Prediction of wear at revolute clearance joints in flexible mechanical systems

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Abstract

In this paper, a numerical approach is proposed to predict wear at the revolute clearance joint in flexible mechanical systems, which integrates wear prediction with flexible multibody dynamics. In the approach, the flexible component is modeled based on absolute nodal coordinate formulation, and the contact force in the joint is calculated by a continuous contact model. The comparison of the wear predicted at the clearance joint in rigid and flexible planar slider-crank mechanism demonstrates that the proposed approach can be used to model and predict wear at revolute clearance joint in flexible multibody systems, and the wear result predicted is slightly reduced after taking components’ flexibility into account.

Keywords: Wear prediction; Clearance joint; Flexible mechanism; Absolute nodal coordinate formulation (ANCF)

1. Introduction

Wear at the joint is inevitable and often a critical factor influencing the service life. In order to better design the flexible mechanical systems with maximized service life, it is absolutely necessary to develop methods and tools to predict the wear at the clearance joint in a flexible mechanism.

Several researchers have studied the flexible mechanism with clearance joint. Chunmei et al. [1-3] studied the effects of the flexibility and clearance on the behavior of a flexible four-bar mechanism. Tian et al.[4] studied planar flexible multibody systems with clearance based on the absolute nodal coordinate formulation. The above researchers neglected the influence of wear at the clearance joint. This problem has been recognized recently and some works with the consideration of wear are further developed only in the rigid multibody systems by Flores et al [5-7].

As mentioned above, few studies focus on the wear prediction at the clearance joint in a flexible mechanism. Therefore, the primary purpose of this study is to predict the wear at the clearance joint in flexible multibody systems. An iterative wear prediction procedure using the FEM is employed to calculate the wear.

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2. Modeling a flexible mechanism systems with clearance joint

2.1 Planar ANCF-based locking-free shear deformation beam element

A planar ANCF-based locking-free shear deformation beam element in the global coordinate frame is shown in Fig.1(a). The vector of nodal coordinates of the beam element can be expressed as

\[ \mathbf{e} = \begin{bmatrix} (r^i)^T, (r^j)^T, (r^k)^T, (r^l)^T, (r^m)^T, (r^n)^T \end{bmatrix}^T \]

where \( r^m (m = i, j, k) \) is the position coordinates, and \( r^n = \left[ \partial r^m / \partial y, \partial r^m / \partial y \right] \) determines the beam cross section.

The displacement field of the element under the global coordinate system can be expressed as

\[ \mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} a_0 + a_1 x + a_2 y + a_3 x y + a_4 x^2 + a_5 x^2 y \\ b_0 + b_1 x + b_2 y + b_3 x y + b_4 x^2 + b_5 x^2 y \end{bmatrix} = \mathbf{S} \mathbf{e} \]

where \( x \) and \( y \) denote the coordinates in the element local coordinate system. \( \mathbf{S} \) is the shape function of the element. According to the conventional finite element method, the shape function takes the following form:

\[ \mathbf{S} = \begin{bmatrix} s_1 \mathbf{I}_2 \\ s_2 \mathbf{I}_2 \\ s_3 \mathbf{I}_2 \\ s_4 \mathbf{I}_2 \\ s_5 \mathbf{I}_2 \\ s_6 \mathbf{I}_2 \end{bmatrix} \]

\[ s_1 = 1 + 2 \xi^2 - 3 \xi \]

\[ s_2 = l \eta (1 + 2 \xi^2 - 3 \xi) \]

\[ s_3 = 4 \xi - 4 \xi^2 \]

\[ s_4 = l \eta (4 \xi - 4 \xi^2) \]

\[ s_5 = 2 \xi^2 - \xi \]

\[ s_6 = l \eta (2 \xi^2 - \xi) \]

where \( \mathbf{I}_2 \) is a \( 2 \times 2 \) identity matrix, \( \xi = x / l \), \( \eta = y / l \) and \( l \) is the length of undeformed element.

