Cooperative Guidance for Multimissile Salvo Attack

Zhao Shiyu*, Zhou Rui

School of Automation Science and Electrical Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

Received 16 January 2008; accepted 6 May 2008

Abstract

Cooperative guidance problems of multiple missiles are considered in this article. A cooperative guidance scheme, where coordination algorithms and local guidance laws are combined together, is proposed. This scheme actually builds up a hierarchical cooperative guidance architecture, which may provide a general solution to the multimissile cooperative guidance problems. In the case of salvo attacks which require missiles to hit the target simultaneously, both centralized and distributed coordination algorithms are derived based on the impact-time-control guidance (ITCG) law. Numerical simulations are performed to demonstrate the effectiveness of the proposed approaches.

Keywords: cooperative; guidance; impact time constraints; multiple missiles; salvo attack

1 Introduction

Nowadays most of the important strategic and tactical military targets, such as airports and warfare ships, have been equipped with missile defense systems which pose great challenges for missiles to accomplish their missions. In the case of single missile attacks, maneuver at the terminal guidance section proves an effective way to enhance survivability against the threat of missile defense systems[1]. In recent years, salvo attack of multiple missiles has been devised as another effective countermeasure to survive under threat of interceptors[2-3]. In a salvo attack scenario, multiple missiles are required to hit the target simultaneously to introduce a many-to-one engagement situation for missile defense systems. Even if several missiles are intercepted, the target can still be destroyed by the remaining ones.

Numerous publications have addressed the cooperative control of multivehicle systems in such applications as underwater vehicles[4], ground robots[5], and unmanned aerial vehicles (UAVs)[6] in recent years. However, less attention has been paid to multimissile cooperative attacks, typically salvo attacks.

An impact-time-control guidance (ITCG) law, which can control the impact time of guidance was proposed in Ref.[2] and applied to salvo attacks. However, simply applying ITCG to salvo attacks as in Ref.[2] suffers from a disadvantage where the impact time must be preprogrammed manually into all missiles before they are launched. Moreover, there is no communication among the missiles during the guidance. Therefore, the approach to salvo attack simply based on ITCG is an open-loop and a static guidance strategy, which cannot be viewed as a genuine multimissile cooperative attack.

Motivated by this concern, this article introduces coordination algorithms into the guidance of multiple missiles. Each missile is governed by a local guidance law, and multiple missiles are coor-
ordinated by coordination algorithms. Both centralized and distributed coordination algorithms based on ITCG are derived. The guidance of multiple missiles with coordination algorithms can be regarded as a two-level hierarchical architecture, which may provide a general approach to cooperative guidance problems.

2 Hierarchical Cooperative Guidance Architecture

Cooperative control and guidance of multiple UAVs\[6-9\] have been investigated in a number of literatures. UAVs and missiles are both high-speed aerial vehicles, the guidance of which shares some similarities. However, the approaches to the guidance of UAVs mainly include task assignment, cooperative path planning, and path tracking\[7-9\], which are not applicable to the guidance of missiles where guidance laws should be employed.

Previous guidance laws are all designed for single missiles. How to apply these guidance laws to the guidance of multiple missiles is a critical problem that needs to be solved. The approach to the problem in this article is to employ the coordination variable strategy\[9\].

Information exchange is essential for coordination in a team of vehicles. A coordination variable represents the minimal amount of information needed to attain an ad hoc cooperation objective. In salvo attack problems, missiles are required to arrive at the target simultaneously. To succeed in fulfilling this cooperative-time mission, the crucial timing information such as estimated time of arrival (ETA) must be shared jointly. The ETA of each missile can be chosen as the coordination variable.

Multiple missiles are coordinated by coordination algorithms with each missile under the control of the local guidance law ITCG. The coordination variable is the bond that links all missiles together. Generally speaking, this approach aims to introduce coordination strategies into the guidance of multiple missiles, which can be described as a two-level hierarchical architecture (see Fig.1).

3 Coordination Algorithms Based on ITCG

In this section, a centralized coordination algorithm based on ITCG is proposed for salvo attacks. Then this centralized algorithm is decentralized by using consensus protocols. As the main results in this article are partly on the basis of ITCG, the guidance law proposed in Ref.[2] will be presented first.

