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# Gauge invariant Lagrangian construction for massive higher spin fermionic fields

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**Abstract**

We formulate a general gauge invariant Lagrangian construction describing the dynamics of massive higher spin fermionic fields in arbitrary dimensions. Treating the conditions determining the irreducible representations of Poincaré group with given spin as the operator constraints in auxiliary Fock space, we built the BRST charge for the model under consideration and find the gauge invariant equations of motion in terms of vectors and operators in the Fock space. It is shown that like in massless case [I.L. Buchbinder, V.A. Krykhtin, A. Pashnev, Nucl. Phys. B 711 (2005) 367, hep-th/0410215], the massive fermionic higher spin field models are the reducible gauge theories and the order of reducibility grows with the value of spin. In compare with all previous approaches, no off-shell constraints on the fields and the gauge parameters are imposed from the very beginning, all correct constraints emerge automatically as the consequences of the equations of motion. As an example, we derive a gauge invariant Lagrangian for massive spin 3/2 field.


## 1. Introduction

Higher spin field theory is one of the actively developing trends of modern theoretical physics. The various approaches to higher spin problem and current results are widely discussed in the literature (see e.g. [1] for reviews and [2,3] for recent developments for massive and massless case, respectively). In this note we extend the new gauge invariant approach, developed in the papers [5–7] for massless higher spin fermionic fields and for massive higher spin bosonic fields (see discussion of motivations and features of this approach in [6]), to the massive higher spin fermionic field models.

The main idea of the approach under consideration is treatment of the conditions determining the irreducible representation of the Poincaré group with given spin in terms of operator constraints in some auxiliary Fock space and use of the BRST-BFV construction [8] to restore a Lagrangian of the theory on the base of given system of constraints. Such an approach was firstly realized in open string theory [9] and then applied to higher spin field theory [10]. Our aim is to extend the recent results obtained in [5,6] for massless fermionic and massive bosonic higher spin field theories respectively to massive fermionic theories.

The massive fermionic higher spin theories have the specific differences with the massive bosonic ones [6] and demand a separate consideration. In contrast to the bosonic case and similar to the massless fermionic case [5] we get the fermionic operators in the algebra of constraints and corresponding them the bosonic ghosts. Due to the presence of the bosonic ghosts the resulting theory is a gauge theory where an order of reducibility grows with the spin of the field. Except the presence of bosonic ghosts in the fermionic theory there is one more special aspect. Unlike the bosonic case, in the fermionic theory we have two Hermitian constraints thus there is a problem in constructing an appropriate scalar product. It means, we cannot obtain a consistent Lagrangian for fermions by the same method which was used in the bosonic theory since such a method automatically

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leads to second order equations of motion for fermionic fields what contradicts to the spin-statistic theorem. To overcome this problem we partially fix the gauge and partially solve some field equations removing the ghost fields corresponding to the Hermitian constraints and thus derive the correct Lagrangian.

The Letter is organized as follows. In Section 2 we introduce the operator algebra generated by primary constraints which define irreducible representation of the Poincaré group with fixed arbitrary half-integer spin. In Section 3 we formulate a new representation of the operators and a corresponding new scalar product which overcome a problem arising at naive use of BRST construction (see a discussion of this problem in [5, 6]). Then, in Section 4 we construct the BRST operator and derive the Lagrangian for a massive field with given half-integer spin, the corresponding gauge transformations and equations of motion in terms of vectors in Fock space. In Section 5 we illustrate the procedure of Lagrangian construction by finding the gauge Lagrangian for massive field with spin 3/2. Section 6 summarizes the results obtained.

## 2. Algebra of the constraints

It is well known that the totally symmetrical tensor–spinor fields $\Phi_{\mu_1\cdots\mu_n}$ (the Dirac index is suppressed), describing the irreducible massive spin $s = n + 1/2$ representations of the Poincaré group must satisfy the following conditions (see e.g. [4])

$$\left(i\gamma^\nu\partial_\nu - m\right)\Phi_{\mu_1\cdots\mu_n} = 0, \tag{1}$$

$$\gamma^\mu\Phi_{\mu\mu_2\cdots\mu_n} = 0. \tag{2}$$

Here $\gamma^\mu$ are the Dirac matrices

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}, \quad \eta_{\mu\nu} = (+, -, \ldots, -). \tag{3}$$

To avoid a manipulations with a number of indices it is convenient to introduce auxiliary Fock space generated by creation and annihilation operators $a^+_\mu$, $a_\mu$ with vector Lorentz index $\mu = 0, 1, 2, \ldots, D-1$ satisfying the commutation relations

$$\left[a_\mu, a^+_\nu\right] = -\eta_{\mu\nu}. \tag{4}$$

These operators act on vectors in the Fock space

$$|\Phi\rangle = \sum_{n=0}^{\infty}\Phi_{\mu_1\cdots\mu_n}(x)a^{+\mu_1}\cdots a^{+\mu_n}|0\rangle. \tag{5}$$

