SCIENCE DIRECT

PHYSICS LETTERS B

Physics Letters B 593 (2004) 165–174

www.elsevier.com/locate/physletb

Seesaw enhancement of bi-large mixing in two-zero textures

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Received 10 January 2004; received in revised form 29 February 2004; accepted 25 March 2004

Available online 28 May 2004

Editor: T. Yanagida

Abstract

The seesaw enhancement of the bi-large mixings are discussed for the two-zero textures of the neutrino mass matrix. There are no large mixings in both Dirac neutrino mass matrix m_D and right-handed Majorana neutrino mass matrix M_R, however, the bi-large mixing is realized via the seesaw mechanism. We present twelve sets of m_D and M_R for the seesaw enhancement and discuss the related phenomena, the $\mu \to e + \gamma$ process and the leptogenesis. The decay rate of $\mu \to e + \gamma$ is enough suppressed due to zeros in the Dirac neutrino mass matrix. Six m_D lead to the lepton asymmetry, which can explain the baryon number in the universe. Other six m_D are the real matrices, which give no CP asymmetry. Modified Dirac neutrino mass matrices are also discussed.

The texture with zeros of the neutrino mass matrix have been discussed [1–4] to explain neutrino masses and mixings [5], which have been presented by the recent neutrino experiments [6–9]. It was found that the two-zero textures are consistent with the experimental data in the basis of the diagonal charged lepton mass matrix [10]. Consequently, the neutrino mass matrix does not display the hierarchical structure as seen in the quark mass matrix [11–17].

Since the two-zero textures of Ref. [10] are given for the light effective neutrino mass matrix M_ν, one needs to find the seesaw realization [18] of these textures from the standpoint of the model building. We have examined the seesaw realization of the neutrino mass matrix with two zeros [19]. Without fine tunings between parameters of the Dirac neutrino mass matrix m_D and the right-handed Majorana neutrino one M_R, we obtained several textures of m_D for the fixed M_R [19]. Among them, there are textures of m_D and M_R which have hierarchical masses without large mixings. These present the seesaw enhancement of mixings, because there is no large mixings in m_D and M_R, but the bi-large mixing is realized via the seesaw mechanism. The seesaw enhancement are important in the standpoint of the quark–lepton unification, in which quark masses are hierarchical and quark mixings are very small.[1]

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doi:10.1016/j.physletb.2004.03.093

The general discussions of the seesaw enhancement were given in the case of two flavors [20,21]. Specific cases were discussed in the case of three flavors [22,23] because it is very difficult to get general conditions for the seesaw enhancement of the bi-large mixing.

However, the two-zero texture of the neutrino mass matrix M_ν are helpful to study the seesaw enhancement of the bi-large mixing. In this Letter, we present sets of m_D and M_R to give the seesaw enhancement in the two-zero textures of M_ν and discuss the related phenomena, the $\mu \to e + \gamma$ process and the leptogenesis [24].

There are fifteen two-zero textures for the neutrino mass matrix M_ν, which have five independent parameters. Among these textures, seven acceptable textures with two independent zeros were found for the neutrino mass matrix [10], and they have been studied in detail [12,13,16,17]. Especially, the textures A_1 and A_2 of Ref. [10], which correspond to the hierarchical neutrino mass spectrum, are strongly favored by the recent phenomenological analyses [11,12,17]. Therefore, the two textures are taken in order to discuss the seesaw enhancement.

Putting data of neutrino masses and mixings [25],

$$\begin{aligned} &0.35 \leqslant \tan^2\theta_{\rm sun} \leqslant 0.54, && 6.1\times10^{-5} \leqslant \Delta m^2_{\rm sun} \leqslant 8.3\times10^{-5}\ {\rm eV}^2, \quad 90\%\ {\rm C.L.},\\ &0.90 \leqslant \sin^2 2\theta_{\rm atm}, && 1.3\times10^{-3} \leqslant \Delta m^2_{\rm atm} \leqslant 3.0\times10^{-3}\ {\rm eV}^2, \quad 90\%\ {\rm C.L.}, \end{aligned} \tag{1}$$

the relative magnitude of each entry of the neutrino mass matrix is roughly given for the textures A_1 and A_2 as follows:

$$M_\nu \simeq m_0 \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \quad \text{for } A_1, \qquad m_0 \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{for } A_2, \tag{2}$$

