Synthesis of robust controller-regulators for omnidirectional mobile robot with irregular movement

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Abstract

In this work an analytical synthesis method of TS fuzzy controller, which provides robustness according to stability degree for dynamic object having parametric uncertainty with irregular movement has been proposed. The effectiveness of the proposed method has been directly shown by the example of analytically synthesis of fuzzy regulator for omnidirectional (OD) mobile robot.

\begin{flushleft}
1. Introduction
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Many technical devices, such as manipulation robots\textsuperscript{1,3}, mechatronic modules, technical processes as the dynamic control objects have uncertainties with irregular movement. These control objects are often described in the form of complex nonlinear differential equations\textsuperscript{1-5}. Furthermore, on control objects of this type the mathematical models could have certain errors due to mutual transformations and simplifications. By this reason, via existing methods, for example, the method of linear matrix inequalities (LMI), automatic control system synthesized for dynamic object which is written with nonlinear mathematical models on the bases of Lyapunov quadratic function $V(t) = x^T(t)Px(t)$, here $P$ – is some positive quadratic matrix) sometimes do not meet the requirements of quality indicators or robustness\textsuperscript{3-9}.

In order to control dynamic objects (mobile robot), that is described by linear and uncertain models, the synthesis of the regulators is based on application of fuzzy logic theory. Since synthesized regulators are based on the

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knowledge base, their application for other objects (robots) are limited. In addition, to determine the stability with quality indicators of ACS and the relationship between parameters of the objects in these systems is very difficult or almost impossible.

Considering mentioned, in this paper the developed analytically parametric synthesis method of controller is proposed. This method provides high stability and accuracy of the projected ACS for the objects. As a dynamic object, the mobile robot is described by means of multivariable differential equation, which leads to mathematical models of TS type.

Application of analytically parametrically synthesized TS fuzzy regulator for controlling OD mobile robot and simulation of ACS confirmed the effectiveness of the proposed analytical synthesis method. So that the control system of the mobile robot rolling in all directions provides possible maximum stability degree and quickly work at the expense of speed and coordinates.

2. Statement of the synthesis problem of robust controller for an uncertainty dynamic object (OD mobile robot) given by multiply fuzzy TS model

In many manipulation and mobile robots dynamic movement of kinematic units is described by multivariable nonlinear differential equations of n-th order. In most cases, non-linear mathematical models of robots can be transformed into TS fuzzy model.

In the stability conditions, which meets requirements, is mainly used in the solution of synthesis problems of TS fuzzy controller and it is not detected as a high of stability degree. Furthermore, in the reported methods during the synthesis of the controller it is impossible to consider requirements of other quality criteria of the control systems. For example, in the method of linear matrix inequalities LMI which is used at the decision of control synthesis problem - controller for nonlinear dynamic objects only the stability of system is considered. In the method of LMI the synthesis tasks are sequential iterative procedure, and it is impossible to take into account other quality system indicators. Also in the method of the LMI control the determined parameters of controller can be sufficiently large and be of different signs, which is not always feasible in practice. Otherwise, the tuning parameters of controller are not limited, and this is unacceptable from the point of view of practical realization.

In general, some of the dynamic objects with uncertainty irregular movement such as robots can be described in the form of fuzzy differential equations:

\[
\dot{x}(t) = f(x(t), u(t), p), \quad y(t) = \phi(x(t), u(t)) \quad x \in \mathbb{R}^{n_x}, \quad u \in \mathbb{R}^{n_u}, \quad t \in [t_0, t_f], \quad p \in P.
\] (2.1)

Here, \(x, u, p\) - being respectively, \(n_x = \text{dim } x\) - dimensional, \(n_u = \text{dim } u\) - dimensional, \(n_p = \text{dim } p\) - dimensional uncertainty parameters are vector which characterizes the technical parameters (sometimes partially influences of external perturbations) of the object; \(y\) is \(n_y = \text{dim } y\) - dimensional output variable; \(P\) - describes the range of the variation of parameters \(p\); \([t_0, t_f]\) - the time interval of control process.

