Predictive supersymmetry from criticality

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Abstract

Motivated by the absence of any direct signal of new physics so far, we present a simple supersymmetric model in which the up-type Higgs mass-squared parameter $m_{H_u}^2$ crosses zero at a scale close to the weak scale. Such a theory may be motivated either by the conventional naturalness picture or by the landscape picture with certain assumptions on prior probability distributions of parameters. The model arises from a simple higher dimensional setup in which the gauge and Higgs fields propagate in the bulk while the matter fields are on a brane. The soft supersymmetry breaking parameters receive contributions from both moduli and anomaly mediations, and their weak scale values can be analytically solved for in terms of a single overall mass scale $M$. The expected size for $M$ depends on whether one adopts the naturalness or landscape pictures, allowing for the possibility of distinguishing between these two cases. We also present possible variations of the model, and discuss more general implications of the landscape picture in this context.

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1. Introduction

Weak scale supersymmetry is an extremely attractive idea. It is based on a beautiful theoretical construction of enlarging the spacetime structure to anticommuting variables, and is supported indirectly by the successful unification of gauge couplings at high energies \cite{1}. It also stabilizes the large hierarchy between the weak and the Planck scales due to a cancellation between the standard model and its superpartner contributions to the Higgs potential. In fact, this latter property has been one of the strongest motivations for weak scale supersymmetry.

From the experimental point of view, the most exciting aspect of weak scale supersymmetry is the existence of various superpartners at the TeV scale. Can we predict the spectrum of these superparticles? We already know, from the absence of a large new contribution to flavor changing neutral current and $CP$-violating processes, that the superparticle spectrum must have a certain special structure, such as flavor universality. Moreover, non-discovery of both superparticles and a light Higgs boson at LEP II puts strong constraints on the spectrum. This typically leads to fine-tuning of order a few percent in reproducing the correct scale for electroweak symmetry breaking, and is called the supersymmetric fine-tuning problem (for a recent analysis, see \cite{2}). It seems plausible that successfully addressing this problem provides a key to the correct theory at the TeV scale, and to a fundamental mechanism or principle behind it.

There are two different approaches towards the supersymmetric fine-tuning problem. A conventional approach is to search for a model that is “natural”. In the context of the minimal supersymmetric standard model (MSSM), this amounts to looking for a model in which the supersymmetry breaking mass-squared parameter for the up-type Higgs field, $m_{H_u}^2$, is somehow suppressed at
towards larger values, the relation $
u \tilde{\mu}$ probability distributions of various couplings and top Yukawa coupling, to $|\mu|$, that the requirement of weak scale is a result of environmental selection. In this case, if $yt$ leads to a nontrivial tension between the values of $m_{\tilde{\mu}}$ and other generic supersymmetry breaking squared masses $\tilde{m}^2$—typically it requires a small hierarchy between $|m_{\tilde{\mu}}^2|$ and $\tilde{m}^2$. In the context of gravity mediation—arguably the “simplest” mediation of supersymmetry breaking—this implies that we must find a model in which the “tree-level” and “radiative” contributions to $m_{\tilde{\mu}}^2$ cancel to a large extent, either “accidentally”, as in the scenario of [3], or by some mechanism, as in the model of [4,5].

An alternative approach towards the problem appears if we live in “the multiverse”, rather than the universe. Motivated partly by Weinberg’s successful “prediction” of the observed value of the cosmological constant [6], and partly by the suggestion that string theory has an exponentially large number of discrete nonsupersymmetric vacua [7], it has become increasingly plausible that our universe is only one among a tremendous number of various universes, in which physical constants can take vastly different values. This “landscape” hypothesis may lead to a significant change in our notion of naturalness, and it is reasonable to consider the supersymmetric fine-tuning problem in this context. It has recently been argued that the landscape picture may lead to a small hierarchy between the Higgs mass-squared parameter and the scale of superparticle masses $\tilde{m}$ under certain assumptions on the probability distributions of various couplings and $\tilde{m}$ [8]. Specifically, under the existence of statistical “pressures” pushing $\tilde{m}$ towards larger values, the relation $v^2 \sim \tilde{m}^2/8\pi^2$ may be obtained from environmental selection, where $v$ is the electroweak scale. Moreover, if the parameter $\mu$ also scans independently with $\tilde{m}$ and if the holomorphic supersymmetry breaking Higgs mass-squared parameter, $\mu B$, is sufficiently small at a high scale, then we obtain $v^2 \sim |\mu|^2 \sim m_{\tilde{\mu}}^2 \sim \tilde{m}^2/8\pi^2$.

It is interesting that the two different pictures described above can both lead to a scenario in which the supersymmetry breaking parameter $m_{\tilde{\mu}}^2$ crosses zero at a scale not much different from the weak scale. In fact, the two pictures may not be totally unrelated. Suppose, for example, that the ultraviolet theory at the gravitational or unification scale gives universal scalar squared masses $m_0^2$ ($> 0$), as in the minimal supergravity scenario [11]. In this case, the parameter $m_{\tilde{\mu}}^2$ crosses zero at a renormalization scale of order the weak scale, as long as the gaugino masses are small compared with $|m_0^2|^{1/2}$. This phenomenon is known as focus point behavior, and this class of theories was claimed to be natural [3], since $|m_{\tilde{\mu}}^2|$ is relatively small at the weak scale and thus no strong cancellation is required between the two terms in the right-hand side of Eq. (1). An immediate criticism of this argument, based on the conventional viewpoint, is that if the value of the top Yukawa coupling, $y_t$, were different, then the property of $|m_{\tilde{\mu}}^2|$ being small at the weak scale would be destroyed—in other words, the fractional sensitivity of the weak scale, $v$, to a variation of the top Yukawa coupling, $\partial \ln v^2/\partial \ln y_t$, is very large. This criticism, however, is not appropriate if the property of $|m_{\tilde{\mu}}^2| \ll \tilde{m}^2$ at the weak scale is a result of environmental selection. In this case, if $y_t$ were changed, the scale of supersymmetry breaking masses, $\tilde{m}$, would also be changed in such a way that $|m_{\tilde{\mu}}^2| \sim \tilde{m}^2/8\pi^2 \ll \tilde{m}^2$ at the “new” weak scale $\sim |m_{\tilde{\mu}}^2|$. As a result, we always find $|m_{\tilde{\mu}}^2| \ll \tilde{m}^2$ at the “weak scale” regardless of the value of $y_t$. The observed value of $y_t$ will then be determined as a result of (another) environmental selection, presumably a combination of the consideration in [12] and others.

