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(2,1)-Total labelling of trees with 3,4 are not in D△

Haina Sun

Department of Fundamental Courses ,Ningbo Institute of Technology,Zhejiang Univ,Ningbo 315100,China.

Abstract

Let T be a tree with maximum degree $\Delta \geq 4$. Let $D_\Delta(T)$ denote the set of integers k for which there exist two distinct vertices of maximum degree of distance at k in T. It was known that $\Delta+1 \leq \lambda_2^t(T) \leq \Delta+2$. In this paper, we prove that if $3,4 \notin D_\Delta(T)$, then $\lambda_2^t(T) = \Delta+1$.

Keywords: $(2,1)$-Total labelling; Tree; Maximum degree; Combinatorial problems.

1.Introduction

Motivated by the Frequency Channel Assignment problem, Griggs and Yeh [1] introduced the $L(2,1)$-labelling of graphs. This notion was subsequently generalized to the $L(p,q)$-labelling problem of graphs. Let p and q be two nonnegative integers. An $L(p,q)$*-labelling* of a graph G is a function f from its vertex set $V(G)$ to the set $\{0,1,\cdots,k\}$ such that $|f(x)-f(y)| \geq p$ if x and y are adjacent, and $|f(x)-f(y)| \geq q$ if x and y are at distance 2. The $L(p,q)$*-labelling number* $\lambda_{p,q}(G)$ of G is the smallest k such that G has an $L(p,q)$-labelling f with $\max\{f(v) | v \in V(G)\} = k$. In particular, we simply write $\lambda(G) = \lambda_{2,1}(G)$.

A star is a tree that consists of Δ leaves (A leaf is a $1-$vertex) and a $\Delta-$vertex. A generalized star is a tree that all vertices are leaves except that two adjacent vertices. Obviously, a star is also a generalized star and the $L(2,1)-$total labelling number of a generalized star is $\Delta+1$. Let M denote a

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generalized star, a tree of order 8 consisting two adjacent 4-vertices and four leaves. Let $K_{1,4}$ denote the star of order 5. Clearly, both $K_{1,4}$ and M are type 1. Recent research see references [2,3,4,5,6].

2.Trees with $3,4 \notin D_{\Delta}$

Given an edge $e = vu \in E(T)$, we use $T_v(e)$ to represent the subtree of T which is rooted at the vertex v and contains the edge e.

Theorem. If T is a tree with $\Delta \geq 4$ and $3,4 \notin D_{\Delta}(T)$, then T is Type 1.

Proof. The proof is proceeded by induction on $|T|$. The theorem holds clearly if $|T| = 5$. Let T be a tree with $|T| \geq 6$, $\Delta \geq 4$ and $3,4 \notin D_{\Delta}(T)$. If T is a generalized star, it is easy to construct a $(2,1)$-total labelling of T using the label set $B = \{0,1,\cdots,\Delta+1\}$. Thus, assume that T is not a generalized star.

If T contains a leaf v adjacent to a minor vertex u, then $T - v$ has a $(2,1)$-total labelling f using $B = \{0,1,\cdots,\Delta+1\}$, by the induction hypothesis (or using the fact that every tree T^* has $\lambda_2^t(T^*) \leq \Delta(T^*) + 2$). Since $deg(u) \leq \Delta - 1$, there exist at most $\Delta - 2 + 3 = \Delta + 1$ forbidden labels for the edge vu and at most four forbidden labels for the vertex v. By $|B| = \Delta + 2$, we can first extend f to vu and then to v. Hence, assume that no leaf is adjacent to a minor vertex.

First, suppose that $\Delta \geq 5$. T contains a configuration: i.e., a minor vertex v adjacent to $deg(v) - 1$ major handles $x_1, x_2, \cdots, x_{deg(v)-1}$ and the other vertex y. Let

$$T' = T - \bigcup_{i=1}^{deg(v)-1} (L(x_i) \cup \{x_i\}).$$

By the induction hypothesis, T' has a $(2,1)$-total labell -ing f using $B = \{0,1,\cdots,\Delta+1\}$. If $f(v) \notin \{0, \Delta+1\}$, then f can be extended into a $(2,1)$-total labelling of. T Assume that $f(v) \in \{0, \Delta+1\}$. Relabel v with a label from $B \setminus \{0, \Delta+1, f(y), f(vy)-1, f(vy), f(vy)+1\}$. As $|B \setminus \{0, \Delta+1, f(y), f(vy)-1, f(vy), f(vy)+1\}|, \geq |B| - 6 = \Delta + 2 - 6 \geq 5 + 2 - 6 = 1$ such a relabelling is feasible. f can be extended to T. Next, suppose that $\Delta = 4$. For each possible case, we construct a subtree T' and let f be a $(2,1)$-total labelling of T' using the label set $B = \{0,1,\cdots,5\}$. Afterwards, we are going to extend f to the whole tree T by a series of labelling or relabelling.

1.If T contains a path $x_1x_2x_3x_4$ such that d(x_2)=2 and x_1 is a major handle.

We set $T' = T - u - L(u)$. If $f(v) \notin \{0,5\}$, that f can be extended to T. So assume that $f(v) \in \{0,5\}$, say $f(v) = 0$. We first label u with 5, and then label uv with a label in $\{2,3\} \setminus \{f(vw)\}$, where w is the neighbor of v different from u. f can be extended to T.

