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# On Steam Pipe Network Modeling and Flow Rate Calculation

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**Abstract**

The paper demonstrates the method to set up the pipe network hydraulic-thermal synthetic mode by applying hydraulic and thermal models of single pipe, and proposes the algorithm based on searching for the problem that iterative calculation sometimes cannot derive convergent reasonable result as well. Compared the calculated values with the measurements, it shows the validation of the model and effectiveness of the algorithm.

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Key Words: Steam Pipe Network; Modeling; Flow Rate Calculation

## 1. Introduction

For the reason that the status parameters of steam will largely vary along with phase changing in the process of steam transmission, the density, viscosity and fiction coefficient will largely change. Only by combining hydraulic and thermal calculation can the calculated mass flow rates in steam pipes be derived with higher accuracy[1]. The International Association for the Property of Water and Steam released IAPWS-IF97 equations in 1997[2], which provides the base for hydraulic and thermal calculation. Sylvie and Wishsanuruk drew the conclusion that the structure of tree is better than circle and rectangle [3]. Tian applied the laws of conservation of mass, momentum and energy to solve the equation[4]; Zhang proposed the method of hydraulic and thermal coupling calculation [1]. However, the validation of the models should be further researched and verified.

Due to the nonlinearity and complexity of the hydraulic-thermal model, iterative calculating was applied in ordinary algorithm. Reference [5] testified the condition of convergent iterative, however it didn't suggest how to preset the initial values. If the initialized values are unreasonably preset, the calculation may fall into failure.

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The paper builds up the hydraulic-thermal synthetic model, and solves the problem of calculating the flow rate in each pipe in case of the outer nodes' temperature and pressure values being known.

## 2. The Static Model of Steam Pipe Network

To deduce the model, the real conditions are simplified as following:

①The steam in the pipes is axial-direction one dimensional flow；②The network is composed of nodes and branches(pipes). The nodes can be divided into intermediate nodes and outer nodes.③There is no condensate or secondary steam and the effect is neglected.

### 2.1. The hydraulic and thermal model of single pipe.

- The hydraulic model of single pipe

By the law of momentum conservation and transferring the metric units into engineering units, the hydraulic model can be written as[1]:

$$P_1^2 - P_2^2 = 1.25 \times 10^8 \frac{\lambda q^2 P_1 T_2 Z_2 L}{D^5 \rho_m T_1 Z_1} \tag{1}$$

In the equation：

$P_1, P_2$ —the input and output pressure of the steam in the pipe, Mpa;

$T_1, T_2$ —the input and output absolute temperature, K;

$Z_1, Z_2$ —compressibility factor of the input and output steam;

$\lambda$ —frictional factor;

$q$ —the mass flow rate, t/h;

$L$ —the length of the pipe, m;

$D$ —the inner diameter of the pipe, mm;

$\rho_m$ —the weighted mean density of the steam in the pipe, kg/m³.

Usually, there's little difference between the input and output products of compressibility factor and temperature in the same pipe. As considering the elbows, reducer extenders, the joints and other friction factors,  the equivalent coefficient $\eta$ is added to the equation. Equation (1) is changed to (2) :

$$P_1^2 - P_2^2 = 1.25 \times 10^8 \frac{\lambda q^2 P_1 (1 + \eta) L}{D^5 \rho_m} \tag{2}$$

In the equation, $\eta$ —equivalent length coefficient；

Thus:    $$q = \frac{D^5 \rho_m}{1.25 \times 10^8 \lambda q P_1 (1+\eta) L} P_1^2 - P_2^2 = C_P (P_1^2 - P_2^2) \tag{3}$$

$$C_P = \frac{D^5 \rho_m}{1.25 \times 10^8 \lambda q P_1 (1 + \eta) L} \tag{4}$$

In accordance with Альтщуль equation, frictional factor $\lambda$：

$$\lambda = 0.11 \times (\frac{\Delta}{D} + \frac{68}{\mathrm{Re}})^{0.25} \tag{5}$$

In the equation, $\Delta$ —the equivalent absolute roughness, mm;

$\mathrm{Re}$ —the Reynolds number of the steam in the pipe;

After transferring the units of $D, q$ to mm and t/h (ton per hour), Reynolds number is written as:

$$\mathrm{Re} = \frac{Du \rho_m}{\mu} = 354 \frac{q}{D \mu} \tag{6}$$

In the equation, $u$ —the characteristic velocity of the steam in this pipe, m/s

$\mu$ —the mean dynamic viscosity coefficient. It's determined by equation (10) [6].

