Relating different approaches to nuclear broadening

Jörg Raufeisen

Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Received 9 January 2003; received in revised form 31 January 2003; accepted 11 February 2003

Editor: M. Čvetić

Abstract

Transverse momentum broadening of fast partons propagating through a large nucleus is proportional to the average color field strength in the nucleus. In this work, the corresponding coefficient is determined in three different frameworks, namely, in the color dipole approach, in the approach of Baier et al. and in the higher twist factorization formalism. This result enables one to use a parametrization of the dipole cross section to estimate the values of the gluon transport coefficient and of the higher twist matrix element, which is relevant for nuclear broadening. A considerable energy dependence of these quantities is found. In addition, numerical calculations are compared to data for nuclear broadening of Drell–Yan dileptons, \( J/\psi \) and \( \Upsilon \) mesons. The scale dependence of the strong coupling constant leads to measurable differences between the higher twist approach and the other two formalisms.

© 2003 Elsevier Science B.V. Open access under CC BY license.

PACS: 24.85.+p; 13.85.Qk

Keywords: Dipole cross section; Higher twists; Nuclear broadening

1. Introduction

A fast parton (quark or gluon) propagating through nuclear matter accumulates transverse momentum by multiple interactions with the soft color field of the nucleus. At not too high energies, this phenomenon is experimentally accessible by measuring nuclear broadening of Drell–Yan (DY) dileptons or of \( J/\psi \) and \( \Upsilon \) mesons produced in proton–nucleus (\( pA \)) collisions. Nuclear broadening is defined as the increase of the mean transverse momentum squared of the produced particle compared to proton–proton (\( pp \)) collisions, i.e.,

\[
\delta \langle p_T^2 \rangle = \langle p_T^2 \rangle_{pA} - \langle p_T^2 \rangle_{pp}.
\]  

During the past decade, at least three different theoretical approaches have been developed to describe this effect, namely, the color dipole approach [1,2], the approach of Baier et al. [3] and the higher twist factorization formalism [4,5] (see also [6] for earlier work). This enormous interest in a QCD based description of nuclear effects is mainly motivated by the experimental program at the relativistic heavy ion collider (RHIC). Data from heavy ion collisions at RHIC require a profound theoretical understanding of nuclear effects in terms of QCD for a reliable interpretation (see, e.g., [7]).
In each of the three approaches [1–5], broadening is proportional to a nonperturbative parameter, which has to be determined from experimental data. This often limits the predictive power of the theory. Moreover, one would like to know, how different theoretical formulations of transverse momentum broadening relate to one another, and to what extend they represent the same (or different) physics. The purpose of this Letter is to present relations between the nonperturbative parameters, thereby illuminating the connection between these seemingly very different approaches. Since the nonperturbative input to the dipole approach [1,2] is known from processes other than nuclear broadening, one can then obtain independent estimates for the parameters of the other two approaches and calculate δ proton (p_T^2) in a parameter free way. In addition, we study the energy dependence of the nonperturbative parameters, which has to be known if one wants to extrapolate results from fixed target energies to RHIC and the large hadron collider (LHC).

We shall now briefly summarize the basic formulae for nuclear broadening. In the color dipole approach [1,2], transverse momentum broadening of an energetic parton propagating through a large nucleus is given by

$$\delta(p_T^2)^{\text{dipole}} = 2\rho_A L C_R(0),$$

where $\rho_A = 0.16 \text{ fm}^{-3}$ is the nuclear density, and $L = 3R_A/4$ is the average length of the nuclear medium traversed by the projectile parton before the hard reaction occurs ($R_A$ is the nuclear radius). The index $R$ refers to the color representation of the projectile parton, $R = F$ for a quark and $R = A$ for a gluon. The nonperturbative physics is parametrized in the quantities

$$C_F(0) = \frac{d}{dr_T^2} \sigma_{N q}^N(r_T) \bigg|_{r_T \to 0},$$

$$C_A(0) = 9C_F(0)/4.$$  

(3)

Here, $\sigma_{N q}^N(r_T)$ is the cross section for scattering a color singlet quark–antiquark ($q\bar{q}$) pair with transverse separation $r_T$ off a nucleon $N$. This dipole cross section arises from a complicated interplay between attenuation and multiple rescattering of the incident parton [2].

