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ORIGINAL ARTICLE

General traveling wave solutions of the strain wave equation in microstructured solids via the new approach of generalized (G'/G)-expansion method

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Abstract The new approach of generalized (G'/G)-expansion method is significant, powerful and straightforward mathematical tool for finding exact traveling wave solutions of nonlinear evolution equations (NLEEs) arise in the field of engineering, applied mathematics and physics. Dispersive effects due to microstructure of materials combined with nonlinearities give rise to solitary waves. In this article, the new approach of generalized (G'/G)-expansion method has been applied to construct general traveling wave solutions of the strain wave equation in microstructured solids. Abundant exact traveling wave solutions including solitons, kink, periodic and rational solutions have been found. These solutions might play important role in engineering fields.

1. Introduction

Microstructured materials like alloys, crystallites, ceramics, and functionally graded materials have gained wide application. The modeling of wave propagation in such materials should be able to account for various scales of microstructure [1–3]. The existence and emergence of solitary waves in complicated physical problems apart from the model equations of mathematical physics should be analyzed with sufficient correctness. It has recently become more attractive to obtain exact solutions of nonlinear partial differential equations through computer algebra that facilitate complex and tedious algebraic computations. Evaluating exact and numerical solutions, in particular, traveling wave solutions, of nonlinear equations in mathematical physics play an important role in soliton theory [4,5]. There are several studies where the governing equations for waves in microstructured solids have been derived and solitary waves are analyzed [6,7]. Mathematical modeling of physical and engineering problems is innately governed by nonlinear partial differential equations and hence investigation of exact solutions of nonlinear partial differential equations is

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very much important. The exact solutions of these equations give information about the structure of these problems. To this end, in the past several decades various effective methods have been developed and established to solve and understand the mechanisms of these phenomena. Among them the notable are, the modified simple equation method [8–12], the (G'/G)-expansion method [13–21], the Adomian decomposition method [22], the Lie group symmetry method [23], the homotopy analysis method [24,25], the first integration method [26], the inverse scattering method [27], the theta function method [28,29], the tanh-function method [30,31], the extended tanh-function method [32,33], the homogeneous balance method [34], the Jacobi elliptic function method [35,36], the Hirota's bilinear method [37], the sine-cosine method [38], etc.

Recently, Naher and Abdullah [39] presented an effective and straightforward method, called the new approach of generalized (G'/G) expansion method to obtain exact traveling wave solutions of NLEEs. In this article, we put forth the new approach of generalized (G'/G) expansion method to construct exact traveling wave solutions including solitons, kink, periodic and rational solutions to the strain wave equation in microstructured solids.

2. Description of the new generalized (G'/G)-expansion method

Consider the general nonlinear partial differential equation

$$H(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \ldots), \tag{1}$$

where $u = u(x,t)$ is an unknown function, H is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of the method are as follows:

Step 1: Combining the real variables x and t by a compound variable ξ, we suppose that

$$u(x,t) = u(\xi), \quad \xi = x \pm Vt, \tag{2}$$

where V is the speed of the traveling wave. The transformation (2) transforms Eq. (1) into an ordinary differential equation (ODE) for $u = u(\xi)$

$$F(u, u', u'', u''', \ldots), \tag{3}$$

where F is a function of $u(\xi)$ and its derivatives.

Step 2: According to possibility, Eq. (3) can be integrated term by term one or more times, yields constant(s) of integration. The integral constant may be zero, for simplicity.

Step 3: Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\xi) = \sum_{i=0}^{N} d_i (p+M)^i + \sum_{i=1}^{N} e_i (p+M)^{-i}, \tag{4}$$

where either d_N or e_N may be zero, but both of them could be not zero at a time. d_i $(i = 0, 1, 2, \ldots, N)$ and e_i $(i = 1, 2, \ldots, N)$ and p are constants to be determined later and $M(\xi)$ is

$$M(\xi) = (G'/G) \tag{5}$$

where $G = G(\xi)$ satisfies the following auxiliary nonlinear ordinary differential equation:

$$h_1 G G'' - h_2 G G' - h_3 (G')^2 - h_4 G^2 = 0, \tag{6}$$

where the prime stands for derivative with respect to ξ; h_1, h_2, h_3 and h_4 are real parameters.

Step 4: To determine the positive integer N, taking the homogeneous balance between the highest order nonlinear terms and the derivatives of the highest order come out in Eq. (3).

Step 5: Substitute Eqs. (4) and (6) including Eq. (5) into Eq. (3) and the value of N obtained in Step 4, we obtain polynomials in $(p+M)^N$ $(N = 0, 1, 2, \ldots)$ and $(p+M)^{-N}$ $(N = 0, 1, 2, \ldots)$. Then, we collect each coefficient of the resulted polynomials to zero yields a set of algebraic equations for d_i $(i = 0, 1, 2, \ldots, N)$ and e_i $(i = 1, 2, \ldots, N)$, p and V.

Step 6: Suppose that the value of the constants d_i $(i = 0, 1, 2, \ldots, N)$, e_i $(i = 1, 2, \ldots, N)$, p and V can be found by solving the algebraic equations obtained in Step 5. Since the general solution of Eq. (6) is well known, inserting the values of d_i, e_i, p and V into Eq. (4), we obtain more general type and new exact traveling wave solutions of the nonlinear partial differential Eq. (1).

