Analyses of Self-Focusing Phenomenon and Temperature Rise in Fused Silica by Ultrashort Pulse Laser Irradiation

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Abstract

When an ultrashort pulse laser is irradiated into the inside of a permeable medium, the refractive index changes dependently on the electric field strength. As a result, a self-focusing phenomenon occurs and self-focusing can lead to a filamentation. In the light absorption medium, the absorbed light energy changes to the thermal energy after ultrashort pulse laser irradiation. Then melting or partial ablation phenomena occur near the focusing area in the medium. Therefore, internal modification, lap welding of permeable materials and so on can be achieved. In this study, a paraxial wave equation including light absorption term was solved by numerical calculation using the fast Fourier transform for irradiation of a Kerr medium by an ultrashort pulse laser. Using the absorbed energy distribution as the initial condition, heat transfer analysis in the material was conducted. The influences of the nonlinear index intensity coefficient and the absorption coefficient on the temperature distribution in the material were investigated.

1. Introduction

When an ultrashort pulse laser is irradiated into the inside of a permeable medium, the refractive index changes dependently on the electric field strength. As a result, a self-focusing phenomenon occurs and self-focusing can lead to a filamentation. In the light absorption medium, the absorbed light energy changes to the thermal energy after ultrashort pulse laser irradiation. Then melting or partial ablation phenomena occur near the focusing area in the medium. Therefore, internal modification, lap welding of permeable materials and so on can be achieved. In this study, a paraxial wave equation including light absorption term was solved by numerical calculation using the fast Fourier transform (FFT) for irradiation of a Kerr medium by an ultrashort pulse laser. The study on birefringent modifications in the optical glass conducted by Kudriasov et al. [1] is referred to, and self-focusing and filamentation phenomena are analyzed in fused silica. Using the absorbed energy distribution as the initial condition, heat transfer analysis in the fused silica was conducted. The influences of the nonlinear index intensity coefficient and the absorption coefficient on the temperature distribution in the material were investigated. The validity of the analysis method and the analyzed results are confirmed by comparing the experimental results by Kudriasov et al.

2. Paraxial Wave Equation Including Light Absorption Term in a Kerr Medium

The wave equation in the nonlinear medium is expressed as follows:

\[
\nabla^2 E - \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} = \mu \frac{\partial^2 P_{\text{NL}}}{\partial t^2}
\]  

(1)
where $E$ is the electric field, $v$ is the phase velocity in the medium, $\mu$ is the magnetic permeability of the medium, and $P_{\text{NL}}$ is the nonlinear dielectric polarization. As the nonlinear dielectricity appears the third nonlinearity in the Kerr medium, it is expressed as follows:

$$P_{\text{NL}} = \varepsilon_0 \chi^{(3)} E^3$$

(2)

where $\varepsilon_0$ is the permittivity of vacuum, and $\chi^{(3)}$ is the third-order nonlinear susceptibility. On the other hand, the variation of the refractive index in Kerr medium is expressed by

$$\Delta n = n_2 I$$

(3)

where $I$ is the light intensity and $n_2$ is the nonlinear index intensity coefficient and is given by

$$n_2 = \frac{3 \chi^{(3)} \eta}{4n}$$

(4)

We assume that the electric field $E(x, y, z, t)$ comprises a complex envelope $E_c(x, y, z)$ riding on a carrier of frequency $\omega$ and wave number $k$ that propagates in $+z$ direction:

$$E(x, y, z, t) = \text{Re} \left\{ E_c(x, y, z) \exp \left[ i (\omega t - k z) \right] \right\}$$

(5)

Substituting Eq. (5) into Eq. (1) and assuming that $E_c(x, y, z)$ is a slowly varying function of $z$ (the direction of propagation), we obtain the paraxial wave equation,

$$\frac{\partial E_c}{\partial z} = - \frac{\alpha}{2} E_c$$

(6)

Furthermore, $E_c(x, y, z)$ is attenuated in the light absorption medium, and is expressed by

$$E_c(x, y, z + dz) = E_c(x, y, z) \exp \left( - \frac{\alpha}{2} dz \right)$$

(7)

where $\alpha$ is absorption coefficient and is expressed by

$$\alpha(t) = \alpha_0 + \beta I$$

(8)

where $\alpha_0$ is the linear absorption coefficient and $\beta$ is the nonlinear absorption coefficient. In the very small area in the medium, $\alpha(t)$ can be considered as constant, then Eq. (7) is given by a solution of a differential equation

$$\frac{\partial E_c}{\partial z} = - \frac{\alpha}{2} E_c$$

(9)

Superposing Eqs. (6) and (9), the paraxial wave equation to be solved here is given by

$$\frac{\partial E_c}{\partial z} = \frac{1}{2k} \left( \frac{\partial^2 E_c}{\partial x^2} + \frac{\partial^2 E_c}{\partial y^2} \right) - \frac{\alpha}{2} i \Delta n_0 E_c$$