In the approach of Baier et al. [3] (BDMPS approach hereafter), broadening is related to the transverse momentum in a random walk through the nuclear medium, thereby undergoing multiple soft rescatterings. In the higher twist approach, the (anti)quark medium, thereby undergoing multiple soft rescatterings. In the higher twist approach, the (anti)quark

$$\delta(p_T^2)^{\text{BDMPS}} = \hat{q}_R L.$$  

(4)

In this approach, all nonperturbative physics is contained in $\hat{q}_R$, which is a measure for the strength of the interaction between the projectile quark and the target.

The dipole and the BDMPS approach quite obviously describe the same physics, see Ref. [8]. Both, $C_R(0)$ and $\hat{q}_R$ can be related to the gluon density of a nucleon. By comparing the corresponding expressions in Ref. [9] (for $C_R(0)$) and Ref. [3] (for $\hat{q}_R$), one obtains [10],

$$\hat{q}_R = 2\rho_A C_R(0).$$

(5)

Thus, $\delta(p_T^2)^{\text{dipole}} = \delta(p_T^2)^{\text{BDMPS}}$.

The relation to the higher twist factorization formalism [4,5] is less clear. In the dipole and in the BDMPS approach, the projectile parton acquires transverse momentum in a random walk through the nuclear medium, thereby undergoing multiple soft rescatterings. In the higher twist approach, the (anti)quark from the projectile proton exchanges only one additional soft gluon with the nucleus before the Drell-Yan process is used to probe the nucleus (A is the atomic mass of the nucleus) [4],

$$\delta(p_T^2)^{\text{HT}} = \frac{4\pi^2}{3} \alpha_s(M^2) \lambda_{LQS}^2 A^{1/3}.$$  

(6)

The quantity $\lambda_{LQS}$ originates from a model of the soft-hard twist-4 matrix element [5], $T_{qG}(x_2) \approx \lambda_{LQS}^2 A^{1/3} f_{q/A}(x_2)$, where $f_{q/A}(x_2)$ is the density of quarks with momentum fraction $x_2$ in the nucleus. The

![Fig. 1. Twist-4 contribution to nuclear broadening. The projectile antiquark carries momentum fraction $x_1$ of its parent hadron and undergoes one soft rescattering before it annihilates with a quark from the nucleus. The Drell–Yan process is used to probe the transverse momentum of the antiquark.](image-url)
strong coupling constant \( \alpha_s \) enters at the characteristic hard scale of the process that probes the transverse momentum of the incident parton, i.e., the dilepton mass \( M^2 \). Thus, in the higher twist approach, \( \alpha_s \) is small, even though the exchanged gluon is soft. The smallness of \( \alpha_s \) is crucial for the applicability of the QCD factorization theorem.

2. Relating the dipole approach to the higher twist formalism

In order to relate all three approaches, one clearly cannot simply set equal the broadening given in Eqs. (2), (4) and (6), and then read off a relation between \( C_R(0) \), \( \hat{q}_R \) and \( \lambda^2_{\text{LQS}} \). Instead, one has to find a relation between these three quantities in an independent way, and only after that, one can check whether all three approaches predict the same (or different) \( \hat{q}(p_T^2)^R \). It is then possible to use a model for the dipole cross section to estimate \( \hat{q}_R \) and \( \lambda^2_{\text{LQS}} \), since \( \sigma^N_{q\bar{q}} \) is known much better than these two quantities.

The plan is to relate \( C_R(0) \) and \( \lambda^2_{\text{LQS}} \) to the quantity

\[
[F^2] = \frac{1}{2\pi P^+} \int dy^- \langle N | F_a^{+\omega}(y^-) F_{a,\omega}(0) | N \rangle, \tag{7}
\]

which measures the average color field strength experienced by the projectile parton. In Eq. (7), \( P^+ \) is the light-cone momentum of the nucleus \( |A \rangle \) per nucleon. The index \( \omega \) runs over the two transverse directions, and \( F_a^{+\omega} \) is the gluon field strength operator. Since we are dealing with nonperturbative quantities, the result will of course be model dependent.

