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Quenching parameter in a holographic thermal QCD

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Abstract

We have calculated the quenching parameter, $\hat{q}$ in a model-independent way using the gauge–gravity duality. In earlier calculations, the geometry in the gravity side at finite temperature was usually taken as the pure AdS black hole metric for which the dual gauge theory becomes conformally invariant unlike QCD. Therefore we use a metric which incorporates the fundamental quarks by embedding the coincident D7 branes in the Klebanov–Tseytlin background and a finite temperature is switched on by inserting a black hole into the background, known as OKS-BH metric. Further inclusion of an additional UV cap to the metric prepares the dual gauge theory to run similar to thermal QCD. Moreover $\hat{q}$ is usually defined in the literature from the Glauber model perturbative QCD evaluation of the Wilson loop, which has no reasons to hold if the coupling is large and is thus against the main idea of gauge–gravity duality. Thus we use an appropriate definition of $\hat{q}$: $\hat{q}L^- = 1/L^2$, where L is the separation for which the Wilson loop is equal to some specific value. The above two refinements cause $\hat{q}$ to vary with the temperature as T^4 always and to depend linearly on the light-cone time L^- with an additional $(1/L^-)$ correction term in the short-distance limit whereas in the long-distance limit, $\hat{q}$ depends only linearly on L^- with no correction term. These observations agree with other holographic calculations directly or indirectly.

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1. Introduction

In the initial stage of ultrarelativistic heavy-ion collisions energetic partons in the form of jets are produced from the hard collisions. After receiving a large transverse momentum, these jets plough through the fireball for a transitional period of about a few fm$/c$ and will thus loose energy due to the interaction of the hard partons with the medium constituents, known as the jet quenching. As a result the yield of hadrons with high transverse momentum (p_T) is shown to be significantly suppressed in comparison with the cumulative yields of nucleon–nucleon collisions. There are mainly two contributions to the energy loss of the partons in the medium: one is due to the radiation emitted by the decelerated color charges, *i.e.* bremsstrahlung of gluons [1–3] and the other one is due to the collisions among the partons in the medium [4].

The experimental discoveries at RHIC revealed that the matter produced is a strongly coupled quark–gluon plasma (sQGP) unlike weakly interacting gas of partons expected from the naive asymptotic freedom, *for example*, the observed elliptic flow, the quenching of jets while traversing through the medium etc. The jet quenching is parametrized by the quenching parameter, $\hat{q}$, which is defined by the average transverse momentum square transferred from the traversing parton per unit mean free path. The extracted values of this transport coefficient in relativistic heavy-ion collisions by the JET collaboration [5] range from 1 to 25 GeV^2/fm, which are much larger than those estimated from the perturbative QCD calculations. This hints some non-perturbative mechanisms which may contribute to the jet quenching mechanism. Thus it is worthwhile to calculate the possible values of $\hat{q}$ in the strong coupling limit. The first principle lattice QCD however, cannot be applied for this purpose, which requires the real-time dynamics.

The simplest gauge–gravity duality [6–8] between the type IIB superstring theory formulated on AdS$_5 \times S^5$ space and $\mathcal{N} = 4$ supersymmetric Yang–Mills theory (SYM) in four dimensions provides a robust tool to explore the thermodynamical and transport properties of sQGP. Although the underlying dynamics, QCD is different from $\mathcal{N} = 4$ SYM but the correspondence seems feasible because some of the properties of all strongly interacting systems show some universality behavior. One of the notable observation is the universal value $(1/4\pi)$ for the η/s ratio for the quantum field theories having a holographic description [9] and thus it gives a lower bound to the ratio for sQGP. Motivated by these similarities between the $\mathcal{N} = 4$ SYM and the corresponding theory of supergravity, the jet-quenching parameter, $\hat{q}$ was related to the expectation value of the Wilson loop $W^A[\mathcal{C}]$ in adjoint representation due to the Eikonal approximation [10]:

$$\langle W^A[\mathcal{C}]\rangle \approx \exp\left(-\frac{1}{4\sqrt{2}}\hat{q}L_-L^2\right), \tag{1}$$

where $\mathcal{C}$ is a rectangular contour of size $L \times L_-$, with the sides, having the length L_- run along the light-cone. There were other calculations of $\hat{q}$ [11,30,12] using a very different setup and arriving at different conclusions. In the context of relativistic heavy ion collisions, the effects of finite 't Hooft coupling (λ) as well as chemical potential on $\hat{q}$ was studied in [31,40,39] and the jet stopping in strongly-coupled QCD-like plasmas with gravity duals have also been studied using the string α' expansion in AdS/CFT [34,35].

However, since $\hat{q}$ is related to the transverse momentum (p_T) broadening so to calculate the mean p_T, we need to Fourier transform (FT) of the Wilson loop

$$W(p_T) = \int d^2L\, e^{ip\cdot L}\, W(L). \tag{2}$$

The above FT emerges if we intend to calculate the particle production in the scattering of a quark on a target, and the target will be the medium in the jet quenching problem. It turns out that the above FT is proportional to the quark production cross section, $W(p_T) \sim d\sigma/d^2p$ [13, 14]. Let us explore the subtleties which might help us to search for the correct definition of $\hat{q}$. For example, if we define $\hat{q}$ as $\langle p_T^2 \rangle / L^-$, as some authors do. So we would then need to find $\langle p_T^2 \rangle$. But this seems easy because $\langle p_T^2 \rangle \sim \nabla_\perp^2 W(L)$ at $L = 0$. This seems consistent with getting the coefficient of the L^2 term in the exponent, as in (1). However, since our aim is to model QCD and in QCD at high p_T perturbative physics works, and $d\sigma/d^2p \sim 1/p_T^4$, so $\langle p_T \rangle$ is infinite (irrespective of what happens at lower p_T). In other words one cannot trust $W(L)$ from AdS at very small L. A way out is to define $\hat{q}$ as $\langle p_T \rangle^2 / L^-$. Since $\langle p_T \rangle$ is finite even in perturbative QCD, this definition is safe. To find $\langle p_T \rangle$ we need the typical momentum scale of $W(L)$ and if one knows $W(p_T)$, then one should be able to find $\langle p_T \rangle$ exactly. Otherwise one could argue that $\langle p_T \rangle$ is given by the saturation scale Q_s, as the only scale available in the problem at high enough energy. Hence the standard prescription of finding Q_s by requiring the Wilson loop, $W(L = 1/Q_s)$ to be a constant, should probably give one a good estimate of $\langle p_T \rangle$.

