value with higher LHCb statistics and at the next run of the LHC.

Finally, the quite rare processes in the third pair ($B^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$), which are due to exchange amplitudes or final state rescattering, have been predicted to have extremely small branching fractions [31,32]. Asymmetry measurements in these decays which would test the U-spin prediction

$$\Delta\Gamma\big(B_s \to \pi^+\pi^-\big) = -\Delta\Gamma\big(B^0 \to K^+K^-\big) \tag{8}$$

are quite challenging.

We will now use U-spin symmetry to obtain relations among the three pairs of processes in Table 1. We start by noting that the initial states $B^0 \sim \bar{b}d$ and $B_s \sim \bar{b}s$ are members of a U-spin doublet, while the $\Delta S = 1$ and $\Delta S = 0$ parts of the Hamiltonian, $H_{\rm eff}^{\Delta S=1} \sim (\bar{b}s)$, $-H_{\rm eff}^{\Delta S=0} \sim -(\bar{b}d)$, transform like a U-spin doublet *when a minus sign is assigned to the $\Delta S = 0$ part*. The neutral final states involving the U-spin doublets (π^-, K^-) and $(K^+, -\pi^+)$ are superpositions of U-spin singlet ($U = 0$) and triplet ($U = 1$) states. We denote $\Delta S = 0$ and $\Delta S = 1$ decay amplitudes into singlet and triplet states by $A_0^{d,s}$ and $A_1^{d,s}$, respectively. Each of these amplitudes consists of two terms with specific CKM factors (occasionally being referred to as "tree" and "penguin" amplitudes) [16],

$$\begin{aligned} A^d_{0,1} &= V^*_{ub}V_{ud}A^u_{0,1} + V^*_{cb}V_{cd}A^c_{0,1}, \\ A^s_{0,1} &= V^*_{ub}V_{us}A^u_{0,1} + V^*_{cb}V_{cs}A^c_{0,1}. \end{aligned} \tag{9}$$

A straight-forward U-spin decomposition gives:

(a) $A\big(B^0 \to K^+\pi^-\big) = A_1^s,$
$\quad A\big(B_s \to \pi^+K^-\big) = A_1^d,$

(b) $A\big(B^0 \to \pi^+\pi^-\big) = \frac{1}{2}A_1^d + \frac{1}{2}A_0^d,$

Table 2
Magnitudes of amplitudes and their charge-conjugates for processes in Table 1 calculated using Eq. (12).

Decay mode	$\lvert A\rvert(10^{-3})$	$\lvert\bar{A}\rvert(10^{-3})$
$B^0 \to K^+\pi^-$	4.60 ± 0.06	4.24 ± 0.06
$B_s \to \pi^+K^-$	2.00 ± 0.12	2.61 ± 0.15
$B^0 \to \pi^+\pi^-$	1.90 ± 0.08	2.56 ± 0.07
$B_s \to K^+K^-$	4.90 ± 0.49	5.00 ± 0.49
$B^0 \to K^+K^-$	0.35 ± 0.07	0.35 ± 0.07
$B_s \to \pi^+\pi^-$	0.85 ± 0.08	0.85 ± 0.08

$$\begin{aligned}
&A\big(B_s \to K^+K^-\big) = \frac{1}{2}A_1^s + \frac{1}{2}A_0^s,\\
\text{(c)}\quad &A\big(B^0 \to K^+K^-\big) = -\frac{1}{2}A_1^d + \frac{1}{2}A_0^d,\\
&A\big(B_s \to \pi^+\pi^-\big) = -\frac{1}{2}A_1^s + \frac{1}{2}A_0^s.
\end{aligned} \tag{10}$$

The identical U-spin structures of amplitudes within each of the three pairs of processes (a), (b) *and* (c), *each involving a U-spin reflection* $d \leftrightarrow s$, *lead to the three asymmetry relations* (1), (6) *and* (8). For instance, the CP rate differences $\Delta\Gamma(B^0 \to K^+\pi^-)$ and $\Delta\Gamma(B_s \to \pi^+K^-)$ involve the same amplitude factor, $\mathrm{Im}[(A_1^c)^* A_1^u]$, multiplying equal CKM factors with opposite signs, $4\,\mathrm{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*) = -4\,\mathrm{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*)$ [16].

