TDOA measurement based GDOP analysis for radio source localization

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Abstract

The revolution brought by GPS has lead to the development of various positioning applications. These applications use measurements (travel time of signal or time of flight) in determining the position. The time of flight requirement in GPS has restricted its use in positioning of unknown objects. Whereas, localization of an unknown enemy Radio Source (URS) such as enemy radar system, tracking of Unmanned Aerial Vehicle (UAV) etc., have high demand in the field of defence in a country like India, they require a new type of measurement technique called Time difference of Arrival (TDOA). There are various factors that affect the position accuracy including amount of measurement noise, algorithm employed for positioning and sensor URS geometry. The sensor-URS geometry is one of the most predominant factors in determining the accuracy estimate and is referred to as Geometry Dilution of Precision (GDOP). This is a well defined problem in positioning systems that use GPS/Time of arrival (TOA) measurements. However, it needs to be refined for URS localization systems/TDOA measurements. This paper mainly focuses on explaining and deriving the concepts of GDOP in relation to TDOA measurement based URS localization systems. For a comprehensive understanding, an illustrative example of localizing an URS with TDOA measurements is explained and discusses the effect of sensor geometry with the help of GDOP profiles. In addition, this paper explains the process of identifying an optimal sensor configuration for URS localization systems. For the purpose of simulation, five sensors arranged in two different configurations are considered. A target surveillance area of 3600 Sq-Kms with 169 target zones is used in generation of GDOP profiles over the Indian subcontinent.

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1. Introduction

Tracking and monitoring an URS is a critical task in the field of defence for safe guarding the country from enemy attacks, guiding military troops in the enemy territory etc. The essential requirements for such positioning systems are set of sensors and a central processing unit. These systems localize the position of an URS relative to the origin defined with reference to the sensor network location. The Radio frequency signal emitted from URS reaches the sensors at different times. These time differences are used in determining the position of URS and are called TDOA measurements. For example, in order to find the three dimensional position of an URS shown in Fig.1, the system needs to have three or more sensors and a central processing system preinstalled at known locations.

1.1. URS localization with TDOA measurements

Localization of the URS shown in Fig.1 with TDOA measurement technique is explained in this section. The scenario uses five sensors with S₀ as the reference sensor (origin) and R₀,R₁,...,R₄ represent the range between URS and iᵗʰ Sensor,Sᵢ where i=0,1,2,3 and 4.

Let the TDOA measurement observed between reference sensor, S₀ and the iᵗʰ sensor, Sᵢ is given as

\[ TDOA_{0i} = t_0 - t_i \]  \hspace{1cm} (1)

Where, \( t_0 \) = Arrival time at S₀ sensor, \( t_i \) = Arrival time at Sᵢ sensor and i=1, 2, 3 and 4.

Hence, the observed range difference of arrival between Sensor, S₀ and the iᵗʰ sensor, Sᵢ is referred to as RDOA₀ᵢ and is calculated by multiplying the TDOA₀ᵢ with the velocity of signal (c) (Eq.2)

\[ RDOA_{0i} = c \times TDOA_{0i} = c \times (t_0 - t_i) = R_0 - R_i \]  \hspace{1cm} \text{where, } i=1, 2, 3 \text{ and } 4 \]  \hspace{1cm} (2)
Here, \( R_0 = \sqrt{(X_0 - X_{Rs})^2 + (Y_0 - Y_{Rs})^2 + (Z_0 - Z_{Rs})^2} \) and \( R_1 = \sqrt{(X_1 - X_{Rs})^2 + (Y_1 - Y_{Rs})^2 + (Z_1 - Z_{Rs})^2} \)

(3)

Where, \((X_{Rs}, Y_{Rs}, Z_{Rs})\) defines the position coordinates of URS, \((X_0, Y_0, Z_0)\) and \((X_i, Y_i, Z_i)\) define the reference sensor and \(i^{th}\) sensor position coordinates respectively.

