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In Contango Versus Backwardation, The Truth May Not Be In Convenience: Disequilibrium States and the Spot-Forward Balance in Commodity Markets

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Abstract

The notion of a stochastic 'convenience yield' to explain variations and reversals in the spot-forward premium is a common rationalisation in commodity market research. However, such variations may arise from causes more intrinsically related to the structure and cash flows of the extended commodity markets. An instance is where the market can be subject to disequilibrium phases, characterised by rationing or clearing impediments that interfere with arbitrage. These are likely to arise when market inventory is in short supply, so that disequilibrium switches can be based on the inventory /sales ratio.

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1. Introduction

The relationship between spot and forward commodity prices has in recent years been thrown into the broader spotlight of public attention. In part this arose with high profile corporate hedging crises; often precipitated by a switch from a normal state of backwardation, where forward prices are lower than spot prices, to contango, for a discussion see Culp/Miller [7], Edwards/Canter[8] , Mello/Parsons[16], or for the general empirics of basis variation in the oil market, Litzenberger/Rabinowitz[15].

Nomenclature

S_0	the current (time 0) spot price.
F_0	the current price for forward (period 1) delivery.
r_g	the commodity loan rate, where this exists.
r_c	the spot - forward balance rate (defined in section 3).
r	the current borrowing rate for money; determined exogenously to the commodity market.

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A second major impetus has been the rise of China as an industrial power, together with a search by hedge funds for alternative investments after the global finance crisis and the enhanced spectrum of tradable products (e.g. carbon emission rights). Commodity trading has become big business, while commodity spot and forward prices have become the subject of day to day media commentary. Switches between contango and backwardation have become more frequent (cf. Bowden/Posch[4]).

The present contribution adds a further dimension to the discussion of the convenience yield, that of disequilibrium dynamics. In a perfect world, the actions of agents with different motivations are reconciled with efficient pricing mechanisms by means of an institutional framework that may be implicit as well as explicit. A commodity market can be regarded as a compound structure with multiple clearing prices for each constituent market. Thus in addition to spot and forward prices for the commodity itself, there is an implicit market for the lending and borrowing of the commodity. Though not explicitly studied here, the market for gold comes as close as anything to such an ideal framework. The gold loan rate adjusts to meet the demands of arbitrageurs who borrow gold to sell now to take advantage of a futures price seen as too cheap (section 2). The convenience that arises to users from having gold on hand, rather than lending it, becomes factored in the gold loan rate, as does the storage cost for the gold inventories necessary for such lending operations. For this special case, the convenience yield becomes a well defined objective reality, and the gold loan rate can exceed the interest rate as the cost of financial carry, leading to observed backwardation. Commercial gold storage with transferable (and lendable) titles has become big business with major repositories in Singapore, Switzerland and the US. More recently the copper market has been evidencing similar formalised storage facilities in China and elsewhere.

The scheme of the paper is as follows. Section 2 establishes a stylised market structure within which equilibrium or disequilibrium regimes are to be embedded. Agents are identified in terms of their activities and motivations. Discussion turns in section 3 to disequilibrium states, where the apparent commodity loan rate, backed out from the spot-forward balance as the 'convenience yield', differs from the imputed equilibrium rate. The final section 5 concludes with a discussion and overview. An empirical implementation of our model is beyond the scope of this paper, see Bowden/Posch[4] for more details.

2. Agents, structures, and full market equilibrium

Disequilibrium states can be understood as departures from a background state of equilibrium. Both depend upon an understanding of the market structure. The present section outlines a representative commodity market structure and the imputed equilibrium price variables that result. Development is on a general level, the object being to describe how predetermined or exogenous variables might drive the equilibrium solution.

2.1. *Market structure*

A commodity market is typically a compound structure made up of three markets or submarkets: for the physical commodity, for forward delivery (or the equivalent non-deliverable), and for commodity loan agreements, which doubles as an implied storage market. All three markets are linked by flows. At any moment in time, the total expressed supply into the spot market is given by the sum of supply from opening stock, new producer supply, and also arbitrage supply if spot and forward prices are seen as out of line. Likewise, the total spot demand depends upon the flow of industrial user demand, any demand excess to current carryover, and arbitrage operations or arbitrage settlement demand outstanding from the previous period.

The respective markets are in turn populated by different types of economic agents, with the proviso that any particular player may at the same or different times adopt more than one position, or act with multiple motivations.