These vectors are the functionals of the higher spin fields and can be called the higher spin functionals. The conditions (1), (2) are realized as the constraints on the vectors (5) in the form

$$T'_0|\Phi\rangle = 0, \qquad T'_1|\Phi\rangle = 0, \tag{6}$$

where

$$T'_0 = \gamma^\mu p_\mu + m, \tag{7}$$

$$T'_1 = \gamma^\mu a_\mu, \tag{8}$$

with $p_\mu = -i\frac{\partial}{\partial x^\mu}$. If constraints (6) are fulfilled for the general state (5) then conditions (1), (2) are fulfilled for each component $\Phi_{\mu_1\cdots\mu_n}(x)$ in (5) and hence vectors (5) under relations (6) describe the free arbitrary higher spin fermionic fields.

Important element of general BRST construction is finding a closed algebra of all constraints on the field $|\Phi\rangle$ (5) generated by their (anti)commutators. Finding such an algebra in the case under consideration is similar to that in the massless fermionic case [5]. In the massless case the constraints which are analog of $T'_0$, $T'_1$ should be treated as fermionic. In the massive case the constraint $T'_0$ (7) looks like a sum of two terms: one is fermionic massless Dirac operator $i\gamma^\mu p_\mu$ like in the massless case and another one is bosonic mass operator $m$. This situation looks contradictory. In order to avoid the contradiction ones act as follows. We introduce another constraints $T_0$, $T_1$ on the state vector (5) instead of $T'_0$, $T'_1$

$$T_0 = \tilde{\gamma}^\mu p_\mu + \tilde{\gamma}m, \tag{9}$$

$$T_1 = \tilde{\gamma}^\mu a_\mu, \tag{10}$$

where $\tilde{\gamma}^\mu$, $\tilde{\gamma}$ is a set of $D+1$ Grassmann odd objects which obey the following gamma-matrix-like conditions

$$\left\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\right\} = 2\eta^{\mu\nu}, \qquad \left\{\tilde{\gamma}^\mu, \tilde{\gamma}\right\} = 0, \qquad \tilde{\gamma}^2 = -1. \tag{11}$$

We have no necessity to realize the above objects in explicit form since for construction of the final Lagrangians we will switch from "gamma-matrix-like objects" (11) to the "true" gamma-matrices by the relation

$$\gamma^\mu = \tilde{\gamma}^\mu\tilde{\gamma} = -\tilde{\gamma}\tilde{\gamma}^\mu. \tag{12}$$

One can check that the relations (11) lead to the relations (3) for "true" gamma-matrices (12). Then ones can show that $T_0$, $T_1$ and $T'_0$, $T'_1$ related by the following conditions

$$T_0 = \tilde{\gamma}T'_0, \qquad T_1 = \tilde{\gamma}T'_1. \tag{13}$$

Thus Eqs. (6) are equivalent to the equations

$$T_0|\Phi\rangle = 0, \qquad T_1|\Phi\rangle = 0 \tag{14}$$

and therefore we can treat just the operators $T_0$ (9) and $T_1$ (10) as primary constraints. Commutators of these constraints generate all other constraints on the ket-vector state (5). Thus we get three more constraints[1]

$$L_0|\Phi\rangle = L_1|\Phi\rangle = L_2|\Phi\rangle = 0, \tag{15}$$

where

$$L_0 = -p^2 + m^2, \qquad L_1 = a^\mu p_\mu, \qquad L_2 = \frac{1}{2}a_\mu a^\mu. \tag{16}$$

Our purpose is to construct the Lagrangian for the massive fermionic higher spin fields generalizing the method of [5–7]. This method is based on finding an Hermitian BRST operator on the base of some system of operator constraints forming a closed algebra invariant under Hermitian conjugation. Hence, in our case we should look for a set of operator constraints which is invariant under Hermitian conjugation. Let us define a scalar product in the Fock space taking bra-vector state in the form

$$\langle\tilde{\Phi}| = \sum_{n=0}^{\infty}\langle 0|a^{\mu_1}\cdots a^{\mu_n}\Phi^+_{\mu_1\cdots\mu_n}(x)\tilde{\gamma}^0 \tag{17}$$

[1] Algebra of the operators will be written down in an explicit form later. See Table 1 at page 388.