From the point of view of model-building, i.e. searching for the model describing physics above the TeV scale, we may then be motivated to look for a model in which $|m_{\tilde{\mu}}^2|$ is suppressed compared with $\tilde{m}^2$ at the weak scale, i.e. $|m_{\tilde{\mu}}^2|$ crosses zero at a scale close to the weak scale. If this property arises without a strong cancellation between the “tree-level” and “radiative” contributions to $|m_{\tilde{\mu}}^2|$, then we can consider that the model is natural in the conventional sense. Even if it arises due to a strong cancellation, however, the model may still be interesting since it can arise as a result of environmental selection under certain circumstances. Note that the requirement of $|m_{\tilde{\mu}}^2|$ being suppressed at the weak scale is different from the one that the Higgs mass-squared parameter, $m_{\tilde{\mu}}^2 \simeq m_{\tilde{\mu}}^2 + |\mu|^2$, is suppressed at the weak scale, which should always be the case. We are requiring that the cancellation (if any) must take place “inside” $m_{\tilde{\mu}}^2$, and not between $m_{\tilde{\mu}}^2$ and $|\mu|^2$.

Since the condition of $|m_{\tilde{\mu}}^2| \ll \tilde{m}^2$ at the weak scale gives only one constraint on the large number of soft supersymmetry breaking masses, we clearly need other guiding principles to narrow down the possibilities and obtain predictions on the superparticle masses. Without having a detailed knowledge of physics at the gravitational or unification scale, we simply take the viewpoint that the physics at that scale should be “simple”—sufficiently simple that the resulting supersymmetry breaking masses also take a simple form. This clearly makes sense if we take the conventional “universe” picture, and may also be supported by the absence of large supersymmetric flavor-changing and CP-violating contributions (which would arise if the superparticle masses were chaotic).

\footnote{This conclusion depends on the probability distributions of parameters. For example, if certain couplings do not “scan”, the low-energy theory may be split supersymmetry [9], or simply the standard model [10]. The assumption here corresponds to an independent scanning of $\tilde{m}$ and the supersymmetric couplings. It is interesting that supersymmetry may still play an important role in addressing the gauge hierarchy problem even in the existence of a landscape of vacua, under certain mild assumptions.}
In the context of the “multiverse” (or landscape) picture, we merely hope that such a “simple” model is statistically preferred by the vacuum counting in the fundamental theory. In practice, if a sufficiently “simple” model defined at the high energy scale gives \( |m_H^2| \ll \hat{m}^2 \) at the weak scale, we consider it interesting regardless of the level of cancellation occurring in \( m_H^2 \).

In this Letter we present an example of such models. The model is very simple, and arises as a low-energy effective theory of higher dimensional theories in which the standard model gauge and Higgs fields propagate in the bulk while matter fields are confined on a \((3+1)\)-dimensional brane. The compactification scale is of the order of the unification scale, and the low-energy effective theory below this scale is simply the MSSM. Upon stabilizing a volume modulus by a simple gaugino condensation, the superparticle masses in the low-energy theory receive contributions from both moduli and anomaly mediations. We find that this model gives vanishing \( m_H^2 \) at a scale (very) close to the weak scale, satisfying the criterion described above. All the supersymmetry breaking parameters, except for the holomorphic Higgs mass-squared parameter, are predicted (essentially) in terms of a single overall mass parameter \( M \), with the resulting spectrum showing a pattern distinct from conventional supergravity and gauge mediation models. This model gives a “non-hierarchical” spectrum of \( M_3 \sim m_f \) \((= O(M))\), where \( M_3 \) and \( m_f \) represent generic gaugino and sfermion masses, although variations of the model giving the “hierarchical” spectrum of \( M_3 \sim m_f / 4\pi \) \((\sim |\mu|)\) may also be considered. The scale of the overall mass parameter \( M \) depends on which of the naturalness or landscape pictures we take, but will be generally in the range between \( O(\nu) \) and a multi-TeV scale. For the Higgs sector, we simply assume that the required structures for the \( \mu \) and \( \mu B \) parameters are prepared, presumably by statistical preference in the case that the landscape picture is adopted.

The Letter is organized as follows. In the next section we present our model and derive predictions on the supersymmetry breaking masses which are independent of the picture adopted. In Section 3 we discuss the implications of the model in both the “universe” and “multiverse” pictures, and argue that the difference can appear in the size of the overall mass scale for the superparticle masses. In Section 4 we conclude by giving discussions on the issue of obtaining predictions for the superparticle masses in the landscape picture. In particular, we present several possible scenarios arising from a landscape of vacua in the “vicinity” of the particular model in Section 2, and elucidate under what conditions, or with what additional assumptions, the setup can give strong predictions on the superparticle spectrum.

