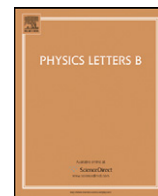


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## Renormalization group equations in resonance chiral theory

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## ABSTRACT

The use of the equations of motion and meson field redefinitions allows the development of a simplified resonance chiral theory Lagrangian: terms including resonance fields and a large number of derivatives can be reduced into corresponding  $\mathcal{O}(p^2)$  resonance operators, containing the lowest possible number of derivatives. This is shown by means of the explicit computation of the pion vector form-factor up to next-to-leading order in  $1/N_C$ . The study of the renormalization group equations for the corresponding couplings demonstrates the existence of an infrared fixed point in the resonance theory. The possibility of developing a perturbative  $1/N_C$  expansion in the slow running region around the fixed point is shown here.

© 2009 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/3.0/).1.  $1/N_C$  expansion in resonance chiral theory

Resonance chiral theory ( $R\chi T$ ) is a description of the Goldstone-resonance interactions within a chiral invariant framework [1,2]. The pseudo-Goldstone fields  $\phi$  are introduced through the exponential realization  $u(\phi) = \exp(i\phi/\sqrt{2}F)$ . The standard effective field theory momentum expansion is not valid in the presence of heavy resonance states and an alternative perturbative counting is required.  $R\chi T$  takes then the formal  $1/N_C$  expansion as a guiding principle [3]: at leading order (LO) the interaction terms in the Lagrangian with a number  $k$  of meson fields (and their corresponding couplings) scale as  $\sim N_C^{1-k/2}$  [3]. For instance, the resonance masses are counted as  $\mathcal{O}(N_C^0)$ , the three-meson vertex operators are  $\mathcal{O}(N_C^{-1/2})$ , etc. The subdominant terms in the Lagrangian will have then subleading  $1/N_C$  scalings with respect to these ones. If our action is now arranged according to the number of resonance fields in the operators, one has

$$\mathcal{L}_{R\chi T} = \mathcal{L}^{\text{GB}} + \mathcal{L}^{R_i} + \mathcal{L}^{R_i R_j} + \mathcal{L}^{R_i R_j R_k} + \dots, \quad (1)$$

where the resonance fields  $R_i$  are classified in  $U(n_f)$  multiplets, with  $n_f$  the number of light quark flavours.

*A priori*,  $\mathcal{L}_{R\chi T}$  might contain chiral tensors of arbitrary order. However, for most phenomenological applications, terms with a large number of derivatives tend to violate the asymptotic short-distance behaviour of QCD Green Functions and form factors [4]. Likewise, it is possible to prove that in the chiral limit the most general  $S-\pi\pi$  interaction is provided by the operator of lowest or-

der in derivatives [5]. A similar proof can be derived for the  $V-\pi\pi$  vertex [6]

The operators of the leading  $R\chi T$  Lagrangian without resonance fields are those from  $\chi PT$  at  $\mathcal{O}(p^2)$  [7],

$$\mathcal{L}_{\text{LO}}^{\text{GB}} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle. \quad (2)$$

The Goldstone fields, given by  $u(\phi)$ , enter in the Lagrangian through the covariant tensors  $u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\}$  and  $\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$ , with  $\ell_\mu$ ,  $r_\mu$  and  $\chi$  respectively the left-current, right-current and scalar-pseudoscalar density sources [1,8]. Likewise, it is convenient to define  $f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$ , with  $F_{L,R}^{\mu\nu}$  the left- and right-field strength tensors [1,8].

In the case of the vector multiplet, one has at LO in  $1/N_C$  the operators [1]

$$\mathcal{L}_{\text{LO}}^V = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle, \quad (3)$$

where the antisymmetric tensor field  $V^{\mu\nu}$  is used in  $R\chi T$  to describe the spin-1 mesons [1,2,7], with the kinetic and mass terms,

$$\mathcal{L}_{\text{Kin}}^V = -\frac{1}{2} \langle V_{\lambda\nu} \nabla^\lambda \nabla_\rho V^{\rho\nu} \rangle + \frac{1}{4} M_V^2 \langle V_{\mu\nu} V^{\mu\nu} \rangle. \quad (4)$$

The covariant derivative is defined through  $\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X]$ , with the chiral connection  $\Gamma_\mu = \frac{1}{2}\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\}$ . Other works have widely studied alternative representations of the vector mesons such as general four-vector formalisms [9,10], the gauged chiral model [11,12] or the hidden local symmetry framework [9,13,14].