The relation between \( C_R(0) \) and \( F^2 \) can be obtained quite straightforwardly. Note that the dipole cross section is related to the gluon density \( xG_N(x) \) of a nucleon by [9],

\[
\sigma^N_{q\bar{q}}(x, r_T) = \frac{\pi^2}{3} \alpha_s r_T^2 xG_N(x), \tag{8}
\]

and in light-cone gauge, the gluon density is given by [11],

\[
xG_N(x) = \int \frac{dy^-}{2\pi P^+} e^{-ixP^+y^-} \langle N | F_a^{+\omega}(y^-) F_{a,\omega}(0) | N \rangle. \tag{9}
\]

What are the relevant scales for \( xG_N(x) \) and \( \alpha_s \)? Obviously, the Fourier modes of the nuclear color field that give the dominant contribution to broadening are of order \( \delta(p_T^2)^R \). Since this is not much larger than \( \Lambda^2_{\text{QCD}} \) in present experiments, the gluon density in this case is not a parton distribution like in deeply inelastic scattering (DIS) and should not be evolved with the QCD evolution equations. Moreover, the soft gluon carries momentum fraction \( x \sim \delta(p_T^2)^R/(x_1S) \) of its parent nucleon, which is essentially zero. Here, \( x_1 \) is the momentum fraction of the projectile parton, and \( S \) is the hadronic center of mass (c.m.) energy. We therefore set \( x \to 0 \) in Eq. (9) and write \( C_F(0) \) as

\[
C_F(0) = \frac{\pi^2}{3} \alpha_s \langle \hat{q}(p_T^2)^R/(F^2) \rangle. \tag{10}
\]

It is important to note that in the dipole approach and in the BDMP approach, the scale of \( \alpha_s \) is the same as in the gluon density. This is the main difference between these two approaches and the higher twist formalism.

The next step is to find an expression for \( \lambda^2_{\text{LQS}} \). We shall follow the model assumptions of Ref. [5],

\[
T_{qG}^{\text{SH}}(x_2) = \int \frac{dy_0^-}{2\pi} e^{i x_2 P^+ y_0^-} \Theta(y_0^- - y_1^-) \Theta(-y_1^-) \times \frac{1}{2} \langle A | \hat{q}(0) \gamma^+ q(y_0^-) F_a^{+\omega}(y_1^-) F_{a,\omega}^{+\omega}(y_1^-) | A \rangle \tag{11}
\]

\[
\approx \int \frac{dy_0^-}{2\pi} e^{-ix_2 P^+ y_0^-} \frac{1}{2} \langle A | \hat{q}(0) \gamma^+ q(y_0^-) | A \rangle \times \langle A | F_a^{+\omega}(y_1^-) F_{a,\omega}^{+\omega}(y_1^-) | A \rangle \tag{12}
\]

\[
\approx \lambda^2_{\text{LQS}} A^{1/3} f_{q/A}(x_2). \tag{13}
\]

Here, \( x_2 \) is the momentum fraction of the quark from the nucleus in Fig. 1. The meaning of the positions \( y_1^- \) on the light-cone are illustrated in Fig. 1 as well. The step functions \( \Theta \) ensure that the soft gluon is exchanged before the annihilation. In Eq. (12), the matrix element is factorized by introducing an approximate unit operator, \( 1 \approx |A| \langle A |/(2P^+ V) \), where \( V \) is the volume of the nucleus [12]. In this step, all correlations between the quark and the gluon in Fig. 1 are neglected. As pointed out in Ref. [5], one has \( |y_0^-| \ll
Because of the rapidly oscillating phase factor in Eq. (12). In addition, $$|y_1^- - y_1'| \ll R_A$$ because of confinement [5]. This allows one to approximate $$\Theta(y_0^- - y_1') \approx \Theta(-y_1') \approx \Theta(-y_1^-)$$ in Eq. (12). With these approximations, the $$y_0^-$$-integral factorizes to give the nuclear quark density $$f_q/A(x_2) \approx Af_q/N(x_2)$$. The integral over the remaining step function yields a factor L, and in the last integration one recovers the right-hand side of Eq. (7), though with $$|N|$$ replaced by $$|A|$$. Assuming that there are no nontrivial nuclear effects on the gluon field, the result reads,

$$\rho_{AL} A^{1/3} = \frac{1}{2} \rho_AL(F^2).$$  \hspace{1cm} (14)

Note that we do not introduce a new model for the QCD (x2). Eq. (14) follows from the model assumptions of Ref. [5].