3.1 ITCG

Consider a planar homing guidance problem as shown in Fig.2. Suppose that the target is stationary and the missile speed \( V \) is constant. \( \alpha \) is the acceleration command applied normally to the velocity vector to change \( \theta \). Other variables in Fig.2 are self-explanatory. On the basis of the linearized kinematic model, the guidance problem with impact time constraint can be transformed into an optimal control problem.

![Fig.2 Homing guidance geometry.](image)
Linearized kinematic model is
\[
\begin{align*}
\frac{dy}{dx} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (a_b + a_t) \\
\frac{d\theta}{dx} &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \theta
\end{align*}
\] (1)

Cost function is
\[
J = \frac{1}{2} \int_0^L a^2(x)dx
\] (2)

Initial constraints are
\[
y(x_0) = 0, \quad \theta(x_0) = \theta_0, \quad x_0 = 0
\] (3)

Terminal constraints are
\[
y(x_f) = y_f, \quad \theta(x_f) = 0
\] (4)

Impact time constraint is
\[
\int_0^x \sqrt{1 + \dot{\theta}^2(x)}dx = VT_d
\] (5)

where \(T_d\) denotes the designated impact time. The acceleration \(a\) consists of two scalar commands, that is, \(a = a_b + a_t\), where \(a_b\) is the main control command for reducing the miss-distance and \(a_t\) an additional command for adjusting the impact time.

Solve the optimal control problem and obtain the ITCG law as
\[
a = a_p - \frac{60V_s^2}{\rho R_{go}^3}(T_d - \hat{T}_{go})
\] (6)

where \(a_p = NV\hat{\lambda}\) denotes the control command produced by proportional navigation guidance (PNG) with navigation constant \(N=3\), the \(\hat{\lambda}\) line-of-sight (LOS) rate, \(R_{go}\) the current range between missile and target, \(\hat{T}_{go}\) the estimation of the time-to-go. The estimation of the time-to-go in Ref.[2] is given by
\[
\hat{T}_{go} = \frac{1 + \varphi^2}{10}R_{go}/V
\] (7)

where \(\varphi\) denotes the angle between LOS and the missile velocity vector.

For more details of the ITCG law, please refer to Ref.[2].

3.2 Centralized coordination algorithm

Suppose \(n\) missiles participate in the salvo attack. From Eq.(6), the control command of missile \(i\) \((i = 1, 2, \cdots, n)\) is
\[
u_i = a_p - a_t(T_d - \hat{T}_{go})
\] (8)

where
\[
a_t = 60V_s^2 / (\rho R_{go}^3)
\] (9)

Choose the total control effort of \(n\) missiles as the cost function
\[
J = \sum_{i=1}^n u_i^2
\] (10)

Then the optimal designated rendezvous time (i.e., impact time) for all missiles is
\[
T_d^* = \arg \min_{T_d} J
\] (11)

Eq.(11) means that the rendezvous time of multiple missiles is chosen as the one that can minimize the total control energy. According to Eqs.(8)-(11), it is easy to obtain
\[
T_d^* = \left( \sum_{i=1}^n a_i^2 T_{go} + \sum_{i=1}^n a_i^2 \right) / \sum_{i=1}^n a_i^2
\] (12)

Denote \(\delta = \sum_{i=1}^n a_i^2 / \sum_{i=1}^n a_i^2\), then Eq.(12) can be expressed as
\[
T_d^* = \sum_{i=1}^n a_i^2 T_{go} / \sum_{i=1}^n a_i^2 + \delta
\] (13)

where \(T_d^*\) is a combination of a weighted average of \(\hat{T}_{go}\) and an additional part \(\delta\). Eq.(9) implies that \(a_t^2 \rightarrow +\infty\) when \(R_{go} \rightarrow 0\). In fact, the additional part \(\delta\) is much smaller than the first part especially when the missile comes close to the target. By ignoring the additional part \(\delta\), a suboptimal designated rendezvous time can be obtained as
\[
T_d^* = \sum_{i=1}^n w_i T_{go} / \sum_{i=1}^n w_i
\] (14)

where \(w_i = [V_s^2 / (\rho R_{go}^3)]^2\). Eq.(14) indicates that the designated impact time is a weighted average of the time-to-go estimation of each missile.