In summary the above definition of $\hat{q}$ in (1) as a coefficient of the L^2 term in the Wilson line correlator may not be correct because the motivation for the definition (1) in [10] comes from the Glauber model perturbative QCD evaluation of the Wilson loop [15,16].[1] Therefore this perturbative expression has no reasons to hold when the coupling is large, which is the main idea of gauge–gravity duality. A more appropriate definition of $\hat{q}$ is then to postulate the equation

$$\hat{q} L^- = 1/L^2 \,, \tag{3}$$

where L is the quark–antiquark separation for which the expectation value of the Wilson loop in adjoint representation is equal to some specific value. The above definition (3) can also be understood as follows: since $\hat{q} L^-$ behaves like the saturation scale squared in small-x physics and the saturation scale is defined by requiring that the expectation value of Wilson loop is equal to some constant at $L = 1/Q_s$ [16,15].

The calculations for $\hat{q}$ discussed so far used the geometry as the pure AdS black hole metric, for which the dual gauge theory is conformally invariant SYM theory *unlike the QCD*. This is one of the central theme of our work. Therefore, the aim of the present paper – to extend/modify the shortcomings of the above-mentioned calculation [10] – is twofold: (i) the first aim is to study the jet quenching in a gravitational background which is dual to a gauge theory with an RG flow that confines in the far IR and is asymptotically free at the far UV. Recently a gravity dual with a black hole and seven branes embedded via Ouyang embedding is constructed [23,19], which resembles the main features of strongly coupled QCD, *i.e.* is almost conformal in the UV with no Landau poles or UV divergences of the Wilson loops, but has logarithmic running of coupling in the IR. Recently one of us have explored the properties of heavy quarkonium bound states with the above geometry and the findings [20,21] can only be understood as the artifact of the correct geometry for real QCD. (ii) The second one is the appropriate definition of $\hat{q}$ as in (3) for which the Wilson loop is equal to some specific value, *say* 1/2. Our work is therefore organized as follows. Section 2 will be devoted to revisit the Ouang–Klebanov–Strassler geometry and its improvements at the UV sector. In Section 3.1, we employ the aforesaid geometry to obtain the renormalized Nambu–Goto action in both short- and long-distance limits. Thereafter we will obtain the quenching parameter in Section 3.2 and will also discuss briefly the results of other calculations. Finally we conclude in Section 4.

1 In fact, it is already incorrect once someone includes perturbative QCD corrections to the Glauber formula.

2. Construction of dual geometry

A conformal gauge theory does not flow with the scale, hence it has a trivial RG flow. The AdS/CFT correspondence conjectures that a conformal theory in four dimensions can be mapped on the boundary of a pure anti-de Sitter space [6]. But if the theory has a non-trivial RG flow like QCD, which is confining in IR and conformal in UV, we cannot describe the full theory on the boundary of some higher dimensional space and hence need to envisage differently at running energy scales. One way out is to embed the D branes in the geometry and as a result the corresponding gauge theory exhibits logarithmic RG flow. Such a construction was done in the Klebanov–Strassler (KS) geometry [17] through a warped deformed conifold with three-form type IIB fluxes and the corresponding dual gauge theory is confining in the far IR limit but is not free at UV limit. The other demerits of the KS geometry are that it is devoid of quarks in the fundamental representation and cannot be generalized to finite temperature.

The inclusion of fundamental matter in string theory is possible by embedding a set of flavor branes in addition to the color branes. The strings connecting to the color and flavor branes in the adjoint representation of $U(N_c)$ group give the gauge particles and the mesons, respectively whereas those connected to both the flavor and color branes in the fundamental representation give the quarks and antiquarks, respectively. In principle one could go to large number of color (N_c) and flavor (N_f) branes in the near horizon limit and translates the branes into fluxes and then construct the gravity background which is holographically dual to gauge theory of quarks and gluons. In practice the back reaction of the probes on the background could be neglected through the probe approximation ($N_f \ll N_c$) and the flavor physics is then extracted by analyzing the effective action which describes the flavor branes in the color background [28,29]. Since the full global solution for the backreaction of D7 branes in the KS background becomes nontrivial so the insertion of the fundamental quarks in the original KS geometry [17] becomes difficult. Peter Ouyang [18] has successfully put the coincident D7 branes into the Klebanov–Tseytlin background [22], known as OKS geometry, which has all the type IIB fluxes switched on including the axio-dilaton and the local metric was then computed by incorporating the deformations of the seven branes by moving them far away from the regime of interest. Hence the axion-dilaton vanishes for the background locally, but there will be non-zero axion-dilaton globally, as a result the local back reactions on the metric modify the warp factors to the full global scenario.

For realizing the finite temperature a black hole is inserted into the OKS background, *i.e.* OKS-BH geometry, where the Hawking temperature corresponds to the gauge theory temperature. Thus the metric in OKS-BH geometry is expressed in terms of warp factor (h) [23]

$$ds^2 = \frac{1}{\sqrt{h}}\Big[-g_1(u)dt^2 + dx^2 + dy^2 + dz^2\Big] + \sqrt{h}\Big[g_2^{-1}(u)du^2 + d\mathcal{M}_5^2\Big] \tag{4}$$

where $g_i(u)$ are the black-hole factors as a function of the extra dimension, u and $d\mathcal{M}_5^2$ is due to the warped resolved–deformed conifold. The gauge theory dual to the metric (4) flows correctly at IR like QCD but the effective degrees of freedom grow indefinitely at UV limit. The situation becomes worse even in the presence of fundamental flavors because its proliferation leads to Landau poles and hence the Wilson loops diverges at UV. To circumvent the problem, one need to add the appropriate UV cap to the AdS–Schwarzschild geometry in the asymptotic UV limit. However, the additional UV caps, in general may deform the IR geometry but the far IR geometry has not been changed because the UV caps correspond to adding the non-trivial irrelevant operators in the dual gauge theory. These operators keep far IR physics completely unchanged, but the physics at not-so-small energies may be changed a bit.