(10)

3. Results and Discussion

In this paper, the study on birefringent modifications in the optical glass conducted by Kudriašov et al. [1] is referenced, and self-focusing and filamentation phenomena are analyzed. The laser beam is focused by a lens whose focal length is 200 mm, and is irradiated on the surface of a fused silica. Laser irradiation conditions are shown in Table 1. The intensity distribution at the surface is assumed to be Gaussian. The pulse shape is assumed to be also Gaussian, because they are not specified in Ref. [1]. Optical and thermal properties of fused silica [2] are shown in Table 2. The variation of the refractive index in Kerr medium is given by Eq. (3) and is dependent on the laser intensity $I$. The nonlinear index intensity coefficient $n_2$ is known as shown in Table 2. The absorption coefficient $\alpha$ also depends on the laser intensity $I$. In Eq. (8), linear absorption coefficient $\alpha_0$ is known, but the nonlinear absorption coefficient $\beta$ is unknown. Therefore the nonlinear absorption coefficient $\beta$ is treated as a parameter in this study. The numerical calculation of Eq. (10) is conducted by using the split-step Fourier method [3] with FFT.

Figure 1 shows the influence of nonlinear absorption coefficient $\beta$ on intensity distribution when time is pulse center ($t = 0$), that is, at the peak intensity. When the nonlinear absorption coefficient $\beta$ is relatively large

<table>
<thead>
<tr>
<th>Table 1. Laser irradiation condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength $\lambda$</td>
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<td>Pulse energy $E_p$</td>
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<td>Pulse width $t_p$</td>
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<td>Beam diameter at surface $2r_c$</td>
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<td>Divergence angle $\theta$</td>
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<th>Table 2. Optical and thermal properties of fused silica</th>
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<td>Refractive index $n$</td>
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<td>Nonlinear index intensity coefficient $n_2$</td>
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<tr>
<td>Linear absorption coefficient $\alpha_0$</td>
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<td>Density $\rho$</td>
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<tr>
<td>Specific heat $c$</td>
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<td>Thermal conductivity $K$</td>
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such as $1 \times 10^{-5}$ μm/W, the laser is absorbed near surface, and the laser beam intensity decreases as it propagates in the medium without self-focusing. Self-focusing can be recognized as the nonlinear absorption coefficient $\beta$ becomes small, because the intensity of laser beam which propagates in the medium is relatively large although the laser absorption near the surface is small. When the nonlinear absorption coefficient $\beta$ is $2 \times 10^{-4}$ μm/W, the laser is not absorbed almost near the surface. Remarkable self-focusing phenomenon occurs and the minimum spot appears at the depth of about 6.4 to 6.5 mm. The peak intensity reaches about 6 TW/cm². As the transparency of the medium increases, the intensity decrease becomes small, the extent of self-focusing becomes large, and the location of the minimum spot moves gradually in the surface direction.

Self-focusing is generated repeatedly when transparency increases sufficiently, such as $\beta = 1 \times 10^{-5}$ μm/W.

Fig. 1. Influence of nonlinear absorption coefficient $\beta$ on intensity distribution ($E_0 = 1.5 \mu J, t = 0 \ s$). The unit of color scale is W/μm², and $10^4$ W/μm² is 1 TW/cm².

Fig. 2. Enlargement of the minimum spot areas in Fig. 1. Aspect ratio is 1:1. The unit of color scale is W/μm², and $10^4$ W/μm² is 1 TW/cm².


\( \mu \text{m/W} \), because the laser beam diverges after self-focusing but its intensity is still large, as a result self-focusing occurs repeatedly until the intensity sufficiently decreases. The peak intensity in Fig. 1(f) is order of 10 TW/cm\(^2\). Figure 2 shows the enlargement of minimum spot areas in Fig. 1. The aspect ratio of these figures is 1:1. We can see that filamentation occurs at these minimum spot areas.

The analysis results are compared with the experimental results by Kudríašov et al. Figure 3 shows the experimental results by Kudríašov. Comparing with the analyzed result (Fig. 1) where pulse energy is 1.5 \( \mu \text{J} \), the minimum spot depth coincides with the experimental result when the nonlinear absorption coefficient \( \beta \) is \( 2 \times 10^{-9} \) \( \mu \text{m}/\text{W} \).

Figure 4 shows the influence of pulse energy \( E_p \) on the intensity distribution when time is pulse center \( (t = 0) \). The nonlinear absorption coefficient \( \beta \) is \( 2 \times 10^{-9} \) \( \mu \text{m}/\text{W} \). When the pulse energy is 0.1 \( \mu \text{J} \), the laser beam diverges in the medium, also as shown in Fig. 3(a). Self-focusing becomes remarkable as the pulse energy becomes large. Self-focusing is generated repeatedly when the pulse energy is very large. Intensity distributions along the central axis in Fig. 4 are shown in Fig. 5. When the intensity is over about 10 TW/cm\(^2\), self-focusing is generated repeatedly.