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The naive dimensional analysis of the operators tells us that the tree-level LO amplitudes will scale like  $\mathcal{M} \sim p^2$  in the external momenta  $p$ . At one loop, higher power corrections  $\mathcal{M} \sim p^4 \ln(-p^2)$  are expected to arise. These logs will come together with ultraviolet (UV) divergences  $\lambda_\infty p^4$ , requiring new operators subleading in  $1/N_C$ , with a larger number of derivatives with respect to the leading order ones. These  $\mathcal{O}(p^4)$  corrections look, in principle, potentially dangerous if the momenta become of the order of the resonance masses. Since there is no characteristic scale  $\Lambda_{R\chi T}$  that suppresses them for  $p \ll \Lambda_{R\chi T}$ , they could become as important as the  $\mathcal{O}(p^2)$  leading order contributions.

In the present case of the  $\pi\pi$  vector form-factor (VFF), in order to fulfill the one-loop renormalization one needs the subleading operators [15]

$$\begin{aligned} \mathcal{L}_{\text{NLO}}^{\text{GB}} &= -i\tilde{L}_9 \langle f_+^{\mu\nu} u_\mu u_\nu \rangle, \\ \mathcal{L}_{\text{NLO}}^V &= X_Z \langle V_{\lambda\nu} \nabla^\lambda \nabla_\rho \nabla^2 V^{\rho\nu} \rangle + X_F \langle V_{\mu\nu} \nabla^2 f_+^{\mu\nu} \rangle \\ &\quad + 2iX_G \langle V_{\mu\nu} \nabla^2 [u^\mu, u^\nu] \rangle. \end{aligned} \quad (5)$$

However, the  $\mathcal{L}_{\text{NLO}}^V$  couplings  $X_{Z,F,G}$  are not physical by themselves: it is impossible to fix them univocally from the experiment. Indeed, since these subleading  $\mathcal{L}_{\text{NLO}}^V$  operators are proportional to the equations of motion, one finds that  $\mathcal{L}_{\text{NLO}}^V$  can be fully transformed into the  $M_V$ ,  $F_V$ ,  $G_V$  and  $\tilde{L}_9$  terms and into other operators that do not contribute to the VFF by means of meson field redefinitions [15,16]. Furthermore, higher derivative resonance operators that could contribute to the VFF at tree-level can be also removed from the Lagrangian in the same way [6].

One of the aims of this Letter is to show how the potentially dangerous higher power corrections arising at next-to-leading order (NLO) [17,18] actually correspond to a slow logarithmic running of the couplings of the LO Lagrangian. We will make use of the equations of motion of the theory and meson field redefinitions to remove analytical corrections going like higher powers of the momenta. This leaves just the problematic log terms  $p^4 \ln(-p^2)$ , which will be minimized by means of the renormalization group equations and transformed into a logarithmic running of  $M_V$ ,  $F_V$ ,  $G_V$  and  $\tilde{L}_9$ .

## 2. The pion vector form-factor

In order to exemplify the procedure, the rest of the Letter is devoted to a thorough study of the pion vector form-factor in the chiral limit:

$$\langle \pi^-(p_1) \pi^0(p_2) | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} (p_1^\mu - p_2^\mu) \mathcal{F}(q^2), \quad (6)$$

with  $q \equiv p_1 + p_2$ .

The renormalized amplitude shows the following general structure in terms of renormalized vertex functions and the renormalized vector correlator,

$$\mathcal{F}(q^2) = \mathcal{F}(q^2)_{1\text{PI}} + \frac{\Phi(q^2)\Gamma(q^2)}{F^2} \frac{q^2}{M_V^2 - q^2 - \Sigma(q^2)}, \quad (7)$$

with  $\Sigma(q^2)$  the vector self-energy, and  $\mathcal{F}(q^2)_{1\text{PI}}$ ,  $\Phi(q^2)$  and  $\Gamma(q^2)$  being provided, respectively, by the 1-particle-irreducible (1PI) vertex functions for  $J_V^\mu \rightarrow \pi\pi$ ,  $J_V^\mu \rightarrow V$  and  $V \rightarrow \pi\pi$  (Fig. 1). Thus, at large  $N_C$ ,  $R\chi T$  yields for the VFF,

$$\mathcal{F}(q^2) = 1 + \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2}. \quad (8)$$

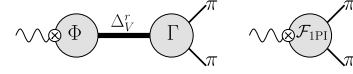


Fig. 1. 1PI-topologies contributing to the pion VFF.