Using the general solution of Eq. (6), we have the following solutions of Eq. (5):

Family 1: When $h_2 \neq 0$, $\psi = h_1 - h_3$ and $\Phi = h_2^2 + 4h_4(h_1 - h_3) > 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi} \frac{A\sinh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right) + B\cosh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)}{A\cosh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right) + B\sinh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)} \tag{7}$$

Family 2: When $h_2 \neq 0$, $\psi = h_1 - h_3$ and $\Phi = h_2^2 + 4h_4(h_1 - h_3) < 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{h_2}{2\psi} + \frac{\sqrt{-\Phi}}{2\psi} \frac{-A\sin\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right) + B\cos\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)}{A\cos\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right) + B\sin\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)} \tag{8}$$

Family 3: When $h_2 \neq 0$, $\psi = h_1 - h_3$ and $\Phi = h_2^2 + 4h_4(h_1 - h_3) = 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{h_2}{2\psi} + \frac{C_2}{C_1 + C_2\xi} \tag{9}$$

Family 4: When $h_2 = 0$, $\psi = h_1 - h_3$ and $\Omega = \psi h_4 > 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Omega}}{\psi} \frac{A\sinh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right) + B\cosh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)}{A\cosh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right) + B\sinh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)} \tag{10}$$

Family 5: When $h_2 = 0$, $\psi = h_1 - h_3$ and $\Omega = \psi h_4 < 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{-\Omega}}{\psi} \frac{-A\sin\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right) + B\cos\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)}{A\cos\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right) + B\sin\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)} \tag{11}$$

3. Application of the method

In this section, we will implement the application of the new approach of generalized (G'/G) expansion method to construct more general traveling wave solutions of the strain wave equation in microstructured solids. The governing equation of the strain waves in microstructured solids is given by

$$u_{tt} - u_{xx} - \varepsilon\alpha_1(u^2)_{xx} - \gamma\alpha_2 u_{xxt} + \delta\alpha_3 u_{xxxx} - (\delta\alpha_4 - \gamma^2\alpha_7)u_{xxtt} + \gamma\delta(\alpha_5 u_{xxxxt} + \alpha_6 u_{xxttt}) = 0. \quad (12)$$

Porubov and Pastrone [40] discussed the bell-shaped and kink-shaped solutions of the strain waves in microstructured solids in engineering applications problem. If $\gamma = 0$, we have the non-dissipative case, and governed by the double dispersive equation (see [41] for details)

$$u_{tt} - u_{xx} - \varepsilon\alpha_1(u^2)_{xx} + \delta\alpha_3 u_{xxxx} - \delta\alpha_4 u_{xxtt} = 0. \quad (13)$$

The balance between nonlinearity and dispersion takes place when $\delta = O(\varepsilon)$. Therefore, Eq. (13) becomes

$$u_{tt} - u_{xx} - \varepsilon\{\alpha_1(u^2)_{xx} - \alpha_3 u_{xxxx} + \alpha_4 u_{xxtt}\} = 0. \quad (14)$$

Now, using the wave transformation (2) into the Eq. (14) yields:

$$(V^2-1)u'' - \varepsilon\alpha_1(u^2)'' + \varepsilon(\alpha_3 - V^2\alpha_4)u^{(4)} = 0, \quad (15)$$

where u'' denotes the second derivative, and $u^{(4)}$ denote the fourth derivative with respect to ξ. Eq. (15) is integrable, therefore, integrating we obtain

$$(V^2-1)u - \varepsilon\alpha_1(u^2) + \varepsilon(\alpha_3 - V^2\alpha_4)u'' + K = 0, \quad (16)$$

where K is an integral constant.

Taking the homogeneous balance between u^2 and u'' in Eq. (14), we obtain $N = 2$. Therefore, the solution of Eq. (16) takes the form:

$$u(\xi) = d_0 + d_1(p+M) + d_2(p+M)^2 + e_1(p+M)^{-1} + e_2(p+M)^{-2}, \quad (17)$$

where d_0, d_1, d_2, e_1 and e_2 are constants to be determined.

Substituting Eq. (17) together with Eqs. (5) and (6) into Eq. (15), the left-hand side is converted into polynomials in $(p + M)^N$ $(N = 0, 1, 2, \ldots)$ and $(p + M)^{-N}$ $(N = 1, 2, \ldots)$. We collect each coefficient of these resulted polynomials to zero, yields a set of simultaneous algebraic equations (for simplicity which are not presented) for d_0, d_1, d_2, e_1, e_2 p, K and V. Solving these algebraic equations with the help of symbolic computation software, we obtain following:

Set 1: $d_1 = 0$, $d_2 = 0$,

$$K = -\frac{1}{4\varepsilon\alpha_1 h_1^4}(-16\varepsilon^2 V^4\alpha_4^2 h_4^2\psi^2 - 8\varepsilon^2\alpha_3^2 h_2^2 h_4\psi + 32\varepsilon^2 V^2\alpha_4\alpha_3 h_4^2\psi^2 - 8\varepsilon^2 V^4\alpha_4^2 h_2^2 h_4\psi + 16\varepsilon^2 V^2\alpha_4 h_2^2\alpha_3 h_4\psi - 16\varepsilon^2\alpha_3^2 h_4^2\psi^2 + V^2 h_1^4 + 2\varepsilon^2 V^2\alpha_4 h_2^4\alpha_3 - \varepsilon^2\alpha_3^2 h_2^4 + h_1^4 - \varepsilon^2 V^4 h_2^4\alpha_4^2 - 2h_1^4 V^2),$$

$$d_0 = -\frac{1}{2\varepsilon\alpha_1 h_1^2}\{12\varepsilon V^2\alpha_4 p^2\psi^2 - 12\varepsilon\alpha_3 p^2\psi^2 + 12\varepsilon V^2\alpha_4 h_2 p\psi - 8\varepsilon^2 V^2\alpha_4 h_4\psi - 12\varepsilon\alpha_3 h_2 p\psi + 8\varepsilon\alpha_3 h_4\psi - V^2 h_1^2 + h_1^2 + \varepsilon V^2\alpha_4 h_2^2 - \varepsilon\alpha_3 h_2^2\}, \quad (18)$$

$$e_1 = \frac{6}{\alpha_1 h_1^2}\{\alpha_3 h_4 h_2 - \alpha_3 h_2^2 p - 2\alpha_3 p^3\psi^2 - 3\alpha_3 h_2 p^2\psi + 2\alpha_3 h_4 p\psi + V^2\alpha_4 h_2^2 p - V^2\alpha_4 h_2 h_4 + 2V^2\alpha_4 p^3\psi^2 + 3V^2\alpha_4 h_2 p^2\psi - 2V^2\alpha_4 h_4 p\psi\},$$

$$e_2 = -\frac{6}{\alpha_1 h_1^2}\{h_4^2(V^2\alpha_4(1+p^2) - \alpha_3) + (2h_4 p^2\psi - p^4\psi^2)(\alpha_3 - V^2\alpha_4) - \alpha_3 h_2^2 p^2 - 2h_2(p^3\psi - p h_4)(\alpha_3 - V^2\alpha_4)\},$$

where $\psi = h_1 - h_3$, h_1, h_2, h_3, h_4 are free parameters and V, p are constants

Set 2:

$$d_1 = \frac{6}{\alpha_1 h_1^2}(2V^2\alpha_4 p\psi^2 + V^2\alpha_4 h_2\psi - 2\alpha_3 p\psi^2 - \alpha_3 h_2\psi), e_1 = 0, e_2 = 0,$$
$$d_2 = -\frac{6}{\alpha_1 h_1^2}(-\alpha_3\psi^2 + V^2\alpha_4\psi^2),$$
$$d_0 = -\frac{1}{2\varepsilon\alpha_1 h_1^2}(12\varepsilon V^2\alpha_4 p^2\psi^2 - 12\varepsilon\alpha_3 p^2\psi^2 + 12\varepsilon V^2\alpha_4 h_2 p\psi - 8\varepsilon^2 V^2\alpha_4 h_4\psi - 12\varepsilon\alpha_3 h_2 p\psi + 8\varepsilon\alpha_3 h_4\psi - V^2 h_1^2 + h_1^2 + \varepsilon V^2\alpha_4 h_2^2 - \varepsilon\alpha_3 h_2^2), \quad (19)$$

$$K = -\frac{1}{4\varepsilon\alpha_1 h_1^4}(-16\varepsilon^2 V^4\alpha_4^2 h_4^2\psi^2 - 8\varepsilon^2\alpha_3^2 h_2^2 h_4\psi + 32\varepsilon^2 V^2\alpha_4\alpha_3 h_4^2\psi^2 - 8\varepsilon^2 V^4\alpha_4^2 h_2^2 h_4\psi + 16\varepsilon^2 V^2\alpha_4 h_2^2\alpha_3 h_4\psi - 16\varepsilon^2\alpha_3^2 h_4^2\psi^2 + V^2 A^4 + 2\varepsilon^2 V^2\alpha_4 h_2^4\alpha_3 - \varepsilon^2\alpha_3^2 h_2^4 + h_1^4 - \varepsilon^2 V^4 h_2^4\alpha_4^2 - 2h_1^4 V^2),$$

where $\psi = h_1 - h_3$, h_1, h_2, h_3, h_4 are free parameters and V, p are constants