Denote $\rho_1, \rho_2$ as the input and output steam densities. According to the property of steam[6], The industrial steam usually belongs to region 2. $\rho_1, \rho_2, \mu, \rho_m$ can be calculated with following equations:

$$\rho_1 = \frac{P_1}{\pi (r_\pi^0 + r_\pi^r) R T_1} \tag{7}$$

$$\rho_2 = \frac{P_2}{\pi (r_\pi^0 + r_\pi^r) R T_2} \tag{8}$$

$$\rho_m = \frac{\rho_1}{3} + \frac{2\rho_2}{3} \tag{9}$$

$$\mu = \Psi(\delta, \tau) \eta^* \tag{10}$$

The unclaimed symbols in equation(7), (8), (10)are defined in reference [6].

- The thermal model of single pipe

By the law of energy conversation, the static thermal model can be written as follows after transferring the metric units to the usual engineering units:

$$T_1 - T_2 = \frac{(1+\beta)q_l L}{1000 \, c_p q \times \frac{1000}{3600}} = \frac{(1+\beta)q_l L}{278 \, c_p q} \tag{11}$$

$$q = \frac{278 \, c_p q^2}{(1+\beta)q_l L}(T_1 - T_2) = C_T (T_1 - T_2) \tag{12}$$

$$C_T = \frac{278 \, c_p q^2}{(1+\beta)q_l L} \tag{13}$$

In upper equations, the definitions and units of $T_1, T_2, L, q$ are the same as eqation (1).(note: changing equation (11) to (12) is to make thermal model has the same pattern with equation (3), and later equations (19)(20)(21) have the uniform pattern)

$\beta$ —the appending heat loss coefficient for the appendix of pipe, valve, and support, the value can be 0.15-0.25[1] depending on pipe laying methods;

$c_p$ —specific heat capacity at constant pressure, kJ/kg. K;

$q_l$ —the amount of heat loss along unit pipe length, W/m.

In term of equation IF-97, The unclaimed symbols are defined in the reference [6].

$$c_p = -R\tau^2 (\gamma_{\tau\tau}^o + \gamma_{\tau\tau}^\gamma) \tag{14}$$

The amount of heat loss along unit pipe length can be calculated using equation (15)

$$q_l = \frac{T_0 - T_a}{\frac{1}{2\pi\varepsilon}\ln\frac{D_o}{D_i} + \frac{1}{\pi D_0 a_w}} \tag{15}$$

In the equation, $T_0, T_a$ —the out surface temperature of the pipe (can be treated as the temperature of steam) and the environment temperature, K;

$D_0, D_i$ —the outer and inner diameters of the steam pipe, mm;

$\varepsilon$ —the coefficient of thermal conductivity of the heat insulated material, W/m.K;

In equation (15), $a_w$ is determined by：

$$a_w = 11.6 + 7\sqrt{v} \tag{16}$$

$v$ —the velocity of environmental air, m/s;

### 2.2. Steam pipe network synthetic model

A.Incidence matrix of pipe network
- Number the nodes and pipes

Draw out pipe network map, and number the nodes of the graph in the order of steam sources, users, three-way nodes. Suppose there are $m_1$ intermediate nodes and $m_2$ outer nodes, there are total $m = m_1 + m_2$ nodes and $p = m - 1$ pipes. Number the pipes in the order of leaf pipes, branch pipes.
- Determine the elements in the incidence matrix $A \in R^{m \times p}$

Suppose the element of the *i*th row and the *j*th column in matrix A is $a_{ij}$, the value is defined as：

$$a_{ij} = \begin{cases} 1 & (flow \quad in \quad from \quad node \quad i) \\ -1 & (flow \quad out \quad from \quad node \quad i) \\ 0 & (unrelated) \end{cases} \tag{17}$$