Although QCD contains an infinite number of hadronic states, only a finite number of them is considered for most phenomenological analyses [4]. We will include in the  $R\chi T$  just the lightest mesons (Goldstones and vectors). Likewise, only the lowest threshold contributions are taken into account in this work – the massless two-Goldstone cut – and loops from higher cuts will be assumed to be renormalized in a  $\mu$ -independent scheme, such that they decouple as far as the total energy remains below their production threshold (see for instance Appendix C.2 in Ref. [18]). In general, all the considerations along the Letter will be restricted to this range. Only at the end we will allow a small digression about speculations and results for our form-factor calculation in the high-energy limit.

The one-loop calculation produces a series of ultraviolet divergences that require of subleading operators in  $1/N_C$  ( $X_Z$ ,  $X_F$ ,  $X_G$ ,  $\tilde{L}_9$ ) to fulfill the renormalization of the vertex functions [15,16]:

$$\begin{aligned} \Sigma(q^2) &= -2q^4 X_Z - \frac{n_f}{2} \frac{2G_V^2}{F^2} \frac{q^4}{96\pi^2 F^2} \ln \frac{-q^2}{\mu^2}, \\ \Gamma(q^2) &= -4\sqrt{2} X_G q^2 \\ &\quad + G_V \left[ 1 - \frac{n_f}{2} \left( 1 - \frac{G_V^2}{F^2} \right) \frac{q^2}{96\pi^2 F^2} \ln \frac{-q^2}{\mu^2} + \Delta_t(q^2) \right], \\ \Phi(q^2) &= F_V - 2\sqrt{2} X_F q^2 - \frac{n_f}{2} \frac{2G_V}{F^2} \frac{q^2}{96\pi^2} \ln \frac{-q^2}{\mu^2}, \\ \mathcal{F}(q^2)_{1\text{PI}} &= 1 + \frac{2q^2 \tilde{L}_9}{F^2} + \Delta_t(q^2) \\ &\quad - \frac{n_f}{2} \left( 1 - \frac{G_V^2}{F^2} \right) \frac{q^2}{96\pi^2 F^2} \ln \frac{-q^2}{\mu^2}, \end{aligned} \quad (9)$$

$n_f$  being the number of light flavours and  $\Delta_t$  the finite and  $\mu$ -independent contribution from the triangle diagram that contains the  $t$ -channel exchange of a vector meson,

$$\Delta_t(q^2) = \frac{n_f}{2} \frac{2G_V^2}{F^2} \frac{M_V^2}{16\pi^2 F^2} \hat{\Delta}_t(q^2/M_V^2), \quad (10)$$

with  $\hat{\Delta}_t(x) = [\text{Li}_2(1+x) - \text{Li}_2(1)](\frac{1}{x} + \frac{5}{2x} + 1) + \ln(-x)(\frac{1}{x} + 2) - \frac{1}{x} - \frac{9}{4}$ , vanishing at zero like  $\hat{\Delta}_t = -\frac{1}{12}x \ln(-x) + \frac{35}{72}x + \mathcal{O}(x^2)$  and growing for large  $x$  like a double log,  $\hat{\Delta}_t \sim -\frac{1}{2} \ln^2 |x|$ . For the energies we are going to study ( $|q^2| \lesssim 1 \text{ GeV}^2$ ), it will have little numerical impact.

The couplings that appear in the finite vertex functions in (9) are the renormalized ones. The NLO running of  $G_V(\mu)$  induces then a residual  $\mu$ -dependence in (9) at next-to-next-to-leading order (NNLO) which allows us to use the renormalization group techniques to resum harmful large radiative corrections. However, the NLO operators  $X_{Z,F,G}$  from (5) are found to be proportional to the equations of motion [15,16]. The physical meaning of this is that these parameters can be never extracted from the experiment in an independent way. The amplitudes rather depend on effective combinations of them and other couplings. Thus, it is possible to transform the renormalized part of these operators into the  $M_V$ ,  $F_V$ ,  $G_V$  and  $\tilde{L}_9$  operators and other terms that do not contribute to the amplitude by means of a convenient meson field redefinition  $V \rightarrow V + \xi(X_Z, X_F, X_G)$  [15,16]:

$$\begin{aligned}
X_{Z,F,G} &\xrightarrow{\xi} 0, \\
\tilde{L}_9 &\xrightarrow{\xi} \tilde{L}_9 + (\sqrt{2}X_F G_V + 2\sqrt{2}F_V X_G - X_Z F_V G_V), \\
F_V &\xrightarrow{\xi} F_V + (2X_Z F_V M_V^2 - 2\sqrt{2}X_F M_V^2), \\
G_V &\xrightarrow{\xi} G_V + (2X_Z G_V M_V^2 - 4\sqrt{2}X_G M_V^2), \\
M_V^2 &\xrightarrow{\xi} M_V^2 + 2X_Z M_V^4.
\end{aligned} \tag{11}$$

Hence, it is possible then to consider a suitable shift that removes the renormalized operators  $X_{Z,F,G}$  from the Lagrangian, encoding their information and running in the remaining  $\tilde{L}_9$ ,  $F_V$ ,  $G_V$  and  $M_V$ . Although this transformation  $\xi$  depends on the renormalization scale  $\mu$  (as it depends on the renormalized  $X_{Z,F,G}$ ), the resulting theory is still equivalent to the original one. The redundant parameters  $X_{Z,F,G}$  are removed for every  $\mu$  from the vector self-energy and vertex functions in (9), inducing in the remaining couplings a running ruled by the renormalization group equations (RGE),

$$\begin{aligned}
\frac{1}{M_V^2} \frac{\partial M_V^2}{\partial \ln \mu^2} &= \frac{n_f}{2} \frac{2G_V^2}{F^2} \frac{M_V^2}{96\pi^2 F^2}, \\
\frac{\partial G_V}{\partial \ln \mu^2} &= G_V \frac{n_f}{2} \frac{M_V^2}{96\pi^2 F^2} \left( \frac{3G_V^2}{F^2} - 1 \right), \\
\frac{\partial F_V}{\partial \ln \mu^2} &= 2G_V \frac{n_f}{2} \frac{M_V^2}{96\pi^2 F^2} \left( \frac{F_V G_V}{F^2} - 1 \right), \\
\frac{\partial \tilde{L}_9}{\partial \ln \mu^2} &= \frac{n_f}{2} \frac{1}{192\pi^2} \left( \frac{F_V G_V}{F^2} - 1 \right) \left( 1 - \frac{3G_V^2}{F^2} \right).
\end{aligned} \tag{12}$$

If one now takes the VFF expression given by (7) and (9) and sets  $\mu^2 = Q^2$  (with  $Q^2 \equiv -q^2$ ), it gets the simple form,

$$\begin{aligned}
\mathcal{F}(q^2) &= -\frac{2Q^2 \tilde{L}_9(Q^2)}{F^2} + [1 + \Delta_t(q^2)] \\
&\times \left[ 1 - \frac{F_V(Q^2) G_V(Q^2)}{F^2} \frac{Q^2}{M_V^2(Q^2) + Q^2} \right],
\end{aligned} \tag{13}$$

with the evolution of the couplings with  $Q^2$  prescribed by the RGE (12). Notice that if the subleading terms  $\tilde{L}_9$  and  $\Delta_t(q^2)$  are dropped, one is left with the resummed expression at leading log for the LO form-factor (8). The residual NNLO dependence could be estimated by varying  $\mu^2$  around  $Q^2$ , in the range between  $Q^2/2$  and  $2Q^2$ , as it is often done in RGE analysis. In this scheme,  $M_V$  would be related to the pole mass through  $M_{V,\text{pole}}^2 = M_V^2(\mu) + \frac{n_f}{2} \frac{2G_V^2}{F^2} \frac{M_V^4}{96\pi^2 F^2} \ln \frac{M_V^2}{\mu^2} = M_V^2(M_V)$ .