Table 1
Algebra of the operators

| | $T_0$ | $T_1$ | $T_1^+$ | $L_0$ | $L_1$ | $L_1^+$ | $L_2$ | $L_2^+$ | $G_0$ | $M_1$ | $M_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_0$ | $-2L_0$ | $2L_1$ | $2L_1^+$ | 0 | 0 | 0 | 0 | 0 | 0 | $2M_2$ | 0 |
| $T_1$ | $2L_1$ | $4L_2$ | $-2G_0$ | 0 | 0 | −(22) | 0 | $-T_1^+$ | $T_1$ | 0 | 0 |
| $T_1^+$ | $2L_1^+$ | $-2G_0$ | $4L_2^+$ | 0 | (22) | 0 | $T_1$ | 0 | $-T_1^+$ | 0 | 0 |
| $L_0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $L_1$ | 0 | 0 | −(22) | 0 | 0 | (23) | 0 | $-L_1^+$ | $L_1$ | 0 | 0 |
| $L_1^+$ | 0 | (22) | 0 | 0 | −(23) | 0 | $L_1$ | 0 | $-L_1^+$ | 0 | 0 |
| $L_2$ | 0 | 0 | $-T_1$ | 0 | 0 | $-L_1$ | 0 | $G_0$ | $2L_2$ | 0 | 0 |
| $L_2^+$ | 0 | $T_1^+$ | 0 | 0 | $L_1^+$ | 0 | $-G_0$ | 0 | $-2L_2^+$ | 0 | 0 |
| $G_0$ | 0 | $-T_1$ | $T_1^+$ | 0 | $-L_1$ | $L_1^+$ | $-2L_2$ | $2L_2^+$ | 0 | 0 | 0 |
| $M_1$ | $2M_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $2M_2$ | 0 |
| $M_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

and postulate the following Hermitian properties[2] for $\tilde{\gamma}^\mu$ and $\tilde{\gamma}$

$$(\tilde{\gamma}^\mu)^+ = \tilde{\gamma}^0\tilde{\gamma}^\mu\tilde{\gamma}^0, \qquad (\tilde{\gamma})^+ = \tilde{\gamma}^0\tilde{\gamma}\tilde{\gamma}^0 = -\tilde{\gamma}. \tag{19}$$

As a result we get two Hermitian constraints $T_0$ and $L_0$ and the three other are non-Hermitian[3]

$$T_1^+ = \tilde{\gamma}^0(T_1)^+\tilde{\gamma}^0 = \tilde{\gamma}^\mu a_\mu^+, \qquad L_1^+ = (L_1)^+ = a^{+\mu}p_\mu,$$
$$L_2^+ = (L_2)^+ = \frac{1}{2}a^{+\mu}a_\mu^+. \tag{20}$$

Therefore we extend the set of constraints adding three new operators $T_1^+$, $L_1^+$, $L_2^+$ to the initial constraints (9), (10), (16) on the ket-state vector (5). As a result a set of the operators $T_0$, $T_1$, $T_1^+$, $L_0$, $L_1$, $L_1^+$, $L_2$, $L_2^+$ is invariant under Hermitian conjugation.

Algebra of the operators $T_0$, $T_1$, $T_1^+$, $L_0$, $L_1$, $L_1^+$, $L_2$, $L_2^+$ is open in terms of (anti)commutators of these operators. According to [5–7] we must include in the set of operators all the operators which are needed for the algebra be closed. These operators are

$$G_0 = -a_\mu^+a^\mu + \frac{D}{2}, \qquad M_1 = \tilde{\gamma}m, \qquad M_2 = -m^2. \tag{21}$$

Since these operators are obtained as (anti)commutators of a constraint on the ket-vector state (5) with a constraint on bra-vector state (17) then operators (21) cannot be considered as constraints neither on the space of bra-vectors nor on the space of ket-vectors. Total algebra of the operators is written in Table 1, where

$$[L_1^+, T_1] = [T_1^+, L_1] = T_0 - M_1, \tag{22}$$
$$[L_1, L_1^+] = L_0 + M_2. \tag{23}$$

The operators $T_0$, $T_1$, $T_1^+$, $M_1$ are fermionic and the operators $L_0$, $L_1$, $L_1^+$, $L_2$, $L_2^+$, $G_0$, $M_2$ are bosonic. All the commutators are graded, i.e. graded commutators between the fermionic operators are anticommutators and graded commutators which include any bosonic operator are commutators. In Table 1 the first arguments of the graded commutators are listed in the left column and the second argument of graded commutators are listed in the upper row. We will call this algebra as free massive half-integer higher spin symmetry algebra.

Method of deriving the higher spin Lagrangians on the base of operator algebra analogous one given by Table 1 was discussed in [5] and [6]. One can show that a naive use of BRST construction leads to some contradictions (see discussions of this point in [5,6]) and is directly applicable only for spin 1/2 case. According to the method developed in [5–7] to overcome these contradictions we should pass to new representation for the operator algebra given in Table 1. This will be done in the next section.