B.The flow rate balance equation of the pipe network

Denote：$P = (P_1^2, P_2^2, ..., P_m^2)^T$, $P_i^2, (i = 1, ..., m)$ the square of the ith node's pressure of, MPa$^2$；

$T = (T_1, T_2, ..., T_m)^T$, $T_i, (i = 1, ..., m)$ the *i*th node temperature, ℃；

$\vec{q} = (q_1, q_2, ..., q_p)^T$, $q_j, (j = 1, ..., p)$ the flow rate of the jth pipe, t/h；

$Q = (Q_1, Q_2, ..., Q_m)^T$, $Q_i, (i = 1, ..., m)$ the flow rate of the *i*th node, unit t/h; For steam souce the value is minus $q_j$, and for the user the value is positive $q_j$, other wise, the value is 0.

$C_p^* = diag(C_{P1}, C_{P2}, ..., C_{Pp}), C_{Pj}, (j = 1, ..., p)$ the parameter of the jth pipe determined by equation (4)；

$C_T^* = diag(C_{T1}, C_{T2}, ..., C_{Tp}), C_{Tj}, (j = 1, ..., p)$ the parameter of the jth pipe determined by equation(13).

In term of the mass conservation law, the total flow rate of each node should be 0, that is：

$$A\vec{q} + Q = 0 \tag{18}$$

According to the hydraulic and thermal equations, the pipe network equation can be written as:

$$\vec{q} = C_p^* A^T P = C_T^* A^T T \tag{19}$$

Substitute (19) into (18) to get:

$$A C_p^* A^T P + Q = 0 \tag{20}$$

$$A C_T^* A^T T + Q = 0 \tag{21}$$

Equation (3), (4), (12), (13), (19), (20), (21) make up the static model of steam pipe network.

## 3. Hydraulic and thermal calculation based on searching

Industrial networks are equipped with temperature, pressure and flow meters at the nodes of steam sources and users except intermediate nodes. The proposed algorithm tries to calculate the flow rate in each pipe with the given condition. The flow rate meter readings are applied to evaluate of the algorithm.

By analyzing the steam pipe network model, one can find the key to the problem is the unknown intermediate nodes' temperature and pressure values coupling hydraulic and thermal calculation.
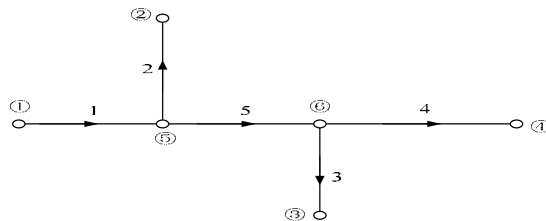


**Fig 1: Layout of steam pipe network**

The method proposed in this paper is based on searching. Now set figure 1 as an example to illustrate this method, the index numbers of the nodes and pipes are shown in the figure:

Step 1：Determine the elements of matrices A, Q as the preliminarily defined.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{pmatrix} \tag{22}$$

$$Q = (-q_1, q_2, q_3, q_4, 0, 0)^T \tag{23}$$

As the previous definitions for equation (18)(19), $P_5, P_6, T_5, T_6, \vec{q}$ are unknown. From equation (4), it's easy to discover frictional factor and density are in connection with $P_5, P_6, T_5, T_6$. Only by presetting $\vec{q}$ can equation (19)(20) be applied for hydraulic iterative calculation. However, in thermal calculation only $c_p$ relates with $P_5, P_6, T_5, T_6$. Changing equation (11) to (24), $\vec{q}$ can be calculated directly.

$$q_i = \frac{(1+\beta)q_{li}L_i}{278c_p\Delta T_i}, (i = 1, 2, ..., 5) \tag{24}$$

In equation (24), $i$—the pipe index, $\Delta T_i$—the temperature difference between input and output steam of the $i$th pipe. So, the thermal model is chosen to calculate first.