The first two RGE refer to  $M_V$  and  $G_V$  and form a closed system with the trajectories given by

$$G_V^2 = \frac{F^2}{3} (1 + \kappa^3 M_V^6), \tag{14}$$

with  $\kappa$  an integration constant. It leads to the solution

$$\frac{1}{M_V^2} + \kappa f(\kappa M_V^2) = -\frac{2n_f}{3} \frac{1}{96\pi^2 F^2} \ln \frac{\mu^2}{\Lambda^2}, \tag{15}$$

with  $f(x) = \frac{1}{6} \ln \frac{x^2+2x+1}{x^2-x+1} - \frac{1}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} - \frac{\pi}{6\sqrt{3}} = \mathcal{O}(x)$ , and  $\Lambda$  an integration constant. Since  $-\frac{2\pi}{3\sqrt{3}} \leq f(x) \leq 0$ , the term  $\kappa f(\kappa M_V^2)$  in (15) becomes negligible for very low momentum,  $\mu \ll \Lambda$ , producing a logarithmic running. The parameters  $M_V$  and  $G_V$  show then an infrared fixed point at  $M_V = 0$  and  $G_V = F/\sqrt{3}$ . The corresponding flow diagram is shown in Fig. 2. The same happens for  $F_V$  and  $\tilde{L}_9$ , which freeze out when  $\mu \rightarrow 0$ .  $F_V$  tends to

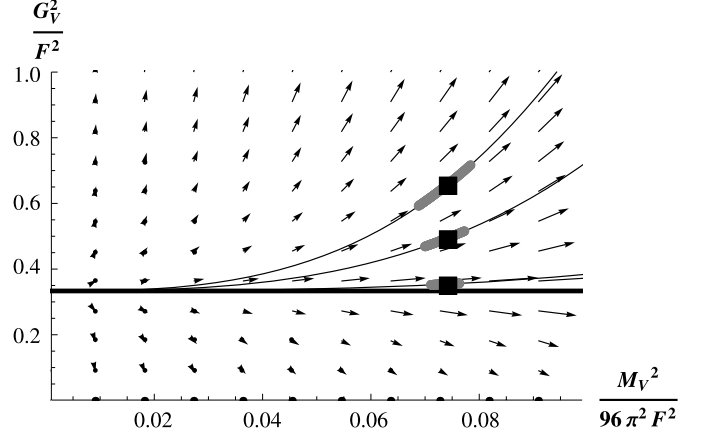


Fig. 2. Renormalization group flow for  $M_V^2$  and  $G_V^2$ . The points  $M_V(\mu_0) = 775$  MeV and  $G_V(\mu_0) = 75, 65, 55$  MeV are plotted with filled squares, together with their trajectories for  $n_f = 2$  (thin black lines). For illustrative purposes, and assuming those as initial conditions for  $\mu_0 = 770$  MeV, we also show their running between  $\mu = 500$  MeV and  $\mu = 1$  GeV (thick gray lines). The horizontal line represents the  $G_V$ -fixed point at  $G_V^2 = F^2/3$ .

the infrared fixed point  $\sqrt{3}F$  (and hence  $F_V G_V \xrightarrow{\mu \rightarrow 0} F^2$ ) and  $\tilde{L}_9(\mu)$  goes to a constant value  $\tilde{L}_9(0)$ .

An analogous renormalization group analysis of the fixed points was also performed in Ref. [14] within a Wilsonian approach in the Hidden Local Symmetry framework [13].

### 3. A digression on high-energy constraints

Although the present computation is only strictly valid below the first two-meson threshold with at least one resonance (since these channels were not included here), one is allowed to speculate about the high-energy behaviour of our expression (13).

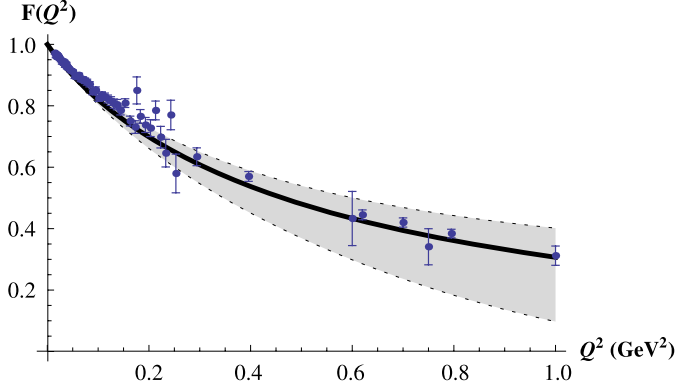
It is remarkable that the value of the resonance couplings at the infrared fixed point,  $F_V G_V = F^2$  and  $3G_V^2 = F^2$ , coincides with those obtained if one demands at large- $N_c$  the proper high energy behaviour of, respectively, the VFF [4,15] and the partial-wave scattering amplitude [19].