### 2. Model

In this section we present a simple model that has the property that the soft Higgs mass squared is vanishing at a scale close to the weak scale. We consider that physics above the unification scale is higher dimensional, and that the standard model gauge and Higgs fields propagate in the bulk while the matter fields are localized on a \((3+1)\)-dimensional brane. The low-energy effective theory is then given by the following 4D supergravity action:

\[
S = \int d^4x \sqrt{-g} \left[ \int d^4\theta \, C^{-1} C (-3(T + T^\dagger) + (T + T^\dagger)H^\dagger H + M^\dagger M) \\
+ \left\{ \int d^2\theta \left( \frac{1}{4} T W^{\alpha \beta} V^\alpha_{\beta} + C^3 W \right) + \text{h.c.} \right\} \right],
\]

where \( C \) is the chiral compensator superfield, \( T \) is the moduli superfield parameterizing the volume of the compact dimensions, and \( g_{\mu\nu} \) is the metric in the superconformal frame. The superfields \( H \) and \( M \) collectively represent the Higgs and matter fields of the MSSM, i.e. \( H = H_u, H_d \) and \( M = Q_i, U_i, D_i, L_i, E_i \) with \( i \) the generation index, and the superpotential \( W \) contains the usual MSSM Yukawa couplings \( W_{\text{Yukawa}} \). This setup naturally arises, for example, if grand unification is realized in higher dimensions above the compactification scale \([13,2]\).

In Eq. (2), we have assumed that moduli fields other than \( T \), e.g. ones parameterizing the shape of the compact dimensions, (if any) are absent in the low-energy theory. We have also assumed that higher order terms, e.g. terms involving powers of \( 1 / (T + T^\dagger) \), are sufficiently suppressed, which is technically natural since the theory is weakly coupled at the compactification scale.

To obtain realistic phenomenology at low energies, the moduli field \( T \) must be stabilized. We assume that the stabilization superpotential for \( T \) takes the simple form arising from a single gaugino condensation. The superpotential \( W \) is then given by

\[
W = W_{\text{Yukawa}} + A e^{-aT} + c,
\]

where \( a \) is a real constant. The parameters \( A \) and \( c \) are constants of order unity and the gravitino mass \((\ll 1)\), respectively (in units of the 4D gravitational constant \( M_{\text{Pl}} \approx 10^{18} \) GeV, which is taken to be 1). These parameters can be taken real in the presence of an approximate shift symmetry for \( \text{Int}T \). Since the superpotential of Eq. (3) stabilizes the modulus \( T \) at a supersymmetry preserving...
D-branes, located at the bottom of a warped throat. (The configuration of the gauge, Higgs and matter fields described before corresponds to identifying them as \( D7 \)-, \( D7 \)- and \( D3 \)-brane fields, respectively.)

The minimization of the potential, derived from Eqs. (2)–(4), leads to supersymmetry breaking (\( F \)-term) expectation values for the compensator \( C \) and the modulus \( T \):

\[
\frac{F_C}{C} = \frac{c}{(T + T^\dagger)^{3/2}} = m_{3/2}^3,
\]

\[
\frac{F_T}{T + T^\dagger} = \frac{2}{a(T + T^\dagger)^{m_{3/2}}} \equiv M_0,
\]

where \( m_{3/2} \) is the gravitino mass. This implies that there is a little hierarchy between the sizes of \( F_C \) and \( F_T \):

\[
\frac{F_C / C}{F_T / (T + T^\dagger)} = \frac{a}{2} (T + T^\dagger) = \ln \left( \frac{M_{Pl}}{m_{3/2}} \right),
\]

so that the supersymmetry breaking parameters in the MSSM receive comparable contributions from both moduli and anomaly mediations [15]. Here, we have recovered the gravitational constant \( M_{Pl} \) in the right-hand side of Eq. (7). Note that the above Eqs. (5)–(7) are valid up to corrections of \( O(1/8\pi^2) = O(1/\ln(M_{Pl}/m_{3/2})) \).

The supersymmetry breaking masses in the present model show the behavior of a reduced effective messenger scale, \( M_{mess} \), due to an interplay between the moduli and anomaly mediated contributions [16] (for a simple proof, see [2]). By solving renormalization group equations at the one-loop level, the soft supersymmetry breaking masses at an arbitrary renormalization scale \( \mu_R \) are given by

\[
M_a(\mu_R) = M_0 \left[ 1 - \frac{b_a}{8\pi^2} \hat{s}_0^2(\mu_R) \ln \left( \frac{M_{mess}}{\mu_R} \right) \right],
\]

\[
m_i^2(\mu_R) = M_0^2 \left[ r_I - 4 \left\{ \gamma_I(\mu_R) - \frac{1}{2} \frac{d\gamma_I(\mu_R)}{d \ln \mu_R} \ln \left( \frac{M_{mess}}{\mu_R} \right) \right\} \ln \left( \frac{M_{mess}}{\mu_R} \right) \right],
\]

\[
A_{IJK}(\mu_R) = M_0 \left[ -(r_I + r_J + r_K) + 2 \left( \gamma_I(\mu_R) + \gamma_J(\mu_R) + \gamma_K(\mu_R) \right) \ln \left( \frac{M_{mess}}{\mu_R} \right) \right].
\]