## 3. New representation of the algebra

The approach developed in [5–7] is based on special representation of the operator algebra generated by constraints. In the case under consideration we have to find another representation of the algebra given in Table 1. This representation is constructed so that all the operators which are not constraints (21) contain an arbitrary parameter or be zero in new representation. To built the new representation we enlarge the representation space by introducing the new creation and annihilation operators: two pairs of bosonic $b_1^+$, $b_1$ and $b_2^+$, $b_2$ and one pair of fermionic $d^+$, $d$ creation and annihilation operators which satisfy the standard commutation relations

$$\{d, d^+\} = 1, \qquad [b_1, b_1^+] = [b_2, b_2^+] = 1. \tag{24}$$

Then construction of the new representation consists in extending of the operator expressions (9), (10), (16), (20), (21) with the help of the additional creation and annihilation operators so that the new expressions of the operators have the desired properties. We will use the following new representation for the algebra

$$T_{0\text{new}} = T_0 = \tilde{\gamma}^\mu p_\mu + \tilde{\gamma}m, \tag{25}$$
$$T_{1\text{new}} = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma}b_1 - d^+b_2 - 2(b_2^+b_2 + h)d, \tag{26}$$
$$T_{1\text{new}}^+ = \tilde{\gamma}^\mu a_\mu^+ - \tilde{\gamma}b_1^+ + 2b_2^+d + d^+, \tag{27}$$
$$L_{0\text{new}} = L_0 = -p^2 + m^2, \tag{28}$$
$$L_{1\text{new}} = a^\mu p_\mu + mb_1, \tag{29}$$

[2] It can be checked that relations (19) lead to usual Hermitian relations for the "true" gamma-matrices (12)

$$(\gamma^\mu)^+ = \gamma^0\gamma^\mu\gamma^0. \tag{18}$$

[3] In what follows we shall write $F^+ = (F)^+$ instead of $F^+ = \tilde{\gamma}^0(F)^+\tilde{\gamma}^0$ and omit $\tilde{\gamma}^0$ in writing Hermitian conjugation.

$$L^+_{1\text{new}} = a^{+\mu} p_\mu + m b^+_1, \tag{30}$$

$$L_{2\text{new}} = \frac{1}{2} a_\mu a^\mu - \frac{1}{2} b_1^2 + \left(d^+ d + b_2^+ b_2 + h\right) b_2, \tag{31}$$

$$L^+_{2\text{new}} = \frac{1}{2} a^+_\mu a^{\mu+} - \frac{1}{2} b_1^{+2} + b_2^+, \tag{32}$$

$$G_{0\text{new}} = -a^+_\mu a^\mu + \frac{D}{2} + d^+ d + b_1^+ b_1 + 2 b_2^+ b_2 + h + \frac{1}{2}, \tag{33}$$

$$M_{1\text{new}} = 0, \tag{34}$$

$$M_{2\text{new}} = 0. \tag{35}$$

Thus we see that the operator $G_0$ which is not a constraint contain an arbitrary parameter $h$, and the two other operators-nonconstraints $M_1$ and $M_2$ are zero. It may be shown that this new representation of the algebra can be obtained from the new representation of the corresponding massless operator algebra [5] by dimensional reduction $R^{1,D} \to R^{1,D-1}$ with the following decomposition

$$p_M = (p_\mu, m), \qquad a^M = \left(a^\mu, b_1\right),$$
$$a^{+M} = \left(a^{+\mu}, b_1^+\right), \qquad \gamma^M = \left(\tilde\gamma^\mu, \tilde\gamma\right), \tag{36}$$

$$M = 0, 1, \ldots, D, \qquad \mu = 0, 1, \ldots, D-1, \qquad \eta^{DD} = -1. \tag{37}$$

It is easy to see, the operators (26), (27) and (31), (32) are not Hermitian conjugate to each other

$$T^+_{1\text{new}} \neq (T_{1\text{new}})^+, \qquad L^+_{2\text{new}} \neq (L_{2\text{new}})^+ \tag{38}$$

if we use the usual rules for Hermitian conjugation of the additional creation and annihilation operators

$$d^+ = (d)^+, \qquad b_2^+ = (b_2)^+. \tag{39}$$

To make them conjugate each other we change the scalar product in the enlarged space $\langle \tilde\Psi_1 | \Psi_2 \rangle_{\text{new}} = \langle \tilde\Psi_1 | K_h | \Psi_2 \rangle$ with

$$K_h = \sum_{n=0}^{\infty} \frac{1}{n!} \left( |n\rangle\langle n| C(n,h) - 2 d^+ |n\rangle\langle n| d C(n+1,h) \right), \tag{40}$$

$$C(n,h) = h(h+1)\cdots(h+n-1),$$
$$C(0,h) = 1, \qquad |n\rangle = \left(b_2^+\right)^n |0\rangle. \tag{41}$$