where m_0 denotes a constant mass and $\lambda \simeq 0.2$. These matrix is given in terms of the Dirac neutrino mass matrix m_D and the right-handed Majorana neutrino mass matrix M_R by the seesaw mechanism as

$$M_\nu = m_D M_R^{-1} m_D^{\rm T}. \tag{3}$$

Zeros in m_D and M_R provide zeros in the neutrino mass matrix M_ν of Eq. (2) as far as we exclude the possibility that zeros are originated from accidental cancellations among matrix elements. In other words, we take a standpoint that the two-zero texture should come from zeros of the Dirac neutrino mass matrix and the right-handed Majorana mass matrix. Possible textures of m_D and M_R were given in Ref. [19]. Among them, we select the set of m_D and M_R, which reproduce the seesaw enhancement of the bi-large mixing.

Let us fix the right-handed Majorana neutrino mass matrix without large mixings. We take simple right-handed Majorana neutrino mass matrix with only three independent parameters. Then, there are ${}_6{\rm C}_3 = 20$ textures. Among them, six textures are excluded because they have a zero eigenvalue, which corresponds to a massless right-handed Majorana neutrino. Other two textures are also excluded because the two-zero textures A_1 and A_2 cannot be reproduced without accidental cancellations. One of the two textures is the diagonal matrix, and another one is the matrix with three zeros in the diagonal elements.

We show twelve real mass matrices with three independent parameters[2] with mass eigenvalues $|M_1| = \lambda^m M_3$ and $|M_2| = \lambda^n M_3$, where M_3 is the mass of the third generation, and m and n are integers with $m > n > 1$:

$$a_i \text{ type} \quad M_R \simeq M_3 \begin{pmatrix} -1 & 0 & \lambda^{\frac{m}{2}} \\ 0 & \lambda^n & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \end{pmatrix}_{a_1}, \quad M_3 \begin{pmatrix} 0 & -\lambda^{\frac{n}{2}} & \lambda^{\frac{m+n}{2}} \\ -\lambda^{\frac{n}{2}} & 1 & 0 \\ \lambda^{\frac{m+n}{2}} & 0 & 0 \end{pmatrix}_{a_2},$$

[1] Although phenomenological analyses of the two-zero textures were given in the diagonal basis of the charged lepton, some authors [15–17] have also studied the two-zero textures of neutrinos in the basis of charged lepton mass matrix with small off-diagonal components.

[2] The classification of M_R, types a_i, b_i, c_i ($i = 1, 2, 3, 4$), follows from Ref. [19].

$$M_3\begin{pmatrix}0 & 0 & \lambda^{\frac{m}{2}}\\ 0 & \lambda^n & 0\\ \lambda^{\frac{m}{2}} & 0 & -1\end{pmatrix}_{a_3}, \quad M_3\begin{pmatrix}0 & 0 & \lambda^{\frac{m+n}{2}}\\ 0 & 1 & -\lambda^{\frac{n}{2}}\\ \lambda^{\frac{m+n}{2}} & -\lambda^{\frac{n}{2}} & 0\end{pmatrix}_{a_4}; \tag{4}$$

$$b_i \text{ type} \quad M_R \simeq M_3\begin{pmatrix}-\lambda^n & \lambda^{\frac{m+n}{2}} & 0\\ \lambda^{\frac{m+n}{2}} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}_{b_1}, \quad M_3\begin{pmatrix}0 & \lambda^{\frac{m+n}{2}} & -\lambda^{\frac{n}{2}}\\ \lambda^{\frac{m+n}{2}} & 0 & 0\\ -\lambda^{\frac{n}{2}} & 0 & 1\end{pmatrix}_{b_2},$$

$$M_3\begin{pmatrix}0 & \lambda^{\frac{m+n}{2}} & 0\\ \lambda^{\frac{m+n}{2}} & -\lambda^n & 0\\ 0 & 0 & 1\end{pmatrix}_{b_3}, \quad M_3\begin{pmatrix}0 & \lambda^{\frac{m+n}{2}} & 0\\ \lambda^{\frac{m+n}{2}} & 0 & -\lambda^{\frac{n}{2}}\\ 0 & -\lambda^{\frac{n}{2}} & 1\end{pmatrix}_{b_4}; \tag{5}$$