(a) Producers or users. Markets exist in the first place because industrial users demand the commodity, with a corresponding supply response from producers.

(b) Physical storers. Storage will refer here to finished inventory, rather than in the production pipeline as goods in progress. It includes producer stockpiles, consumer stock in hand, and to stocks held by independent commercial storage operators and available for purchase or loan.

(c) Speculators. These are agents who take open positions in the forward market, gambles which may or may not have a physical spot leg. They are responsive to their expectations for future prices in relation to current spot prices, or to the term structure of forward prices.

(d) Arbitrageurs. The aim here is to take fully covered positions where spot and forward prices are seen as out of line.

The agents act within the three implied submarkets: spot, forward, and commodity loan markets. Demand and supply in each submarket are linked by arbitrage activity. In the present paper arbitrageurs are seen as subject to potential constraints, which may encompass liquidity shortages, institutional or behavioural barriers. However, in the absence of any such constraints, the textbook arbitrage operation works via all three implied markets. If the forward price is seen as too cheap, then arbitrageurs will borrow the physical at rate r_g and sell it spot at S_0. The borrowed physical commitment is covered by buying on the forward market at F_0. Alternatively if the spot price is seen as too cheap, arbitrageurs will buy the physical spot, using money borrowed at rate r; lend the physical at rate r_g and sell the proceeds forward. The net result is that the three implied prices will adjust to ensure that

$$F_0(1+r_g) = S_0(1+r),$$

which is spot forward parity. Rate r is the carry cost of the physical, while the commodity loan rate r_g is the effective carry cost of the forward. The equilibrium commodity loan rate will reflect storage costs, including the opportunity cost of funding it. It will also encompass the convenience benefit to users from having stock on hand should it be needed to ramp up production. In a perfect market, the commodity loan rate can be equated with the convenience yield, effectively the net price of storage.

2.2. Agent behaviour

Spot market supply at any time t is made up of fresh supply by commodity producers together with a contribution from opening stock (K_t) carried over as inventory from the previous period. Fresh supply is represented as

$$q_t^s = \varphi_s(S_t, z_{st}),$$

where z_s refers to exogenous production drivers. Opening inventory holders can elect to supply directly into the spot market (q_{kt}^s) or hold as carry over stock

$$K_{ct} = (K_t - q_{kt}^s)_+$$

.Decisions by inventory holders to either supply now or carry are driven by current versus expected future spot prices. The carry is also available for commodity loan operations in the current period, so that the commodity loan rate r_g is a further influence on the spot supply or carry decision. In summary,

$$q_{kt}^s = \varphi_k(S_t, S_{t+1}^e, r_g)K_t \text{ with } \varphi_1 > 0; \varphi_2 < 0; \varphi_3 < 0.$$

The complementary carry allocation K_{ct} is increasing in S_{t+1}^e relative to the current price S_t.

Spot market demand likewise stems from two possible motives. Demand by industrial users for current production activities is

$$q_t^d = \varphi_d(S_t, z_{dt}),$$

where z_d incorporates exogenous demand drivers. A second source of spot demand is for carry purposes for next period's desired opening inventory

$$K^d_{t+1} = \kappa(S^e_{t+1}),$$

so that high expected prices induce more investment in stock. Incremental demand will occur only if desired end of period inventory exceeds the carry K_{c0}. In this case,

$$q^d_{kt} = [K^d_{t+1} - K_{ct}]_+ = [\kappa(S^e_{t+1}) - K_{ct}]_+.$$

The forward and commodity loan markets utilise the carry stock K_{ct}. Spot market sales originating in arbitrage must be supplied out of carry stock and returned into carry at the end of the period. Expected future spot prices are an important formative input on the current forward rate F_t, which in turn has a feedback relationship with the commodity loan rate r_{gt}. In turn, the commodity loan rate is determined within the arbitrage submarket. Arbitrage demand and supply, as expressed in the spot market, may be taken as proportional to any perceived spot-forward discrepancies.