Thus, in all three approaches, broadening is related to the quantity $$\langle F^2 \rangle$$, and one finds from Eqs. (2), (4) and (6),

$$\delta\langle |p_T|^2 \rangle_{dipole} = \delta\langle |p_T|^2 \rangle_{BDMPS}$$

$$= \frac{2\pi^2}{3} \alpha_s \langle \delta\langle |p_T|^2 \rangle \rangle_{R} L(F^2),$$

$$\delta\langle |p_T|^2 \rangle_{HT} = \frac{2\pi^2}{3} \alpha_s (M^2) \rho_A L(F^2).$$  \hspace{1cm} (15)

The new result is the coefficient $$2\pi^2\alpha_s \rho_AL/3$$, the proportionality between broadening and the average color field strength in the target was already known before [5,13,14]. It is remarkable, that the only difference between $$\delta\langle |p_T|^2 \rangle_{dipole}$$ and $$\delta\langle |p_T|^2 \rangle_{HT}$$ is the scale of the strong coupling constant. We stress that this difference cannot be dismissed as a higher order correction. Instead, it is the result of different physical pictures of nuclear broadening.

At first sight, the result Eq. (15) may seem puzzling. How can the double scattering approximation yield essentially the same expression for broadening as a resummation of all rescatterings? In fact, it was demonstrated in Refs. [2,14], that double scattering does not lead to an $$A^{1/3}$$-dependence of broadening.

This contradiction can be resolved in the following way: the probability to have $$n$$ interactions of the projectile parton with the medium before the Drell–Yan process takes place is (neglecting correlations) Poisson distributed, $$P_n = (\sigma T_A)^n e^{-\sigma T_A} / n!$$, where $$\sigma$$ is the cross section for a single soft scattering, and $$T_A$$ is the nuclear thickness. Apparently, the $$A$$-dependence of the single scattering probability is quite different from $$A^{1/3}$$. In the dipole and the BDMPS approach, the accumulated transverse momentum is proportional to the mean number of scatterings, i.e., $$\sigma T_A$$, and hence proportional to $$A^{1/3}$$. Therefore, it was concluded in [2] that it is essential to sum all rescatterings in order to get an $$A^{1/3}$$ law. The higher twist approach, however, does not only use the double scattering approximation, it is also an expansion in $$A^1$$ ($$T_A$$). To leading order in this parameter, $$P_1$$ is identical to the mean number of rescatterings. This property of the Poisson distribution is the reason why the two expressions for $$\delta\langle |p_T|^2 \rangle$$ in Eq. (15) can be so similar. In fact, it has been shown recently [15] that Eq. (6) remains valid, if the projectile quark exchanges an arbitrary number of gluons with the target nucleus. It should be stressed at this point, that $$\delta\langle |p_T|^2 \rangle$$ only depends on the average color field strength in the target and is not sensitive to details of the color field. Regarding details of the $$p_T$$ dependence of nuclear effects, one certainly has to expect very different expressions from the dipole approach and the higher twist formalism.

3. Phenomenological applications

One can now choose a particular model of the dipole cross section to get an estimate for $$\hat{q}_R$$ and $$\rho_{LQS}$$. In this Letter, the parametrization of Kopeliovich, Schäfer and Tarasov (KST) [16] will be used, because it is motivated from the phenomenology of soft hadronic interactions. With the KST-parametrization, $$C_R(0) = C_R(0,s)$$ depends on the energy $$E_P$$ of the projectile parton, $$s = 2m_N E_P$$, where $$m_N$$ is the nucleon mass. In all calculation, we also take into account Gribov’s inelastic corrections (i.e., gluon shadowing), as explained in [2]. At fixed target energies, this leads only to a $$\sim10\%$$ reduction of $$C_F(0,s)$$ for a heavy nucleus, but at larger values of $$\sqrt{s}$$, which are relevant for LHC, $$C_F(0,s)$$ is reduced by approximately 1/3.