In addition, the six amplitudes in Eqs. (10) are seen to obey two triangle relations for $\Delta S = 0$ and $\Delta S = 1$ transitions,

$$\begin{aligned}
&A\big(B_s \to \pi^+K^-\big) - A\big(B^0 \to \pi^+\pi^-\big) + A\big(B^0 \to K^+K^-\big) = 0,\\
&A\big(B^0 \to K^+\pi^-\big) - A\big(B_s \to K^+K^-\big) + A\big(B_s \to \pi^+\pi^-\big) = 0.
\end{aligned} \tag{11}$$

Similar relations are obeyed by charged-conjugated amplitudes. These relations have been shown in Ref. [5] to hold under the broader assumption of flavor SU(3) symmetry. We will now check the validity of these triangle relations using current experimental data.

Neglecting the tiny difference between B^0 and B_s lifetimes and omitting phase space factors, we calculate magnitudes for these six amplitudes and their charge-conjugates:

$$|A|^2 = \mathcal{B}(1 - A_{CP}), \qquad |\bar{A}|^2 = \mathcal{B}(1 + A_{CP}). \tag{12}$$

The results are summarized in Table 2. In the absence of asymmetry measurements for the rare decays $B^0 \to K^+K^-$ and $B_s \to \pi^+\pi^-$ we have assumed that these two asymmetries vanish. The magnitudes calculated in Table 2 verify the closure of the two triangles (11) and their charge-conjugates. This behavior predicted in the U-spin symmetry limit is seen to be independent of the values assumed for the asymmetries in $B^0 \to K^+K^-$ and $B_s \to \pi^+\pi^-$.

Neglecting the amplitudes of these two rare processes relative to the other corresponding $\Delta S = 0$ and $\Delta S = 1$ amplitudes, which are almost an order of magnitude larger, would mean taking $A_1^{d,s} = A_0^{d,s}$. In this approximation the two triangles degenerate to straight lines, $A(B^0 \to \pi^+\pi^-) = A(B_s \to \pi^+K^-)$, $A(B_s \to K^+K^-) = A(B^0 \to K^+\pi^-)$.

3. First order U-spin breaking

We will now study first order U-spin breaking corrections in Eqs. (10), searching for possible relations among CP rate asymmetries which would be free of such corrections. U-spin breaking is introduced in hadronic matrix elements by inserting a quark mass term $\mathcal{M}_{\rm Ubrk} \propto \bar{s}s - \bar{d}d$ behaving like $U = 1$, $U_3 = 0$, multiplying the effective Hamiltonian which transforms as a U-spin doublet. Thus the correction operator for $\Delta S = 1, 0$ transitions (corresponding to $U_3 = 1/2, -1/2$) transforms as a direct product $1 \otimes 1/2$ consisting of $U = 1/2$ and $U = 3/2$ operators,

$$\begin{aligned}
&\mathcal{M}_{\rm Ubrk} H_{\rm eff}^{\Delta S=1,0} \propto \pm(1,0) \otimes \left(\frac{1}{2}, \pm\frac{1}{2}\right)\\
&\quad = -\sqrt{\frac{1}{3}}\mathcal{O}^{\frac{1}{2}}_{\pm\frac{1}{2}} \pm \sqrt{\frac{2}{3}}\mathcal{O}^{\frac{3}{2}}_{\pm\frac{1}{2}}.
\end{aligned} \tag{13}$$

The initial $\pm$ signs originate in the signs of $+H_{\rm eff}^{\Delta S=1}$ and $-H_{\rm eff}^{\Delta S=0}$ transforming as two components of a U-spin doublet. Upper and lower indices on operators denote values of U and U_3, respectively. First order corrections in (10) are given in terms of matrix elements of $\mathcal{O}^{\frac{1}{2}}$ and $\mathcal{O}^{\frac{3}{2}}$ for final states with $U = 0, 1$ and $U = 1$, respectively.