Set 3:

$$d_2 = \frac{6\psi^2}{\alpha_1 h_1^2}(\alpha_3 - V^2\alpha_4), \quad e_1 = 0, \quad p = -\frac{h_2}{2\psi}, \quad d_1 = 0,$$
$$d_0 = \frac{1}{2\varepsilon\alpha_1 h_1^2}(-h_1^2 + Vh_1^2 - 8\varepsilon\alpha_3 h_4\psi + 8\varepsilon V^2\alpha_4 h_4\psi - 2\varepsilon\alpha_4 h_2^2 + 2\varepsilon V^2\alpha_4 h_2^2),$$
$$e_2 = -\frac{3}{8\alpha_1 h_1^2\psi^2}(-h_2^4\alpha_3 + h_2^4 V^2\alpha_4 + 8h_2^2 V^2\alpha_4 h_4\psi - 8h_2^2\alpha_3 h_4\psi + 16V^2\alpha_4 h_4^2\psi^2 - 16\alpha_3 h_4^2\psi^2),$$

$$K = -\frac{1}{4\varepsilon\alpha_1 h_1^4}(512\varepsilon^2 V^2\alpha_4\alpha_3 h_4^2\psi^2 - 256\varepsilon^2\alpha_3^2 h_4^2\psi^2 - 256\varepsilon^2 V^4\alpha_4^2 h_4^2\psi^2 - 128\varepsilon^2\alpha_3^2 h_2^2 h_4\psi + 256\varepsilon^2 V^2\alpha_4 h_2^2\alpha_3 h_4\psi - 128\varepsilon^2 V^4\alpha_4^2 h_2^2 h_4\psi - 2h_1^4 V^2 + V^2 h_1^4 + h_1^4 - 16\varepsilon^2\alpha_3^2 h_2^4 - 16\varepsilon^2 V^4 h_2^4\alpha_4^2 + 32\varepsilon^2 V^2\alpha_4 h_2^4\alpha_3), \quad (20)$$

where $\psi = h_1 - h_3, p, h_1, h_2, h_3, h_4$ are free parameters and V is constant.

For set 1, substituting Eq. (18) into Eq. (17) along with Eq. (7) and simplifying, yields following traveling wave solutions (if $A = 0$ but $B \neq 0$; $B = 0$ but $A \neq 0$) respectively:

$$u_{1_1}(\xi) = \frac{l_1}{2\varepsilon\alpha_1 h_1^2} + \frac{6}{\alpha_1 h_1^2}\left\{l_2\left(p + \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi}\coth\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)^{-1} + l_3\left(p + \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi}\coth\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)^{-2}\right\},$$

$$u_{1_2}(\xi) = \frac{l_1}{2\varepsilon\alpha_1 h_1^2} + \frac{6}{\alpha_1 h_1^2}\left\{l_2\left(p + \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi}\tanh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)^{-1} + l_3\left(p + \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi}\tanh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)^{-2}\right\},$$

where

$$l_1 = -\{12\varepsilon V^2\alpha_4 p^2\psi^2 - 12\varepsilon\alpha_3 p^2\psi^2 + 12\varepsilon V^2\alpha_4 h_2 p\psi - 8\varepsilon^2 V^2\alpha_4 h_4\psi - 12\varepsilon\alpha_3 h_2 p\psi + 8\varepsilon\alpha_3 h_4\psi - V^2 h_1^2 + h_1^2 + \varepsilon V^2\alpha_4 h_2^2 - \varepsilon\alpha_3 h_2^2\},$$

$$l_2=\{\alpha_3h_4h_2-\alpha_3h_2^2p-2\alpha_3p^3\psi^2-3\alpha_3h_2p^2\psi+2\alpha_3h_4p\psi+V^2\alpha_4h_2^2p-V^2\alpha_4h_2h_4+2V^2\alpha_4p^3\psi^2+3V^2\alpha_4h_2p^2\psi-2V^2\alpha_4h_4p\psi\},$$

and

$$l_3=-\{h_4^2(V^2\alpha_4(1+p^2)-\alpha_3)+(2h_4p^2\psi-p^4\psi^2)(\alpha_3-V^2\alpha_4)-\alpha_3h_2^2p^2-2h_2(p^3\psi-ph_4)(\alpha_3-V^2\alpha_4)\},$$

If $A=0$ but $B\neq0$; $B=0$ but $A\neq0$, substituting Eq. (18) into Eq. (17), along with Eq. (8) and simplifying, the exact solutions become respectively:

$$u_{1_3}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p+\frac{h_2}{2\psi}+\frac{\sqrt{-\Phi}}{2\psi}\cot\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)^{-1}+l_3\left(p+\frac{h_2}{2\psi}+\frac{\sqrt{-\Phi}}{2\psi}\cot\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)^{-2}\right\},$$

$$u_{1_4}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p+\frac{h_2}{2\psi}-\frac{\sqrt{-\Phi}}{2\psi}\tan\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)^{-1}+l_3\left(p+\frac{h_2}{2\psi}-\frac{\sqrt{-\Phi}}{2\psi}\tan\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)^{-2}\right\},$$

Substituting Eq. (18) into Eq. (17), together with Eq. (9) and simplifying, the obtained solution becomes:

$$u_{1_5}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p+\frac{h_2}{2\psi}+\frac{C_2}{C_1+C_2\xi}\right)^{-1}+l_3\left(p+\frac{h_2}{2\psi}+\frac{C_2}{C_1+C_2\xi}\right)^{-2}\right\},$$

If $A=0$ but $B\neq0$; $B=0$ but $A\neq0$, substituting Eq. (18) into Eq. (17), along with Eq. (10) and simplifying, we obtain following traveling wave solutions respectively:

$$u_{1_6}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p+\frac{\sqrt{\Omega}}{\psi}\coth\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^{-1}+l_3\left(p+\frac{\sqrt{\Omega}}{\psi}\coth\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^{-2}\right\},$$

$$u_{1_7}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p+\frac{\sqrt{\Omega}}{\psi}\tanh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^{-1}+l_3\left(p+\frac{\sqrt{\Omega}}{\psi}\tanh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^{-2}\right\}.$$