Fig. 2 shows the energy dependence of nuclear broadening for quarks and of the three parameters $$C_F(0,s)$$, $$\hat{q}_A = 9\rho_A C_F(0,s)/2$$ and

$$\rho_{LQS} A^{1/3} = \frac{3}{4\pi^2\alpha_s} \langle \delta\langle |p_T|^2 \rangle \rangle^{\langle T_A \rangle} C_F(0,s).$$  \hspace{1cm} (16)
In the dipole approach, broadening only depends on the energy of the parton and not on the mass of the dipole. In the higher twist formalism, however, \( \delta(p_T^2)_{\text{HT}} \) depends on the dipole mass through \( \alpha_s \). As a consequence, for \( W^\pm \) and \( Z^0 \) production in \( pA \) scattering with \( \sqrt{s} = 8.8 \text{ TeV at the LHC} \) \( (x_1 \approx x_2 \approx 0.01) \), one has \( \delta(p_T^2)_{\text{dipole}} \sim 1.5 \text{ GeV}^2 \) for a heavy nucleus with \( A \sim 200 \) and \( \delta(p_T^2)_{\text{HT}} \sim 0.5 \text{ GeV}^2 \). Of course, this estimate assumes that one can still apply these formalisms at \( x_2 \approx 0.01 \). As explained in more detail in Ref. [19], at very low \( x_2 \), the DY cross section is affected by quantum mechanical interferences, and the transverse momentum broadening of the produced boson does not reflect the broadening of the projectile quark any more. Nuclear broadening in DY at very low \( x_2 \) has been calculated in Ref. [19] and is expected to be much larger than at medium-low \( x_2 \geq 0.01 \).

At fixed target energies, however, these interference effects are negligible, and experimental data for broadening in DY can be compared to a calculation of broadening for quarks. The solid curves in Fig. 3 are obtained from

\[
\delta(p_T^2)_{pA} - \delta(p_T^2)_{pD} = (\langle T_A \rangle - \langle T_D \rangle)C_\pi(0, s),
\]

where the mean nuclear thickness is calculated with realistic parametrizations of nuclear densities from Ref. [24]. The dashed curves in Fig. 3 are obtained by rescaling \( \delta(p_T^2)_{pA} \) by the ratio of strong coupling constants, \( \alpha_s(M^2)/\alpha_s(\delta(p_T^2)_{pA}) \). The higher twist formalism is strictly speaking not applicable to light nuclei, since all contributions that are not enhanced by a power of \( A^{1/3} \) are neglected in this approach. Nevertheless, we believe that a calculation with realistic nuclear densities is a reasonable extrapolation to lighter nuclei.

The relevant quark energies for the 800 GeV proton beam at Fermilab are \( 20 \text{ GeV} \leq \sqrt{s} \leq 25 \text{ GeV} \). The lower solid and dashed curves in Fig. 3 are calculated for \( \sqrt{s} = 20 \text{ GeV} \) and the upper ones for \( \sqrt{s} = 25 \text{ GeV} \). For the higher twist calculation, we vary the scale of \( \alpha_s \) in between the \( J/\psi \) and the \( \Upsilon \) mass. This may serve as an estimate of the theoretical uncertainty. As already noted in Ref. [2], the dipole approach overestimates the DY data from E772 [20] by several standard deviations. This large discrepancy cannot be explained by uncertainties in the parametrization of the dipole cross section [2].
However, we point out that the E772 values for broadening [20] were extracted only from DY data with transverse momentum $p_T \lesssim 3$ GeV [25], where the $p_T$-differential DY cross section is still nuclear enhanced, and may, therefore, underestimate the true value of $\delta(p_T^2)^F$. Moreover, the $O(\alpha_s)$ parton model does not describe some of the $p_T$-integrated DY cross sections measured by E772, either [26]. A future analysis [27] based on new E866 data [28], will include DY data with transverse momentum up to $p_T \lesssim 5$ GeV, and may yield values of $\delta(p_T^2)^F$ that are twice as large [29]. One can, therefore, regard the curves in Fig. 3 as predictions.