Recently the IR geometry part has been suitably modified to obtain the desired dual gauge theory by the McGill group [19,23,24], where the metric (4) will receive further corrections, g_{uu}, because the unwarped metric may not remain Ricci flat due to the presence of both axio-dilaton and seven-brane sources, as:

$$ds^2 = \frac{1}{\sqrt{h}}\Big[-g(u)dt^2 + dx^2 + dy^2 + dz^2\Big] + \sqrt{h}\Big[g(u)^{-1}g_{uu}du^2 + g_{mn}dx^m dx^n\Big] \tag{5}$$

where the black hole factors $g_i(u)$ are set as $g_1(u) = g_2(u) \equiv g(u)$ and the corrections g_{uu} are of the form $1/u^n$ and may be written as a series expansion:

$$g_{uu} = 1 + \sum_{i=0}^{\infty} \frac{a_{uu,i}}{u^i}\ , \tag{6}$$

where the coefficients $a_{uu,i}$ are independent of the extra-dimension coordinate u and are solved exactly in [19]. Thus the warp factor, h can be extracted from the above corrections (6) as

$$h = \frac{L^4}{u^4}\left[1 + \sum_{i=1}^{\infty}\frac{a_i}{u^i}\right]$$

where the coefficients a_i are of $\mathcal{O}(g_s N_f)$ and L is the curvature of space. Thus the metric (5) reduces to OKS-BH in the IR limit and becomes $\mathrm{AdS}_5 \times M_5$ in the UV limit, hence describes well both in IR and UV limits. Therefore, with the change of coordinates $z = 1/u$, we can rewrite the metric (5) as

$$\begin{aligned} ds^2 &= g_{\mu\nu} dX^\mu dX^\nu \\ &= A_n z^{n-2}\left[-g(z)dt^2 + d\overrightarrow{x}^2\right] + \frac{B_l z^l}{A_m z^{m+2} g(z)} dz^2 + \frac{1}{A_n z^n} ds^2_{M_5}, \end{aligned} \tag{7}$$

where $ds^2_{\mathcal{M}_5}$ is the metric of the internal space and the coefficients A_n can be obtained from the coefficients a_i in the warp factor (2) as follows:

$$\frac{1}{\sqrt{h}} = \frac{1}{L^2 z^2 \sqrt{a_i z^i}} \equiv A_n z^{n-2} = \frac{1}{L^2 z^2}\left[a_0 - \frac{a_1 z}{2} + \left(\frac{3a_1^2}{8a_0} - \frac{a_2}{2}\right) z^2 + \cdots\right], \tag{8}$$

which gives $A_0 = \frac{a_0}{L^2}$, $A_1 = -\frac{a_1}{2L^2}$, $A_2 = \frac{1}{L^2}\left(\frac{3a_1^2}{8a_0} - \frac{a_2}{2}\right)$ etc. Note that since a_i's for $i \geq 1$ are of $\mathcal{O}(g_s N_f)$ and $L^2 \propto \sqrt{g_s N}$, so in the limit $g_s N_f \to 0$ and $N \to \infty$ all A_i's for $i \geq 1$ are very small. The second term in the metric (7) accommodates the $1/u^n$ corrections in (5) via the series, $B_l z^l$, which is expanded further:

$$B_l z^l = 1 + a_{zz,i} z^i. \tag{9}$$

In a comprehensive study [23], the entire geometry is split into three regions. Apart from the two asymptotic regions at IR and UV denoted as regions I and III, respectively, there is an interpolating region II, where at the outermost boundary the three-forms vanish and the innermost boundary will be the outermost boundary of region I. The background in these three regions and the insertion of additional UV cap are extensively analyzed by the corresponding RG flows and the field theory realizations have been discussed in [25]. Recently another suitable model to study certain IR dynamics of QCD is the Sakai–Sugimoto model [26] in the type IIA string theory, which consists of a set of N wrapped color D4-branes on the circle and the flavor branes D8 and $\bar{D}8$ placed at the anti-nodal points of the circle to conceive the mesonic bound states. In its dual gravity, the wrapped D4-branes are replaced by an asymptotically AdS space, but the eight-branes remain and so does the circular direction. However the Sakai–Sugimoto model does not have a UV completion and had been compared recently with the aforesaid gravity dual in [27]. We shall not go into the complete details here and will use the metric (7) to obtain the Nambu–Goto action and hence the Wilson loop is computed through gauge–gravity correspondence in the next section.

3. Gauge–gravity duality

According to the gauge/gravity prescription [6], the expectation value of the Wilson loop, $W(C)$ in a strongly coupled gauge theory is related to the generating functional of the string in the bulk which has the loop C at the boundary

$$\langle W(C)\rangle \sim Z_{\text{string}} \tag{10}$$

In supergravity limit, the generating functional becomes

$$Z_{\text{string}} = e^{iS_{\text{string}}}, \tag{11}$$

where S_{string} is obtained by extremizing the string action, known as the Nambu–Goto action. So the above correspondence (10) is translated into

$$\langle W(C)\rangle \sim e^{iS_{\text{string}}} \tag{12}$$

Thus we will now evaluate the Nambu–Goto action in the next subsection.

3.1. Nambu–Goto action

By the light-cone transformation,

$$\begin{aligned} dt &= \frac{dx^+ + dx^-}{\sqrt{2}} \\ dx_1 &= \frac{dx^+ - dx^-}{\sqrt{2}} \end{aligned} \tag{13}$$

the metric (7) is rewritten in terms of light-cone coordinates as

$$\begin{aligned} ds^2 = &\left[-\frac{1}{2}A_n z^{n-2} g + \frac{1}{2}A_n z^{n-2}\right]\left[dx^{+2} + dx^{-2}\right] - (1+g)\, A_n z^{n-2} dx^+ dx^- \\ &+ A_n z^{n-2}\left[dx_2{}^2 + dx_3{}^2\right] + \frac{B_n z^n}{A_n z^{n+2} g} dz^2 + \frac{1}{A_n z^n}\, ds_{M_5}{}^2 \end{aligned} \tag{14}$$