If $A=0$ but $B\neq0$; $B=0$ but $A\neq0$, substituting Eq. (18) into Eq. (17), together with Eq. (11) and simplifying, the obtained exact solutions respectively become:

$$u_{1_8}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p+\frac{\sqrt{-\Omega}}{\psi}\cot\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^{-1}+l_3\left(p+\frac{\sqrt{-\Omega}}{\psi}\cot\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^{-2}\right\},$$

$$u_{1_9}(\xi)=\frac{l_1}{2\varepsilon\alpha_1h_1^2}+\frac{6}{\alpha_1h_1^2}\left\{l_2\left(p-\frac{\sqrt{-\Omega}}{\psi}\tan\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^{-1}+l_3\left(p-\frac{\sqrt{-\Omega}}{\psi}\tan\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^{-2}\right\},$$

where $\xi=x-Vt$.

Similarly, for set 2, substituting Eq. (19) into Eq. (17), along with Eq. (7) and simplifying, the traveling wave solutions become (if $A=0$ but $B\neq0$; $B=0$ but $A\neq0$) respectively:

$$u_{2_1}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}\left(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(V^2\alpha_4-\alpha_3)\left(2\varepsilon h_2^2-3\varepsilon\Phi\coth^2\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)\right),$$

$$u_{2_2}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}\left(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(V^2\alpha_4-\alpha_3)\left(2\varepsilon h_2^2-3\varepsilon\Phi\tanh^2\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)\right),$$

If $A=0$ but $B\neq0$; $B=0$ but $A\neq0$, substituting Eq. (19) into Eq. (17), along with Eq. (8) and simplifying, yields exact solutions respectively:

$$u_{2_3}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}\left(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(V^2\alpha_4-\alpha_3)\left(2\varepsilon h_2^2+3\varepsilon\Phi\cot^2\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)\right),$$

$$u_{2_4}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}\left(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(V^2\alpha_4-\alpha_3)\left(2\varepsilon h_2^2+3\varepsilon\Phi\tan^2\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)\right),$$

Substituting Eq. (19) into Eq. (17), along with Eq. (9) and simplifying, the obtained solution becomes:

$$u_{2_5}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}\left(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+\left(V^2\alpha_4-\alpha_3\right)\left(2\varepsilon h_2^2-12\varepsilon\psi^2\left(\frac{C_2}{C_1+C_2\xi}\right)^2\right)\right),$$

If $A=0$ but $B\neq0$; $B=0$ but $A\neq0$, substituting Eq. (19) into Eq. (17), together with Eq. (10) and simplifying, yields following traveling wave solutions:

$$u_{2_6}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(\alpha_3-V^2\alpha_4)\times\left(\varepsilon h_2^2-12\varepsilon\sqrt{\Omega}\left(\left(h_2\coth\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)-\sqrt{\Omega}\coth^2\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)\right)\right),$$

$$u_{2_7}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(\alpha_3-V^2\alpha_4))\times\left(\varepsilon h_2^2-12\varepsilon\sqrt{\Omega}\left(\left(h_2\tanh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)-\sqrt{\Omega}\tanh^2\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)\right)\right),$$

If $A=0$ but $B\neq0$; $B=0$ but $A\neq0$, substituting Eq. (19) into Eq. (17), along with Eq. (11) and simplifying, the exact solutions become:

$$u_{2_8}(\xi)=\frac{1}{2\varepsilon\alpha_1h_1^2}(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(\alpha_3-V^2\alpha_4)\times\left(\varepsilon h_2^2-12\varepsilon\sqrt{\Omega}\left(\left(ih_2\cot\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)+\sqrt{\Omega}\coth^2\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)\right)\right),$$

$$u_{2_9}(\xi)=\frac{1}{2\varepsilon\alpha_1 h_1^2}(8\varepsilon h_4\psi(\varepsilon V^2\alpha_4-\alpha_3)+h_1^2(V^2-1)+(\alpha_3-V^2\alpha_4)\times\left(\varepsilon h_2^2+12\varepsilon\sqrt{\Omega}\left(\left(ih_2\tan\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)-\sqrt{\Omega}\tan^2\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)\right)\right)\Bigg),$$

where $\xi = x - Vt$.