where \( M_a, m_i^2 \) and \( A_{IJK} \) are gaugino masses, non-holomorphic scalar squared masses, and scalar trilinear interactions (with the Yukawa couplings factored out), respectively. The indices \( I, J, K \) run over \( Q_i, U_i, D_i, L_i, E_i, H_u, H_d \), with \( r_I \)'s defined by \( r_Q = r_U = r_D = r_L = r_{E_l} = 0 \) and \( r_{H_u} = r_{H_d} = 1/3 \); \( \hat{s}_0(\mu_R) \) are the running gauge couplings at a scale \( \mu_R \), and \( b_a \) and \( \gamma_I(\mu_R) \) are the beta-function coefficients and the anomalous dimensions, respectively, defined by \( d(1/g_i^2)/d \ln \mu_R = -b_a/8\pi^2 \) and \( d \ln Z_I / d \ln \mu_R = -2\gamma_I \). Where \( Z_I \) is the wavefunction renormalization factor for the field \( I \). The parameter \( M_{mess} \) is given by

\[
M_{mess} = f \left( \frac{M_{Pl}}{m_{3/2}} \right)^{1/2},
\]

where \( M_{Pl} \) represents the compactification scale, which is of the order of the unification scale \( \approx 10^{16} \) GeV, and \( f \) is an \( O(1) \) coefficient depending, e.g., on \( A \) in Eq. (3). The parameter \( M_0 \) is defined in Eq. (6) and represents the overall mass scale for the supersymmetry breaking parameters.

The expressions of Eqs. (8)–(10) show that the supersymmetry breaking masses in this model take a very simple form:

\[
M_1 = M_2 = M_3 = M_0,
\]

\[
m_i^2 = m_i^2 = m_i^2 = m_i^2 = m_i^2 = m_i^2 = m_i^2 = m_i^2 = M_0^2,
\]

\[
A_{ii} = A_{ji} = A_{kj} = -M_0.
\]

\[\text{The notation here follows that of Ref. [5] except that the sign convention for } A_{IJK} \text{ is reversed.}\]
data alone. The value of $\tan \beta$ (a physical threshold at an intermediate scale. superparticle mass scale: $\mu_C$) is determined by the Higgs sector parameters, which are flavor universal in the present model, as $A_U$, $A_D$ and $A_e$. (Our sign convention for the soft supersymmetry breaking parameters follows that of the SUSY Les Houches Accord [17].) Here, we have suppressed possible higher order corrections of $O(M_{mess}^2/8\pi^2)$ in Eq. (13).4 Note that the spectrum of Eqs. (12)–(14) is identical with what would be obtained at the compactification scale in simple moduli mediated (or equivalently Scherk–Schwarz) supersymmetry breaking [18]. The low-energy soft supersymmetry breaking parameters, defined at the weak scale $m_{mess}$, and evolved them down using the one-loop renormalization group equations of the MSSM. (The two-loop renormalization group equations have been used for the supersymmetric parameters.) Note that while the soft supersymmetry breaking parameters are depicted only for $\mu_R \leq M_{mess}$, it should be understood that they are, in fact, generated at a scale of order $M_U$. (The squark and slepton squared masses are negative at scales above $M_{mess}$, but this does not cause a problem since our vacuum is metastable at the time scale of the age of the universe.) Below, we will choose $M_0$ and $M_{mess}$ to be free parameters of our analysis, since these parameters have $O(1)$ uncertainties that cannot be determined from the low-energy data alone. The value of $\tan \beta$ is determined by the Higgs sector parameters, $\mu$ and $\mu_B$, whose origin we leave unspecified.5

A remarkable feature of the superparticle masses in Fig. 1 is that the up-type Higgs mass-squared parameter crosses zero at the superparticle mass scale:

$$m_H^2(\mu_C) = 0 \quad \text{at} \quad \mu_C \simeq M_0.$$

While the precise value of $\mu_C$—the scale where $m_H^2$ crosses zero—depends on the values of $M_{mess}$ and $\tan \beta$, it is of order $M_0$ for a wide range of these parameters. Note that $\mu_C$ does not depend on $M_0$, since the renormalization group equations are homogeneous in $M_0$. (If we take $M_U$ to be a free parameter, instead of $M_{mess}$, then $\mu_C$ depends slightly on $M_0$ for a fixed $M_U$, through a weak dependence of $M_{mess}$ on $M_0$.) In the example of $M_0 = 400$ GeV in Fig. 1, the value of $\mu_C$ is within a factor of 2 from $M_0$ for $M_{mess} \approx (10^9 \sim 10^{10})$ GeV for $\tan \beta \approx (5 \sim 30)$. (In fact, a value of $M_{mess}$ giving $\mu_C$ within a factor of 2 from $M_0$ can be found for

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4 Approximate flavor universality for these corrections must be assumed in the case that $M_0$ is not much larger than a TeV.

5 We note that essentially all the conclusions below also apply in any theory in which the soft supersymmetry breaking masses take the form of Eqs. (12)–(14) at the scale of Eq. (15). These boundary conditions might arise, e.g., in a theory where the fundamental scale is at an intermediate scale or in a theory where there is a physical threshold at an intermediate scale.
The scale $M$ where the top Yukawa coupling, $y_t$, breaking masses, except for the holomorphic Higgs mass-squared parameter $\mu_B$ and $\mu_C$, and we have neglected the small effects from the bottom Yukawa coupling, $A_u$, $A_d$, and $A_e$, which are negative, the absolute values are plotted. The scalar trilinear interaction parameters for the first two generations, $A_u$, $A_d$, and $A_e$, are not shown.