We parametrize the two-dimensional world sheet and their derivatives in terms of the light-cone coordinates

$$\begin{aligned} &\tau = x^-, \quad \sigma = x_2 \in [-\frac{r}{2}, \frac{r}{2}], \\ &x_2 = \text{const}, \;\; x_3 = \text{const}, \;\; z = z(x_2) \\ &\partial_\alpha = \frac{\partial}{\partial \tau}, \;\; \partial_\beta = \frac{\partial}{\partial \sigma}. \end{aligned} \tag{15}$$

With the above parametrization (15), the elements of the induced metric defined by

$$g_{\alpha\beta} = G\mu\nu \frac{\partial x^\mu}{\partial\sigma^\alpha}\frac{\partial x^\nu}{\partial\sigma^\beta} \tag{16}$$

can be read off from the above metric (14)

$$\begin{aligned} g_{--} &= \frac{A_n z^n (1-g)}{2z^2} \\ g_{-2} &= g_{2-} = 0 \\ g_{22} &= \frac{A_n z^n}{z^2} + \frac{B_n z^n}{z^2 A_n z^n g} z'^2 . \end{aligned} \tag{17}$$

Thus the determinant of the induced metric, $g_{\alpha\beta}$ can be calculated

$$\mathbf{det}\; g_{\alpha\beta} = g_{--}g_{22} = \frac{1}{2{z_h}^4}\left[\left(A_n z^n\right)^2 + \frac{(B_n z^n)\, z'^2}{g}\right], \tag{18}$$

hence the Nambu–Goto action can be obtained as

$$\begin{aligned} S &= -\frac{1}{2\pi\alpha'}\int\int d\sigma d\tau \sqrt{-\mathbf{det} g_{\alpha\beta}} \\ &= -\frac{1}{2\pi\alpha'}\int\int d\sigma d\tau \sqrt{-\frac{1}{2{z_h}^4}\left[(A_n z^n)^2 + \frac{(B_n z^n)\, z'^2}{g}\right]}, \end{aligned} \tag{19}$$

where α' ($= \frac{R^2}{\sqrt{\lambda}}$, R is the AdS radius and λ is the 't Hooft coupling) is the string tension. Thus the equation of motion:

$$z'\frac{\partial\mathcal{L}}{\partial z'} - \mathcal{L} = C \tag{20}$$

can be written from the above Lagrangian ($\mathcal{L}$) in (19) as

$$-\left(A_n z^n\right)^2 = C\sqrt{(A_n z^n)^2 + \frac{(B_n z^n)\, z'^2}{g}}, \tag{21}$$

where C is a constant of motion and can be obtained from the condition: $z' = 0$ at $z = z_m$,

$$C^2 = \left(A_n z_m^n\right)^2 \tag{22}$$

After substituting the constant C, the equation of motion becomes finally

$$z'^2 = \frac{(A_n z^n)^2 g}{B_n z^n}\left[\frac{(A_n z^n)^2}{(A_n {z_m}^n)^2} - 1\right] \tag{23}$$

Since the Lagrangian is independent of the time so after integrating over the time-like coordinate (x_-), the action becomes

$$S = -\frac{iL^-}{2\sqrt{2}\pi\alpha' {z_h}^2}\int_{-\frac{L}{2}}^{+\frac{L}{2}} dx_2 \sqrt{(A_n z^n)^2 + \frac{(B_n z^n)\, z'^2}{g}} \tag{24}$$

$$= -\frac{i2L^-}{2\sqrt{2}\pi\alpha' {z_h}^2}\int_0^{z_m} dz \sqrt{\frac{(A_n z^n)^2}{z'^2} + \frac{(B_n z^n)}{g}} \tag{25}$$

We will now substitute z'^2 from the equation of motion (23) to obtain the action. Since $(A_n z^n)^2 \ll (A_n {z_m}^n)^2$ so neglecting the higher-order terms and keeping up to the second-order term, the action (25) is simplified into (without loss of generality, $A_0 = 1$, $A_1 = 0$, and $A_2 = A$ (say), and similarly, $B_0 = 1$, $B_1 = 0$, and $B_2 = B$ (say) in units of L^2 [19][2])

$$S \simeq -\frac{\sqrt{2}L^-}{2\pi\alpha' {z_h}^2\left(1 + A{z_m}^2\right)} \int_0^{z_m} \frac{dz}{\sqrt{g}} \left(1 + \frac{B}{2}z^2\right)\left(1 + Az^2\right) \tag{26}$$

We will now evaluate the Nambu–Goto action by solving the above integral in both short- and long-distance limits:

Case-I: In the short-distance ($z_m \ll z_h$) limit, after performing the integration in (26) the action is written in terms of Gaussian hypergeometric functions

$$\begin{aligned} S &= -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2\left(1 + Az_m^2\right)} \int_0^{z_m} dz \left(\frac{1 + \frac{B+2Az^2}{2} + \frac{ABz^4}{2}}{\sqrt{1 - \frac{z^4}{z_h^4}}}\right) \\ &= -\frac{L^- z_m}{\sqrt{2}\pi\alpha' {z_h}^2(1 + Az_m^2)} \left[{}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{z_m^4}{z_h^4}\right) + \frac{(B + 2A)z_m^2}{6}\, {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; \frac{z_m^4}{z_h^4}\right) \right. \\ &\quad \left. + \frac{ABz_h^4}{6}\left(-\sqrt{1 - \frac{z_m^4}{z_h^4}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{z_m^4}{z_h^4}\right)\right)\right] \end{aligned} \tag{27}$$

On expanding the hypergeometric functions in powers of $(\frac{z_m}{z_h})$

$$\begin{aligned} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{z_m^4}{z_h^4}\right) &= \left(1 + \frac{{z_m}^4}{10{z_h}^4} + \ldots\right), \\ {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; \frac{z_m^4}{z_h^4}\right) &= \left(1 + \frac{3{z_m}^4}{14{z_h}^4} + \ldots\right), \\ {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{z_m^4}{z_h^4}\right) &= \left(1 + \frac{{z_m}^4}{10{z_h}^4} + \ldots\right), \end{aligned} \tag{28}$$

respectively and ignoring the higher-order terms beyond the second power, the action becomes

$$S \overset{z_m \ll z_h}{\simeq} -\frac{L^- z_m}{\sqrt{2}\pi\alpha' z_h^2}\left[1 + \frac{(B - 4A)z_m^2}{6} + \frac{z_m^4}{10z_h^4}\right] \tag{29}$$