The element dynamic equation can be written as [8]

\[ \mathbf{M}_e \ddot{\mathbf{e}} + \mathbf{F}_e = \mathbf{Q}_e \]

Where \( \mathbf{M}_e \) is the mass matrix of beam element, \( \mathbf{F}_e \) is the elastic forces and \( \mathbf{Q}_e \) is the vector of element generalized nodal forces caused by gravity and contact forces.

2.1 Modeling a revolute clearance joint

Fig. 1(b) shows two bodies \( i \) and \( j \) connected by a revolute clearance joint. The eccentricity vector \( \mathbf{e}_{ij} \) and penetration depth \( \delta \) can be calculated as

\[ \mathbf{e}_{ij} = \mathbf{r}_i - \mathbf{r}_j, \quad \delta = e_{ij} - c, \quad e_{ij} = \sqrt{\mathbf{e}_{ij}^T \mathbf{e}_{ij}} \]

where \( \mathbf{r}_i \) and \( \mathbf{r}_j \) are the position vector of \( P_i \) and \( P_j \) in the global coordinate frame, respectively. \( c = R_j - R_i \) is the radial clearance. \( R_i \) and \( R_j \) are the radius of pin and bushing, respectively.

Normal contact force and friction force in revolute clearance joint is calculated as
where \( e_c \) is the restitution coefficient. \( \dot{\delta} \) is penetration velocity. \( \dot{\delta}^{(i)} \) is the initial penetration velocity. \( v_k \) and \( E_k \) are the Poisson’s ratio and Young’s modulus of body \( k \). \( \mu \) is the coefficient of friction.

### 2.2 Computational strategy to solve the dynamics equations of multibody system including flexible beams

Based on the ANCF, the dynamic equation of a constrained flexible body can be assembled [9] as

\[
\mathbf{M}_i^a \ddot{\mathbf{a}}_i + \mathbf{T}_i^a \lambda_i = \mathbf{Q}_i - \mathbf{F}_i \quad (i = 1, n_c)
\]

where \( \mathbf{a} \) is the nodal coordinates of all flexible bodies. \( \mathbf{e}_i \) is the nodal coordinates of the flexible body \( i \), and \( n_c \) is the total number of the flexible bodies. \( \mathbf{M}_i^a \) is the assembled mass matrix of the flexible body \( i \). \( \mathbf{T}_i^a \) and \( \mathbf{Q}_i \) are the system constraint equations and Jacobian matrix, respectively. \( \lambda_i \) is the Lagrange multiplier vector. \( \mathbf{F}_i \) is the elastic force vector and \( \mathbf{Q}_i \) is the general external force vector.

For a multibody system composed of rigid and flexible bodies, the dynamic equations of system can be assembled as

\[
\begin{bmatrix}
\mathbf{M}_a & \mathbf{0} \\
\mathbf{0} & \mathbf{M}_u
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{u}} \\
\ddot{\mathbf{u}}
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{\Phi}_a^u \\
\mathbf{\Phi}_u^u
\end{bmatrix}
\lambda = \begin{bmatrix}
\mathbf{Q}_a \\
\mathbf{Q}_u - \mathbf{F}_u
\end{bmatrix}
\]

where subscript \( a \) and \( u \) denote flexible and rigid bodies, respectively. The detailed description about the rigid bodies can be found in literature [10].

In this paper, the generalized-\( \mathcal{A} \) method [11] is used to solve Eq. (8), which allows an optimal combination of accuracy at low-frequency and numerical damping at high-frequency.

### 3. Wear prediction procedure

#### 3.1 Determination of the wear coefficient using Radial Basis Function Neural Network (RBFNN) technique

A general trend to calculate wear is the use of Archard’s wear model [12] in an iterative procedure. The wear coefficient in this model strongly depends on contact condition. In order to determine the wear coefficient at different contact pressure and sliding velocity, a RBFNN[13] technique is adopted. As shown in Fig.2, RBFNN is a two-layer network. The activations of hidden units are Gaussian functions of the inputs \( \mathbf{x} \), and the final output of the RBFNN is

\[
y(\mathbf{x}) = \sum_{i=1}^{m} w_i \varphi_i(\mathbf{x}), \quad \varphi_i(\mathbf{x}) = \exp \left[ -\frac{||\mathbf{x} - \mathbf{x}_i||^2}{2\sigma_i^2} \right] \quad (i = 1, m)
\]

where \( \mathbf{x}_i \) and \( \sigma_i \) are the center and width of the \( i \) th hidden unit. \( m \) is the number of the hidden layer units, and \( w_i \) is the network weight of the \( i \) th hidden unit.