$\tan \beta \approx (3 \sim 50)$. The mass squared for the right-handed stau, however, becomes negative at the weak scale for $\tan \beta \gtrsim 30$.) These results do not change significantly by including higher order effects, e.g. the two-loop renormalization group effects, or by varying the top quark mass within a $2\sigma$ range of the recently reported value, $m_t = 171.4 \pm 2.1$ GeV [19]. At the leading order, we find from Eq. (9) that the scale $\mu_C$ is given by

$$\mu_C \approx M_{\text{mess}} \exp \left( \frac{8\pi^2(6y_t^2 - 3g_2^2 - \sqrt{64g_3^2y_t^4 - 36y_t^4 + 15g_2^4}}{32g_3^2y_t^2 - 36y_t^4 + 18g_2^2y_t^2 + 3g_2^2} \right) \approx 10^{-7} M_{\text{mess}},$$

where the top Yukawa coupling, $y_t$, and the $SU(3)_C$ and $SU(2)_L$ gauge couplings, $g_3$ and $g_2$, are evaluated at the scale $\mu_R \approx \mu_C$, and we have neglected the small effects from the bottom Yukawa coupling, $y_b$, and the $U(1)_Y$ gauge coupling, $g_1$. To obtain $\mu_C \approx M_0$, a larger $M_0$ requires a larger $M_{\text{mess}} \propto M_0$. For fundamental parameters of the theory, this implies $f \propto M_0$ (see Eq. (11)). Since the superparticle mass scale $M_0$ is close to $\mu_C$, we can evaluate the soft supersymmetry breaking parameters at the superparticle mass scale $M_0$ approximately by substituting Eq. (17) into Eqs. (12)–(14). This gives predictions for all the supersymmetry breaking masses, except for the holomorphic Higgs mass-squared parameter $\mu B$ (and $m_0^2$), in terms of the overall mass scale $M_0$ and the running gauge and Yukawa couplings at that scale. Note that we even do not have to know the value of $M_{\text{mess}}$—for given values of $M_0$ and $\tan \beta$, which we need to obtain the values of the Yukawa couplings, we can predict all the supersymmetry breaking parameters with the assumption of Eq. (16).

In Fig. 2, we present the predicted values of the supersymmetry breaking parameters for $M_0 = 400$ GeV as a function of $\tan \beta$. The left panel shows the predictions of the gaugino masses, $M_\alpha$, the first two generation sfermion masses, $m_{\tilde{f}}$, and the down-type Higgs boson mass, $m_{H_d}$. The right panel shows the third generation scalar trilinear interaction parameters, $A_{t,b,r}$, and the third generation sfermion masses, $m_{\tilde{f}}$. The scalar trilinear interaction parameters for the first two generations, $A_{u,d,e}$, are not shown. The predictions for $M_\alpha$, $m_{\tilde{f}}$, $m_{\tilde{Q}_3}$, $m_{\tilde{U}_3}$, $m_{\tilde{D}_3}$, $A_3$ (and $A_{\tilde{g}}$, which is not shown) are rather insensitive to the value of $\tan \beta$, while those for $m_{H_d}$, $m_{\tilde{d}_3}$, $m_{\tilde{E}_3}$, $A_b$, $A_e$ (and $A_{\tilde{d}}$) have weak sensitivities to $\tan \beta$. (The sensitivity is strong for $m_{\tilde{E}_3}$ for $\tan \beta \gtrsim 30$ where it approaches zero.) For $\tan \beta \sim 10$, the predicted ratios among the soft supersymmetry breaking parameters (including the first two generation scalar trilinear interaction parameters) are given by

$$M_1 : M_2 : M_3 : m_{\tilde{Q}} : m_{\tilde{U}} : m_{\tilde{D}} : m_{\tilde{L}} : m_{\tilde{E}} : m_{\tilde{Q}_3} : m_{\tilde{U}_3} : m_{\tilde{D}_3} : m_{\tilde{L}_3} : m_{\tilde{E}_3} : m_{H_d} : A_u : -A_d : -A_e : -A_t : -A_b : -A_e \approx 0.71 : 0.91 : 1.8 : 1.5 : 1.4 : 1.4 : 0.52 : 0.30 : 1.3 : 1.1 : 1.4 : 0.51 : 0.28 : 1.1 : 2.2 : 2.6 : 1.3 : 1.7 : 2.5 : 1.3.$$

Here, we have presented the numbers in units of $M_0$. Note that these numbers are subject to errors of $O(10\%)$, coming from “higher order” effects, for quantities associated with the colored superparticles. (The errors for quantities that are not associated with the colored superparticles are smaller.) In the case that we take the “universe” picture, these effects include the fact that the superparticle mass scale $M_0$ does not “coincide” with $\mu_C$, although the two are of the same order. This source of errors does not exist if we adopt the “multiverse” picture, where $M_0$ and $\mu_C$ are very close.
The predictions of Eq. (18) have a sensitivity to the value of $M_0$, but only through the running of the gauge and Yukawa couplings. As a result, these predictions, and the predictions for the ratios of the supersymmetry breaking masses obtained from Fig. 2, are valid in a wide range of $M_0$ with only small corrections. In the case that $M_0$ is in a multi-TeV region (as will be the case in the “multiverse” picture; see the next section), the corrections are still smaller than about 10%. For example, the predictions of Eq. (18) change for $M_0 = 3 \text{ TeV}$ to

$$
\begin{align*}
M_1 : M_2 : M_3 & : m_{\tilde{G}} : m_{\tilde{U}} : m_{\tilde{D}} : m_{\tilde{L}} : m_{\tilde{E}} : m_{\tilde{\tilde{G}}} : m_{\tilde{\tilde{U}}} : m_{\tilde{\tilde{D}}} : m_{\tilde{\tilde{L}}} : m_{\tilde{E}} \\
& : m_{\mu_{\tilde{d}}} : -A_{u} : -A_{d} : -A_{e} : -A_{\tau} : -A_{\nu} \\
& \simeq 0.63 : 0.90 : 1.8 : 1.5 : 1.4 : 1.4 : 0.56 : 0.33 : 1.3 : 1.0 : 1.4 : 0.56 : 0.31 \\
& : 1.1 : 2.2 : 2.7 : 1.4 : 1.8 : 2.5 : 1.4,
\end{align*}
$$

but these are not much different from the ones in Eq. (18).