$$c_i \text{ type} \quad M_R \simeq M_3\begin{pmatrix}\lambda^m & 0 & 0\\ 0 & -1 & \lambda^{\frac{n}{2}}\\ 0 & \lambda^{\frac{n}{2}} & 0\end{pmatrix}_{c_1}, \quad M_3\begin{pmatrix}1 & -\lambda^{\frac{n}{2}} & 0\\ -\lambda^{\frac{n}{2}} & 0 & \lambda^{\frac{m+n}{2}}\\ 0 & \lambda^{\frac{m+n}{2}} & 0\end{pmatrix}_{c_2},$$

$$M_3\begin{pmatrix}\lambda^m & 0 & 0\\ 0 & 0 & \lambda^{\frac{n}{2}}\\ 0 & \lambda^{\frac{n}{2}} & -1\end{pmatrix}_{c_3}, \quad M_3\begin{pmatrix}1 & 0 & -\lambda^{\frac{n}{2}}\\ 0 & 0 & \lambda^{\frac{m+n}{2}}\\ -\lambda^{\frac{n}{2}} & \lambda^{\frac{m+n}{2}} & 0\end{pmatrix}_{c_4}, \tag{6}$$

where there are no large mixings in twelve matrices since the mass eigenvalues are supposed to be hierarchical. The minus signs in the matrix elements are taken to reproduce signs in the texture A_1 and A_2 of Eq. (2).

There are several Dirac neutrino mass matrices to give the textures A_1 and A_2 in Eq. (2) [19]. We show Dirac neutrino mass matrices $(m_D)_{a_i}$, $(m_D)_{b_i}$, $(m_D)_{c_i}$ $(i = 1 \sim 4)$ with maximal number of zeros, which have no large mixings, to give the texture A_2.[3] For each matrix of $(M_R)_{a_i}$, $(M_R)_{b_i}$, $(M_R)_{c_i}$ those are given as follows:

$$a_i \text{ type} \quad m_D \simeq m_{D0}\begin{pmatrix}\lambda & 0 & 0\\ 0 & 0 & \lambda^{\frac{m}{2}}\\ 1 & \lambda^{\frac{n}{2}} & 0\end{pmatrix}_{a_1}, \quad m_{D0}\begin{pmatrix}\lambda^{\frac{n}{2}+1} & 0 & 0\\ 0 & 0 & \lambda^{\frac{m}{2}}\\ 0 & 1 & 0\end{pmatrix}_{a_2},$$

$$m_{D0}\begin{pmatrix}0 & 0 & \lambda\\ \lambda^{\frac{m}{2}} & 0 & 0\\ 0 & \lambda^{\frac{n}{2}} & 1\end{pmatrix}_{a_3}, \quad m_{D0}\begin{pmatrix}0 & 0 & \lambda^{\frac{n}{2}+1}\\ \lambda^{\frac{m}{2}} & 0 & 0\\ 0 & 1 & 0\end{pmatrix}_{a_4}; \tag{7}$$

$$b_i \text{ type} \quad m_D \simeq m_{D0}\begin{pmatrix}\lambda^{\frac{n}{2}+1} & 0 & 0\\ 0 & \lambda^{\frac{m}{2}} & 0\\ \lambda^{\frac{n}{2}} & 0 & 1\end{pmatrix}_{b_1}, \quad m_{D0}\begin{pmatrix}\lambda^{\frac{n}{2}+1} & 0 & 0\\ 0 & \lambda^{\frac{m}{2}} & 0\\ 0 & 0 & 1\end{pmatrix}_{b_2},$$

$$m_{D0}\begin{pmatrix}0 & \lambda^{\frac{n}{2}+1} & 0\\ \lambda^{\frac{m}{2}} & 0 & 0\\ 0 & \lambda^{\frac{n}{2}} & 1\end{pmatrix}_{b_3}, \quad m_{D0}\begin{pmatrix}0 & \lambda^{\frac{n}{2}+1} & 0\\ \lambda^{\frac{m}{2}} & 0 & 0\\ 0 & 0 & 1\end{pmatrix}_{b_4}; \tag{8}$$

$$c_i \text{ type} \quad m_D \simeq m_{D0}\begin{pmatrix}0 & \lambda & 0\\ 0 & 0 & \lambda^{\frac{n}{2}}\\ \lambda^{\frac{m}{2}} & 1 & 0\end{pmatrix}_{c_1}, \quad m_{D0}\begin{pmatrix}0 & \lambda^{\frac{n}{2}+1} & 0\\ 0 & 0 & \lambda^{\frac{m}{2}}\\ 1 & 0 & 0\end{pmatrix}_{c_2},$$

