RESEARCH PROBLEMS

With Volume 36 of Discrete Mathematics, a Research Problem Section has been established. Problems in this section are intended to be research level problems rather than standard exercises. People wishing to submit such problems should send them (in duplicate) to:

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The following should be included: (1) The name of the person(s) who originally posed the problem; (2) the name and address of a person willing to act as a correspondent; and (3) references and other pertinent information.

The Editorial Board of Discrete Mathematics invites readers to provide information about solutions, partial results and other pertinent items related to problems posed earlier, if possible indicating the source of the information, for example papers appearing in different journals, preprints, etc. This information will be passed along to readers from time to time in order to keep them apprised of the current status of various problems.

People wishing to provide information about problems that appeared earlier should write to Professor Alspach. People wishing to correspond on technical matters concerning a problem should write to the correspondent.

Problem 41. Posed by Abraham Berman, Aviezri S. Fraenkel and Joseph Kahane.

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Given a rectangular array of heaps of tokens. Two players play alternately in two-dimensional nim. Every move consists of selecting a row or column of non-empty heaps and removing a positive number of tokens from some of the heaps in the selected row or column. In last-player-win (LPW) mode, the player
first unable to move loses, his opponent wins. The outcome is reversed in last-
player-lose (LPL) mode. A favorite special case is when all heaps have size 1
initially. Typical variations are triangular and hexagonal boards. A variation of
the rules is the additional requirement that the removed tokens be in contiguous
heaps.

Computations of the previous-player-winning (P) positions show that they are
sparse. Despite this sparsity, there appears to be a considerable overlap of LPW
and LPL P-positions (somewhat reminiscent of nim). But apart from simple cases
(such as solutions for \( m \times n \) matrices with \( m \leq 2 \)), very little is known about this
class of games. Fremlin [3] computed all the LPW and LPL P-positions which fit
into a 4 \( \times \) 4 square. The special case of Piet Hein's nimbi, played on a hexagonal
board with 12 positions, was solved in [2] by determining 19 LPW P-positions and
19 LPL P-positions, nine of which are common. These suffice for winning both
LPW and LPL modes of play. A somewhat more general setting for these
problems is provided in [1].

References

Centre de Recherches Mathématiques, Université de Montréal, Montréal, Québec, Canada, 1978.

Problem 42. Posed by Aviezri S. Fraenkel and Frank Harary.

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Let \( G \) be a finite graph. Two players play alternately. The first player selects a
vertex \( u_1 \) and forms the set \( S_1 = \{u_1\} \). The second player selects \( u_2 \not\in S_1 \), if any, and
forms the set \( S_2 \) as the union of \( S_1 \) with all vertices on all geodesics between \( u_2 \)
and \( S_1 \), that is, all vertices on shortest paths between \( u_2 \) and \( S_1 \). The first player
now chooses a vertex \( u_3 \not\in S_2 \), if any, and forms the set \( S_3 \) as the union of \( S_2 \) with
all vertices on all geodesics between \( u_3 \) and \( S_2 \). The play continues in the
indicated manner and the first player unable to play is the loser, his opponent the
winner.

Úlehla's result [2] provides a polynomial strategy for the case that \( G \) is a tree:
After the first player picks some vertex \( u_1 \), direct the tree away from \( u_1 \). Now
apply Úlehla's transformation. If the first player can win by picking \( u_1 \) as a first
move, we are done. Otherwise we try the same procedure with the first player
picking some \( u_2 \not= u_1 \). If the first player loses whatever vertex he picks at the
beginning, then the second player can win. Is there a more efficient strategy?

The strategy for several families of graphs is given in [1]. Develop a general theory for geodetic games on graphs. Is there a polynomial strategy for geodetic games played on arbitrary acyclic digraphs?

References


Problem 43. Posed by Moshe Rosenfeld.

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Every cubic 3-connected planar graph admits an even 2-factor, that is, a 2-factor in which each cycle has even length. It can be easily obtained by taking all the edges of two colors in a Tait coloring of the edges. Does there always exist an even 2-factor in which none of the components is a cycle of length 4? It can be shown that the number of components is at least $\frac{1}{6}n$ for some even 2-factor.