As a result we get that the operators (26), (27) and (31), (32) be conjugate each other relatively the new scalar product since the following relation take place

$$K_h T_{1\text{new}} = \left(T^+_{1\text{new}}\right)^+ K_h, \qquad K_h T^+_{1\text{new}} = (T_{1\text{new}})^+ K_h, \tag{42}$$

$$K_h L_{2\text{new}} = \left(L^+_{2\text{new}}\right)^+ K_h, \qquad K_h L^+_{2\text{new}} = (L_{2\text{new}})^+ K_h. \tag{43}$$

Thus we have constructed the representation of the algebra given in Table 1 which possesses the properties formulated in the beginning of this section, i.e., the operators-nonconstraints (21) are zeros or contain an arbitrary parameter. Then the BRST operator will reproduce the proper equations (6) or (14) up to gauge transformation.

## 4. Lagrangian for the free massive fermionic fields

Now we construct Lagrangian for free massive higher spin fermionic fields. For this we construct BRST operator as if all the operators of the algebra are the first class constraints

$$\begin{aligned}\tilde Q = {} & q_0 T_0 + q_1^+ T_{1\text{new}} + q_1 T^+_{1\text{new}} + \eta_0 L_0 + \eta_1^+ L_{1\text{new}} \\ & + \eta_1 L^+_{1\text{new}} + \eta_2^+ L_{2\text{new}} + \eta_2 L^+_{2\text{new}} + \eta_G G_{0\text{new}} \\ & + i\left(\eta_1^+ q_1 - \eta_1 q_1^+\right) p_0 - i\left(\eta_G q_1 + \eta_2 q_1^+\right) p_1^+ \\ & + i\left(\eta_G q_1^+ + \eta_2^+ q_1\right) p_1 + \left(q_0^2 - \eta_1^+ \eta_1\right) \mathcal{P}_0 \\ & + \left(2 q_1 q_1^+ - \eta_2^+ \eta_2\right) \mathcal{P}_G + \left(\eta_G \eta_1^+ + \eta_2^+ \eta_1 - 2 q_0 q_1^+\right) \mathcal{P}_1 \\ & + \left(\eta_1 \eta_G + \eta_1^+ \eta_2 - 2 q_0 q_1\right) \mathcal{P}_1^+ + 2\left(\eta_G \eta_2^+ - q_1^{+2}\right) \mathcal{P}_2 \\ & + 2\left(\eta_2 \eta_G - q_1^2\right) \mathcal{P}_2^+ \end{aligned} \tag{44}$$

and assume that the state vectors $|\chi\rangle$ and the gauge parameters in the enlarged space (including ghosts) are independent of the ghosts corresponding the operators which are not constraints

$$\begin{aligned}|\chi\rangle = {} & \sum_{k_i} (q_0)^{k_1} \left(q_1^+\right)^{k_2} \left(p_1^+\right)^{k_3} (\eta_0)^{k_4} \left(d^+\right)^{k_5} \left(\eta_1^+\right)^{k_6} \left(\mathcal{P}_1^+\right)^{k_7} \\ & \times \left(\eta_2^+\right)^{k_8} \left(\mathcal{P}_2^+\right)^{k_9} \left(b_1^+\right)^{k_{10}} \left(b_2^+\right)^{k_{11}} \\ & \times a^{+\mu_1} \cdots a^{+\mu_{k_0}} \chi^{k_1 \cdots k_{11}}_{\mu_1 \cdots \mu_{k_0}}(x) |0\rangle. \end{aligned} \tag{45}$$

The corresponding ghost number of this vector is 0, as usual. The sum in (45) is assumed over $k_0, k_1, k_2, k_3, k_{10}, k_{11}$ running from 0 to infinity and over $k_4, k_5, k_6, k_7, k_8, k_9$ running from 0 to 1. Let us notice that the new BRST charge (44) is self-conjugate in the following sense $\tilde Q^+ K_h = K_h \tilde Q$, with operator $K_h$ (40).