$$m_{D0}\begin{pmatrix}0 & 0 & \lambda\\ 0 & \lambda^{\frac{n}{2}} & 0\\ \lambda^{\frac{m}{2}} & 0 & 1\end{pmatrix}_{c_3}, \quad m_{D0}\begin{pmatrix}0 & 0 & \lambda^{\frac{n}{2}+1}\\ 0 & \lambda^{\frac{m}{2}} & 0\\ 1 & 0 & 0\end{pmatrix}_{c_4}, \tag{9}$$

[3] For the texture A_1, we easily obtain the Dirac neutrino mass matrices by exchanging the second and third rows.

where m_{D0} denotes the magnitude of the Dirac neutrino mass and complex coefficients of order one are omitted. Although these matrices have no large mixing among three families, the neutrino mass matrix M_ν has the bi-large mixing through the seesaw mechanism. These are so-called seesaw enhancement of the bi-large mixing.

These Dirac matrices are ones with maximal number of zeros. Without changing the mixings and the mass eigenvalues in the leading order, some zeros can be replaced with small *non-zero* entries as follows:

$$
\begin{aligned}
&a_i \text{ type}\quad m_D \simeq m_{\mathrm{D0}} \begin{pmatrix} \lambda & 0 & 0 \\ \lambda^x & \lambda^y & \lambda^{\frac{m}{2}} \\ 1 & \lambda^{\frac{n}{2}} & 0 \end{pmatrix}_{a_1}, \begin{cases} x > 0, \\ y > n/2, \end{cases} \quad m_{\mathrm{D0}} \begin{pmatrix} \lambda^{\frac{n}{2}+1} & 0 & 0 \\ \lambda^x & \lambda^z & \lambda^{\frac{m}{2}} \\ \lambda^y & 1 & 0 \end{pmatrix}_{a_2}, \begin{cases} x > n/2, \\ y > n/2, \\ z > 0, \end{cases} \\
&\qquad m_{\mathrm{D0}} \begin{pmatrix} 0 & 0 & \lambda \\ \lambda^{\frac{m}{2}} & \lambda^x & \lambda^y \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{a_3}, \begin{cases} x > n/2, \\ y > 0, \end{cases} \quad m_{\mathrm{D0}} \begin{pmatrix} 0 & 0 & \lambda^{\frac{n}{2}+1} \\ \lambda^{\frac{m}{2}} & \lambda^z & \lambda^x \\ 0 & 1 & \lambda^y \end{pmatrix}_{a_4}, \begin{cases} x > n/2, \\ y > n/2, \\ z > 0; \end{cases}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&b_i \text{ type}\quad m_D \simeq m_{\mathrm{D0}} \begin{pmatrix} \lambda^{\frac{n}{2}+1} & 0 & 0 \\ \lambda^x & \lambda^{\frac{m}{2}} & \lambda^y \\ \lambda^{\frac{n}{2}} & 0 & 1 \end{pmatrix}_{b_1}, \begin{cases} x > n/2, \\ y > 0, \end{cases} \quad m_{\mathrm{D0}} \begin{pmatrix} \lambda^{\frac{n}{2}+1} & 0 & 0 \\ \lambda^x & \lambda^{\frac{m}{2}} & \lambda^z \\ \lambda^y & 0 & 1 \end{pmatrix}_{b_2}, \begin{cases} x > n/2, \\ y > n/2, \\ z > 0, \end{cases} \\
&\qquad m_{\mathrm{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & \lambda^x & \lambda^y \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{b_3}, \begin{cases} x > n/2, \\ y > 0, \end{cases} \quad m_{\mathrm{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & \lambda^x & \lambda^z \\ 0 & \lambda^y & 1 \end{pmatrix}_{b_4}, \begin{cases} x > n/2, \\ y > n/2, \\ z > 0; \end{cases}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
&c_i \text{ type}\quad m_D \simeq m_{\mathrm{D0}} \begin{pmatrix} 0 & \lambda & 0 \\ \lambda^x & \lambda^y & \lambda^{\frac{n}{2}} \\ \lambda^{\frac{m}{2}} & 1 & 0 \end{pmatrix}_{c_1}, \begin{cases} x > m/2, \\ y > 0, \end{cases} \quad m_{\mathrm{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^x & \lambda^y & \lambda^{\frac{m}{2}} \\ 1 & \lambda^z & 0 \end{pmatrix}_{c_2}, \begin{cases} x > 0, \\ y > n/2, \\ z > n/2, \end{cases} \\
&\qquad m_{\mathrm{D0}} \begin{pmatrix} 0 & 0 & \lambda \\ \lambda^x & \lambda^{\frac{n}{2}} & \lambda^y \\ \lambda^{\frac{m}{2}} & 0 & 1 \end{pmatrix}_{c_3}, \begin{cases} x > m/2, \\ y > 0, \end{cases} \quad m_{\mathrm{D0}} \begin{pmatrix} 0 & 0 & \lambda^{\frac{n}{2}+1} \\ \lambda^x & \lambda^{\frac{m}{2}} & \lambda^y \\ 1 & 0 & \lambda^z \end{pmatrix}_{c_4}, \begin{cases} x > 0, \\ y > n/2, \\ z > n/2, \end{cases}
\end{aligned} \tag{12}
$$