Finally, for set 3, substituting Eq. (20) into Eq. (17), together with Eq. (7) and simplifying, yields following traveling wave solutions (if $A = 0$ but $B \neq 0$; $B = 0$ but $A \neq 0$) respectively:

$$u_{3_1}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}+\frac{3\Phi}{2\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\coth^2\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)+\frac{3l_5}{2\Phi\alpha_1 h_1^2}\tanh^2\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right),$$

$$u_{3_2}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}-\frac{3\Phi}{2\alpha_1 h_1^2}(-\alpha_3+V^2\alpha_4)\tanh^2\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)+\frac{3l_5}{2\Phi\alpha_1 h_1^2}\coth^2\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right),$$

where

$$l_4=-h_1^2+Vh_1^2-8\varepsilon\alpha_3 h_4\psi+8\varepsilon V^2\alpha_4 h_4\psi-2\varepsilon\alpha_4 h_2^2+2\varepsilon V^2\alpha_4 h_2^2,$$

and

$$l_5=-\left(-h_2^4\alpha_3+h_2^4V^2\alpha_4+8h_2^2V^2\alpha_4 h_4\psi-8h_2^2\alpha_3 h_4\psi+16V^2\alpha_4 h_4^2\psi^2-16\alpha_3 h_4^2\psi^2\right),$$

If $A = 0$ but $B \neq 0$; $B = 0$ but $A \neq 0$, substituting Eq. (20) into Eq. (17), along with Eq. (8) and simplifying, we obtain following solutions respectively:

$$u_{3_3}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}-\frac{3\Phi}{2\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\cot^2\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)-\frac{3l_5}{2\Phi\alpha_1 h_1^2}\tan^2\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right),$$

$$u_{3_4}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}-\frac{3\Phi}{2\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\tan^2\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)-\frac{3l_5}{2\Phi\alpha_1 h_1^2}\cot^2\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right),$$

Substituting Eq. (20) into Eq. (17), along with Eq. (9) and simplifying, the obtained solution becomes:

$$u_{3_5}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}+\frac{6\psi^2}{\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\left(\frac{C_2}{C_1+C_2\xi}\right)^2+\frac{3l_5}{8\alpha_1 h_1^2\psi^2}\left(\frac{C_2}{C_1+C_2\xi}\right)^{-2}.$$

If $A = 0$ but $B \neq 0$; $B = 0$ but $A \neq 0$, substituting Eq. (20) into Eq. (17), along with Eq. (10) and simplifying, yields following exact traveling wave solutions respectively:

$$u_{3_6}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}+\frac{6\psi^2}{\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\left(\frac{-h_2}{2\psi}+\frac{\sqrt{\Omega}}{\psi}\coth\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^2+\frac{3l_5}{8\alpha_1 h_1^2\psi^2}\left(\frac{-h_2}{2\psi}+\frac{\sqrt{\Omega}}{\psi}\coth\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^{-2},$$

$$u_{3_7}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}+\frac{6\psi^2}{\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\left(\frac{-h_2}{2\psi}+\frac{\sqrt{\Omega}}{\psi}\tanh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^2+\frac{3l_5}{8\alpha_1 h_1^2\psi^2}\left(\frac{-h_2}{2\psi}+\frac{\sqrt{\Omega}}{\psi}\tanh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)\right)^{-2},$$

If $A = 0$ but $B \neq 0$; $B = 0$ but $A \neq 0$, substituting Eq. (20) into Eq. (17), along with Eq. (11) and simplifying, the obtained exact solutions become respectively:

$$u_{3_8}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}+\frac{6\psi^2}{\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\left(\frac{-h_2}{2\psi}+\frac{\sqrt{-\Omega}}{\psi}\cot\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^2+\frac{3l_5}{8\alpha_1 h_1^2\psi^2}\left(\frac{-h_2}{2\psi}+\frac{\sqrt{-\Omega}}{\psi}\cot\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^{-2},$$

$$u_{3_9}(\xi)=\frac{l_4}{2\varepsilon\alpha_1 h_1^2}+\frac{6\psi^2}{\alpha_1 h_1^2}(\alpha_3-V^2\alpha_4)\left(\frac{-h_2}{2\psi}-\frac{\sqrt{-\Omega}}{\psi}\tan\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^2+\frac{3l_5}{8\alpha_1 h_1^2\psi^2}\left(\frac{-h_2}{2\psi}-\frac{\sqrt{-\Omega}}{\psi}\tan\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)\right)^{-2},$$

where $\xi = x - Vt$.

In this article, we obtain twenty seven solutions with more free parameters by the new approach of generalized (G'/G)-expansion method. Accordingly, the applied method might be an advance and efficient mathematical tool in solving nonlinear equations that arise in the field of engineering problems. If $\gamma \neq 0$, (12) can also be solved by this method.

4. Discussions

The advantages and validity of the method over the generalized and improved (G'/G)-expansion method have been discussed in the following.

4.1. Advantages

The crucial advantage of the new approach against the generalized and improved (G'/G)-expansion method is that the method provides more general and abundant exact traveling wave solutions with much real parameter. The exact solutions of PDEs have its important significance to disclose the internal mechanism of the complex physical phenomena. Apart from the physical application, the close-form solutions of nonlinear evolution equations assist the numerical solvers to compare the accuracy of their results and help them in the stability analysis.

4.2. Validity

In Ref. [15] Akbar et al. used the linear ordinary differential equation as auxiliary equation and the traveling wave solutions are presented in the form, $u(\xi)=\sum_{n=-m}^{m}\frac{e_{-n}}{(d+(G'/G))^n}$, where either e_{-m} or e_m may be zero, but both e_{-m} or e_m cannot be zero at the same time. It is worth mentioning that some of our solutions are coincided with already published results, if parameters taken particular values which validates our methods. Moreover, in Ref. [15] Akbar et al. investigated the well-established strain wave equation in microstructured solids to obtain exact solutions via the generalized and improved (G'/G)-expansion method and achieved only six solutions (see Table 1 below). Moreover, in this article twenty seven solu-

Table 1 Comparison between newly obtained solutions and Akbar et al. [15] solutions.