We finally discuss the Higgs sector of the model. To have the correct electroweak symmetry breaking phenomenology, the $\mu$ and $\mu B$ parameters must be of order the weak scale. In particular, the classical contribution to $B \equiv \mu B / \mu$ of the gravitino mass must be suppressed. Here we simply assume that the value of $B$ is sufficiently suppressed, for example the case that $B$ is somehow dominated by the quantum (anomalous) contribution: $B = 2 M_0 [\gamma_{H_u}(\mu_R) + \gamma_{H_d}(\mu_R)] \ln(M_{\text{mess}}/\mu_R)$. (This expression for $B$ is, in fact, a solution to the one-loop renormalization group equation.) We may also consider the case that $B$ is generated by the expectation value of a singlet field through $W = \lambda S H_u H_d$ (at least in the context of the “universe” picture), whose effect on the evolutions of the Higgs soft masses are suppressed if the value of $\lambda$ is sufficiently small.

3. Implications

We have seen that the model given by Eqs. (2)–(4) provides the predictions of Eq. (18), which depend only very weakly on the values of $\tan \beta$ and $M_0$ (see Fig. 2 and Eq. (19)). The expected range for the overall scale $M_0$, however, differs depending on the scenario we consider. In this section we discuss this issue, as well as other phenomenological implications of the model.

Let us first take the conventional “universe” picture, i.e. the overall scale $M_0$ does not effectively “scan”. In this case, our guiding principle will be “naturalness”, i.e. the observed scale of electroweak symmetry breaking, $v \simeq 174$ GeV, should be a “typical” value in the parameter space of the model. For fixed values of the supersymmetric couplings, this is rather clear in our model because of the suppression of $m_{H_u}^2$ relative to the other soft masses at the weak scale. (We assume that the Higgs sector is arranged such that there is no large $\mu B$ term of order $\mu M_{3/2}$.) Naturalness of the model becomes clearer when compared with other, typical supersymmetry breaking models. Consider, for example, a model in which the supersymmetry breaking parameters of Eqs. (12)–(14) are generated at the unification scale, $M_U \approx 10^{16}$ GeV, as in the pure moduli mediated model of [18]. In this case, the size of the up-type Higgs mass squared $|m_{H_u}^2|$, relative to the other soft masses, is much larger at the weak scale. (The evolutions of soft masses in the two models are depicted in Fig. 3.) An important point is that while $|m_{H_u}^2|$ keeps increasing towards the infrared from the scale $\mu_c$ where $m_{H_u}^2$ crosses zero, dragged by increasing $M_1$ through $g_1$ and $g_7$, the right-handed slepton masses $m_{\tilde{\nu}}$ stay almost constant, as they receive only small contributions through $g_1$. As a consequence, if the crossing scale $\mu_c$ is much larger than the weak scale, we would obtain a hierarchy $|m_{H_u}^2| / m_{\tilde{\nu}}^2 \gg 1$ at the weak scale (see Fig. 3b), leading to fine-tuning between

![Fig. 3. Evolutions of soft supersymmetry breaking masses in the model of Section 2 (the left panel), and in the model where the supersymmetry breaking parameters of Eqs. (12)–(14) are given at the unification scale, $M_U \approx 10^{16}$ GeV (the right panel). Here, we have taken $M_0 = 400$ GeV and $\tan \beta = 10$ in both cases. Solid lines represent the gaugino masses ($M_3$, $M_2$ and $M_1$ from the top), dashed lines the first two generation sfermion masses ($m_{\tilde{G}}, m_{\tilde{U}}, m_{\tilde{D}}, m_{\tilde{L}}$ and $m_{\tilde{E}}$ from the top), and dotted lines the Higgs mass parameter ($m_{H_u}$ and $m_{H_d}$ from the top).](image-url)
we expect that $M$ requires a somewhat small value of $B$ line). The $\mu$ obtained $dP$ the statistical pressure of way. As discussed in Ref. [8], this leads to a small hierarchy between the weak scale and the scale of the sfermion masses. With $\text{breaking parameters}$ (including the ones generated through direct interactions with the uplifting sector, if any) scale in a similar general the distribution of $\text{in a multi-TeV region. This implies that in this picture all the superparticles, other than the higgsinos, as well as all the $CP$-odd, the

$$|\mu|^2$$ terms in Eq. (1) under the LEP II constraint of $m_\pi \gtrsim 100 \text{ GeV}$. Our model avoids this because $\mu_C$ is close to the weak scale (see Fig. 3a).

Since there is no particular reason that $\mu_C$ is extremely close to the scale of superparticle masses, $|\mu_C - M_0|/M_0 \ll 1$, we expect that there is some discrepancy between the two quantities, e.g. $|\ln(\mu_C/M_0)| = O(1)$. The value of $m_{H/H_0}$ at the weak scale is then not much smaller than $m_\pi^2$, so that the overall scale $M_0$ is not much larger than the weak scale to avoid fine-tuning in Eq. (1). We typically expect $400 \text{ GeV} \lesssim M_0 \lesssim 1 \text{ TeV}$, where the lower bound comes from $m_\pi \gtrsim 100 \text{ GeV}$. With these values of $M_0$, the physical mass for the lightest neutral Higgs boson, $M_{\text{lightest neutral Higgs boson}}$, can easily exceed the experimental lower bound of $M_{\text{lightest neutral Higgs boson}} \gtrsim 114 \text{ GeV}$ [20]. This is because our model provides a relatively large value of $A_t$, at the weak scale, so that we can avoid the Higgs-boson mass bound with relatively small top squark masses. (The importance of large $A_t$, in reducing fine-tuning was particularly emphasized in Refs. [2, 5].)