where x, y and z are positive integers. These Dirac neutrino mass matrices are asymmetric ones. However, only the b_3 and b_4 textures in Eq. (11) are adapted to the symmetric texture in the $SO(10)$-like GUT if y, m and n are relevantly chosen [14,15]. For example, in the b_3 case, taking $y = n/2$ and $m = n + 2$, the symmetric mass matrix is given, especially, putting $n = 8$, we have the hierarchical mass matrix such like the up-quark mass matrix.

Let us discuss these obtained textures of m_D in the $\mu \to e + \gamma$ decay and the leptogenesis. It is well known that the Yukawa coupling of the neutrino contributes to the lepton flavor violation (LFV). Many authors have studied the LFV in the minimal supersymmetric standard model (MSSM) with right-handed neutrinos assuming the relevant neutrino mass matrix [26–31]. In the MSSM with soft breaking terms, there exist lepton flavor violating terms such as off-diagonal elements of slepton mass matrices and trilinear couplings (A-term). It is noticed that large neutrino Yukawa couplings and large lepton mixings generate the large LFV in the left-handed slepton masses. For example, the decay rate of $\mu \to e + \gamma$ can be approximated as follows:

$$
\Gamma(\mu \to e + \gamma) \simeq \frac{e^2}{16\pi} m_\mu^5 F \left| \frac{(6 + 2a_0^2) m_{\mathrm{S0}}^2}{16\pi^2} (Y_\nu Y_\nu^\dagger)_{21} \ln \frac{M_X}{M_R} \right|^2, \tag{13}
$$

where the neutrino Yukawa coupling matrix Y_ν is given as $Y_\nu = m_D/v_2$ (v_2 is a VEV of Higgs) at the right-handed mass scale M_R, and F is a function of masses and mixings for SUSY particles. In Eq. (13), we assume the universal scalar mass (m_{S0}) for all scalars and the universal A-term ($A_f = a_0 m_{\mathrm{S0}} Y_f$) at the GUT scale M_X. Therefore, the branching ratio $\mu \to e + \gamma$ depends considerably on the texture m_D [29–31].

The magnitude of $(m_D m_D^\dagger)_{21}$ is a key ingredient to predict the branching ratio of the $\mu \to e + \gamma$ process.[4] Many works have shown that this branching ratio is too large [29,30]. The conditions for $(m_D m_D^\dagger)_{21}$ were given in Ref. [32] as follows:

$$H_{21} \leqslant 10^{-2} \tan^{-1/2}\beta \left(\frac{m_{S0}}{100\ \mathrm{GeV}}\right)^2 \left(\frac{Br(\mu \to e\gamma)}{1.2 \times 10^{-11}}\right)^{-1/2},$$
$$H_{31} H_{23} \leqslant 10^{-1} \tan^{-1/2}\beta \left(\frac{m_{S0}}{100\ \mathrm{GeV}}\right)^2 \left(\frac{Br(\mu \to e\gamma)}{1.2 \times 10^{-11}}\right)^{-1/2}, \tag{14}$$

where

$$H_{ij} = \sum_k (m_D)_{ik} \left(m_D^\dagger\right)_{kj} \ln \frac{M_X}{M_{Rk}}. \tag{15}$$