Comparing the analysis results of Fig. 4 with the experimental results shown in Fig. 3(b) by Kudríašov et al., the minimum spot depth coincides with the turning point at each pulse energy in the experiments. Figure 6 shows the intensity distribution along the central axis at various times during the pulse duration when the pulse energy is 1.5 \( \mu \text{J} \). Self-focusing starts at a deeper position and the minimum spot area turns around at the pulse center. Figure 7 shows Intensity distributions at time \( \pm 10 \), \( \pm 20 \) and \( \pm 30 \) fs during pulse duration.

The intensity distributions at various depths at pulse center are shown in Fig. 8. Self-focusing phenomenon can be recognized clearly. The wave front rounds the minimum spot point, therefore light wave interferes. As a result, a diffraction pattern is generated just after the minimum spot point as shown in Fig. 8(f).

Furthermore, the now pulse width is 130 fs, which is very short. Therefore, it is considered that the absorbed laser energy changes to the heat energy after the pulse end. That is, the absorbed laser energy in the pulse duration is stored in the medium without heat conduction. Heat conduction analysis is conducted assuming thermal diffusion occurs after the pulse end.

Temperature distributions at the pulse end are shown in Fig. 9 when the pulse energy is 1.5 \( \mu \text{J} \). Comparing with Fig. 1, the temperature distribution is affected mostly by the intensity distribution at the pulse center.

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Fig. 3. Experimental results by Kudríašov et al. [1]

Fig. 4. Influence of pulse energy \( E_p \) on intensity distribution \( (\beta = 2 \times 10^{-9} \mu \text{m}/\text{W}, t = 0) \) s. The unit of color scale is W/µm\(^2\); and 10\(^4\) W/µm\(^2\) is 1 TW/cm\(^2\).
Fig. 5. Intensity distributions along the central axis in Fig. 4.

Fig. 6. Intensity distribution along the central axis at various time during pulse duration when the pulse energy is 1.5 \( \mu J \).

(a) \( t = \pm 10 \) fs  
(b) \( t = \pm 20 \) fs  
(c) \( t = \pm 30 \) fs

Fig. 7. Intensity distributions at time \( \pm 10, \pm 20 \) and \( \pm 30 \) fs during pulse duration. The unit of color scale is W/\( \mu m^2 \), and \( 10^9 \) W/\( \mu m^2 \) is 1 TW/cm².

(a) Position of Figs. (b) to (f)  
(b) \( z = 0 \) mm  
(c) \( z = 2 \) mm  
(d) \( z = 4 \) mm  
(e) \( z = 6 \) mm  
(f) \( z = 8 \) mm

Fig. 8. Intensity distributions at various depths at pulse center. The unit of color scale is W/\( \mu m^2 \), and \( 10^9 \) W/\( \mu m^2 \) is 1 TW/cm².

The minimum spot position moves according to the pulse shape as shown in Fig. 6. As a result, high temperature region spreads a little in the depth direction. Heat conduction is very small after the pulse end.
The maximum temperature distributions are shown in Fig. 10 when the pulse energy is 1.5 μJ. The maximum temperature distribution is almost the same as the temperature distribution at pulse end (see Fig. 9). It is concluded that the processed area, such as void formation, fusion, is determined almost by the temperature distribution at pulse end.

4. Conclusion

A paraxial wave equation including light absorption term was solved by numerical calculation with the fast Fourier transform (FFT) in a Kerr medium irradiated by an ultrashort pulse laser. The study on birefringent modifications in the optical glass conducted by Kudriašov et al. is referenced, and self-focusing and filamentation phenomena are analyzed in fused silica. Using the absorbed energy distribution as the initial condition, heat transfer analysis in the fused silica was conducted. The obtained results are summarized as follows:

(1) Self-focusing becomes remarkable as the nonlinear absorption coefficient becomes small or pulse energy increases. Self-focusing is generated repeatedly when transparency increases sufficiently or pulse energy becomes very large.

(2) When the nonlinear absorption coefficient of fused silica is $2 \times 10^{-9}$ μm/W, the minimum spot depth coincides with the experimental results. When the intensity is over about 10 TW/cm$^2$, self-focusing is generated repeatedly.

(3) The minimum spot position moves according to the pulse shape. The high temperature region spreads a little in the depth direction.

(4) The maximum temperature distribution is almost the same as the temperature distribution at pulse end. Processed area, such as void formation, fusion, is determined almost by the temperature distribution at pulse end.

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