Before training the network, the wear tests should be carried out using pin-on-disc technique. Then, the network is trained according to the tests data. During the process of wear prediction, the wear coefficient can be obtained by taking the current contact pressure and sliding velocity as the inputs to the trained network.
3.2 Archard’s wear model

For a periodical wear case, periodical external forces are applied on the contact bodies and then the finite-element-method is used to calculate contact status. By dividing a complete cycle of the relative motion between the bodies into \( n \) steps, the wear depth can be expressed as

\[
h_{j}^{\alpha} = h_{j-1}^{\alpha} + A_{j} \sum_{i=1}^{n} k_{j,i}^{\alpha} p_{j,i}^{\alpha} s_{j,i}^{\alpha}
\]

where the superscript \( \alpha \) indicates the node of the FEM on the contact surface. The subscripts \( i \) and \( j \) refer to the current step and cycle, respectively. \( s_{j,i}^{\alpha} \) is incremental sliding distance. As stated above, in order to get the current wear coefficient \( k_{j,i}^{\alpha} \) can be obtained from the trained RBFNN according to the contact pressure \( p_{j,i}^{\alpha} \) as well as the sliding velocity. \( A_{j} \) is the current extrapolation factor used to reduce the computation cost in repeated wear analysis[7].

Wear should be reflected by updating the geometry of the contact bodies. This may be achieved by moving the contact surface in the direction of the surface normal by an amount equal to the incremental wear depth.

4. Integration of wear model into dynamic analysis

The flowchart integrating wear prediction with flexible multibody dynamics is shown in Fig.3.

In the first stage, dynamic analysis of the multibody systems is performed to determine the reaction force \( F \) and incremental sliding distance \( S \). The sliding velocity \( V \) can be determined by the kinematic analysis of the systems with the assumption that all joints are ideal and all components are rigid.

In the next stage, based on the results obtained above, joint wear can be calculated with alterable wear coefficient and reflected by updating the geometric profile. And the evolved geometry due to wear should be reflected in the dynamic analysis. This can be done by changing the value of the clearance \( c \) rather than keeping it as a constant.
Then a new dynamic analysis for the next cycle will be performed until the desired number of cycles has been completed.

In order to avoiding distorted finite element meshes, it is necessary to re-mesh the model when the cumulative displacement of any contact node reaches a certain percentage of the corresponding surface element height [14].

5. Results and discussion

5.1 Determination of wear coefficient by RBFNN

Pin-on-disc tests are carried out on CETR UMT Multi-Specimen Test System. The discs were made of 42CrMo with a hardness of 60 HRC. The pins were made of CuZn25A16Fe3Mn3 hardened to 235 HB. Because the material of pin is softer than that of disc, only the pin’s wear coefficient is measured.

Fig.4 shows the wear coefficient predicted by the trained RBFNN according to tests data. “•” denote the tests data. The curved surface is the results of the RBFNN trained according to tests data. It can be found that the prediction results agree well with tests data.

5.2 Wear prediction on a flexible slider-crank mechanism with a revolute clearance joint

In this work a flexible slinder crank mechanism with a revolute clearance joint shown in Fig.5 (a), is chosen. The crank and the slider are rigid. The length, mass and the moment of inertia of crank are 0.1 m, 0.3Kg and 1E5 Kgm², respectively. The mass of the slider is 0.5Kg. The connecting rod is assumed to be flexible with the length of 0.34m, and its material density and Young’s module are 2.66Kg/m³ and 211GPa, respectively. It is divided into 4 beam elements with cross section of 18×5mm². All joints are ideal except one between the connecting rod and slider. The width of the joint is 1.8E-2m, and its dimension and the material properties are listed in Table 1. The friction coefficient is 0.1. The crank is the driving link and rotates at a constant angular velocity of 1200r/min. This wear
simulation is run for 400,000 cycles. In order to take account of both efficiency and precision, the values of $R$ and $A_j$ in Eq. (10) are chosen as 200 and 5000, respectively.