These conditions give constraints for the magnitude of $(m_D m_D^\dagger)_{ij}$ and M_3. Zeros in Dirac mass matrices m_D may lead to $(m_D m_D^\dagger)_{ij} = 0$ and then it suppress the $\mu \to e + \gamma$ decay. Actually, all m_D in Eqs. (7), (8) and (9) give $(m_D m_D^\dagger)_{21} = 0$ and $(m_D m_D^\dagger)_{31} (m_D m_D^\dagger)_{23} = 0$.[5] Even if non-zero terms λ^x, λ^y, λ^z are taken as seen in Eqs. (10), (11) and (12), $(m_D m_D^\dagger)_{21}$ and $(m_D m_D^\dagger)_{31} (m_D m_D^\dagger)_{23}$ are suppressed as far as $x, y, z \gg 1$. Then, the branching ratio is safely predicted to be below the present experimental upper bound 1.2×10^{-11} [33] due to zeros.

Let us examine our textures in the leptogenesis [34–36], which is based on the Fukugita–Yanagida mechanism [24]. The CP-violating phases in the Dirac neutrino mass matrix are key ingredients for the leptogenesis while the right-handed Majorana neutrino mass matrix are taken to be real in Eqs. (4), (5) and (6). Although the non-zero entries in the Dirac neutrino mass matrix are complex, three phases are removed by the redefinition of the left-handed neutrino fields. There is no freedom of redefinition for the right-handed ones in the basis with the real M_R. We should move to the diagonal basis of the right-handed Majorana neutrino mass matrix in order to calculate the magnitude of the leptogenesis. Then, the Dirac neutrino mass matrices $\bar{m}_D$ in the new basis is given as follows:

$$\bar{m}_D = P_L m_D O_R, \tag{16}$$

where P_L is a diagonal phase matrix and O_R is the orthogonal matrix which diagonalizes M_R as $O_R^{\mathrm{T}} M_R O_R$ in Eqs. (4), (5) and (6). Since the phase matrix P_L can remove one phase in each row of m_D, three phases disappear in $\bar{m}_D$.

As a typical example, we show the case of the b_3 texture in Eq. (5). By taking three eigenvalues of M_R as follows:[6]

$$M_1 = \lambda^m M_3, \qquad M_2 = -\lambda^n M_3. \tag{17}$$

We obtain the orthogonal matrix O_R as

$$O_R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \tan^2\theta = \lambda^{m-n}. \tag{18}$$

---

[4] $m_D m_D^\dagger$ does not depend on the basis of the right-handed sector.

[5] For the texture A_1 case, the some Dirac mass matrices give non-zero $(m_D m_D^\dagger)_{21}$, which leads to the constraint for M_3.

[6] The minus sign of M_2 is necessary to reproduce M_R in Eq. (5). This minus sign is transfered to m_D by the right-handed diagonal phase matrix $diag(1, i, 1)$.

Then the Dirac mass matrices m_D of b_3 in Eq. (8) can be parameterized in the new basis as follows:

$$\bar{m}_D = m_{D0}\begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & \lambda^{\frac{n}{2}} e^{i\rho} & 1 \end{pmatrix} O_R, \tag{19}$$

where only one phase ρ remains. The magnitude of m_{D0} is determined by the relation $m_{D0}^2 \simeq m_0 M_3$, where $m_0 \simeq \sqrt{\Delta m_{\rm atm}^2}/2$.

We examine the lepton number asymmetry in the minimal SUSY model with the right-handed neutrinos. In the limit $M_1 \ll M_2, M_3$, the lepton number asymmetry ϵ_1 (CP asymmetry) for the lightest heavy Majorana neutrino (N_1) decays into $l^{\mp}\phi^{\pm}$ [37] is given by

$$\epsilon_1 = \frac{\Gamma_1 - \bar{\Gamma}_1}{\Gamma_1 + \bar{\Gamma}_1} \simeq -\frac{3}{8\pi v_2^2}\left(\frac{{\rm Im}[\{(\bar{m}_D^\dagger \bar{m}_D)_{12}\}^2]}{(\bar{m}_D^\dagger \bar{m}_D)_{11}}\frac{M_1}{M_2} + \frac{{\rm Im}[\{(\bar{m}_D^\dagger \bar{m}_D)_{13}\}^2]}{(\bar{m}_D^\dagger \bar{m}_D)_{11}}\frac{M_1}{M_3}\right), \tag{20}$$

where $v_2 = v\sin\beta$ with $v = 174$ GeV. The lepton asymmetry Y_L is related to the CP asymmetry through the relation

$$Y_L = \frac{n_L - n_{\bar{L}}}{s} = \kappa\frac{\epsilon_1}{g_*}, \tag{21}$$

where s denotes the entropy density, g_* is the effective number of relativistic degrees of freedom contributing to the entropy and κ is the so-called dilution factor which accounts for the washout processes (inverse decay and lepton number violating scattering). In the MSSM with right-handed neutrinos, one gets $g_* = 232.5$.