Equation 2 is nonlinear and can be solved using a closed-form solution\(^3\) or can be linearised and approximated to 1st order Taylor’s series\(^4\). The resultant linearised equation is a differential equation represented in matrix notation, \((B \times \delta P_{Rs} \delta RD OA_{0i})\) given in Eq. 4.

\[
\begin{bmatrix}
B_{X1} & B_{Y1} & B_{Z1} \\
B_{X2} & B_{Y2} & B_{Z2} \\
\vdots & \vdots & \vdots \\
B_{X4} & B_{Y4} & B_{Z4}
\end{bmatrix}
\begin{bmatrix}
\delta X_{Rs} \\
\delta Y_{Rs} \\
\delta Z_{Rs}
\end{bmatrix}
= \begin{bmatrix}
\delta RD OA_{01} \\
\delta RD OA_{02} \\
\vdots \\
\delta RD OA_{04}
\end{bmatrix}
\]

(4)

Where,

\[
B_{X1} = \frac{\dot{X}_{Rs} - X_i}{R_i}, \quad B_{Y1} = \frac{\dot{Y}_{Rs} - Y_i}{R_i}, \quad B_{Z1} = \frac{\dot{Z}_{Rs} - Z_i}{R_i}
\]

\[
\delta RDOA_{0i} = RDOA_{0i} - \hat{RDOA}_{0i}
\]

Here \((\dot{X}_{Rs}, \dot{Y}_{Rs}, \dot{Z}_{Rs})\) represents the URS position estimate, the vector \(\delta P_{Rs} = (\delta X_{Rs}, \delta Y_{Rs}, \delta Z_{Rs})\) represents the change in the estimate position or error in the estimated position, vector \(\delta RD OA_{0i}\) represents the error in measurements, \(\hat{RDOA}_{0i}\) is the estimated range difference of arrival between \(S_0\) and \(S_i\) sensors and B matrix is the measured matrix.

When the matrix B in Eq.4 is a square matrix (i.e. no.of measurements equal to no.of unknowns), then the URS estimated position, \((\dot{X}_{Rs}, \dot{Y}_{Rs}, \dot{Z}_{Rs})\) is updated with a change in the estimate position, \(\delta P_{Rs}\) which is computed using Eq.5 and this process is repeated till a threshold is reached. If the matrix B is not square (i.e. over determined system as shown in Fig.1), then Eq.6 is used to update URS estimated position, \((\dot{X}_{Rs}, \dot{Y}_{Rs}, \dot{Z}_{Rs})\) and the above process is repeated\(^5\).

\[
\delta R_{Rs} = B^{-1} \times \delta RDOA_{0i}
\]

(5)

\[
\delta R_{Rs} = (B^T B)^{-1} B^T \times \delta RDOA_{0i}
\]

(6)

The accuracy in the estimated position, \((\dot{X}_{Rs}, \dot{Y}_{Rs}, \dot{Z}_{Rs})\) of URS is influenced by various system and environment factors like uncertainty in measurements, system noise and sensor-URS geometry. However, the effect due to the noise and other factors can be reduced by employing various error mitigation techniques\(^6\) and positioning algorithms\(^7\). The only way in avoiding the effect due to geometry is with proper placement of sensors\(^8\). This task can be performed by analyzing the GDOP profiles for various sensor configurations.

2. Effect of sensor geometry on URS position accuracy

Position accuracy is defined as the degree of closeness of estimated position to the true position\(^9\) or is defined as the error in estimated position\(^10\). It depends on sensor geometry and measurement error. Equation 7 describes their relationship where, UERE represents Root Mean Square Error in range (RMSE) or User Estimated Range Error.

\[
\text{Estimated URS Position Accuracy} = \text{Sensor to URS geometry} \times \text{UERE}
\]

(7)
Recalling Eq.4, the vector, $\delta RDOA_0$, which represents error in range (RDOA) measurements, is the UERE term in Eq.7 for TDOA systems. In general, it is considered that measurement and position errors are random variables with zero mean and variance 1 (standard normal distribution). Hence, their RMSE is computed from the error covariance matrices and Eq.7 is modified accordingly and is given in Eq.8.