While explaining the meaning of parametric uncertainty of the object in synthesizing control-regulator, the vector \(p\) is usually described as a fuzzy number- set \(\tilde{p} = [\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_{n_p}]\) or as an interval of numbers-set:

\[
p_i = \text{Supp } \tilde{p}_i = [p^{l_i}_i, p^{r_i}_i], \quad i = 1, n_p.
\] (2.2)

Here \(p^{l_i}_i, p^{r_i}_i\) - are respectively the left and right boundaries of interval numbers. If in the model (2.1) function \(f(\bullet)\) is known, the change in state coordinates of the object will be depend on the variation of parameters.

In the design-synthesis of control systems range of varied vectors which is included parameters \(p\) are accepted as nominal values. This is subjective assessment of the designer. Approximation of fuzzy and uncertain interval parameters is only authentic with nominal values.

Uncertainty dynamic objects can be described using various mathematical models, such as
\[
\dot{x}(t) = A\dot{x}(t) + B\ddot{u}(t)
\]
\[
\ddot{y}(t) = C\ddot{x}(t)
\]  
(2.3)

or

\[
\dot{x}(t) = \hat{A}x + \hat{B}u
\]
\[
y(t) = \tilde{C}x
\]  
(2.4)

The model presented in the form of (2.3), A, B, and C - is matrix of conventional coefficients with corresponding dimension. In the model (2.4) \( \hat{A}, \hat{B}, \tilde{C} \) - are the matrices of fuzzy numbers with appropriate dimension. Describing this type of objects with ordinary differential equations the parameters of controller with interval number which provides stable action of control system, is possible to synthesize analytically by the method proposed in 10.

A number of uncertain nonlinear dynamic objects, including manipulators and robots such as OD mobile robots, can be represented as the following fuzzy TS model:

\[
R_i: \text{If } z_1(t) \text{ is } M^i_1 \text{ and } z_2(t) \text{ is } M^i_2 \text{ and } z_n(t) \text{ is } M^i_n \text{ Then } \]
\[
\dot{x} = A_i x(t) + B_i u(t), \ y(t) = C_i x(t), \ i = 1, q
\]  
(2.5)

Where \( M^i_j \) - is fuzzy term set of the j-th state variable of the object and has the membership function \( \mu^i_j \); and \( A_i, B_i \) and \( C_i \) - are relatively the matrices of parameter s with appropriate i-th linguistic rules (A - \( n \times n \), B - \( n \times m \) , C - \( r \times n \) dimensional matrix); \( i \) – is serial number of the linguistic rules.

State vectors and output variables of the uncertainty and nonlinear object, the model of (2.5) can be described by differential equation as follows with sufficient precision:

\[
\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x) (A_i x(t) + B_i u(t)), \ y(t) = \sum_{i=1}^{q} \mu_i(x) C_i x(t).
\]  
(2.6)

Here, \( \mu_i(x) \) - functions defined as follows 1-3:

\[
\mu_i(x) = \frac{\omega_i(x)}{\sum_{i=1}^{q} \omega_i(x)} = \frac{\omega_i(x)}{\sum_{i=1}^{q} \mu_i^j}
\]  
(2.7)

Another key feature of the controlling object (ex, OD mobile robot) is that the system must provide a tracking of state variables (speed) and output (astatic due to error) variables. To meet this requirement, the r structure of control-regulator will be formed as follows:

\[
R_{ct}: \text{If } x_1(t) \text{ is } M^1_1 \text{ AND } x_2(t) \text{ is } M^1_2 \text{ AND } \ldots \text{ AND } x_n(t) \text{ is } M^1_n, \text{ Then } u_{ct} = K_1 x(t) + G e(t), \ i = 1, q
\]  
(2.8)

where \( e(t) \) is deviation (error) of the output vector of the object from \( y_{task}(t) \) task values of y(t) variable vectors:

\[
\dot{e}(t) = y_{task}(t) - \sum_{i=1}^{n} \mu_i(x) C_i x(t)
\]  
(2.9)
According to equations (2.6), (2.8) and (2.9) the mathematical model of closed-loop control system can be represented as follows:

\[
x(t) = \sum_{i=1}^{q} \mu_i(x) A_i x(t) + \sum_{i=1}^{q} \mu_i(x) B_i \left( \sum_{j=1}^{q} \mu_j(x)(K_j x(t) + G_j e(t)) \right) \tag{2.10}
\]