The produced lepton asymmetry Y_L is converted into a net baryon asymmetry Y_B through the $(B+L)$-violating sphaleron processes. One finds the relation [38]

$$Y_B = \xi Y_{B-L} = \frac{\xi}{\xi - 1}Y_L, \qquad \xi = \frac{8N_f + 4N_H}{22N_f + 13N_H}, \tag{22}$$

where N_f and N_H are the number of fermion families and Higgs doublets, respectively. Taking into account $N_f = 3$ and $N_H = 2$ in the MSSM, we get

$$Y_B = -\frac{8}{15}Y_L. \tag{23}$$

On the other hand, the low energy CP violation, which is a measurable quantity in the long baseline neutrino oscillations [39], is given by the Jarlskog determinant $J_{\rm CP}$ [40], which is calculated by

$$\det\left[M_\ell M_\ell^\dagger, M_\nu M_\nu^\dagger\right] = -2i J_{\rm CP}\left(m_\tau^2 - m_\mu^2\right)\left(m_\mu^2 - m_e^2\right)\left(m_e^2 - m_\tau^2\right)\left(m_3^2 - m_2^2\right)\left(m_2^2 - m_1^2\right)\left(m_1^2 - m_3^2\right), \tag{24}$$

where M_ℓ is the diagonal charged lepton mass matrix, and m_1, m_2, m_3 are neutrino masses.

Since the CP-violating phase is only ρ, we can find a link between the leptogenesis (ϵ_1) and the low energy CP violation ($J_{\rm CP}$) in our textures of the Dirac neutrinos. By using the Dirac neutrino mass matrix in Eq. (19), we get

$$\epsilon_1 \simeq -\frac{3m_{D0}^2}{8\pi v_2^2}\lambda^m \sin 2\rho \simeq -8.8\times 10^{-17} M_1 \sin 2\rho, \qquad J_{\rm CP} \simeq \frac{1}{64}\lambda^2\frac{\Delta m_{\rm atm}^2}{\Delta m_{\rm sol}^2}\sin 2\rho, \tag{25}$$

where M_1 is given in the GeV unit and $\tan\beta \geqslant 10$ is taken. It is remarked that ϵ_1 only depends on M_1 and the phase ρ, and the relative sign of ϵ_1 and $J_{\rm CP}$ is opposite. Taking the experimental data $\Delta m_{\rm sol}^2/\Delta m_{\rm atm}^2 \simeq \lambda^2$ and $\sin 2\rho \simeq 1$, we predict $J_{\rm CP} \simeq 0.01$, which is rather large and then is favored for the future experimental measurement.

The five cases of the Dirac neutrino mass matrix (a_1, a_3, b_1, c_1, c_3) in Eqs. (7), (8) and (9) lead to same results in Eq. (25). In other six cases of the Dirac neutrino mass matrix ($a_2, a_4, b_2, b_4, c_2, c_4$), the CP-violating phases are removed because of only three non-zero entries. Then, we get $\epsilon_1 = 0$, but the same result in Eq. (25) for $J_{\rm CP}$.

If we use the modified Dirac neutrino mass matrices in Eqs. (10), (11) and (12), new CP-violating phases appear. However, the contribution to ϵ_1 is a next-leading one as far as $x \gg 1$, $y \gg 1$, $z \gg 1$.