The finite element model of the clearance joint is shown in Fig.7. Pin is modeled as a rigid element. The bushing is modeled by eight-node quadrilateral elements. All translational degrees of freedom of the nodes on the outer surface of the bushing are constrained. Joint reaction forces are applied on the pilot point that is attached to the pin.

<table>
<thead>
<tr>
<th></th>
<th>Bushing</th>
<th>Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial inner radius (mm)</td>
<td>10</td>
<td>9.5</td>
</tr>
<tr>
<td>Outer radius (mm)</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>Material</td>
<td>CuZn25A16Fe3Mn3</td>
<td>42CrMo</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>90</td>
<td>207</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>Hardness</td>
<td>235 HB</td>
<td>60 HRC</td>
</tr>
</tbody>
</table>

Fig. 6 shows the results from the initial dynamics after the transient phase has died out. In Fig.6 (a), the plots of the contact reaction force are shown in three different cases: (1) the connecting rod is modeled as a rigid body (RCR) with clearance joint, (2) the connecting rod is modeled as a flexible body (FCR) with clearance joint, (3) RCR with ideal joint. It can be seen that, at the dead-positions of the connecting rod, the force exhibits high frequency oscillation when the clearance exists. Compared with the case of RCR with clearance, the flexible rod reduces the impacts by more than half. So the flexible component acts as a suspension for the mechanism. This phenomenon was also found by Khemili [3].

Fig. 6(b) shows the pin center paths of clearance joint for a complete crank revolution. From the Fig. 6(b), we recognize three different modes of responses: free flight, impact with rebound and continuous contact. The contact regions concentrate on different modes of responses: free flight, impact with rebound and continuous contact. The contact regions concentrate on both the left and the right sides of the bushing, which means that wear will occur mostly in these two regions.

![Fig. 6.(a) Reaction forces in the clearance joint. (b) The paths of pin center of FCR.](image)

![Fig. 7(a) Comparison of the wear on the bushing for the RCR and FCR after 400,000 crank cycles. (b) Comparison of the change in the maximum wear depth for the RCR and FCR.](image)
Fig. 7(a) shows the comparison of the wear depth for the two cases after 400,000 crank cycles. It is clear that the wear result is slightly higher at most of regions of the bushing inner surface when the rod is modeled as a rigid body. Fig. 7(b) shows the growing trend of the maximum wear depth at the clearance joint with the increase of the crank revolutions. It can be found that the predicted wear result in the RCR case is slightly higher than that in the FCR case. Moreover, as the increase of the crank revolutions, the difference of the result between the two cases will be more and more obvious. The reason of above results might stem from the two types of model. When the rod is modeled as a rigid body, the value of impact in the dynamic results is much larger than that of case that rod is modeled as a flexible body. This will subsequently lead to a larger wear result and clearance, which in turn go on to increase the impact value.

6. Concluding remarks

This paper aims to predict the wear at revolute clearance joint in a flexible mechanism. An integrated approach for wear prediction is presented by combing the procedures of multibody dynamics and wear prediction. A planar slider-crank mechanism including a revolute clearance joint is used as a numerical example. It is found that, when the components’ flexibility is taken into account, the value of impact simulated in the clearance joint is greatly decreased, and the predicted wear result is also slightly reduced in comparison to that of a rigid system, which means that the flexible component not only acts as a suspension for the mechanism, but also alleviates wear at the clearance joint.

Acknowledgment

The authors would be most grateful to be supported by the NSF of China (51175332, 51205247, 50905891), The Key Project of State Key Laboratory of Mechanical System and Vibration (MSVZD201104) and the 973 National Basic Research Priorities Program of China (2009CB724302).

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