Likewise, it is also interesting to note that the requirement that our one-loop form factor (13) vanishes when  $Q^2 \rightarrow \infty$  [4,20,21] leads to these same solutions: the constraints  $F_V G_V = F^2$  and  $3G_V^2 = F^2$  are required to freeze out the running of  $\tilde{L}_9$  and  $F_V G_V$  and to kill the  $q^2 \ln(-q^2)$  and  $q^0 \ln(-q^2)$  short-distance behaviour; additionally,  $\tilde{L}_9 = 0$  is needed in order to remove the remaining  $\mathcal{O}(q^2)$  terms at  $q^2 \rightarrow \infty$ .

The reason for this interplay between short distances and fixed points is that in our case the massless logarithms come always together with the UV-divergence  $\lambda_\infty$  in the form  $[\lambda_\infty + \ln \frac{Q^2}{\mu^2}]$ . In similar terms, these logs are related to the one-loop spectral function  $\text{Im} \mathcal{F}(q^2)$ . When only the two-Goldstone channel  $\phi\phi$  is open, in the chiral limit, the optical theorem states

$$\begin{aligned}
\text{Im} \mathcal{F}_{\pi\pi} &= \sum_{\phi\phi} T_{\pi\pi \rightarrow \phi\phi}^* \mathcal{F}_{\phi\phi} \\
&\stackrel{q^2 \rightarrow \infty}{=} \frac{n_f}{2} \left[ \frac{q^2}{96\pi F^2} \left( 1 - \frac{3G_V^2}{F^2} \right) + \mathcal{O}(q^0) \right] \\
&\times \left[ \left( 1 - \frac{F_V G_V}{F^2} \right) + \mathcal{O}(q^{-2}) \right],
\end{aligned} \tag{16}$$

with  $\mathcal{F}_{\phi\phi}$  the vector form-factor with  $\phi\phi$  in the final state and  $T_{\pi\pi \rightarrow \phi\phi}$  the  $I = J = 1$  partial-wave scattering amplitude. If the VFF spectral function is demanded to vanish at high energies then



**Fig. 3.** Illustrative comparison of the VFF at NLO and Euclidean data ( $Q^2 = -q^2 > 0$ ) [22]. We have used  $M_V(\mu_0) = 775$  MeV,  $F_V(\mu_0)G_V(\mu_0) = 3G_V(\mu_0)^2 = F^2$  and  $\bar{L}_9(\mu_0) = 0$  for  $\mu_0 = 770$  MeV (solid line). The mild relevance of  $G_V$  within the loops in the Euclidean range is represented by the gray band, which shows the VFF for a large variation of the input  $G_V(\mu_0)^2$ , in the range from zero up to  $F^2$ , while  $F_V(\mu_0)G_V(\mu_0)$ ,  $M_V(\mu_0)$  and  $\bar{L}_9(\mu_0)$  are kept the same as before.

one necessarily needs  $1 - \frac{3G_V^2}{F^2} = 0$  and  $1 - \frac{F_V G_V}{F^2} = 0$ . These conditions eliminate the  $\mathcal{O}(q^2 \ln(-q^2))$  and  $\mathcal{O}(q^0 \ln(-q^2))$  logarithms and their accompanying  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^0)$  UV-divergences. Regarding the tree-level contribution to the VFF,  $\mathcal{F}(q^2)^{\text{tree}} \stackrel{q^2 \rightarrow \infty}{\approx} \frac{2\bar{L}_9}{F^2} q^2 + (1 - \frac{F_V G_V}{F^2}) + \mathcal{O}(q^{-2})$ , one finds then that there is no running for  $\bar{L}_9$  nor for  $F_V G_V$ . The freezing in the running of the remaining combination,  $G_V^2$ , is due to the  $\mathcal{O}(q^0)$  behaviour of the one-loop  $T_{\pi\pi \rightarrow \pi\pi}$  spectral function at  $q^2 \rightarrow \infty$ ,

$$\text{Im} T_{\pi\pi \rightarrow \pi\pi} = \sum_{\phi\phi} |T_{\pi\pi \rightarrow \phi\phi}|^2 \stackrel{q^2 \rightarrow \infty}{\approx} \frac{n_f}{2} \left[ \frac{q^2}{96\pi F^2} \left( 1 - \frac{3G_V^2}{F^2} \right) + \mathcal{O}(q^0) \right]^2, \quad (17)$$

after imposing the former constraint  $1 - \frac{3G_V^2}{F^2} = 0$ . The  $\mathcal{O}(q^2)$  logs and the accompanying UV-divergences are absent in the  $\pi\pi$  partial-wave amplitude. Hence, no running is induced in the corresponding  $\mathcal{O}(q^2)$  combination of couplings that is relevant for the scattering amplitude, which in  $R\chi T$  happens to be  $G_V^2$ .