In order to calculate the baryon asymmetry, we need the dilution factor involves the integration of the full set of Boltzmann equations [41]. A simple approximated solution which has been frequently used is given by [42]

$$\kappa = 0.3\left(\frac{10^{-3}\ \mathrm{eV}}{\tilde{m}_1}\right)\left(\ln\frac{\tilde{m}_1}{10^{-3}\ \mathrm{eV}}\right)^{-0.6} \quad \left(10^{-2}\ \mathrm{eV} \leqslant \tilde{m}_1 \leqslant 10^3\ \mathrm{eV}\right), \tag{26}$$

where

$$\tilde{m}_1 = \frac{(\bar{m}_D^\dagger \bar{m}_D)_{11}}{M_1}. \tag{27}$$

By using this approximate dilution factor and Eqs. (21) and (22), we can estimate Y_B in our textures as follows:

$$Y_B \simeq -2.3 \times 10^{-3}\epsilon_1\kappa. \tag{28}$$

It is noticed that Y_B and J_{CP} are same sign since ϵ_1 has minus sign.

The WMAP has given the new result [43]

$$\eta_B = 6.5^{+0.4}_{-0.3} \times 10^{-10}(1\sigma), \tag{29}$$

which leads to

$$Y_B \simeq \frac{1}{7}\eta_B. \tag{30}$$

In our textures, we have $(\bar{m}_D^\dagger \bar{m}_D)_{11} = m_{\mathrm{D0}}^2\lambda^m$, which gives $\tilde{m}_1 = \frac{1}{2}\sqrt{\Delta m^2_{\mathrm{atm}}} \simeq 0.022$. Then we get the dilution factor $\kappa \simeq 7 \times 10^{-3}$. Putting the observed value into Eq. (28), we get

$$M_1 \sin 2\rho \simeq 6 \times 10^{10}\ \mathrm{GeV}. \tag{31}$$

This result means that M_1 is should be larger than 6×10^{10} GeV in order to explain the baryon number in the universe. This value is consistent with previous works [34–36].

It is important to present the discussion from the standpoint of the GUT, which is given after Eq. (11). Taking $n = 8$ and $m = 6$ in the b_3 case of Eq. (11) as in the previous discussion, one obtains $M_3 \sim 10^{15}$ GeV and $M_1 \sim 10^8$ GeV taking account of $\Delta m^2_{\mathrm{atm}} \simeq 2 \times 10^{-3}\ \mathrm{eV}^2$. This result does not satisfy the condition of Eq. (31). However, the simple $SO(10)$ fermion mass relation may be consistent with the leptogenesis in the case of the more complicated texture of M_R, which leads to the two-zero texture A_2, as seen in the work of [44]. Details are presented in the preparing paper including the degenerate case of M_R in the simple $SO(10)$ approach [45].

We add the discussion of another important problem. In the framework of supersymmetric thermal leptogenesis, there is cosmological gravitino problems. The gravitino with a few TeV mass does not favor $M_1 \geqslant 10^{10}$ GeV [46], because M_1 should be lower than the maximum reheating temperature of the universe after inflation. In order to keep the thermal leptogenesis in the SUSY model, we may consider the gravitino with $O(100)$ TeV mass, which is derived from the anomaly mediated SUSY breaking mechanism [47].

Summary is given as follows. We have discussed the textures with the seesaw enhancement. These textures are important in the standpoint of the quark–lepton unification, in which quark masses are hierarchical and quark mixings are very small. It is very difficult to get general conditions for the seesaw enhancement of the bi-large mixing, however, the two-zero texture of the left-handed neutrino mass matrix M_ν are helpful to study the seesaw enhancement of the bi-large mixing. Once the basis of the right-handed Majorana neutrino mass matrix is fixed, one can find some sets of m_D and M_R, which have hierarchical masses without large mixings, to give the two-zero textures A_1 and A_2 without fine tuning among parameters of these matrices. These sets present the seesaw enhancement of the bi-large mixing, because there is no large mixings in m_D and M_R, but bi-large mixing is realized via the seesaw mechanism. We present twelve sets of m_D and M_R for the seesaw enhancement. Then, the decay rate of $\mu \to e + \gamma$ is enough suppressed due to zeros in the Dirac neutrino mass matrix. Six sets lead to the lepton asymmetry, which depends on only M_1 and the phase ρ. Putting the observed value of baryon number in the universe, $M_1 \simeq 6 \times 10^{10}$ GeV is obtained. It is remarked that J_{CP} is the same sign as the Y_B, and its magnitude is predicted to be $\simeq 0.01$. Other six ones provide the real Dirac neutrino mass matrices, which give no CP asymmetry. Study of modified right-handed Majorana neutrino mass matrices is important for realistic model buildings based on the quark–lepton unification.

Acknowledgements

This research was supported in part by the National Science Foundation in the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan (No. 12047220).

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