In addition to the extremal surface constructed above for the Nambu–Goto action, there is another trivial one given by the two disconnected world sheets, placed one at $x_2 = +\frac{L}{2}$ and another at $x_2 = -\frac{L}{2}$. The action for these two surfaces is

$$S_0 = -\frac{2}{2\pi\alpha'} \int dz dx^- \sqrt{-g_{--}g_{zz}}$$

[2] Such a choice is of course consistent with supergravity solution for the background we used in our work.

$$= -\frac{iL^-}{\sqrt{2}\pi\alpha' z_h^2}\int_0^{z_m} dz\,\frac{1+\frac{B}{2}z^2}{\sqrt{1-\frac{z^4}{z_h^4}}} \tag{30}$$

$$= -\frac{iL^- z_m}{\sqrt{2}\pi\alpha' z_h^2}\left[{}_2F_1\left(\frac{1}{4},\frac{1}{2},\frac{5}{4};\frac{z_m^4}{z_h^4}\right)+\frac{Bz_m^2}{6}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};\frac{z_m^4}{z_h^4}\right)\right] \tag{31}$$

Expanding the above hypergeometric functions in powers of $(\frac{z_m}{z_h})$

$${}_2F_1\left(\frac{1}{4},\frac{1}{2},\frac{5}{4};\frac{z_m^4}{z_h^4}\right)=\left(1+\frac{{z_m}^4}{10{z_h}^4}+...\right), \tag{32}$$

$${}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};\frac{z_m^4}{z_h^4}\right)=\left(1+\frac{3{z_m}^4}{14{z_h}^4}+...\right), \tag{33}$$

respectively and ignoring the higher-order terms beyond the second power, the action to be subtracted (S_0) becomes

$$S_0 \overset{z_m \ll z_h}{\simeq} -\frac{iL^- z_m}{\sqrt{2}\pi\alpha' z_h^2}\left[1+\frac{Bz_m^2}{6}+\frac{z_m^4}{10z_h^4}+\cdots\right] \tag{34}$$

Therefore the renormalized action is obtained by subtracting the action (34) for the two disconnected surfaces from (29)

$$\begin{aligned} S_I &\overset{z_m \ll z_h}{\simeq} S - S_0 \\ &= -\frac{L^- z_m}{\sqrt{2}\pi\alpha' {z_h}^2}\left[\left(1+\frac{(B-4A)\,z_m^2}{6}+\frac{z_m^4}{10z_h^4}\right)-i\left(1+\frac{Bz_m^2}{6}+\frac{z_m^4}{10z_h^4}\right)\right] \end{aligned} \tag{35}$$

Case II: In the long-distance limit ($z_m \gg z_h$) the integral in the action (26) is split into integrations:

$$\begin{aligned} S &= -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2(1+A{z_m}^2)}\left[\int_0^{z_h} dz\frac{(1+\frac{Bz^2}{2})(1+Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}}+\int_{z_h}^{z_m} dz\frac{(1+\frac{Bz^2}{2})(1+Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}}\right] \\ &\equiv \mathrm{I}+\mathrm{II}\,, \end{aligned} \tag{36}$$

where the first integral (I) becomes

$$\begin{aligned} \mathrm{I} &= -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2(1+A{z_m}^2)}\left[\int_0^{z_h} dz\frac{(1+\frac{Bz^2}{2})(1+Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}}\right] \\ &\simeq -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2}\left[1.3z_h+0.3(B+2A){z_h}^3+0.22AB{z_h}^5\right] \end{aligned} \tag{37}$$

and the second integral (II) becomes, after neglecting the higher-order terms in powers of $(\frac{z_h}{z_m})$ and keeping up to the second order

$$\begin{aligned}
\mathrm{II} &= -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2(1+A{z_m}^2)}\left[\int_{z_h}^{z_m} dz \frac{(1+\frac{Bz^2}{2})(1+Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}}\right] \\
&\simeq \frac{-iL^-}{\sqrt{2}\pi\alpha' {z_h}^2}(1.14 z_h + 0.5(B+2A) z_m {z_h}^2 + 0.17 AB {z_m}^3 {z_h}^2)
\end{aligned} \tag{38}$$

Therefore the Nambu–Goto action in this limit becomes

$$\begin{aligned}
S &\overset{z_m \gtrapprox z_h}{=} -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2}\Big[(1.3 z_h + 0.3(B+2A){z_h}^3 + 0.22 AB {z_h}^5) \\
&+ \quad i(1.14 z_h + 0.5(B+2A) z_m {z_h}^2 + 0.17 AB {z_m}^3 {z_h}^2)\Big]
\end{aligned} \tag{39}$$

Similarly the action to be subtracted (30) in this limit can be written as

$$S_0 = -\frac{iL^-}{\sqrt{2}\pi\alpha' z_h^2}\left[\int_0^{z_h} dz \frac{1+\frac{B}{2}z^2}{\sqrt{1-\frac{z^4}{z_h^4}}} + \int_{z_h}^{z_m} dz \frac{1+\frac{B}{2}z^2}{\sqrt{1-\frac{z^4}{z_h^4}}}\right] \tag{40}$$

After integrating and keeping the terms up to the second-order, the action, S_0 for two disconnected surfaces becomes

$$S_0 \overset{z_m \gg z_h}{\simeq} -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2}\Big[-1.14 z_h - 0.3 B {z_h}^3 + i(1.3 z_h + 0.3 B {z_h}^3)\Big] \tag{41}$$

Therefore, the renormalized action is given by

$$\begin{aligned}
S_I &\overset{z_m \gtrapprox z_h}{=} S - S_0 \\
&= \quad -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2}\Big[2.44 z_h + 0.5 B z_m {z_h}^2 + i(-0.16 z_h + 0.5(B+2A) z_m {z_h}^2)\Big]
\end{aligned} \tag{42}$$

3.2. Jet quenching parameter

We will now obtain the quenching parameter, $\hat{q}$ for which the expectation value of the Wilson loop in the adjoint representation is equal to some specific value, say, C,

$$\langle W_A \rangle = e^{i2S_I} = C \tag{43}$$

In our problem, $\langle W \rangle$ becomes complex-valued, which is a feature previously encountered in [15] as well. Since $\langle W \rangle$ is the S-matrix for a quark dipole-medium scattering, it is allowed to be complex. If we were calculating Q_s we would need the imaginary part of the forward scattering amplitude: since $S = 1 + iT$, then $\Im T = 1 - \Re S = 1 - \Re\langle W \rangle$. This was exactly done in [15]. Therefore we redefined $\hat{q}$ in (3), where L is the separation at which the real part of the Wilson loop is constant (C).