2.If T contains a path $x_1x_2x_3x_4$ such that d(x_2)=3, x_1 is a major handle and

$x_2\ y_1\ y_2$ is a path, where y_1 is a neighbor of x_2 with y_1 is a major handle.We set $T' = T - \{x_1, x_2\} - L(x_1) - L(x_2)$.

(2.1) Assume that $deg(x_3) = 2$. Let $x'_3 \neq x$ denote the second neighbor of x_3. By the symmetry of labels in B, let $f(x'_3) \in \{0,1,2\}$. In order to extend f to T, we only need to handle the following cases

(2.1a) $f(x'_3) = 0$. It follows that $f(x_3 x'_3) \in \{2,3,4,5\}$.

If $f(x_3 x'_3) = 2$, we relabel x_3, x_3x, x with $4,1,3$, respectively.

If $f(x_3 x'_3) = 3$, we relabel x_3, x_3x, x with $5,1,4$, respectively.

If $f(x_3 x'_3) = 4$, we relabel x_3, x_3x, x with $1,5,3$, respectively.

If $f(x_3 x'_3) = 5$, we relabel x_3, x_3x, x with $1,4,2$, respectively.

(2.1b) $f(x'_3) = 1$. It follows that $f(x_3 x'_3) \in \{3,4,5\}$.

If $f(x_3 x'_3) = 3$, we relabel x_3, x_3x, x with $5,2,4$, respectively.

If $f(x_3 x'_3) = 4$, we relabel x_3, x_3x, x with $2,5,3$, respectively.

If $f(x_3 x'_3) = 5$, we relabel x_3, x_3x, x with $0,4,2$, respectively.

(2.1c) $f(x'_3) = 2$. It follows that $f(x_3 x'_3) \in \{0,4,5\}$.

If $f(x_3 x'_3) = 0$, we relabel x_3, x_3x, x with $3,5,1$, respectively.

If $f(x_3 x'_3) = 4$, we relabel x_3, x_3x, x with $1,5,3$, respectively.

If $f(x_3 x'_3) = 5$, we relabel x_3, x_3x, x with $1,4,2$, respectively.

(2.2) Assume that $deg(x_3) = 4$. $f(x_3) \in \{0,5\}$, say $f(x_3) = 0$. Thus, $f(x_3x) \in \{2,3,4,5\}$. If $f(x_3x) = 2$, we relabel x with 4. If $f(x_3x) \in \{3,4,5\}$, we relabel x with 1. f can be extended to T.

3.If T contains a path $x_1x_2x_3x_4$ such that x_1 is a major handle and x_2 is a weak major handle.We set $T' = T - \{u_1, u_2\} - L(u_1) - L(u_2)$. By symmetry, we may assume that $f(v) \in \{0,1,2\}$. If $f(v) = 0$, the proof is similar to Case (2.2).

Assume that $f(v) = 1$. Then, $f(vu) \in \{3,4,5\}$. If $f(vu) \in \{4,5\}$, then we relabel u with 2. If $f(vu) = 3$, it follows that $f(vv_1) \in \{4,5\}$. We first exchange the labels of vu and vv_1, and then relabel u with 2 and v with 0. f can be extended to T. Assume that $f(v) = 2$. Then, $f(vu) \in \{0,4,5\}$. If $f(vu) = 0$, we relabel u with 3. If $f(vu) \in \{4,5\}$, we relabel u with 1. f can be extended to T.

4.If T contains path $x_1x_2x_3x_4$ such that x_1 is a major handle and x_2 is $\Delta-$degree, $x_2\ y_1\ y_2$ is a path, where y_1 is a neighbor of x_2 with y_1 is a major handle, the neighbor of x_2 other than x_1x_3 and y_1 is a leaf.The proof is divided into the following two subcases:

(4.1) $deg(z) = 4$. We set

$T' = T - \{u, v, u_1, u_2, v_1, v_2\} - L(u_1) - L(u_2) - L(v_1) - L(v_2)$., $f(z) \in \{0,5\}$, say $f(z) = 0$. So, $f(zw) \in \{2,3,4,5\}$. it suffices to construct the following labelling:
If $f(zw) = 2$, we relabel w, wu, wv, u, v with $4,0,1,3,3$, respectively.
If $f(zw) = 3$, we relabel w, wu, wv, u, v with $1,4,5,2,2$, respectively.
If $f(zw) = 4$, we relabel w, wu, wv, u, v with $2,0,5,3,1$, respectively.
If $f(zw) = 5$, we relabel w, wu, wv, u, v with $2,0,4,3,1$, respectively.
(4.2) $deg(z) \le 3$. We set
$T' = T - \{u_1, u_2\} - L(u_1) - L(u_2) + wy$, where
$y \notin V(T)$ is a new vertex. It is easy to see that $|T'| < |T|$ and $\Delta(T') = \Delta(T) = 4$. Since $deg(z) \le 3$, $1 \notin D_\Delta(T')$. By the induction hypothesis, T' has a $(2,1)$ -total labelling f using $B = \{0,1,\cdots,5\}$. Since w is of degree 4 in T' , $f(w) \in \{0,5\}$, say $f(w) = 0$. Thus, $f(wu) \in \{2,3,4,5\}$. Remove y and extend f to T in this way: If $f(wu) = 2$, we relabel u with 4 ; If $f(wu) \in \{3,4,5\}$, we relabel u with 1 .

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