Fig.3 illustrates the cooperative guidance architecture with centralized coordination strategies. Centralized coordination manager (CCM) collects essential information from each missile, applies the coordination algorithm Eq.(14) and then broadcasts the rendezvous time to all missiles. Multiple missiles are capable of hitting the target simultaneously by the guidance from the ITCG, with an identical designated impact time. In practice, the cooperative guidance with centralized coordination can be implemented by using the leader-follower scheme[10], where the leading missile holds much more computational and communicational resources when compared to the followers. Then the CCM can be placed...
on the leading missile.

Fig.3  Cooperative guidance architecture with centralized coordination.

3.3 Distributed coordination algorithm

Consider the case where each missile is only able to communicate with its nearest neighbors because of communication limitation. In this case, absence of the centralized controller makes it necessary to design a distributed coordination algorithm to achieve the agreement on the rendezvous time.

In recent years, consensus problems of multi-agent systems have attracted a great deal of attention\[11\]-\[12\]. This is partly because of the fact that consensus protocols can build up distributed cooperative control schemes readily. Consensus is achieved in a multiagent system if an agreement is reached on certain quantities of interest. In the salvo attack problems considered in this article, multiple missiles need to come to a consensus on the rendezvous time. Therefore it is natural to design a distributed coordination algorithm by using consensus protocols.

Now the objective is to design a distributed algorithm, which can drive multiple missiles asymptotically to reach an agreement on the desired rendezvous time $T_d^i$ in Eq.(14). Note that $T_d^i$ is a weighted average of the time-to-go of each missile. The following result\[12\] would be useful for decentralizing algorithm Eq.(14).

**Lemma 1** Assume that a network has a fixed topology $G=(V,E,A)$, which is a strongly connected graph. If the node dynamics are

$$\dot{x}_i = \sum_{j=1}^{n} a_{ij} (x_j - x_i) \quad (\gamma_i > 0, \ i = 1,2,\cdots,n) \quad (15)$$

where $x_i$ denotes the state of the $i$th node, $a_{ij}$ the entry of the adjacency matrix $A$ of $G$, $\gamma_i$ a positive weight. Subsequently, an agreement is globally and asymptotically reached, where the group decision value will be

$$\alpha = \frac{\sum_i x_i(0)}{\sum_i \gamma_i} \quad (16)$$

Suppose that $n$ missiles attack a single target. The impact time of the missile $i (i = 1,2,\cdots,n)$ is denoted by $T_d^i$. The missile $i$ holds a variable $x_i$ that represents its own understanding of the rendezvous time. From Lemma 1, algorithm Eq.(14) can be decentralized into a distributed one, as follows

$$\dot{x}_i = c_i \sum_{j \in N_i} (x_j - x_i) \quad (17)$$

where $c_i = 1/\gamma_i = [(a_{pR}R_{go})/V_i^3]^2$ and $N_i$ denotes the set of neighbors of missile $i$. Algorithm Eq.(17) can be rewritten into the matrix form as

$$\dot{x} = -CLx \quad (18)$$

where $x = [x_1 \ x_2 \ \cdots \ x_n]^T$, $C = \text{diag}(c_1,c_2,\cdots,c_n)$, $L$ denotes the Laplacian matrix of the graph that represents the communication topology of multiple missiles. According to Lemma 1, the distributed coordination algorithm Eq.(17) guarantees that the multiple missiles will reach an agreement globally and asymptotically on the designated rendezvous time $T_d^i$.

Fig.4 illustrates the cooperative guidance architecture with distributed coordination strategies, where CCM in the centralized case is replaced by many decentralized coordination managers (DCMs), which are distributed in each missile. Here DCM $i$ applies the distributed algorithm Eq.(17).

Fig.4  Cooperative guidance architecture with distributed coordination.