Solutions obtained in this article	Akbar et al. [15] solutions
If $h_1 = 1$, $h_3 = 2$, $h_2 = 3\lambda$, $h_4 = 2\lambda^2$, $\varepsilon = 1$, $\Omega = \lambda^2$ and $u(x,t) = u_{2_1}(\xi)$ becomes: $u_{2_1}(\xi) = \frac{3\lambda^2}{2\alpha_1}(\alpha_3 - V^2\alpha_4)\coth^2\left(\frac{\lambda}{2}\xi\right) + \frac{1}{2\alpha_1}\{-2\lambda^2(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$	If $A = 0$, $\mu = 0$, $d = \frac{\lambda}{2}$, $\varepsilon = 1$ and $u(x,t) = u_{1_1}(\xi)$, solution $u_{2_1}(\xi)$ becomes: $u_{1_1}(\xi) = \frac{3\lambda^2}{2\alpha_1}(\alpha_3 - V^2\alpha_4)\coth^2\left(\frac{\lambda}{2}\xi\right) + \frac{1}{2\alpha_1}\{-2\lambda^2(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$
If $h_1 = 1$, $h_3 = 2$, $h_2 = 3\lambda$, $h_4 = 2\lambda^2$, $\varepsilon = 1$, $\Omega = -\lambda^2$ and $u(x,t) = u_{2_3}(\xi)$ becomes: $u_{2_3}(\xi) = \frac{3\lambda^2}{2\alpha_1}(\alpha_3 - V^2\alpha_4)\cot^2\left(\frac{i\lambda}{2}\xi\right) + \frac{1}{2\alpha_1}\{-2\lambda^2(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$	If $A = 0, \mu = 0, d = \frac{\lambda}{2}$, $\varepsilon = 1$ and $u(x,t) = u_{1_2}(\xi)$ becomes: $u_{1_2}(\xi) = -\frac{3\lambda^2}{2\alpha_1}(\alpha_3 - V^2\alpha_4)\cot^2\left(\frac{i\lambda}{2}\xi\right) - \frac{1}{2\alpha_1}\{-2\lambda^2(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$
If $h_1 = 1$, $h_3 = 0$, $h_2 = 2\lambda$, $h_4 = -\lambda^2$, $\varepsilon = 1$ and $u(x,t) = u_{2_5}(\xi)$, becomes: $u_{2_5}(\xi) = \frac{6}{\alpha_1}(\alpha_3 - V^2\alpha_4)\left(\frac{C_2}{C_1 + C_2\xi}\right)^2 - \frac{1 - V^2}{2\alpha_1}$	If $\mu = 0$, $d = \frac{\lambda}{2}$, $\varepsilon = 3$ and $u(x,t) = u_{1_3}(\xi)$, becomes: $u_{1_3}(\xi) = \frac{6}{\alpha_1}(\alpha_3 - V^2\alpha_4)\left(\frac{B}{A + B\xi}\right)^2 - \frac{1 - V^2}{2\alpha_1}$
If $h_1 = 1$, $h_3 = 0$, $h_2 = -\lambda$, $h_4 = 0$, $\Omega = \lambda^2$ and $u(x,t) = u_{1_1}(\xi)$ becomes: $u_{1_1}(\xi) = \frac{6d^2}{\alpha_1}(\alpha_3 - V^2\alpha_4) \times (d-\lambda)^2 \times \left(d - \frac{\lambda}{2} + \frac{\lambda}{2}\coth\left(\frac{\lambda}{2}\xi\right)\right)^{-2} + \frac{6d}{\alpha_1}(2d-\lambda)(d-\lambda)(\alpha_3 - V^2\alpha_4) \times \left(d - \frac{\lambda}{2} + \frac{\lambda}{2}\coth\left(\frac{\lambda}{2}\xi\right)\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\{\varepsilon\lambda^2(V^2\alpha_4 - \alpha_3) + 12\varepsilon d(d-\lambda)(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$	If $A = 0$, $\mu = 0$ and $u(x,t) = u_{2_1}(\xi)$, becomes: $u_{2_1}(\xi) = \frac{6d^2}{\alpha_1}(\alpha_3 - V^2\alpha_4) \times (d-\lambda)^2 \times \left(d - \frac{\lambda}{2} + \frac{\lambda}{2}\coth\left(\frac{\lambda}{2}\xi\right)\right)^{-2} + \frac{6d}{\alpha_1}(2d-\lambda)(d-\lambda)(\alpha_3 - V^2\alpha_4) \times \left(d - \frac{\lambda}{2} + \frac{\lambda}{2}\coth\left(\frac{\lambda}{2}\xi\right)\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\{\varepsilon\lambda^2(V^2\alpha_4 - \alpha_3) + 12\varepsilon d(d-\lambda)(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$
If $h_1 = 1$, $h_3 = 0$, $h_2 = -\lambda$, $h_4 = 0$, $\Omega = -\lambda^2$ and $u(x,t) = u_{1_3}(\xi)$ becomes $u_{1_3}(\xi) = \frac{6d^2}{\alpha_1}(\alpha_3 - V^2\alpha_4) \times (d-\lambda)^2 \times \left(d - \frac{\lambda}{2} + \frac{i\lambda}{2}\cot\left(\frac{i\lambda}{2}\xi\right)\right)^{-2} + \frac{6d}{\alpha_1}(2d-\lambda)(d-\lambda)(\alpha_3 - V^2\alpha_4) \times \left(d - \frac{\lambda}{2} + \frac{i\lambda}{2}\cot\left(\frac{i\lambda}{2}\xi\right)\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\{\varepsilon\lambda^2(\alpha_3 - V^2\alpha_4) + 12\varepsilon d(d-\lambda)(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$	If $A = 0$, $\mu = 0$ and $u(x,t) = u_{2_2}(\xi)$, becomes: $u_{2_2}(\xi) = \frac{6d^2}{\alpha_1}(\alpha_3 - V^2\alpha_4) \times (d-\lambda)^2 \times \left(d - \frac{\lambda}{2} + \frac{i\lambda}{2}\cot\left(\frac{i\lambda}{2}\xi\right)\right)^{-2} + \frac{6d}{\alpha_1}(2d-\lambda)(d-\lambda)(\alpha_3 - V^2\alpha_4) \times \left(d - \frac{\lambda}{2} + \frac{i\lambda}{2}\cot\left(\frac{i\lambda}{2}\xi\right)\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\{\varepsilon\lambda^2(\alpha_3 - V^2\alpha_4) + 12\varepsilon d(d-\lambda)(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$