$$\text{Estimated URS Position Accuracy} = \text{Geometry} \times \text{Trace} \left( \mathbb{E} \left[ \left( \delta RDOA - \mathbb{E} \left[ \delta RDOA \right] \right) \left( \delta RDOA - \mathbb{E} \left[ \delta RDOA \right] \right)^\top \right] \right).$$  \hspace{1cm} (8)

It is understood from Eq.7 that even though there is no uncertainty in the measurements (i.e. UERE=Identity Matrix), Position accuracy is affected due to geometry. This is illustrated in Fig.2a and Fig.2b and the further decay in the Position accuracy due to the combined effect of measurement error and geometry is shown in Fig.2c and Fig.2d. This phenomenon is called GDOP. Equation 7 also depicts that the estimated position accuracy increases with decreased GDOP coefficient and vice-versa. So identification of an optimal sensor geometry/configuration is essential to increase the position accuracy and this paper tries to achieve this end. As the concept of GDOP in GPS is applicable to a TDOA-system, circles are used instead of hyperbolas for clear understanding.

Fig.2. Effect of Sensor Geometry on Position Accuracy (a) High Precision Sensors at long distance; (b) Low Precision Sensors at Short distance; (c) High Precision Sensors at long distance and uncertain measurements; (d) Low Precision Sensors at Short distance and uncertain measurements.

### 3. Coefficient of GDOP for TDOA base URS systems

As discussed in the previous section, GDOP is defined as the coefficient that provides the effect of sensor URS geometry on the relationship between RMSE in position estimate, $\delta P_{Rs}$ to RMSE in measurements, $\delta RDOA_0$, and is given in Eq.8.

Computation of GDOP implies determining the effect of Sensor Geometry on the position accuracy, $\delta P_{Rs}$, and hence this paper starts with Position Error Covariance matrix in deriving it. The term, $\text{COV}_{\delta P_{Rs}}$ in Eq.9 represents the position error covariance matrix.

$$\text{COV}_{\delta P_{Rs}} = \mathbb{E} \left( \left( \delta P_{Rs} - \mathbb{E} \left[ \delta P_{Rs} \right] \right) \left( \delta P_{Rs} - \mathbb{E} \left[ \delta P_{Rs} \right] \right)^\top \right).$$  \hspace{1cm} (9)

Here, $\mathbb{E}$ is the expectation or mean operator and on assuming that mean of position error is zero, Eq.8 is written as

$$\text{COV}_{\delta P_{Rs}} = \mathbb{E} \left( \delta P_{Rs} \delta P_{Rs}^\top \right).$$  \hspace{1cm} (10)

On substituting Eq.6 in Eq.10, the Covariance matrix is given as

$$\text{COV}_{\delta P_{Rs}} = \mathbb{E} \left( \left( B^T B \right)^{-1} B^T \delta RDOA_0 \delta RDOA_0^\top \right).$$
\[ \text{COV}_{r_s} = E\left( (B^T B)^{-1} B^T \delta RDOA_{0i}^T \delta RDOA_{0i} \right) \]  \hspace{1cm} \text{(11)}

As the elements in matrix B are measured values, the expectation operator is only applied to measurement error matrix, \( \delta RDOA_{0i} \) and Eq.11 is represented as

\[ \text{COV}_{r_s} = (B^T B)^{-1} B^T \left( (B^T B)^{-1} B^T \right)^T E\left( \delta RDOA_{0i} \delta RDOA_{0i}^T \right) \]  \hspace{1cm} \text{(12)}

Here, \( E(\delta RDOA_{0i} \delta RDOA_{0i}^T) \) represents the measurement error Covariance matrix, \( \text{COV}_{\delta RDOA_{0i}} \) with an assumption that mean of the measurement error is zero. It is obvious from Eq.4 that B is the only matrix that holds the information of sensor URS geometry and therefore is also called as Geometry matrix. On further simplification Eq.12 can be rewritten as

\[ \text{COV}_{r_s} = (B^T B)^{-1} B^T \left( (B^T B)^{-1} B^T \right)^T \text{COV}_{\delta RDOA_{0i}} \]  \hspace{1cm} \text{(13)}