Thus decomposing the renormalized action, S_I into the real and imaginary parts, the real part of the expectation value of Wilson loop is

$$\begin{aligned}
\Re\langle W_A \rangle &= \Re\left[e^{i(2\Re S_I + 2i\Im S_I)}\right] \\
&= e^{-2\Im S_I}\left[\cos(2\Re S_I)\right] = C
\end{aligned} \tag{44}$$

Now we will evaluate the quenching parameter for both long- and short-distance limits, using the actions in the respective limits.

Case I: Short-distance limit ($z_m \ll z_h$)

To write the action as a function of the separation L, we first express z_m in terms of L. For that we rewrite the equation of motion (23) in this limit ($z_m \ll z_h$)

$$z'^2 = -\frac{(A_n z^n)^2 g}{B_n z^n} \tag{45}$$

because $(A_n z^n)^2$ is much less than $(A_n {z_m}^n)^2$. Integrating both sides of the equation of motion (45)

$$\int_0^{z_m} dz \frac{\sqrt{B_n z^n}}{(A_n z^n)\sqrt{g}} = i \int_{-L/2}^{0} dx_2 \tag{46}$$

the separation (L) becomes

$$\begin{aligned}\frac{iL}{2} &= \int_0^{z_m} dz \frac{(1+0.5Bz^2)(1-Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}} \\ &= z_m + \frac{z_m^5}{10 z_h^4} + 0.17(B-2A)z_m^3\left(1+\frac{3z_m^4}{14z_h^4}\right) \\ &- 0.17ABz_h^4 z_m\left(-\sqrt{1-\frac{z_m^4}{z_h^4}} + {}_2F_1\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{z_m^4}{z_h^4}\right)\right)\end{aligned} \tag{47}$$

Inverting the series and ignoring the higher-order terms we can express z_m as a function of L as

$$z_m = \frac{Li}{2}\left(1+\frac{(B-2A)}{24}L^2\right) \tag{48}$$

Thus the renormalized action can be expressed in terms of the separation (L) by replacing z_m as a function of L into (35). Ignoring the higher-order terms, the renormalized action is then given by

$$\begin{aligned}S_I = &-\frac{\sqrt{2}L^-}{2\pi\alpha' {z_h}^2}\frac{Li}{2}\left(1+\frac{(B-2A)L^2}{24}\right)\left[\left(1+\frac{(B-4A)L^2}{24}+\frac{L^4}{160z_h^4}\right)\right. \\ &\left. - i\left(1+\frac{BL^2}{24}+\frac{L^4}{160z_h^4}\right)\right]\end{aligned} \tag{49}$$

Now the imaginary and real parts of the renormalized action can be separated, respectively as

$$\Im S_I = -\frac{\sqrt{2}L^-}{2\pi\alpha' {z_h}^2}\frac{L}{2}\left(1+\frac{(B-2A)L^2}{24}\right)\left(1+\frac{(B-4A)L^2}{24}+\frac{L^4}{160z_h^4}\right) \tag{50}$$

and

$$\Re S_I = -\frac{\sqrt{2}L^-}{2\pi\alpha' {z_h}^2}\frac{L}{2}\left(1+\frac{(B-2A)L^2}{24}\right)\left(1+\frac{BL^2}{24}+\frac{L^4}{160z_h^4}\right). \tag{51}$$

Thus the gauge–gravity prescription (44) is reduced into

$$
\begin{aligned}
C &= (1-2\Im S_I)\left(1-2\,(\Re S_I)^2\right) \\
&= \left[1+\frac{L^-L}{\sqrt{2}\pi\alpha' {z_h}^2}\left(1+\frac{(B-2A)\,L^2}{24}\right)\left(1+\frac{(B-4A)\,L^2}{24}+\frac{L^4}{160z_h^4}\right)\right] \\
&\times\left[1-\frac{L^{-2}L^2}{4\pi^2\alpha'^2 z_h^4}\left(1+\frac{(B-2A)\,L^2}{12}\right)\left(1+\frac{BL^2}{12}+\frac{L^4}{80z_h^4}\right)\right]
\end{aligned}
\tag{52}
$$

Let the first and the second term in the square bracket in the above equation (52) be denoted by I and II, respectively

$$
\begin{aligned}
\mathrm{I} &\equiv \left[1+\frac{L^-L}{\sqrt{2}\pi\alpha' {z_h}^2}\left(1+\frac{(B-4A)\,L^2}{24}+\frac{L^4}{160z_h^4}-\frac{(B-2A)\,L^2}{24}\right)\right] \\
&= \left[1+\frac{L^-L}{\sqrt{2}\pi\alpha' {z_h}^2}\left(1-\frac{AL^2}{12}+\frac{L^4}{160z_h^4}\right)\right]
\end{aligned}
\tag{53}
$$

$$
\mathrm{II} \equiv \left[1-\frac{L^{-2}L^2}{4\pi^2\alpha'^2 z_h^4}\left(1+\frac{(B-A)\,L^2}{6}+\frac{L^4}{80z_h^4}\right)\right]
\tag{54}
$$

Therefore the product of the terms I and II in (52) yields

$$
C=\left[1-pL-qL^2-rL^3-sL^4-tL^5-uL^6+\text{higher order terms}\right],
\tag{55}
$$

where

$$
\begin{aligned}
p &\equiv -\frac{L^-}{\pi\sqrt{2}\alpha' {z_h}^2} \\
q &\equiv \frac{L^{-2}}{4\pi^2\alpha'^2 {z_h}^4} \\
r &\equiv \frac{AL^-}{12\pi\sqrt{2}\alpha' {z_h}^2}
\end{aligned}
\tag{56}
$$

By inverting the equation and ignoring the higher-order terms, the separation (L) is given by

$$
L=\frac{1-C}{p}-\frac{q\,(1-C)^2}{p^2}+\frac{(1-C)^3\left(2q^2-pr\right)}{p^5}
\tag{57}
$$