In Fig. 3, the VFF (13) is compared with Euclidean data in the range  $Q^2 = 0-1$  GeV<sup>2</sup> [22], with the values  $M_V = 775$  MeV,  $F_V = 3G_V = \sqrt{3}F$ ,  $\bar{L}_9 = 0$  for  $\mu_0 = 770$  MeV. Although our expression neglects contributions from higher channels, these values produce a fair agreement with the data in Fig. 3. Nevertheless, the non-zero pion mass is responsible of a 20% decreasing in the  $\rho$  width [23] and an accurate description of both the spacelike and timelike data requires the consideration of the pseudo-Goldstone masses. The residual NNLO dependence was estimated by varying the scale  $\mu^2$  between  $Q^2/2$  and  $2Q^2$  in (9), finding a shift of less than 0.3% for the inputs under consideration.

#### 4. Perturbative regime in the $1/N_C$ expansion

Independently of any possible high energy matching [4,21], what becomes clear from the RGE analysis is the existence of a region in the RGE space of parameters (around the infrared fixed point at  $\mu \rightarrow 0$ ) where the loops produce small logarithmic corrections. Although we start with a formally well defined  $1/N_C$  expansion, this is the range where the perturbative description actually makes sense for the renormalized  $R\chi T$  amplitude. In an analogous way, although the fixed order perturbative QCD cross-section calculations are formally correct for arbitrary  $\mu$  (and independent

of it), perturbation theory can only be applied at high energies. In our case, the parameter that actually rules the strength of the resonance-Goldstone interaction in the RGE of Eq. (12) is

$$\alpha_V = \frac{n_f}{2} \frac{2G_V^2}{F^2} \frac{M_V^2}{96\pi F^2}, \quad (18)$$

which goes to zero as  $\mu \rightarrow 0$ . Thus, although the formal expansion parameter of the theory is  $1/N_C$ , this is the actual quantity that appears in the calculation suppressing the subleading contributions. Since at lowest order  $\alpha_V$  is just the ratio of the vector width and mass,  $\Gamma_V/M_V \simeq 0.2$  [23,24], a  $1/N_C$  expansion of  $R\chi T$  is meaningful as far as the concerning resonance is narrow enough (as it happens here).

In the case of broad states or more complicate processes, the identification of the parameter that characterizes the strength of the interaction can be less intuitive. Nonetheless, perturbation theory will be meaningful in  $R\chi T$  as far as there is an energy range where this strength-parameter becomes small, bringing along a slow running for the resonance couplings in the problem.

#### 5. Conclusions

Although, *a priori*,  $R\chi T$  needs of higher derivative operators at NLO, not all the new couplings are physical. The combination of meson field redefinitions and renormalization group equations allows us to develop an equivalent theory without redundant operators and where undesirable higher power corrections are absent.

The study of the running of the couplings entering in the pion vector form-factor shows the existence of an infrared fixed point. The couplings enjoy a slow logarithmic running in the low-energy region around  $\mu \rightarrow 0$ , where the resonance-Goldstone strength parameter  $\alpha_V$  is small enough. It is in this range of momenta that perturbation theory in  $1/N_C$  makes sense for  $R\chi T$ .

The physical amplitudes are then understood in terms of renormalized resonance couplings which evolve with  $\mu$  in the way prescribed by the RGE. A perturbative description of the observable will be possible as far as the loops keep their running slow.

These considerations are expected to be relevant for the study of other QCD matrix elements. In particular, they may play an important role in the case of scalar resonances. The width and radiative corrections are usually rather sizable in the spin-0 channels. The possible presence of fixed points and slow-running regions in other amplitudes (e.g. the pion scalar form-factor) will be studied in future analyses.

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