In fact, centralized approaches always have a better performance compared with the distributed ones. This is because the centralized algorithm can
produce the group decision value immediately, whereas, the distributed one will spend infinite time before $T_d$ converges to $T_d^\dagger$.

4 Simulation Results

To demonstrate the performance of the proposed approaches to multimissile cooperative guidance, a salvo attack scenario is performed. Suppose three missiles attack a stationary target at (0, 0) with the initial conditions shown in Table 1.

**Table 1 Scenario for salvo attack**

<table>
<thead>
<tr>
<th>Missile</th>
<th>Position/m</th>
<th>Heading angle(°)</th>
<th>Speed/(m·s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(−6 894, −5 785)</td>
<td>70</td>
<td>280</td>
</tr>
<tr>
<td>2</td>
<td>(−3 249, −8 927)</td>
<td>95</td>
<td>320</td>
</tr>
<tr>
<td>3</td>
<td>(2 329, −8 693)</td>
<td>135</td>
<td>260</td>
</tr>
</tbody>
</table>

The simulation results will be presented in three cases: ① a salvo attack with preprogrammed designated impact time; ② a salvo attack with centralized coordination; and ③ a salvo attack with distributed coordination.

In the first case, the designated impact time 38 s, is preprogrammed manually into each missile. Fig.5(a) shows the trajectories of three missiles and Fig.5(b) the histories of time-to-go and designated impact time. The dotted lines in Fig.5(a) present the trajectories of three missiles by PNG. The impact time for three missiles by PNG is 32.99 s, 30.22 s, and 35.53 s, respectively. The dispersion of the impact time by PNG is about 5.3 s, whereas, ITCG can drive the three missiles to hit the target simultaneously at the designated impact time.

In the case of a salvo attack with centralized coordination algorithm, the designated impact time of each missile is produced by algorithm Eq.(14) automatically. Fig.6(a) shows the trajectories of three missiles and Fig.6(b) the histories of time-to-go and designated impact time.

In the third case, distributed coordination algorithm Eq.(17) is applied. According to Lemma 1, a feasible communication topology is chosen for the multimissile system as shown in Fig.7. Then the distributed coordination algorithm Eq.(17) can be expressed as

\[
\begin{align*}
\dot{x}_1 &= c_1(x_2 - x_1) \\
\dot{x}_2 &= c_2((x_1 - x_2) + (x_3 - x_2)) \\
\dot{x}_3 &= c_3(x_2 - x_3)
\end{align*}
\]

![Fig.5](image1.png) **Case 1: guidance with preprogrammed designated impact time.**
The missile trajectories and the histories of time-to-go and designated impact time in the third case are shown in Fig. 8(a), (b), (c), and (d), respectively. The designated impact time of each missile is continuous piecewise. Fig. 8(e) presents the histories of $c_i (i = 1, 2, 3)$, where $c_i$ is defined in Section 3.3. The histories of $w_i$ in Eq.(14) are shown in Fig. 8(f), which indicate that Missile 3 plays the most important role in determining the rendezvous time. The terminal dispersion of impact time here is about 0.3 s, which is a little larger than that in the centralized case. This is partly because of the fact that an agreement cannot be reached in finite time. On the other hand, Fig. 8(e) shows that $c_i$ in Eq.(17) converges to zero gradually. This indicates that the convergence speed of the distributed coordination algorithm Eq.(17) gradually reduces to almost zero, so the coordination is hardly effective in the end.
5 Conclusions

The main contribution of this article is to introduce coordination algorithms into the guidance for multiple missiles. Furthermore, genuine autonomous cooperative guidance for multiple missiles is developed. It is noticeable that coordination algorithms here are designed based on local guidance laws, the properties of which would exert direct influence on the performance of cooperative guidance. In the future, studies on the development of new ideal guidance laws to control the impact time and new cooperative guidance strategies should be expected.

References


Biography:
Zhao Shiyu  Born in 1984, he received B.S. degree from Beijing University of Aeronautics and Astronautics (BUAA) in 2006. He is currently a graduate student in the Department of Automatic Control at BUAA. His research interests include guidance & control of aerial vehicles, and cooperative control of multi-agent systems.
E-mail: zsybeijing@gmail.com