We define first order U-spin breaking corrections to decay amplitudes,

$$\begin{aligned}
&\epsilon_1^{s,d} \equiv \sqrt{\frac{1}{3}}\langle U = 0|\mathcal{O}^{\frac{1}{2}}_{\pm\frac{1}{2}}\left|U = \frac{1}{2}\right\rangle,\\
&\epsilon_2^{s,d} \equiv \sqrt{\frac{1}{3}}\langle U = 1|\mathcal{O}^{\frac{1}{2}}_{\pm\frac{1}{2}}\left|U = \frac{1}{2}\right\rangle,\\
&\epsilon_3^{s,d} \equiv \sqrt{\frac{2}{3}}\langle U = 1|\mathcal{O}^{\frac{3}{2}}_{\pm\frac{1}{2}}\left|U = \frac{1}{2}\right\rangle.
\end{aligned} \tag{14}$$

These corrections have CKM structures similar to (9) [16],

$$\begin{aligned}
&\epsilon_i^d = V_{ub}^* V_{ud}\epsilon_i^u + V_{cb}^* V_{cd}\epsilon_i^c,\\
&\epsilon_i^s = V_{ub}^* V_{us}\epsilon_i^u + V_{cb}^* V_{cs}\epsilon_i^c \quad (i = 1, 2, 3).
\end{aligned} \tag{15}$$

A straight-forward U-spin decomposition including these corrections gives:

$$\begin{aligned}
\text{(a)}\quad &A\big(B^0 \to K^+\pi^-\big) = A_1^s - \epsilon_2^s - \frac{1}{2}\epsilon_3^s,\\
&A\big(B_s \to \pi^+K^-\big) = A_1^d + \epsilon_2^d + \frac{1}{2}\epsilon_3^d,\\
\text{(b)}\quad &A\big(B^0 \to \pi^+\pi^-\big) = \frac{1}{2}A_1^d + \frac{1}{2}A_0^d + \frac{1}{2}\epsilon_1^d + \frac{1}{2}\epsilon_2^d - \frac{1}{2}\epsilon_3^d,\\
&A\big(B_s \to K^+K^-\big) = \frac{1}{2}A_1^s + \frac{1}{2}A_0^s - \frac{1}{2}\epsilon_1^s - \frac{1}{2}\epsilon_2^s + \frac{1}{2}\epsilon_3^s,\\
\text{(c)}\quad &A\big(B^0 \to K^+K^-\big) = -\frac{1}{2}A_1^d + \frac{1}{2}A_0^d + \frac{1}{2}\epsilon_1^d - \frac{1}{2}\epsilon_2^d + \frac{1}{2}\epsilon_3^d,\\
&A\big(B_s \to \pi^+\pi^-\big) = -\frac{1}{2}A_1^s + \frac{1}{2}A_0^s - \frac{1}{2}\epsilon_1^s + \frac{1}{2}\epsilon_2^s - \frac{1}{2}\epsilon_3^s.
\end{aligned} \tag{16}$$

We note that at this order in U-spin breaking the U-spin structures of two amplitudes within any given pair are not identical as required for obtaining relations between corresponding CP rate asymmetries. *While the leading terms within each pair of amplitudes have the same U-spin structures and equal signs, the first order U-spin breaking corrections have the same structures but opposite signs*. Consequently two CP rate differences for a given pair of processes now involve equal CKM factors with opposite signs, $4\,\mathrm{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*) = -4\,\mathrm{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*)$, which are however multiplied by *different* amplitude factors.

Denoting by δ the difference between these first order amplitude factors, we now have [33]

$$\Delta\Gamma\big(B^0 \to f\big) + \Delta\Gamma(B_s \to U_r f) = 4\,\mathrm{Im}\big(V_{ub}^* V_{us} V_{cb} V_{cd}^*\big)\delta, \tag{17}$$

where $U_r f$ is a final state obtained from f by U-spin reflection, $U_r: d \leftrightarrow s$. δ vanishes in the U-spin symmetry limit. Expressions of

δ for the above three pairs of processes are readily obtained from Eqs. (16):

$$\text{(a)}\quad \delta_a = 2\,\mathrm{Im}\left[A_1^{c*}\left(\epsilon_2^u + \frac{1}{2}\epsilon_3^u\right)\right] - [c \leftrightarrow u],$$
$$\text{(b)}\quad \delta_b = \frac{1}{2}\,\mathrm{Im}\left[\left(A_1^{c*} + A_0^{c*}\right)\left(\epsilon_1^u + \epsilon_2^u - \epsilon_3^u\right)\right] - [c \leftrightarrow u],$$
$$\text{(c)}\quad \delta_c = \frac{1}{2}\,\mathrm{Im}\left[\left(-A_1^{c*} + A_0^{c*}\right)\left(\epsilon_1^u - \epsilon_2^u + \epsilon_3^u\right)\right] - [c \leftrightarrow u]. \tag{18}$$