Lagrangian for the massive fermionic field with spin $s = n + 1/2$ is constructed as follows (see details in [5])[4]

$$\begin{aligned}\mathcal{L}_n = {} & {}_n\langle \tilde\chi_0^0 | K_n \tilde T_0 | \chi_0^0 \rangle_n + \frac{1}{2} {}_n\langle \tilde\chi_0^1 | K_n \{\tilde T_0, \eta_1^+ \eta_1\} | \chi_0^1 \rangle_n \\ & + {}_n\langle \tilde\chi_0^0 | K_n \Delta Q_n | \chi_0^1 \rangle_n + {}_n\langle \tilde\chi_0^1 | K_n \Delta Q_n | \chi_0^0 \rangle_n, \end{aligned} \tag{46}$$

where $\tilde T_0 = T_0 - 2 q_1^+ \mathcal{P}_1 - 2 q_1 \mathcal{P}_1^+$ and $\{A, B\} = AB + BA$. In Eq. (46) $|\chi_0^0\rangle_n$, $|\chi_0^1\rangle_n$ are coefficients of $|\chi\rangle$ (45) standing at $(q_0)^0 (\eta_0)^0$ and $(q_0)^1 (\eta_0)^0$ respectively which subject to the condition

$$\sigma |\chi_0^i\rangle_n = \left(n + \frac{D-3}{2}\right) |\chi_0^i\rangle_n, \quad i = 0, 1,$$
$$gh\left(|\chi_0^i\rangle_n\right) = -i, \tag{47}$$

$$\begin{aligned}\sigma = {} & -a^+_\mu a^\mu + \frac{D+1}{2} + d^+ d + b_1^+ b_1 + 2 b_2^+ b_2 - i q_1 p_1^+ \\ & + i q_1^+ p_1 + \eta_1^+ \mathcal{P}_1 - \eta_1 \mathcal{P}_1^+ + 2 \eta_2^+ \mathcal{P}_2 - 2 \eta_2 \mathcal{P}_2^+, \end{aligned} \tag{48}$$

$\Delta Q_n$ is the part of $\tilde Q$ (44) which independent of the ghosts $\eta_G$, $\mathcal{P}_G$, $\eta_0$, $\mathcal{P}_0$, $q_0$, $p_0$ and the substitution $-h \to n + (D-3)/2$ is done, $K_n$ is operator (40) where the substitution $-h \to n + (D-3)/2$ is done.

4 The Lagrangian is defined as usual up to an overall factor.

Lagrangian (46) is invariant under the gauge transformation

$$\delta|\chi_0^0\rangle_n = \Delta Q_n|\Lambda_0^0\rangle_n + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\Lambda_0^1\rangle_n, \tag{49}$$

$$\delta|\chi_0^1\rangle_n = \tilde{T}_0|\Lambda_0^0\rangle_n + \Delta Q_n|\Lambda_0^1\rangle_n, \tag{50}$$

which are reducible

$$\delta|\Lambda^{(i)0}{}_0\rangle_n = \Delta Q_n|\Lambda^{(i+1)0}{}_0\rangle_n + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\Lambda^{(i+1)1}{}_0\rangle_n,$$
$$|\Lambda^{(0)0}{}_0\rangle_n = |\Lambda_0^0\rangle_n, \tag{51}$$

$$\delta|\Lambda^{(i)1}{}_0\rangle_n = \tilde{T}_0|\Lambda^{(i+1)0}{}_0\rangle_n + \Delta Q_n|\Lambda^{(i+1)1}{}_0\rangle_n,$$
$$|\Lambda^{(0)1}{}_0\rangle_n = |\Lambda_0^1\rangle_n, \tag{52}$$

with finite number of reducibility stages $i_{\max} = n-1$ for spin $s = n + 1/2$. In the above formulae the gauge parameters $|\Lambda^{(k)i}{}_0\rangle_n$ are subject to the conditions which are analogous to the conditions on $|\chi_0^0\rangle_n$, $|\chi_0^1\rangle_n$. Namely, the gauge parameters are independent on the ghosts $\eta_G$, $\mathcal{P}_G$, $\eta_0$, $\mathcal{P}_0$, $q_0$, $p_0$ and the following conditions are fulfilled

$$\sigma|\Lambda^{(k)i}{}_0\rangle_n = \left(n + \frac{D-3}{2}\right)|\Lambda^{(k)i}{}_0\rangle_n,$$
$$gh(|\Lambda^{(k)i}{}_0\rangle_n) = -(i+k+1). \tag{53}$$

Let us show that Lagrangian (46) reproduces the correct equations (14) on the physical field. As we already mention the massive theory looks like-dimensional reduction of the corresponding massless theory which presented in [5]. First of all we remove all the auxiliary fields associated with the ghost fields and associated with the additional creation operators $d^+$, $b_2^+$ analogously to Section 6 of [5]. After that the only auxiliary fields we get are ones associated with operator $b_1^+$. After this we get the equations of motion

$$(T_0 + \tilde{L}_1^+\tilde{T}_1)|\Psi\rangle_n = 0, \qquad (\tilde{T}_1)^3|\Psi\rangle_n = 0, \tag{54}$$

which are invariant under the gauge transformation

$$\delta|\Psi\rangle_n = \tilde{L}_1^+|\Lambda\rangle_{n-1}, \qquad \tilde{T}_1|\Lambda\rangle_{n-1} = 0. \tag{55}$$