For the considered URS localization system, the size of matrix B in Eq.4 is \( 4 \times 3 \) and hence the resultant matrix in Eq.13, \( (B^T B)^{-1} \) is a \( 3 \times 3 \) matrix (i.e Square). The trace of this matrix defines the GDOP coefficient, with estimated position error variances provided by diagonal elements.

\[ \begin{bmatrix}
\Var_X & \Cov_{XY} & \Cov_{XZ} \\
\Cov_{YX} & \Var_Y & \Cov_{YZ} \\
\Cov_{ZX} & \Cov_{ZY} & \Var_Z
\end{bmatrix}
\]

On comparing Eq.13 with Eq.8, the effect of sensor geometry or the Coefficient of GDOP can be calculated as

\[ \text{GDOP} = \text{trace} \left( (B^T B)^{-1} \right) = \sqrt{\Var_X + \Var_Y + \Var_Z} \]  \hspace{1cm} \text{(14)}

The off diagonal elements of the above matrix are considered independent and are zero.

4. Results and discussion

Two different sensor configurations are used in this section to find the optimal sensor configuration that has low geometrical effect on the precision of estimated URS position. For generating the GDOP profile, a surveillance area of 3600 Sq-Kms is considered with sensors placed as shown in Fig.3a and Fig.3b. In the considered area, 169 target zones are selected and GDOP for pentagon and trapezoidal configurations is determined for every target zone. The areas highlighted in both the figures separate the zones having GDOP less than 10 from other zones. The observed maximum RMS error in the Zones (GDOP<10) is 10.2 m and 7.58 m for trapezoidal and pentagon configurations respectively. The details of maximum and minimum GDOPS and mean error for 169 selected zone areas are given in Table 1. All reported values and plots are computed with measurement noise of 2ns, 4ns, 2ns and 4ns in four TDOA measurements. The simulation constraints used in plotting GDOP profiles for both the configurations are given below.

**Simulation constraints for Pentagon:**

<table>
<thead>
<tr>
<th>Area Considered</th>
<th>: 3600 Sq-Kms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude of URS</td>
<td>: 7Kms.</td>
</tr>
<tr>
<td>TDOA1 measurement error</td>
<td>: 2n sec</td>
</tr>
</tbody>
</table>
TDOA2 measurement error : 4n sec  
TDOA3 measurement error : 2n sec  
TDOA4 measurement error : 4n sec  
Zone area selection distance : 5Kms.

Sensor Coordinates in meters:  
X Coordinates = [0, -19021, -11756, 11756, 19021]  
Y Coordinates = [-20000, -6180, 16180, 16180, -6180]  
Z Coordinates = [25, 35, 40, 30, 45]  
Reference Sensor Coordinates = [0, -20000, 25]

*Simulation constraints for Trapezoid:*  
Area Considered : 3600 Sq-Kms.  
Altitude of URS : 7Kms.  
TDOA1 measurement error : 2n sec  
TDOA2 measurement error : 4n sec  
TDOA3 measurement error : 2n sec  
TDOA4 measurement error : 4n sec  
Zone area selection distance: 5Kms.  
Sensor Coordinates in meters:  
X Coordinates = [15000, -7500, 7500, 15000, 0]  
Y Coordinates = [0, 15000, 18750, 0, 0]  
Z Coordinates = [25, 35, 40, 30, 45]  
Reference Sensor Coordinates = [0, -20000, 25]
5. Conclusions

A new method of determining the Sensor-URS geometry (GDOP) effect on the Unknown radio source position’s accuracy over the Indian subcontinent is presented. In addition, the GDOP profile in the considered surveillance area for pentagon and trapezoidal sensor configurations are presented, which is easily extended to any other configurations. For the measurement noise sequence (2ns, 4ns, 2ns, 4ns) with GDOP value less than 10, the observed maximum RMSE values for both the configurations are less than 10.2 m. It shows that both the configurations are suitable for real time URS localization with TDOA measurements. In addition, the GDOP profile
for trapezoidal sensor configuration shows that it is an optimal sensor configuration for systems operating with a Field of View (FOV) of 180°.

References