That is, all three asymmetry relations (1), (6) and (8) obtain first order U-spin breaking corrections given by Eqs. (17) and (18). Furthermore, these corrections do not cancel in any arbitrary linear combination of the three sums of CP rate differences (17) because, as one can see by inspection, there exists no linear combination of δ_a, δ_b and δ_c that vanishes identically.

As mentioned in Section 2 U-spin breaking corrections in amplitudes are enhanced by a factor four in (5):

$$1 + \frac{\Delta(B_s \to \pi^+K^-)}{\Delta(B^0 \to K^+\pi^-)} = \frac{4\,\mathrm{Im}[A_1^{c*}(\epsilon_2^u + \frac{1}{2}\epsilon_3^u)] - [c \leftrightarrow u]}{\mathrm{Im}(A_1^{c*}A_1^u) - [c \leftrightarrow u]}. \tag{19}$$

Thus current measurements imply that suitably normalized U-spin breaking corrections are at most of order several percent,

$$\frac{\mathrm{Im}[A_1^{c*}(\epsilon_2^u + \frac{1}{2}\epsilon_3^u)] - [c \leftrightarrow u]}{2\,\mathrm{Im}(A_1^{c*}A_1^u)} = 0.03 \pm 0.04. \tag{20}$$

One may also consider *differences* of CP rate differences for each one of the three U-spin pairs,

$$\Delta\Gamma\left(B^0 \to f\right) - \Delta\Gamma(B_s \to U_r f) = 4\,\mathrm{Im}\left(V_{ub}^*V_{us}V_{cb}V_{cd}^*\right)\sigma. \tag{21}$$

First order U-spin breaking corrections do cancel in σ which depends only on U-spin invariant amplitudes,

$$\text{(a)}\quad \sigma_a = -2\,\mathrm{Im}\left(A_1^{c*}A_1^u\right),$$
$$\text{(b)}\quad \sigma_b = \frac{1}{2}\,\mathrm{Im}\left[\left(A_1^{c*} + A_0^{c*}\right)\left(A_1^u + A_0^u\right)\right],$$
$$\text{(c)}\quad \sigma_c = \frac{1}{2}\,\mathrm{Im}\left[\left(A_1^{c*} - A_0^{c*}\right)\left(A_1^u - A_0^u\right)\right]. \tag{22}$$

These three quantities are linearly independent. Thus in general *one is unable to form a linear combination of all six rate asymmetries which would vanish to first order in U-spin breaking*.

We checked that first order U-spin breaking cannot be avoided also in relations between CP rate asymmetries for the following three U-reflected pairs of processes obeying the U-spin theorem [16], $(B^0 \to K^0\pi^0, B_s \to \bar{K}^0\pi^0)$, $(B^0 \to K^0\bar{K}^0, B_s \to \bar{K}^0K^0)$ and $(B^+ \to K^0\pi^+, B^+ \to \bar{K}^0K^+)$.

In the approximation of neglecting the small amplitudes for $B^0 \to K^+K^-$ and $B_s \to \pi^+\pi^-$, which we have seen is equivalent to taking $A_1^{d,s} = A_0^{d,s}$ or $A_1^{c,u} = A_0^{c,u}$, one has $\sigma_a = -\sigma_b$, namely

$$\Delta\Gamma\left(B^0 \to K^+\pi^-\right) - \Delta\Gamma\left(B_s \to \pi^+K^-\right)$$
$$= \Delta\Gamma\left(B_s \to K^+K^-\right) - \Delta\Gamma\left(B^0 \to \pi^+\pi^-\right). \tag{23}$$

This relation holds experimentally largely because of the current large error on $A_{CP}(B_s \to K^+K^-)$ [27]. It will be interesting to watch the effect of improving this asymmetry measurement on the validity of this equality neglecting rescattering in comparison with that of (6) neglecting U-spin breaking.