In Fig. 4 we plot $M_{\text{Higgs}}$, calculated using FeynHiggs 2.4.1 [21], as a function of $M_0$ for $\tan \beta = 5$ (dotted line) and $\tan \beta = 10$ (solid line). The $\mu$ parameter is chosen to be $\mu = 150 \text{ GeV}$ arbitrarily, but the dependence of the result on $\mu$ is very weak. From the figure, we expect that $M_{\text{Higgs}} \lesssim 120 \text{ GeV}$. The value of $B$ is given by $B \approx M_0^2/\mu \tan \beta$, so that the preferred $\tan \beta$ range of $5 \lesssim \tan \beta \lesssim 20$ requires a somewhat small value of $B$ of order $0.1 M_0^2/\mu$. The sensitivity of the weak scale to variations of the supersymmetric parameters is also not so large in this model, since there is no superparticle that has a particularly large mass compared with others. The lightest supersymmetric particle (LSP) is either (a neutral component of) the higgsino or the right-handed stau. In the former case the LSP may be the dark matter of the universe if it is produced nonthermally. In the latter case it will have to decay into some neutral particle, e.g. the axino—the supersymmetric partner of the axion, which may compose the dark matter.

Let us now turn to the case that we adopt the “multiverse”, or the “landscape”, picture. More precisely, we now assume that the overall supersymmetry breaking parameter $M_0$ has different values in different “parts” of the multiverse, or in different vacua of the theory, with larger values preferred by some positive power $n$: $dP \propto dM_0^n$, where $P$ is the probability distribution function. In general the distribution of $M_0$ depends on the structure of the supersymmetry breaking (uplifting) sector and the sector that produces the constant term $c$ in the superpotential, and the assumption of $dP \propto dM_0^n$ corresponds typically to tree-level supersymmetry breaking (since tree-level supersymmetry breaking naturally prefers larger breaking scales). Note that since environmental selection “locks” the value of $d$ in Eq. (4) as $d \approx |c|^2$, through the condition for the cosmological constant being small [6], all the supersymmetry breaking parameters (including the ones generated through direct interactions with the uplifting sector, if any) scale in a similar way. As discussed in Ref. [8], this leads to a small hierarchy between the weak scale and the scale of the sfermion masses. With the statistical pressure of $dP \propto dM_0^n$, the sfermion masses $m$ are pushed towards larger values, but not beyond the scale where the Higgs mass-squared parameter $m_H^2$ crosses zero, since a larger $m$ would lead to the recovery of electroweak symmetry in the Higgs sector, a situation hostile to the existence of observers. This leads to $|m_H^2| = O(\tilde{m}^2/8\pi^2) \ll \tilde{m}^2$, since $m_H^2$ becomes zero around the scale $\tilde{m}$. Moreover, if the parameter $\mu$ scans independently, and if the parameter $B$ is sufficiently small at a high scale, then we obtain $|\mu|^2 = O(\tilde{m}^2/8\pi^2) \ll \tilde{m}^2$ and thus also $|m_{H_0}^2| = O(\tilde{m}^2/8\pi^2) \ll \tilde{m}^2$—the property we found in our model (see Eq. (16)). In order for this argument to be significant, the model must satisfy the conditions for $\mu$ and $B$ given above, which we assume to be the case. An implication of this picture is that the overall scale parameter $M_0$ is expected to be somewhat larger than the weak scale: $M_0^2/8\pi^2 \sim |m_H^2| \sim |\mu|^2 \sim v^2$. The precise hierarchy depends on the strength of the pressure $n$, but we generically expect $M_0$ to be in a multi-TeV region. This implies that in this picture all the superparticles, other than the higgsinos, as well as all the $CP$-odd, the
heavier \( CP \)-even, and the charged Higgs bosons, \( A^0, H^0 \) and \( H^\pm \), have masses in this region \((\sim M_0 \gtrsim 1 \text{ TeV})\). The ratios between the superparticle masses are still given by Eq. (19), and the three Higgs boson masses are given by \( m_{A^0} \sim m_{H^0} \sim m_{H^\pm} \sim m_{H_d} \).

The spectrum just described can lead to quite distinct phenomenology. For example, if \( M_0 \) is somewhat large, e.g. \( M_0 \gtrsim 2 \text{ TeV} \), all the superparticles and heavier Higgs bosons are beyond the discovery reach of the LHC, except for the higgsinos. Thus, the LHC will effectively see the (one Higgs doublet) standard model, plus possibly the higgsinos. Discoveries of superparticles, however, may be possible if \( M_0 \) is lower. The LSP is the neutral component of the higgsinos, which may be the dark matter of the universe. For example, if \( M_0 \simeq 3 \text{ TeV} \), the gravitino mass is \( m_{3/2} \simeq 100 \text{ TeV} \), and the moduli field mass is \( m_T = O(1000–10000 \text{ TeV}) \). The moduli field is expected to dominate the energy density of the early universe, and then it decays into the superparticles and gravitinos, which in turn decay into the LSP. With these masses, the constraint from big-bang nucleosynthesis can be avoided (see, e.g. [22]) and the LSP may compose the dark matter, presumably with some (small) amount of dilution of its energy density. Alternatively, the LSP may decay into a lighter particle, e.g. the axino.

4. Discussions: Predictions from the landscape?

Since it has been difficult to find ways of experimentally “testing” the landscape picture, it is important to consider what implications it can have on the low-energy spectrum and what predictions we can get from it when combined with additional assumptions.