If $h_1 = 1$, $h_3 = 0$, $h_2 = -\lambda$ and $h_4 = 0$ and $u(x,t) = u_{1_5}(\xi)$ becomes: $u_{1_5}(\xi) = \frac{6d^2}{\alpha_1}(\alpha_3 - V^2\alpha_4) \times (d-\lambda)^2 \times \left(d - \frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\right)^{-2} + \frac{6d}{\alpha_1}(2d-\lambda)(d-\lambda)(\alpha_3 - V^2\alpha_4) \times \left(d - \frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\{\varepsilon\lambda^2(\alpha_3 - V^2\alpha_4) + 12\varepsilon d(d-\lambda)(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$	If $\mu = 0$ and $u(x,t) = u_{2_2}(\xi)$ becomes: $u_{2_3}(\xi) = \frac{6d^2}{\alpha_1}(\alpha_3 - V^2\alpha_4) \times (d-\lambda)^2 \times \left(d - \frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\right)^{-2} + \frac{6d}{\alpha_1}(2d-\lambda)(d-\lambda)(\alpha_3 - V^2\alpha_4) \times \left(d - \frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi}\right)^{-1} - \frac{1}{2\varepsilon\alpha_1}\{\varepsilon\lambda^2(\alpha_3 - V^2\alpha_4) + 12\varepsilon d(d-\lambda)(\alpha_3 - V^2\alpha_4) + 1 - V^2\}$
$u_{1_2}(\xi) = \frac{l_1}{2\varepsilon\alpha_1 h_1^2} + \frac{6}{\alpha_1 h_1^2}\left\{ l_2\left(p + \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi}\tanh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)^{-1} + l_3\left(p + \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi}\tanh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)\right)^{-2}\right\}$	No solution found corresponding to this solutions.
$u_{1_4}(\xi) = \frac{l_1}{2\varepsilon\alpha_1 h_1^2} + \frac{6}{\alpha_1 h_1^2}\left\{ l_2\left(p + \frac{h_2}{2\psi} - \frac{\sqrt{-\Phi}}{2\psi}\tan\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)^{-1} + l_3\left(p + \frac{h_2}{2\psi} - \frac{\sqrt{-\Phi}}{2\psi}\tan\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)\right)^{-2}\right\}$	No solution found corresponding to this solutions.

tions of the strain wave equation in microstructured solids are constructed by applying the new approach of generalized (G'/G)-expansion method.

Similarly, no solution is found corresponding to the solutions $u_{1_5}(\xi)$, $u_{1_6}(\xi)$, $u_{1_9}(\xi)$, $u_{2_2}(\xi)$, $u_{2_4}(\xi)$, $u_{2_6}(\xi)$, $u_{2_9}(\xi)$, $u_{3_2}(\xi)$ and $u_{3_9}(\xi)$ in the Ref. [15].

5. Conclusion

In this article, the new approach of generalized (G'/G)-expansion method has successfully been implemented to investigate the nonlinear partial differential equation, namely, the strain wave equation in microstructured solids. Abundant exact traveling wave solutions including solitons, kink, periodic and rational solutions are attained. It is worth mentioning that some of newly obtained solutions are identical to already published results. It has been shown that the applied method is effective and more wide-ranging than the generalized and improved (G'/G)-expansion method because it gives many new solutions. Therefore, the method can be applied to study many other nonlinear partial differential equations which frequently arise in engineering, mathematical physics and other scientific real time application fields.

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