2.3. Full equilibrium

A full equilibrium path will be defined as one along which spot forward parity continually applies, with no dynamic influences from current or outstanding arbitrage operations. Adopting more general time notation, let S^*_t, F^*_t, r^*_{gt} denote such a path. The spot market balance reduces to

$$\varphi_k(S^*_t, S^{*e}_{t+1}, r^*_{gt})K^*_t + \varphi_s(S^*_t, z_{st}) = \varphi_d(S^*_t, z_{dt}) + [\kappa(S^{*e}_{t+1}) - K^*_{ct}]_+ \quad (1)$$

where

$$K^*_{ct} = (1 - \varphi_{k*})K^*_t$$

and inventory accumulation is given by

$$K^*_{t+1} = K^*_{ct} + [\kappa(S^{*e}_{t+1}) - K^*_{ct}]_+ .$$

Equilibrium forward prices are assumed to be driven by expected future prices and by one or more risk factors collectively denoted by π_t:

$$F^*_t = \phi(S^{*e}_{t+1}, S^*_t, \pi_t) \quad (2).$$

Along the equilibrium path, spot-forward parity applies

$$F^*_t(1 + r^*_{gt}) = S^*_t(1 + r_t) \quad (3).$$

Equations (1)-(3) determine the equilibrium time path for the spot, forward prices and the commodity loan rate.

A final remark concerns risk. In full equilibrium states, risk is identified with general volatility in future spot prices, as it affects producers and users. In practice, risk can also enter via a short squeeze where potential inventory sellers or lenders withhold stock to exploit known or suspected short forward positions. This is more likely to arise where market inventory is in short supply. States of this kind are the subject of the next section.

3. Disequilibrium states

The treatment that follows utilises the no-arbitrage equilibrium as a benchmark solution. Once this is established, the effect of spot-forward disequilibrium can be developed as a modification to the benchmark solution. Disequilibrium itself is identified as a failure of spot-forward arbitrage associated with deficiencies or impediments in the associated commodity loan market. In some contexts, established loan markets may not exist. However even where they do exist, clearing may be incomplete or imperfect, problems often associated with the lack of a formal traded market or open outcry, in this case for loans as bilateral OTC agreements. An arbitraging borrower might be willing to pay more, only to find that his or her customary sources have already committed all available loan stock, perhaps at a lower rate. This might particularly arise where inventory available to lend is tight and search becomes difficult. Likewise, an arbitrager who has purchased stock, with the intention of lending it, may well find that nobody wants to borrow. Such a situation might arise if there is already a lot of inventory in the market.

3.1. Non-equilibrating commodity loan rates

For the above reasons, search and clearing impediments for available inventory or inventory loans may create situations where from time to time, $r_{gt} \neq r_{gt}^*$. In turn, this will induce out of equilibrium behaviour for spot and forward prices. Thus:

$$\frac{S_t - S_t^*}{S_t^*} = \zeta(r_{gt} - r_{gt}^*)$$
$$\frac{F_t - F_t^*}{F_t^*} = \psi(r_{gt} - r_{gt}^*) \tag{4}$$

with $\zeta(0) = \psi(0) = 0; \quad \zeta > -1, \psi > -1; \quad \zeta' < 0, \psi' > 0$. To see the effect of these conditions, suppose that there is loan rationing, so that $r_{gt} < r_{gt}^*$. Arbitrageurs cannot borrow all they want at the prevailing rate r_{gt} . This implies a corresponding constraint on buying forward (so that $F_t < F_t^*$) and on selling spot (so that $S_t > S_t^*$). Hence the function ψ passes upwardly through the origin. A similar analysis applies to the spot disequilibrium; in this case the schedule ζ has a negative slope through the origin.

For further reference, three contingencies may occur:

Regime A: $r_{gt} < r_{gt}^*$ and $\Delta_t = F_t(1 + r_{gt}) - S_t(1 + r_t) < 0$

Regime E: $r_{gt} = r_{gt}^*$ and $\Delta_t = F_t(1 + r_{gt}) - S_t(1 + r_t) = 0$ (5a)

Regime B: $r_{gt} > r_{gt}^*$ and $\Delta_t = F_t(1 + r_{gt}) - S_t(1 + r_t) > 0$.

Regime A is where the forward price is seen as too cheap. Arbitrageurs will want to want to borrow stock, sell it into the spot market and cover by buying forward. But if the commodity loan rate is slow to equilibrate, they will be rationed in their ability to borrow the necessary physical stock. Regime C is the complementary disequilibrium, where the forward price is seen as too expensive. Regime E is the equilibrium state as in section

2. For future reference it may also be noted that for:

Regime A: $\zeta > 0, \psi < 0; \quad (\zeta - \psi)/(1+\psi) > 0$; (5b)

Regime B: $\zeta < 0, \psi > 0; \quad (\zeta - \psi)/(1+\psi) < 0$.