Here $T_0$ is given by (9) and

$$|\Psi\rangle_n = \sum_{k=0}^{n}(b_1^+)^k a^{+\mu_1}\cdots a^{+\mu_{n-k}}\psi_{\mu_1\cdots\mu_{n-k}}(x)|0\rangle, \tag{56}$$

$$|\Lambda\rangle_{n-1} = \sum_{k=0}^{n-1}(b_1^+)^k a^{+\mu_1}\cdots a^{+\mu_{n-k-1}}\lambda_{\mu_1\cdots\mu_{n-k-1}}(x)|0\rangle, \tag{57}$$

$$\tilde{L}_1^+ = a^{+\mu}p_\mu + mb_1^+, \qquad \tilde{T}_1 = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma}b_1. \tag{58}$$

Now we get rid of the auxiliary fields associated with $b_1^+$. First we note that the second equation in (55) tells us that $\lambda_{\mu_1\cdots\mu_{n-1}}$ in (57) is unconstrained and the other coefficients in (57) $\lambda_{\mu_1\cdots\mu_{n-k-1}}$ are expressed through it $\lambda_{\mu_1\cdots\mu_{n-k-1}} \sim \tilde{\gamma}^{\mu_1}\cdots\tilde{\gamma}^{\mu_k}\lambda_{\mu_1\cdots\mu_{n-1}}$. Then we decompose equation (54) in power series of $b_1^+$ and using the on-shell gauge transformations we remove first $\psi$ with the help of $\tilde{\gamma}^{\mu_1}\cdots\tilde{\gamma}^{\mu_{n-1}}\lambda_{\mu_1\cdots\mu_{n-1}}$, then $\psi_\mu$ with the help of $\tilde{\gamma}^{\mu_1}\cdots\tilde{\gamma}^{\mu_{n-2}}\lambda_{\mu_1\cdots\mu_{n-1}}$, and so on till $\psi_{\mu_1\cdots\mu_{n-1}}$. Thus we removed all the auxiliary fields and after this we find that the physical field $\psi_{\mu_1\cdots\mu_n}$ obey Eqs. (1), (2).

Now we turn to examples.

## 5. Examples

### 5.1. Spin 1/2

We begin with the simplest case of spin $s = 1/2$ and show that Lagrangian (46) reproduces Dirac Lagrangian. In this case there exists the field $|\chi_0^0\rangle$ only which obeys condition (47)

$$|\chi_0^0\rangle_0 = \psi(x)|0\rangle, \qquad {}_0\langle\tilde{\chi}_0^0| = \langle 0|\psi^+\tilde{\gamma}^0. \tag{59}$$

Substituting (59) in (46) ones get

$$\mathcal{L}_0 = {}_0\langle\tilde{\chi}_0^0|K_0T_0|\tilde{\chi}_0^0\rangle_0 = -\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi. \tag{60}$$

Here we used the definition for the "true" gamma-matrices (12) and introduced the Dirac conjugate spinor

$$\bar{\psi} = \psi^+\gamma^0. \tag{61}$$

Thus we see that Lagrangian (46) reproduces the Dirac Lagrangian up to an overall factor.

### 5.2. Spin 3/2

Our aim is to construct Lagrangian in terms of "true" gamma-matrices (12). To do this we assign a definite Grassmann parity to the fields and the gauge parameters. For example we can take the field $|\chi\rangle$ to be even (odd) then the gauge parameter $|\Lambda^{(0)}\rangle$ will be odd (even), then the gauge parameter $|\Lambda^{(1)}\rangle$ will be even (odd), and so on. For this purpose we will use $\tilde{\gamma}$ to keep the proper Grassmann parity of the fields $|\chi_0^i\rangle$ and the gauge parameters $|\Lambda^{(k)i}{}_0\rangle$.

In the case of $s = 3/2$ we have the following expressions for the fields $|\chi_0^0\rangle$, $|\chi_0^1\rangle$ and the gauge parameter $|\Lambda_0^0\rangle$ which obey the above Grassmann parity conditions and conditions (47) and (53), respectively

$$|\chi_0^0\rangle_1 = [-ia^{+\mu}\psi_\mu(x) + d^+\tilde{\gamma}\psi(x) + b_1^+\varphi(x)]|0\rangle, \tag{62}$$

$$\langle\tilde{\chi}_0^0| = \langle 0|[\psi_\mu^+(x)ia^{+\mu} + \psi^+(x)\tilde{\gamma}d + \varphi^+(x)b_1]\tilde{\gamma}^0, \tag{63}$$

$$|\chi_0^1\rangle_1 = [\mathcal{P}_1^+\tilde{\gamma}\chi(x) - ip_1^+\chi_1(x)]|0\rangle, \tag{64}$$

$$\langle\tilde{\chi}_0^1| = \langle 0|[\chi_1^+(x)ip_1 + \chi^+(x)\tilde{\gamma}\mathcal{P}_1]\tilde{\gamma}^0, \tag{65}$$

$$|\Lambda_0^0\rangle_1 = [\mathcal{P}_1^+\lambda(x) - ip_1^+\tilde{\gamma}\lambda_1(x)]|0\rangle. \tag{66}$$