Therefore the quenching parameter, $\hat{q}$ is obtained from the definition (3):

$$
\begin{aligned}
\hat{q} &= \frac{1}{L^-L^2} \\
&= \frac{L^-}{2\pi^2\alpha'^2 z_h^4(1-C)^2}\left[1-\frac{L^-(1-C)}{\sqrt{2}\pi\alpha' z_h^2}-(1-C)^2\left(1+\frac{A\pi^2\alpha'^2 z_h^4}{3L^{-2}}\right)\right],
\end{aligned}
\tag{58}
$$

which finally results into for $C=\frac{1}{2}$,

$$
\hat{q}=\frac{2L^-}{\pi^2\alpha'^2 z_h^4}\left[\frac{3}{4}-\frac{L^-}{2\sqrt{2}\pi\alpha z_h^2}-\frac{A\pi^2\alpha'^2 z_h^4}{12L^{-2}}\right]
\tag{59}
$$

Case II: In the long-distance limit ($z_m \gg z_h$), let us first express the separation (L) as a function of z_m. Therefore, we split up the limits of integration to the equation of motion (23) and then integrate it to yield L as a function of z_m:

$$\begin{aligned}\frac{iL}{2} &= \int_0^{z_m} dz \frac{(1+0.5Bz^2)(1-Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}} \\ &= \int_0^{z_h} dz \frac{(1+0.5Bz^2)(1-Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}} + \int_{z_h}^{z_m} dz \frac{(1+0.5Bz^2)(1-Az^2)}{\sqrt{1-\frac{z^4}{{z_h}^4}}} \\ &= 1.3z_h + 0.15(B-2A){z_h}^3 - 0.22AB{z_h}^5 \\ &+ i\left[1.14z_h + 0.5(B-2A)z_m - 0.17AB{z_m}^3\right]\end{aligned} \tag{60}$$

Inverting the series and ignoring the higher-order terms we express z_m in terms of L as

$$z_m = \frac{L - 2.28z_h + i2.6z_h}{(B-2A){z_h}^2} \tag{61}$$

Now the (renormalized) action (42) in this limit can be expressed as a function of L:

$$\begin{aligned}S_I = &-\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2}\left[2.44z_h + 0.5B{z_h}^2\left(\frac{L-2.28z_h+i2.6z_h}{(B-2A){z_h}^2}\right) - 0.16iz_h\right. \\ &\left.+\, i0.5(B+2A){z_h}^2\left(\frac{L-2.28z_h+i2.6z_h}{(B-2A){z_h}^2}\right)\right].\end{aligned} \tag{62}$$

Ignoring the higher-order terms, we get the action as a function of L,

$$S_I \overset{z_m \gg z_h}{=} -\frac{L^-}{\sqrt{2}\pi\alpha' {z_h}^2}\left[\frac{0.5BL}{(B-2A)} + i\frac{0.5(B+2A)L}{(B-2A)}\right] \tag{63}$$

Now the real and the imaginary parts of renormalized action can be separated, respectively as

$$\Re S_I = -\frac{BL^-L}{2\sqrt{2}\pi(B-2A)\alpha' {z_h}^2} \tag{64}$$

$$\Im S_I = -\frac{(B+2A)L^-L}{2\sqrt{2}\pi(B-2A)\alpha' {z_h}^2} \tag{65}$$

Thus the gauge–gravity correspondence (44) in this limit is translated into:

$$C = e^{\left[\frac{(B+2A)L^-L}{\sqrt{2}\pi\alpha'(B-2A)z_h^2}\right]} \cos\left[\frac{BL^-L}{\sqrt{2}\pi\alpha'(B-2A)z_h^2}\right] \tag{66}$$

Defining

$$\begin{aligned}a &\equiv \frac{(B+2A)L^-}{\sqrt{2}\pi\alpha'(B-2A)z_h^2} \\ b &\equiv \frac{BL^-}{\sqrt{2}\pi\alpha'(B-2A)z_h^2}\ ,\end{aligned} \tag{67}$$

the above equation (66) has been inverted to give rise the expression for the dipole separation (L) as

$$L = \frac{C-1}{a}\left[1 + \frac{(a^2-b^2)(1-C)}{2a^2} + \frac{(2a^4-3a^2b^2+3b^4)(1-C)^2}{6a^4} \right.$$
$$\left. + \frac{(6a^6-11a^4b^2+16a^2b^4-15b^6)(1-C)^3}{24a^6} + \cdots \right] \tag{68}$$

Using the numerical values of A and B in [19] ($A = B = 0.124$), the expressions for a and b in Eq. (67) can be rewritten as

$$a = -\frac{3\pi T^2 L^-}{\sqrt{2}\alpha'} \quad \text{and} \quad b = -\frac{\pi T^2 L^-}{\sqrt{2}\alpha'}, \tag{69}$$

and hence the separation becomes

$$L = \frac{\sqrt{2}(1-C)\alpha'}{3\pi T^2 L^-}\left[1 + \frac{4(1-C)}{9} + \frac{23(1-C)^2}{81} + \frac{301(1-C)^3}{1458} + \cdots \right] \tag{70}$$

Thus the quenching parameter $\hat{q}$ is obtained from (3) by substituting the square of the separation (70) for $C = 1/2$

$$\hat{q} = \frac{102 T^4}{\alpha'^2} L^-, \tag{71}$$

which is seen to be linear in L^-.

In the study of DIS on a large nucleus in AdS/CFT set up [15], although authors did not calculate $\hat{q}$ directly but if we translate their calculation of the saturation scale, Q_s into our calculation, we could use $\hat{q} = Q_s^2/L^-$. The way Q_s depends on L is, in turn, dependent on which complex branch is chosen. In particular they took $Q_s \sim A^{1/3} \sim L^-$, since $L \sim A^{1/3}$. Hence in both cases $\hat{q}$ comes out to be $\sim L^-$, which appears to be in agreement with our calculation. Since they assumed $L^-(\sim A^{1/3})$ to be large enough, they need not keep the inverse powers of L^-. We even checked with their shock-wave metric [15], where $\hat{q}$ is $\sim L^-$ for large L^- and in agreement with our result in the respective limit.