Turning next to the amplitude triangle relations (11) we observe that both relations are violated by purely $\Delta U = 3/2$ first order U-spin breaking corrections:

$$A\left(B_s \to \pi^+K^-\right) - A\left(B^0 \to \pi^+\pi^-\right)$$
$$+ A\left(B^0 \to K^+K^-\right) = \frac{3}{2}\epsilon_3^d,$$
$$A\left(B^0 \to K^+\pi^-\right) - A\left(B_s \to K^+K^-\right)$$
$$+ A\left(B_s \to \pi^+\pi^-\right) = -\frac{3}{2}\epsilon_3^s. \tag{24}$$

4. Three-body B^+ decays to charged pions and kaons

The LHCb collaboration reported CP asymmetry measurements in all four decay modes of three-body B^+ decays to charged pions and kaons, $B^+ \to K^+K^+K^-$, $K^+\pi^+\pi^-$, $\pi^+\pi^+\pi^-$, $\pi^+K^+K^-$ [34,35]. These processes may be divided into two pairs involving U-spin reflected final states obeying relations between total CP rate asymmetries similar to (1),

$$\Delta\Gamma\left(B^+ \to K^+K^+K^-\right) = -\Delta\Gamma\left(B^+ \to \pi^+\pi^+\pi^-\right),$$
$$\Delta\Gamma\left(B^+ \to K^+\pi^+\pi^-\right) = -\Delta\Gamma\left(B^+ \to \pi^+K^+K^-\right). \tag{25}$$

These predictions have been analyzed recently in Ref. [21] and were found to agree reasonably well with the LHCb measurements, in particular with respect to relative signs of $\Delta S = 0$ and $\Delta S = 1$ asymmetries and their magnitudes which currently involve sizeable errors. We will now study U-spin breaking corrections in (25) in a manner similar to our study of two-body decays in the previous two sections.

We start by observing that the initial B^+ state is a U-spin scalar. Each one of the four final states consisting of three members of U-spin doublets can be decomposed into two doublets, depending on whether the two positively charged mesons combine to $U = 0$ or $U = 1$, and one triplet state. Only the doublets contribute to the $\Delta U = 1/2$ transitions in the U-spin symmetry limit. Two amplitudes, $\mathcal{A}_0$ and $\mathcal{A}_1$, *defined by the U-spin of the two positively charged mesons*, depend also on the three meson momenta, p_1, p_2, p_3, defining a point in the Dalitz plane.

Using these notations one obtains [36],

$$\text{(d)}\quad A\left(B^+ \to K^+(p_1)K^+(p_2)K^-(p_3)\right) = 2\mathcal{A}_1^s(p_1, p_2, p_3),$$
$$A\left(B^+ \to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\right) = 2\mathcal{A}_1^d(p_1, p_2, p_3),$$
$$\text{(e)}\quad A\left(B^+ \to K^+(p_1)\pi^+(p_2)\pi^-(p_3)\right)$$
$$= \mathcal{A}_1^s(p_1, p_2, p_3) - \mathcal{A}_0^s(p_1, p_2, p_3),$$
$$A\left(B^+ \to \pi^+(p_1)K^+(p_2)K^-(p_3)\right)$$
$$= \mathcal{A}_1^d(p_1, p_2, p_3) - \mathcal{A}_0^d(p_1, p_2, p_3), \tag{26}$$

where factors $1/\sqrt{2}$, and $1/\sqrt{6}$ have been absorbed in $\mathcal{A}_0^{d,s}$ and $\mathcal{A}_1^{d,s}$ which involve CKM factors as in (9). *The identical U-spin structures of amplitudes within each of the two U-spin reflected pairs of processes* (d) *and* (e) *lead upon phase space integration to the two asymmetry relations* (25).