Taking into account (2.9) and carrying out a few simple changes in (2.10) a mathematical model of closed-loop fuzzy control system can be written as follows:

\[
B(t) = \sum_{i=1}^{q} \mu_i \sum_{j=1}^{q} \mu_j \tilde{A}_{ij} (p) \tilde{x}(t) + N y_{ex} (t), \quad y(t) = \sum_{i=1}^{q} \mu_i \tilde{C}_i \tilde{x}(t) \tag{2.11}
\]

Here

\[
\tilde{A}_{ij}(p) = \begin{bmatrix} A_i(p) + B_i(p)K_i & B_i(p)G_j \\ -C_i & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 \\ -E \end{bmatrix}, \quad \tilde{C}_i = [C_i, 0], \quad \tilde{x}(t) = \begin{bmatrix} x \\ e \end{bmatrix}, \quad i, j = 1, q
\]

\[
A_i(p) = A_i + \Delta A_i, \quad B_i(p) = B_i + \Delta B_i, \quad 0 \leq \Delta A_i \ll A_{i,\text{nom}}, \quad 0 \leq \Delta B_i \ll B_{i,\text{nom}} \tag{2.12}
\]

\[
x(t), \quad 2n_x, \quad \text{dimension,} \quad y_{ex}(t) - n_y = n_i, \quad \text{dimension vectors, respectively, and; E - the single matrix with} \quad n_x \times n_y, \quad \text{dimension; 0 - zero matrix with} \quad n_x \times n_i, \quad \text{dimension. Taking into account the expressions (2.5), (2.8) and (2.9).}
\]

\[
\Delta A_i \text{ and } \Delta B_i \text{ characterize taking into account parametric uncertainty of the object. In some cases, one can take the values of parametric variations equal to 0.2-2.0% of nominal values.}
\]

**Statement of the synthesis problem.** It is required that the uncertainty multiply control system (2.6)-(2.9) should have astatic features with respect of controlling variables and should meet the requirements of robustness due to stability degree with respect of state (intermediate) variables. Therefore the problem of controller synthesis can be summarized as follows. For an uncertainty object, which could be expressed by (2.6), it is necessary to synthesize such TS robust fuzzy controller that multiply and multidimensional closed-loop system should follow problem effects of output coordinates with high accuracy, otherwise ought to meet possible maximum stability degree due to state coordinates under condition of astatic character

\[
\lim_{t \to \infty} e(t) = 0 \tag{2.13}
\]

and parametric uncertainty (2.12) due to error.

\[
J_1 = \max_{K,G} (-Re(\tilde{A}_i(p), K_i, G_i)) = \delta_i + \max_{K,G} (-Re(\tilde{A}_i(p), K_i, G_i)) \tag{2.14a}
\]

\[
J_2 = \min_{K,G} \|e(A_i, K_i, G_i)\|, \tag{2.14b}
\]

\[
J_3 = \max_{K,G} \left\{ l_i \frac{-Re(\tilde{A}_i(p), K_i, G_i))}{\|l_i(\tilde{A}_i(p), K_i, G_i))\|} \right\}, \tag{2.14c}
\]

\[
K_i \in K, \quad G_i \in G, \quad i = 1, q
\]

Here \( \delta_i > 0, \forall i \in 1, q \) describes the value of the possible maximum value of stability degree of the system in each case, that is in accordance with linguistic rules.