Step 2: Thermal calculation

It can be inferred by the flow direction of steam that：

$$P_2 < P_5 < P_1, \max(P_3, P_4) < P_6 < P_5 \tag{25}$$

$$T_2 < T_5 < T_1, \max(T_3, T_4) < T_6 < T_5 \tag{26}$$

Presume：   $P_5 = \frac{1}{2}(P_2 + P_1); P_6 = \frac{1}{2}(P_5 + \max(P_3, P_4)) \tag{27}$

Set the searching start value for $T_5$, $T_6$ as $T_1$ and $\max(T_3, T_4)$ respectively, and set the searching step size as h then begin searching $T_5$, $T_6$ in the range determined by (26). For each searching step size, calculate $c_p$ (equation (14)) and flow rate of each pipe (equation (24),(25)), and test whether $\|A\vec{q} + Q\| \le \xi_1$ is satisfied. If the inequality is not satisfied, update $T_5$, $T_6$ with one step to continue calculation, or retain the temperature value and the corresponding flow rates.

Step 3: Hydraulic calculation

Just as the thermal calculation, first presume the temperature of the intermediate nodes as the value searched by the former step, and set the start point and step size for the two intermediate pressures. Then begin searching in the region determined by inequality (25) with the step size to calculate the density (equation (9)), frictional factor ((5), (6), (10)), and flow rate ((3), (19)) of each pipe. When calculating the flow rate, presetting the initial values as the result of step 2 will reduce the iterative calculation time. Test whether $\|A\vec{q} + Q\| \le \xi_2$ is satisfied, If the inequality is not satisfied, update $P_5$, $P_6$ with one step size to continue calculation, or retain the pressure values and the corresponding flow rates.

Step 4: Thermal model Verification

Substitute the intermediate nodes' temperature (determined by step 2), pressure and flow rate values (determined by step3) into thermal model to verify the inequality $\|AC_T^*A^TT + Q\| \le \xi_3$. If it is satisfied, the calculation ends with the result of step 3. If not, presume the intermediate pressure values as the result of step 3, reduce step size and go to step 2 and start another calculation cycle.

In the algorithm, the value $\xi_1, \xi_2, \xi_3$ can be set as 0.05-0.3 depending on the demand for errors and calculation stability. The flow chart of the calculation procedure can be depicted with figure 2.
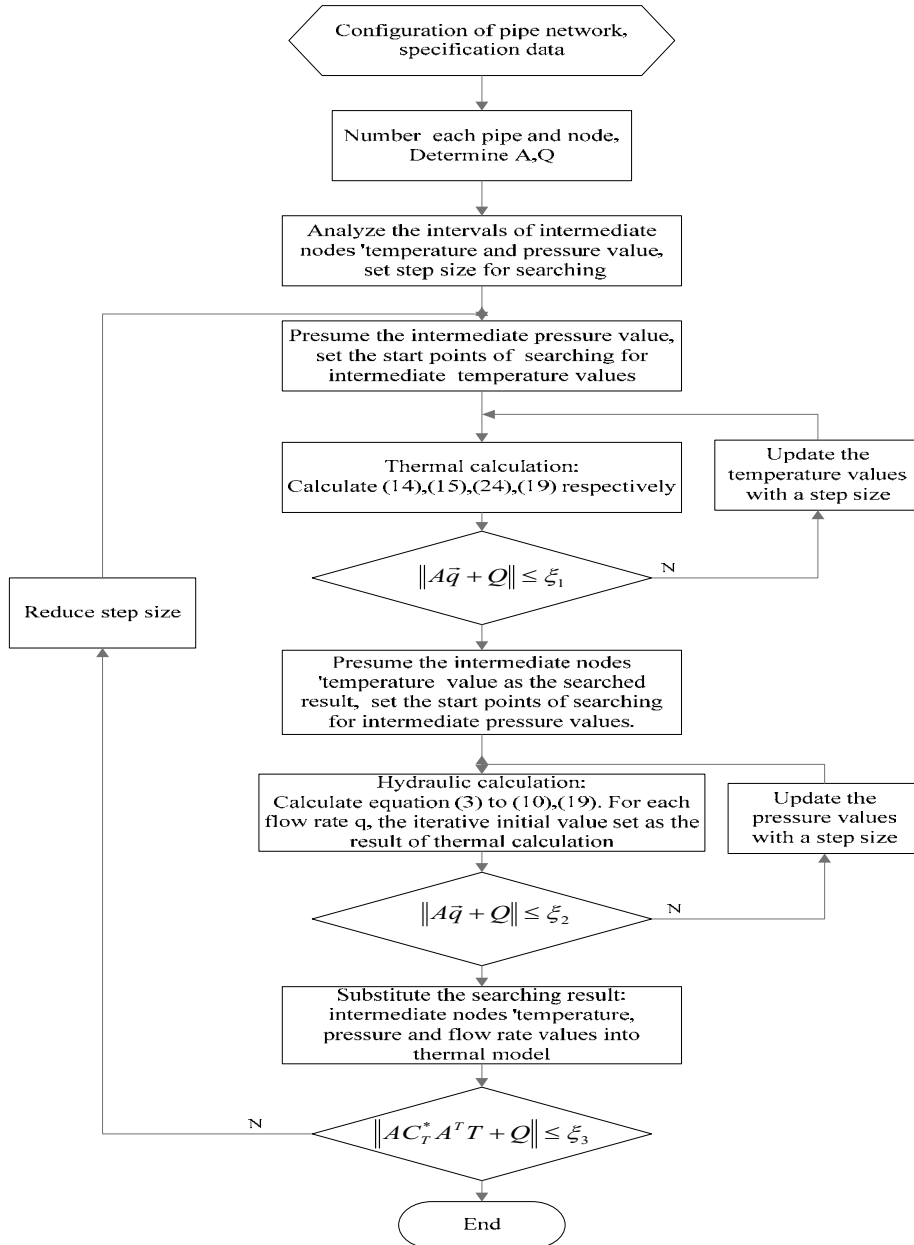
**Fig. 2 The flow chat of flow rate calculation**