3.2. The forward premium

The forward premium, represented here as $(F - S)/F$, is a common object of observation and analysis in the commodity markets. In the implicit form

$$(1 + r_{ct})F_t = (1 + r_t)S_t \tag{6a}$$

the 'convenience yield' r_{ct} has a proxy relationship with the forward premium, though with a negative sign:

$$r_{ct} = r_t - (\frac{F_t - S_t}{S_t})/[1 + (\frac{F_t - S_t}{S_t})] \approx r_t - \frac{F_t - S_t}{F_t}. \tag{6b}$$

If market inventory is low, then its shadow price should be high, with a higher 'convenience yield'. In such a circumstance the observed forward premium would be lower.

However, the term 'convenience yield' can be misleading without further qualification. For as defined in expression (6), r_{ct} corresponds in form to a commodity loan rate. As such, it can inherit the separate influences of a market price, such as disequilibrium constraints in the processes setting the observed market price. For such reasons, we will refer to r_{ct} in more neutral terms as the 'spot-forward balance rate'.

If r_{gt} is the currently observed commodity loan rate, then expressions (5) and (6) imply that:

$$r_{ct} = r_{gt} - \frac{\Delta_t}{F_t}.$$

In regime A, $\Delta_t < 0$. Thus the spot-forward balance rate exceeds the apparent commodity loan rate. To this extent, the spot forward balance rate can be regarded as a shadow price that more closely reflects unsatisfied supply of inventory for arbitrage purposes.

Finally, combining expressions (4) and (6) connects the spot-forward balance rate with the equilibrium loan rate, as:

$$1 + r_{ct} = (1 + r^*_{gt})\frac{1 + \zeta_t}{1 + \psi_t}. \tag{7}$$

This can be further consolidated by making use of a summary decomposition of the equilibrium loan rate. Denote the time t expected rate of change in equilibrium spot prices as

$$s^e_t = \frac{S^{*e}_{t+1} - S^*_t}{S^*_t}. \tag{8a}$$

Consistent with expression (2), suppose also that

$$F^*_t = \frac{S^{*e}_{t+1}}{1 + \pi_{ft}}. \tag{8b}$$

In expression (8b), π_{ft} is a risk factor expressed in discount form (it can be of either sign). Higher values correspond to a higher risk premium. Using equations (8a,b) in conjunction with (3) gives

$$1 + r^*_{gt} = \frac{(1 + r_t)(1 + \pi_{ft})}{1 + s^e_t}. \tag{9}$$

Combining (9) with (7) gives the spot forward balance as

$$1 + r_{ct} = \frac{(1 + r_t)(1 + \pi_{ft})}{1 + s^e_t}(1 + \frac{\zeta_t - \psi_t}{1 + \psi_t}); \tag{10a}$$

or to a first approximation,

$$r_{ct} \approx r_t - s^e_t + \pi_{ft} + \frac{\zeta_t - \psi_t}{1 + \psi_t}. \tag{10b}$$

Expression (10b) can form the basis of an estimating equation, with the substitution of suitable proxies for the right hand terms. Operational aspects are discussed in the next section.

4. Concluding remarks

The notion of a stochastic 'convenience yield' has become a much used device in commodity modelling. However, it is subject to criticism on the grounds of incomplete specification as to origins or causes. It might well arise for reasons that have little to do with the convenience of anybody in particular. A similar point is made by Jarrow[14], who identifies embedded scarcity and usage options in the structure of the market's cash flows. The present paper is to this extent similar, that it seeks an explanation in the way the markets work and the cash flows involved. The convenience yield can best be regarded as a balancing item that originates in the structure and operations of the market. As such there is a case for renaming it as the 'spot forward balance rate' or a similar label more neutral as to implied causation.

The spot forward balance rate may encompass a number of conceptually distinct items. In particular, it can be expected to arise when inventory is tight in relation to sales. It may be worse when markets are more illiquid or storage is difficult, as with lead. And to the extent that it can be regarded as the disequilibrium outcome of a market imperfection, the variation in the spot forward balance is episodic in nature. That may mean it is less amenable to representations in terms of standard stochastic models. Our results confirm these conjectures. Episodic disequilibrium phases and their representations provide a rationale for the diminishing marginal effect as inventories rise, noted by Fama/French [9,10] and others.

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