From other perspective of jet quenching phenomena, by comparing the medium induced energy loss and the p_T-broadening in perturbative QCD with that of the trailing string picture of conformal theory in [30], they also have used $Q_s \sim L^-$, such that $\hat{q} = Q_s^2/L^- \sim L^-$ is in agreement with everything else we obtained so far in our calculations.

4. Results and discussions

We have calculated the quenching parameter, $\hat{q}$ in the holographic set-up of gauge–gravity duality, where the dual gauge theory at finite temperature is more closer to thermal QCD than the $\mathcal{N} = 4$ SYM theory usually used in the literature. Moreover we use a more appropriate definition of $\hat{q}$ compatible with the strong coupling limit of gauge–gravity duality, for which the real part of the Wilson loop expectation value is equal to some specific value (1/2). We have found that in both short- and long-distance limit, $\hat{q}$ depends linearly on L^-. However, in short-distance limit we obtain $1/L^-$ and L^{-2} correction terms.

It is however worth to mention here that it is not clear what one should do with $\hat{q}$ found in a non-perturbative AdS calculation. Since the energy loss calculations are usually done using the perturbative approximation, one cannot simply take a non-perturbative $\hat{q}$ and plug it into the perturbative energy loss expression. But then there is nothing else one can do. This is why people calculated drag force on a heavy quark without looking for $\hat{q}$ [36–38] or the instantaneous energy loss suffered by light quarks in AdS directly [32,33]. It would be interesting to see whether the drag calculation would give the same $\hat{q}$ as the one we have obtained. As far as we remember, the drag calculation in [30] obtained both $\hat{q}$ and Q_s which are in qualitative agreement with what we have gotten.

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References

[1] R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne, D. Schiff, Nucl. Phys. B 483 (1997) 291.
[2] B.G. Zakharov, JETP Lett. 65 (1997) 615.
[3] M. Gyulassy, P. Levai, I. Vitev, Nucl. Phys. B 594 (2001) 371.
[4] M.G. Mustafa, M.H. Thoma, Acta Phys. Hung. A 22 (2005) 93.
[5] K.M. Burke, et al., Phys. Rev. C 90 (2014) 014909.
[6] J.M. Maldacena, Phys. Rev. Lett. 80 (1998) 4859.
[7] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105.
[8] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253;
E. Witten, Adv. Theor. Math. Phys. 2 (1998) 505.
[9] D.T. Son, A.O. Starinets, Annu. Rev. Nucl. Part. Sci. 57 (2007) 95.
[10] H. Liu, K. Rajagopal, U.A. Wiedemann, Phys. Rev. Lett. 97 (2006) 182301.
[11] A.H. Mueller, Nucl. Phys. B 335 (1990) 115.
[12] J. Casalderrey-Solana, D.C. Gulhan, J.G. Milhano, D. Pablos, K. Rajagopal, Nucl. Phys. A 932 (2014) 421;
J. Casalderrey-Solana, D.C. Gulhan, J.G. Milhano, D. Pablos, K. Rajagopal, J. High Energy Phys. 1410 (2014) 19;
J. Casalderrey-Solana, D.C. Gulhan, J.G. Milhano, D. Pablos, K. Rajagopal, J. High Energy Phys. 1509 (2015) 175.
[13] F. D'Eramo, H. Liu, K. Rajagopal, Phys. Rev. D 84 (2011) 065015.
[14] R. Abir, Phys. Lett. B 748 (2015) 467.
[15] J.L. Albacete, Y.V. Kovchegov, A. Taliotis, J. High Energy Phys. 0807 (2008) 074.
[16] Y.V. Kovchegov, E. Levin, Quantum Chromodynamics at High Energy, 1st edition, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, 2012.
[17] I.R. Klebanov, M.J. Strassler, J. High Energy Phys. 0008 (2000) 052.
[18] P. Ouyang, Nucl. Phys. B 699 (2004) 207.
[19] M. Mia, K. Dasgupta, C. Gale, S. Jeon, Phys. Rev. D 82 (2010) 026004.
[20] Binoy Krishna Patra, H. Khanchandani, Phys. Rev. D 91 (2015) 066008.
[21] Binoy Krishna Patra, H. Khanchandani, Lata Thakur, Phys. Rev. D 92 (2015) 085034.
[22] I.R. Klebanov, A.A. Tseytlin, Nucl. Phys. B 578 (2000) 123.
[23] M. Mia, K. Dasgupta, C. Gale, S. Jeon, Nucl. Phys. B 839 (2010) 187.
[24] M. Mia, K. Dasgupta, C. Gale, S. Jeon, Phys. Lett. B 694 (2011) 460.
[25] F. Chen, L. Chen, K. Dasgupta, M. Mia, O. Trottier, Phys. Rev. D 87 (2013) 041901.
[26] T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843.
[27] K. Dasgupta, C. Gale, M. Mia, M. Richard, O. Trottier, J. High Energy Phys. 1507 (2015) 122.
[28] A. Karch, E. Katz, J. High Energy Phys. 0206 (2002) 043.
[29] T. Sakai, J. Sonnenschein, J. High Energy Phys. 0309 (2003) 047.
[30] F. Dominguez, C. Marquet, A.H. Mueller, B. Wu, B.W. Xiao, Nucl. Phys. A 811 (2008) 197.
[31] N. Armesto, J.D. Edelstein, J. Mas, J. High Energy Phys. 0609 (2006) 039.

[32] A. Ficnar, J. Noronha, M. Gyulassy, J. Phys. Conf. Ser. 446 (2013) 012002.
[33] S. Martins, C.B. Mariotto, J. Phys. Conf. Ser. 706 (2016) 052035.
[34] P. Arnold, P. Szepietowski, D. Vaman, G. Vong, J. High Energy Phys. 1302 (2013) 130.
[35] P. Arnold, P. Szepietowski, D. Vaman, J. High Energy Phys. 1207 (2012) 024.
[36] S.S. Gubser, Phys. Rev. D 74 (2006) 126005.
[37] S.S. Gubser, Phys. Rev. D 76 (2007) 126003.
[38] C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, L.G. Yaffe, J. High Energy Phys. 0607 (2006) 013.
[39] S. Li, K.A. Mamo, H. Yee, arXiv:1605.00188 [hep-ph].
[40] Feng-Li Lin, Toshihiro Matsuo, Phys. Lett. B 641 (2006) 45–49.