## 4. Comparisons of Calculation results and Measurements

The specifications of each pipe are shown in table 1. The parameters in the model and algorithm are initially set as such: $\eta = 0.2$, $\Delta = 0.2$, $\beta = 0.15$, $\varepsilon = 0.035$, $\xi_{1,2,3} = 0.3$.

Substitute the specification data in Table 1 and the outer nodes' temperature and pressure data in Table 2 to the algorithm variables to calculate flow rate values. The measured data, the calculated data and the relative difference between them are listed in Table 2. It shows the largest difference is less than

6%. Actually, the neglected factors in model, the parameter errors and measurement errors add to the difference. The comparison testifies the reliability of the model and algorithm.

As for the larger scale steam network which has more than 3 intermediate nodes, it's difficult to apply the algorithm directly because there are more variables to be searched. However, the pipe network can be divided into several smaller networks from the nodes with known temperature and pressure.

**Table1: the Specifications of the Pipes**

| Pipe No. | Length (m) | Inner Diameter (mm) | Outer Diamete r(mm) | heat insulation layer Thickness (mm) |
|---|---|---|---|---|
| 1 | 600 | 700 | 720 | 110 |
| 2 | 2300 | 600 | 630 | 105 |
| 3 | 700 | 400 | 426 | 95 |
| 4 | 1300 | 500 | 529 | 100 |
| 5 | 1200 | 600 | 630 | 105 |

**Table2: the Measurement Data and Calculated Data.**

| Outer node No. | Pressure(MPa) | Temperature (℃) | Measured q(t/h) | Calculated q（t/h） | the relative difference |
|---|---|---|---|---|---|
| 1 | 0.705 | 286 | 117 | 121 | 3.4% |
| 2 | 0.679 | 195 | 19 | 20 | 5.2% |
| 3 | 0.563 | 246 | 19 | 19 | 0 |
| 4 | 0.445 | 256 | 79 | 82 | 3.8% |

## 5. Conclusions

The article demonstrates the procedure of setting up the synthetic model of steam pipe network based on the hydraulic and thermal model of single pipe, and proposes an searching algorithm to calculate the flow rates avoiding iterative calculation methods which sometimes cannot get convergent results.

Compared with the measured values, the calculated values deviate from the measured ones less than 6%. The differences are caused by the accuracy of instrument, the errors of model parameters and simplified condition. The results show the validation of the model and effectiveness of the algorithm.

The proposed model and algorithm can be applied to simulate the running of static steam pipe network, diagnose the measured data and aid the network design or innovation.

## Acknowledgements

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