In this Letter we discussed a framework which may either arise from the naturalness consideration in the conventional picture or from the landscape picture under certain circumstances, and presented an example model which leads to strong predictions of the superparticle masses. The essential ingredients of the framework were:

(i) The up-type Higgs mass-squared parameter \( m_{H_u}^2 \) crosses zero at a scale close to the superparticle mass scale.

(ii) The structure of the theory at the unification (or compactification) scale is “simple” as far as the observable sector is concerned.

The reason that this can lead to strong predictions, despite the fact that each ingredient is not necessarily giving a very strong constraint, is that a generic theory satisfying (ii) does not typically lead to the property of (i), so that the combination of these two criteria can be a very strong constraint on models. The model we presented has a fairly simple structure at the unification scale, arising from a simple setup in higher dimensions, and yet gives a vanishing \( m_{H_u}^2 \) at a scale close to the weak scale. All the supersymmetry breaking parameters, except for the holomorphic Higgs mass-squared parameter \( \mu B \) (and \( m_{H_t}^2 \)), are predicted in terms of a single overall mass scale \( M_0 \) (and \( \tan \beta = \langle H_u \rangle / \langle H_d \rangle \)). The parameter \( M_0 \) is expected to be in a multi-TeV region if it scans with a preference towards larger values.

On the other hand, it is clear that the specific model discussed above is not a unique model satisfying (i) and (ii). For example, we can consider the situation in which the scalar masses are approximately universal at a high scale, with somewhat suppressed gaugino masses. As observed in [3], this leads to suppressed \( m_{H_u} \) at the weak scale. In fact, this situation can be realized in the setup of Eqs. (3), (4) if the supersymmetry breaking (uplifting) sector gives universal scalar masses through direct interactions with the observable sector fields.\(^6\) In the context of the landscape picture, with a statistical pressure acting towards larger values for the supersymmetry breaking parameters, this can lead to relatively degenerate scalar masses in a multi-TeV region and gaugino masses in a few hundred GeV region, with the relative gaugino masses still given by Eq. (18). The higgsino masses are comparable to the gaugino masses, and the value of \( \tan \beta \) will be relatively large of \( O(10) \), for an unsuppressed \( B \) parameter. (The possibility of a relatively unsuppressed \( B \) parameter is an advantage of unsuppressed scalar masses.) In either of these models, the spectrum of superparticles is special such that it leads to a suppressed value of \( m_{H_u}^2 \) at the weak scale, which appears to us as a result of an accidental cancellation due to the specific values of the observed gauge and Yukawa couplings.

While the conditions of (i) and (ii) are keys to obtain strong predictions for the superparticle spectrum, neither is a necessary consequence of the landscape picture. Indeed, it is possible that environmental selection leads to the Higgs mass-squared parameter being small due to cancellation between \( m_{H_u}^2 \) and \( \mu^2 \), and not just inside \( m_{H_u}^2 \), in which case (i) is not necessarily satisfied. Moreover, a simple ultraviolet structure may not be preferred by the statistics of landscape vacua, and the condition (ii) may also be violated. In these cases we lose a strong constraint on the superparticle spectrum, reducing predictivity, but may still get an interesting pattern for the spectrum. For example, landscape statistics may prefer the case in which the supersymmetry breaking (uplifting) sector gives somewhat random scalar masses through direct interactions with the observable sector, in the setup of Eqs. (3), (4). (Flavor universality may still have to be assumed unless these masses are very large.) With the statistical pressure of pushing the overall mass scale to larger values, we find the Higgs mass-squared parameter somewhat suppressed compared with the scalar superparticle masses. The spectrum will then contain the scalar superparticles and the higgsinos in a multi-TeV region, whose masses do not obey simple relations. The gaugino masses, however, may still be of order a few hundred GeV and obey Eq. (18) in the case that the direct

\(^6\) While completing this Letter, we received Ref. [23] which discusses scenarios with similar spectra, but not in the context of the landscape picture, i.e. the picture of scanning parameters.
effect from the supersymmetry breaking sector is suppressed in the gauge kinetic functions. Deviations from Eq. (18), however, can also occur, e.g., if the moduli-stabilization and uplifting sectors deviate from the minimal form of Eqs. (3), (4), in which case the gaugino masses unify at a scale that is not necessarily the intermediate scale of Eq. (15), or if the direct effect is not suppressed in the gauge kinetic functions, in which case the gaugino masses are of order a multi-TeV. The value of $\tan \beta$ will generically be of $O(1)$ for an unsuppressed value for the $B$ parameter.

To summarize, we have argued that both the conventional naturalness picture and the landscape picture (with certain assumptions) may point to a scenario in which $m^2_{H_u}$ crosses zero near the weak scale. Combining this constraint with a simple ultraviolet structure can lead to a highly predictive superparticle spectrum, an example being the model presented in Section 2. The model predicts all the supersymmetry breaking parameters, except for the holomorphic Higgs mass-squared parameter $\mu B$ (and $m^2_{H_u}$), in terms of a single overall mass scale $M_0$ (with a weak dependence on $\tan \beta$). This parameter is expected to be of order a few hundred GeV if it does not scan but in a multi-TeV region if it does scan, allowing for the possibility of experimentally distinguishing between these two cases. We have also discussed implications of the landscape picture on the supersymmetry breaking masses in a general setup arising from Eqs. (3), (4) with possible additional interactions. Depending on the form of these interactions, strong predictions on the entire superparticle masses may be lost, but some predictions, such as those on the gaugino masses, may still be preserved. It is interesting that the experimental observation of one of these spectra may hint at possible statistical pressures acting on parameters of the theory, and thus the gross structure of vacua in the “vicinity” of our own one.

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