Considering the control system, it is possible to make up equation characteristic with linguistic rules,
linguistic rules can be written by the following equations (for simplicity, the number of indices of linguistic rules 
object-an OD mobile robot 5. The mobile robot (OD) has four wheels, three state
(3.5) of the system it is shown that the coordinate
and to determine analytically $K_i$ and $G_i, i = \overline{1,q}$ parameters (matrix of tuning coefficients) of robust TS fuzzy
controller on the bases of conditions meeting the sustainability of the movement by the method reported in

3. The decision of synthesis problem of robust control

A solution method of the synthesis problem of fuzzy TS controller is shown by example of the concrete control
object-an OD mobile robot3. The mobile robot (OD) has four wheels, three state
variable control actions $u(t) = B\tilde{u}(t), \tilde{u}(t) = [\tilde{u}_1(t) \tilde{u}_2(t) \tilde{u}_3(t)]$ and is expressed in the form of a nonlinear
model. After carrying out simple transformations, the mobile robot can be written by uncertain nonlinear model:

$$R_1: If \quad \phi_i(t) \quad is \quad M^i \quad Then, \quad \dot{x} = A_i x(t) + B_i \tilde{u}(t), \quad y(t) = C_i x(t), \quad i = \overline{1,q}$$

$$A_1 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} a_1 & -a_2 & 0 \\ a_2d & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} a_1 & a_2 & 0 \\ -a_2d & a_1 & 0 \\ 0 & 0 & a_3 \end{bmatrix}, \quad B_i = I, C_i = I$$

Here $I$ is a three-dimensional unit matrix, $a_1, a_2, a_3$ - determined coefficients depending on the physical and
geometrical parameters of a mobile robot:

$$a_1 = -2J/(mr^2 + 2l_w), \quad a_2 = 2l_w/(mr^2 + 2l_w), \quad a_3 = -4cL^2/(4l_wml^2 + l_vr^2)$$

Parametric uncertainty is mainly associated with $a_2\phi_\omega(t) = a_2d$ the model in (3.1), i.e.,

$$a_2 = a_2 \phi_{nom}(t) + \Delta a_2 = a_2 d + \Delta a_2, \quad \Delta a_2 \in [0, \sigma], \sigma = 0.01 a_2 \phi_{nom}(t)$$

(3.3)

Considering the structure of the object (3.1) ($x_i$ (independent of state variables $x_1, x_2$ and (2.8) the structure of
the control (regulation) for both of linguistic rules can be determined as follows:

$$\tilde{u}_1(t) = k_{11}\dot{x}_1(t) + k_{12}\dot{x}_2(t) + g_{11}(y_{1tap} - c_{11}x_1)$$

$$\tilde{u}_2(t) = k_{21}\dot{x}_1(t) + k_{22}\dot{x}_2(t) + g_{22}(y_{1tap} - c_{22}x_2)$$

$$\tilde{u}_3(t) = k_{33}\dot{x}_3(t) + g_{33}(y_{1tap} - c_{33}x_3). \quad i = \overline{1,q}$$

Then taking into account the mentioned features and (3.1) - (3.4) the free movement of the system for each
linguistic rules can be written by the following equations (for simplicity, the number of indices of linguistic rules
$i = \overline{1,q}$ are not specified):

$$\dot{x}_1(t) = (a_1 - k_{11})\dot{x}_1 - (a_2d + \Delta a_2 + k_{12})\dot{x}_2 + g_{11}x_1$$

$$\dot{x}_2(t) = (a_2d + \Delta a_2 - k_{21})\dot{x}_1 + (a_1 - k_{22})\dot{x}_2 + g_{22}x_2$$

$$\dot{x}_3(t) = -k_{33}\dot{x}_3 + g_{33}x_3$$

(3.5)