Substituting (62)–(65) into (46) ones find Lagrangian (up to an overall factor) for spin-3/2 field

$$\begin{aligned}\mathcal{L}_1 = {} & \bar{\psi}^\mu[(i\gamma^\sigma\partial_\sigma - m)\psi_\mu - i\gamma_\mu\chi_1 - \partial_\mu\chi] \\ & + \bar{\varphi}[(-i\gamma^\mu\partial_\mu + m)\varphi + m\chi - \chi_1] \\ & + (D-1)\bar{\psi}[(i\gamma^\mu\partial_\mu + m)\psi + \chi_1] \\ & + \bar{\chi}[(i\gamma^\mu\partial_\mu + m)\chi - \chi_1 + \partial^\mu\psi_\mu + m\varphi] \\ & + \bar{\chi}_1[-\chi + i\gamma^\mu\psi_\mu - \varphi + (D-1)\psi]\end{aligned} \tag{67}$$

and substituting (62), (64), (66) into (49), (50) ones find gauge transformations for the fields

$$\delta\psi_\mu = \partial_\mu\lambda + i\gamma_\mu\lambda_1, \qquad \delta\psi = \lambda_1, \qquad \delta\varphi = m\lambda + \lambda_1, \tag{68}$$

$$\delta\chi = (i\gamma^\mu\partial_\mu - m)\lambda - 2\lambda_1, \tag{69}$$

$$\delta\chi_1 = -(i\gamma^\mu\partial_\mu + m)\lambda_1. \tag{70}$$

Here we have used (12) and (61) and that $K_h d^+|0\rangle = -2hd^+|0\rangle$ with $-2h \to (D-1)$.

One can easy check that removing $\varphi$ and $\psi$ with the help of their gauge transformation using the gauge parameters $\lambda$ and $\lambda_1$ respectively we get that $\chi_1 = 0$ and then $\chi = 0$ as consequences of the equations of motion. After this the equations on the physical field $\psi_\mu$ coincide with the equations which define the irreducible representation with spin $s = 3/2$.

Now we transform (67) to Rarita–Schwinger Lagrangian. For this purpose we remove $\varphi$ and $\psi$ with the help of their gauge transformations using the parameters $\lambda$ and $\lambda_1$, respectively. After this we integrate over $\chi_1$ and then over $\chi$ (or the same we use algebraic equation of motion $\chi = i\gamma^\mu\psi_\mu$ ) and get

$$\mathcal{L}_{\rm RS} = \bar\psi^\mu\big(i\gamma^\sigma\partial_\sigma - m\big)\psi_\mu - i\bar\psi^\mu\big(\gamma^\nu\partial_\mu + \gamma_\mu\partial^\nu\big)\psi_\nu + \bar\psi_\mu\gamma^\mu\big(i\gamma^\sigma\partial_\sigma + m\big)\gamma^\nu\psi_\nu. \tag{71}$$

This is the Rarita–Schwinger Lagrangian in $D$ dimensions.

## 6. Summary

We have developed the operator approach to derivation of gauge invariant Lagrangians for fermionic massive higher spin models in arbitrary-dimensional Minkowski space. The approach is based on realization of the conditions determining the fermionic higher spin fields as the operator constraints acting in auxiliary Fock space, finding the closed algebra of the constraints and construction of the BRST operator (or to be more precise, BRST-BFV operator [8]) in this space.

We found that the model under consideration is a reducible gauge theory and the order of reducibility linearly grows with the value of spin. It is shown that the BRST operator generates the consistent Lagrangian dynamics for fermionic fields of any value of spin in space of arbitrary dimension. As an example of general scheme we obtained the gauge invariant Lagrangian and the gauge transformations for the massive fields with spin-3/2 in the explicit form. Derivation of gauge invariant Lagrangians for any other spins on the base of our general approach is a purely technical problem.

The main results of Letter are given by the relations (46), where Lagrangian for the field with arbitrary half-integer spin is given, and (49)–(52) where the gauge transformations for the fields and the gauge parameters are written down. The formulation does not assume to impose any off-shell constraints on the fields and the gauge parameters from the very beginning. All the constraints emerge as the consequences of the equations of motion and gauge fixing. The approach can also be applied to Lagrangian construction for fermionic higher spin fields in AdS space and for the fermionic fields with mixed symmetry.

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