In order to introduce first order U-spin breaking in (26) we calculate additional contributions to decay amplitudes from the U-spin breaking operator $M_{\mathrm{Ubrk}}H_{\mathrm{eff}}$ given in (13). Corrections include matrix elements of this operator for U-spin doublet final states $|(U = 1/2)_0\rangle$ and $|(U = 1/2)_1\rangle$, in which the two positively charged mesons combine to $U = 0$ and $U = 1$, and for a U-spin triplet final state $|U = 3/2\rangle$. Defining

$$\mathcal{E}_1^{s,d} \equiv \frac{1}{\sqrt{6}}\left\langle\left(U = \frac{1}{2}\right)_0\right|\mathcal{O}_{\pm\frac{1}{2}}^{\frac{1}{2}}|U = 0\rangle,$$
$$\mathcal{E}_2^{s,d} \equiv \frac{\sqrt{2}}{3}\left\langle\left(U = \frac{1}{2}\right)_1\right|\mathcal{O}_{\pm\frac{1}{2}}^{\frac{1}{2}}|U = 0\rangle,$$

$$\mathcal{E}_3^{s,d} \equiv \frac{\sqrt{2}}{3}\left\langle U = \frac{3}{2}\middle|\mathcal{O}^{\frac{3}{2}}_{\pm\frac{1}{2}}|U=0\right\rangle, \tag{27}$$

where $\mathcal{E}_i^{s,d}$ have CKM structures as in (15), we calculate

$$\begin{aligned}
&\text{(d)}\quad A\left(B^+ \to K^+K^+K^-\right) = 2\mathcal{A}_1^s - \mathcal{E}_2^s + \mathcal{E}_3^s,\\
&\qquad\; A\left(B^+ \to \pi^+\pi^+\pi^-\right) = 2\mathcal{A}_1^d + \mathcal{E}_2^d - \mathcal{E}_3^d,\\
&\text{(e)}\quad A\left(B^+ \to K^+\pi^+\pi^-\right) = \mathcal{A}_1^s - \mathcal{A}_0^s + \mathcal{E}_1^s - \frac{1}{2}\mathcal{E}_2^s - \mathcal{E}_3^s,\\
&\qquad\; A\left(B^+ \to \pi^+K^+K^-\right) = \mathcal{A}_1^d - \mathcal{A}_0^d - \mathcal{E}_1^d + \frac{1}{2}\mathcal{E}_2^d + \mathcal{E}_3^d.
\end{aligned} \tag{28}$$

We note that, just as in two-body decays, the U-spin breaking terms within each pair of three-body processes have the same U-spin structures but opposite signs. Consequently the asymmetry relations (25) are violated by first order U-spin breaking corrections in a form analogous to Eq. (17). Using arguments similar to those associated with Eqs. (18) and (22) for two-body decays, we conclude that these corrections cannot be avoided by considering arbitrary linear combinations of these four asymmetries.

Before concluding we note that while we were writing-up this work a paper appeared [37], in which SU(3) symmetry amplitude relations and SU(3) breaking corrections in these three-body decay amplitudes have been studied under several additional assumptions with which we do not completely agree. Among these assumptions are: (a) Two equalities, $A(B^+ \to \pi^+\pi^+\pi^-) = A(B^+ \to \pi^+K^+K^-)$ and $A(B^+ \to K^+K^+K^-) = A(B^+ \to K^+\pi^+\pi^-)$, claimed to follow from SU(3) symmetry. (b) The absence of SU(3) breaking corrections in $A(B^+ \to \pi^+\pi^+\pi^-)$ and $A(B^+ \to K^+K^+K^-)$. Point (a) is in clear contradiction with Eqs. (26) and Bose symmetry [36] while point (b) requires $\mathcal{E}_2^{u,c} = \mathcal{E}_3^{u,c}$ in Eqs. (28).

5. Conclusion

We have summarized the current experimental status of U-spin relations predicted among CP rate asymmetries in B decays to two charged pseudoscalar mesons, noticing a rather small U-spin breaking in $\Delta\Gamma(B_s \to K^-\pi^+) = -\Delta\Gamma(B^0 \to K^+\pi^-)$. Introducing an $\bar{s}s - \bar{d}d$ quark mass term for U-spin breaking, we have performed a general analysis of first order U-spin breaking corrections in two-body B^0 and B_s decays and in three-body B^+ decays involving charged pions and kaons. We have shown that these corrections cannot be made to cancel by a judicious choice of a linear combination of several CP rate asymmetries.

Acknowledgements

I am grateful to the CERN Theory Division for its kind hospitality and wish to thank Bhubanjyoti Bhattacharya, Tim Gershon, David London and Jonathan Rosner for helpful comments.

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