It can be assumed that $|k_{12}| = |k_{21}|$ and $|g_{11}| = |g_{22}|, c_{11} = c_{22} = c_{33} = 1 \cdot$ In accordance with the model
(3.5) of the system it is shown that the coordinate $x_3$ does not directly depend on the other coordinates and in this
case, one of the characteristic equation of the system is represented by the equations of the fourth order and the other one by the second order:

\[ s^4 + s^3[k_{11i} + k_{22i} - 2a_1] + s^2[(k_{11i} - a_i)(k_{22i} - a_i) - 2g_{11i} + (a_2d + \Delta a_{2i})^2 - k_{12i}^2] + \]
\[ + s[2a_ig_{11i} - (k_{11i} + k_{22i})] + g_{1i}^2 = 0, \]
\[ s^2 + sk_{33i} - g_{33i} = 0, \]
\[ i = \overline{1, q} \] (3.6)

In the system in which the characteristic equation was written by (3.6) model, the expressions depending on the distribution of the roots of the parameters are determined by the proposed analytical synthesis method. For example, it is advisable to consider the analytic solution of the problem in two options (negative real and complex roots). As in1,2, analytical expressions are determined in the condition of equating characteristic parametric equation \( \Phi(\lambda) = 0 \) and characteristic equation \( \mathcal{C}(\lambda) = 0 \) is expressed with the roots of its derivatives \( \Phi^\prime(\lambda) \) and \( \Phi^\prime\prime(\lambda) \) derivatives for parameters \( K^i \) and \( G^i \) of controller due to state variables and error.

In the first option, when the roots of the characteristic equation are negative and equal, the expressions depending on the stability degree of tuning parameters of the controller and parameters of the object are described as follows:

\[ J^i + \delta = -\frac{k_{11i}a_1}{2}; \]
\[ g_{11}^i = (-k_{11i}^i - a_1^i)^2/4 \]
\[ k_{11}^i = k_{22}^i = -2(J^i + \delta^i) - a^i \quad g_{22}^i = (-k_{22}^i - a_1^i)^2/4 \]
\[ k_{11}^i = k_{21}^i = 0 \quad g_{33}^i = (-k_{33}^i + a_3^i)^2/4 \] (3.7)

In the second option, when roots of the characteristic equation of the system are complex, is adopted as follows:

\[ \lambda_{r=1}^i = -(J^i + \delta_r) \pm j(J^i + \delta_r)\frac{1}{m_{ri}}, \quad i = \overline{1, q} \quad r=1. \]

After certain operations, we determine the following analytical expressions:

\[ k_{11}^i = -2(J^i + \delta^i) - a_1^i \]
\[ g_{11}^i = (-k_{11}^i + a_1^i)^2(1 + \frac{1}{m_{11}^i})/4 \]
\[ k_{22}^i = -2(J^i + \delta^i) - a_1^i \]
\[ g_{22}^i = (-k_{22}^i + a_1^i)^2(1 + \frac{1}{m_{22}^i})/4 \] (3.8)
\[ k_{33}^i = -2(J^i + \delta^i) - a_1^i \]
\[ g_{33}^i = (-k_{33}^i + a_3^i)^2(1 + \frac{1}{m_{33}^i})/4 \]

Due to the state variables, in determining of the regulatory parameters (i.e., in the determination of \( K^i \) and \( G^i \) matrices) the use of expression (3.7) or (3.8) is defined depending on the requirements of the stability degree and the mathematical models of the relevant linguistic rules of the object.

4. Experimental realization

Let’s consider an example that experimentally demonstrates the effectiveness of synthesis method of robust control for nonlinear and uncertainty object. Assume that the physical and geometrical parameters of the control object (mobile robot) is described below: \( m = 16.9 \text{ kg} \), \( L = 0.193 \text{ m} \), \( r = 0.04 \text{ m} \),

\[ l_v = 0.2518 \text{ km}^2, \quad l_w = 1.1 \cdot 10^{-4} \text{ km}^2, \quad J = 8.1633 \cdot 10^{-4} \text{ km}^2/\text{s} \] (4.1)
According to these values of parameters of (3.1) object-mobile robot are determined on the basis of (3.2):
\[ a_1 = -0.0599, \ a_2 = 0.0081, \ a_3 = -0.2901, \ \Delta a_2 \in [0, \sigma], \ \sigma \equiv 0.01 \ a_2 \phi_{nom}(t) = 0.00012. \]