It is interesting to note that, while E772 only used dileptons with $p_T \lesssim 3$ GeV, the transverse momentum imbalance in photoproduction of dijets was measured by E683 [30] only for jets with $p_T > 3$ GeV. It has been argued in [31], that the unusually large effect observed by E683 is (in part) caused by this restriction on $p_T$. In fact, a value of $\lambda_{LQS}^2 \approx 0.1$ GeV$^2$ is needed to accommodate the E683 result [5]. The analysis presented in this Letter clearly favors a much lower value, which is more consistent with the DY data.

The calculations in the dipole approach for broadening of gluons agree quite well with $J/\psi$ and $\Upsilon$ data, which are underestimated by the higher twist formalism, see Fig. 3 (right). Of course, broadening for gluons is equal to broadening in $J/\psi$ and $\Upsilon$ production, only if final state effects are negligible. This assumption is justified by the observation that broadening is very similar (within errorbars) for $J/\psi$ and $\Upsilon$ mesons.

4. Summary

In this Letter, we quantitatively related the color dipole approach [1,2] to the higher twist factorization formalism [4,5], and studied transverse momentum broadening of fast partons propagating through cold nuclear matter. In both approaches, broadening is proportional to the average color field strength experienced by the projectile parton [5,13,14]. We find that the corresponding coefficients differ only by the scale of the strong coupling constant. While broadening is an entirely soft process in the dipole approach, the extension of the QCD factorization theorem to twist-4 is justified by the smallness of $\alpha_s$. In the higher twist formalism, $\alpha_s$ enters at the typical hard scale of the process that probes the transverse momentum of the projectile parton. The equivalence between the dipole and the BDMPS approach [3] was already known before [8].

Since the dipole cross section is much better constrained by data than $\lambda_{LQS}^2$ and $\hat{q}_R$, one is now able to obtain new estimates for the latter two quantities. So far, $\lambda_{LQS}^2$ could be determined only from the same data the higher twist approach is supposed to describe [4], and estimates for $\hat{q}_R$ were based mostly on physical intuition [18]. With the KST-parametrization of the dipole cross section [16], which we use, broadening is a function of the energy of the projectile parton, as one would expect from a soft process. In the higher twist approach, there is an additional scale dependence through $\alpha_s$. To our best knowledge, this is the first time...
that quantitative results for the energy dependence of \( \lambda_{LQS}^2 \) and \( \hat{q}_R \) are presented. It will be necessary to take this energy dependence into account, when applying the higher twist formalism and the BDMPS approach at RHIC or even at LHC energies.

At fixed target energies, numerical calculations in the dipole approach exceed results obtained in the higher twist formalism by a factor of \( \sim 2 \). Most importantly, the uncertainty bands of both approaches do not overlap, if one varies the remaining free parameters within reasonable limits. Available data, however, do not yet allow to rule out one of the theories. Though the dipole approach describes \( J/\psi \) and \( \Upsilon \) data well, this agreement has to be interpreted with great care, since final state effects are not taken into account by the theory. We argue, however, that the similarity between broadening for \( J/\psi \) and \( \Upsilon \) mesons indicates that final state effects are rather small. The higher twist approach underestimates broadening for \( J/\psi \) and \( \Upsilon \) mesons. Broadening for \( \mathrm{DY} \), on the other hand, is overestimated in the dipole approach, while the higher twist formalism reproduces these data well. However, the small values of \( \delta \langle p_T^2 \rangle_F \) measured by E772 may be the result of a too low \( p_T \) cut imposed on the data. A reevaluation of the E772 data in question, as well as new results from E866 measurements, are expected soon [27]. This new analysis will probably yield significantly larger broadening for \( \mathrm{DY} \) dileptons [25,29]. We stress that no parameter in our calculations has been adjusted to fit the data. Thus, the curves presented here can be regarded as predictions.

Acknowledgements

I am indebted to Rainer Fries, Mikkel Johnson, Boris Kopeliovich, Pat McGaughey and Joel Moss for valuable discussion and to Mike Leitch for providing the \( J/\psi \)-broadening data. This work was supported by the US Department of Energy at Los Alamos National Laboratory under Contract No. W-7405-ENG-38.

References