On the basis of the proposed method, if the value of a special number (frequency) of synthesized control system is adopted as \( \omega \equiv 15 \) \( rad/s \) \( \mu m = 0.5 \), then for the first linguistic rules the parameters of TS fuzzy controller will by the following:
\[ k_{11}^1 = -21.9, \ k_{22}^1 = k_{11}^1 = -21.9, \ k_{13}^1 = -20.1, \ k_{12}^1 = k_2^1 = 0, \ f_1^1 + \delta_1 \equiv 11, \ g_{11}^2 = g_{11}^1 = 239, \ g_{13}^3 = 201, \]
For the second linguistic rules:
\[ k_{11}^2 = -21.9, \ k_{22}^2 = k_{11}^2 = -27.9, \ k_{13}^2 = -28.6, \ k_{21}^2 = -k_{12}^2 = -20.6, \]
\[ g_{22}^2 = g_{11}^2 = g_{33}^2 = 196, \]  
(4.2)

For the third linguistic rules matrix K and G generally correspond to the second rule. Only \( k_{21}^3 \) is equal to 20.6. Thus, the mathematical model can be written as (2.8), i.e,
\[ x_{i} (t) = M_{i}^{1}. \]
Then, \( u^i = K^i x(t) + G^i e(t), \ i=1, q \)
(4.3)

According to status and error the following values for matrices of tuning parameters of TS fuzzy controllers are defined as follows:
\[ K^1 = \begin{bmatrix} -21.9 & 0 & 0 \\ 0 & -21.9 & 0 \\ 0 & 0 & -20.1 \end{bmatrix}, \ K^2 = \begin{bmatrix} -27.9 & 20.6 & 0 \\ -20.6 & -27.9 & 0 \\ 0 & 0 & -28.6 \end{bmatrix}, \]
\[ K^3 = \begin{bmatrix} -27.9 & -20.6 & 0 \\ -20.6 & -27.9 & 0 \\ 0 & 0 & -28.6 \end{bmatrix}, \ G^1 = \begin{bmatrix} 239 & 0 & 0 \\ 0 & 239 & 0 \\ 0 & 0 & 201 \end{bmatrix}, \ G^2 = G^3 = \begin{bmatrix} 196 & 0 & 0 \\ 0 & 196 & 0 \\ 0 & 0 & 196 \end{bmatrix} \]  
(4.4)

\( M_{i}^{1} \) fuzzy terms, the universe for \( \phi_{e} (t) \) is \([-d, d] = [-1.5, 1.5] \).

It should be noted that for the global stability of the above synthesized TS type fuzzy control system requires a positive matrix P satisfying LMI:
\[ A_{ij} P + P A_{ij}^T < 0, \quad A_{ij} = \begin{bmatrix} A_i + B_i K_i & B_i G_i \\ -C_i & 0 \end{bmatrix}, \quad C_1 = C_2 = C_3 = 1 \]

The simulation results of the synthesized control systems, i.e. transition processes are shown in Figure 3. When OD mobile robot in MATLAB computer simulation (based on Fuzzy Logic Toolbox and Simulink package) as defined effects are used \( x_1 = -2 \cos \alpha t, \ x_2 = 2 \sin \alpha t, \ x_3 = \exp(-10-t)+0.3t \) as well as the initial state values \((x_1(0), x_2(0), x_3(0)) = (3,3,1)\). Control of the mobile robot provides stability, at the same time, high-quality performance. For example, the regulation period is less than 0.3 seconds.

5. Conclusion

Proposed analytical method for parametric synthesis of TS fuzzy controller, for example, a mobile robot provides robustness, high accuracy and dynamic performance at the expense of state coordinates and output variable.

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