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ORIGINAL ARTICLE

Variational iteration method for flow of non-Newtonian fluid on a moving belt and in a collector

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Abstract In this paper, the thin film of a non-Newtonian fluid namely, a Sisko fluid on a vertical moving belt and this fluid in a collector is investigated. Sisko fluid's behavior is expressed by non-linear equation. At the first problem, we consider a container having a non-Newtonian fluid in it. A wide moving belt passes through this container and moves vertically upward with constant velocity. The graphical representation of the velocity v against the horizontal distance x shows that the velocity increases as the non-Newtonian effect increases. Physics of the second problem includes a moving flat plate with constant velocity. The flat plate is cooled with a kind of oil through which its properties follow the Sisko fluid model. We obtain the velocity gradient with difference values of b and k coefficient, in Collector. By the use of velocity gradient, the pressure gradient can be predicted. Predicting the pressure can help to analyze the extra stresses in the collector. The variational iteration method (VIM) is used to solve this non-linear equation analytically. Comparison of the result obtained by the present method with numerical solution shows the accuracy, reliable and fast convergence of this method for nonlinear problems.

1. Introduction

Most low molecular weight substances such as organic and inorganic liquids, solutions of low molecular weight inorganic salts, molten metals and salts, and gases exhibit Newtonian flow characteristics, i.e., at constant temperature and pressure, and in simple shear, the shear stress (σ) is proportional to the rate of shear ($\dot{\gamma}$) and the constant of proportionality is the familiar dynamic viscosity (η). Such fluids are classically known as the Newtonian fluids. However Many substances of industrial significance, especially of multi-phase nature (foams, emulsions, dispersions and suspensions, slurries, for instance) and polymeric melts and solutions (both natural and manmade) do not conform to the Newtonian postulate

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of the linear relationship between (σ) and ($\gamma^{\cdot}$) in simple shear, for instance. Accordingly, these fluids are variously known as non-Newtonian, non-linear, complex, or rheological complex fluids.

The trend for examining non-Newtonian fluid has been increased with respect to importance of this kind of fluid in industry and so on. Various workers in the field cite a wide variety of applications in rheological problems in biological sciences, geophysics, chemical and petroleum industries [1]. Several others [2–7] investigated the analytic solutions for flow of non-Newtonian fluids under various assumptions. Siddiqui et al. [8] discussed the thin film flows of Sisko fluid on a moving belt and Moallemi et al. [9] discussed Sisko fluid flow in collector. The equations of modeling non-Newtonian Sisko fluid are non-linear equations.

Many different methods have recently introduced to solve nonlinear problems, such as the homotopy analysis method [10], the Adomian's decomposition method (ADM) [11,12], the homotopy perturbation method [13–17], and variational iteration method (VIM). The VIM is strongly and simply capable of solving a large class of linear or nonlinear differential equations without the tangible restriction of sensitivity to the degree of the nonlinear term and also it reduces the size of calculations besides, its interactions are direct and straightforward.

The VIM was first proposed by He [18,19] and is systematically illustrated in [20] and used to give approximate solutions of the problem of seepage flow in porous media with fractional derivatives. The VIM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations. In this method, general Lagrange multipliers are introduced to construct correction functional for the problems. The multipliers can be identified optimally via variational theory. It has been used to solve effectively, easily and accurately a large class of nonlinear problems with approximation [21]. It was shown by author [22] that this method is more powerful than existing techniques such as the Adomian method.

Taza Gul et al. [23] studied the unsteady thin film flow of a second grade fluid over a vertical oscillating belt. The governing equation for velocity field with appropriate boundary conditions is solved analytically using Adomian decomposition method (ADM). Expressions for velocity field have been obtained. Optimal asymptotic method (OHAM) has also been used for comparison. The effects of Stocks number, frequency parameter and pressure gradient parameters have been sketched graphically and discussed.

Noreen sher Akbar [24] investigated the mathematical modeling and analysis of blood flow in a tapered artery with stenosis. His analysis was carried out in the presence of heat and mass transfer. Constitutive equation of Carreau fluid is invoked in the mathematical formulation. Graphical illustrations associated with the tapered arteries namely converging, diverging and non-tapered arteries are examined for different parameters of interest. Noreen and Butt [25] devoted to a study of the peristaltic motion of a Casson fluid of a non-Newtonian fluid accompanied in a horizontal tube. To characterize the non-Newtonian fluid behavior, they considered the Casson fluid model. Suitable similarity transformations are utilized to transform the governing partial differential momentum into the non-linear ordinary differential equations. Also they investigated peristaltic mechanisms in a two dimensional nonuniform channel filled with Herschel–Bulkley fluid under the assumptions of long-wavelength and low-Reynolds-number approximation [26]. The influence of magnetic field on peristaltic flow of a Casson fluid model is considered in another study of Akbar [27]. The governing coupled equations were constructed under long wavelength and low Reynold's number approximation and Exact solutions were evaluated for stream function and pressure gradient in his study. Taza et al. [28] studied the influence of heat transfer analysis of thin film flows of a third grade fluid in the presence of magneto hydrodynamic (MHD) on a vertical moving belt. The momentum and energy equations are solved analytically by using the Adomian decomposition method (ADM) in their work. Optimal Homotopy Asymptotic Method (OHAM) is also used for comparison. Khan et al. [29] studied problem of thin layer third order fluid flow past a vertical lubricating and porous belt that was modeled by a system of nonlinear differential equations studied in the presence of heat. The nonlinear differential equations for the fields of velocity and temperature, are solved analytically by using Optimal Homotopy Asymptotic Method (OHAM) in their study and in another work of Taza et al. [30], the influence of temperature dependent viscosity on thin film flow of a magneto hydrodynamic (MHD) third grade fluid past a vertical belt is investigated. In this paper we use the variational iteration method (VIM) to investigate this film flow of a non-Newtonian Sisko fluid [31] on a vertically moving belt. Results and discussion are given in Section 7. The conclusions are summarized in Section 8.

2. Governing equations

The fundamental equations governing the motion of an incompressible fluid, neglecting the thermal effects are given by

$$\operatorname{div} V = 0 \tag{1}$$

$$\rho\left[\frac{\partial V}{\partial t} + (V\cdot\nabla)V\right] = \nabla\cdot T + \rho b \tag{2}$$

where ρ is the density, T is the Cauchy stress tensor, V is the velocity vector, t is the time and ρb are body forces per unit mass. The Cauchy stress tensor T, is given by

$$T = -pI + S \tag{3}$$

where p is the pressure, I the unit tensor and S the extra stress tensor.

In this paper, we are dealing with non-Newtonian Sisko fluid. For a Sisko fluid the extra stress tensor is defined by [23]

$$S = \left[a + b\left|\sqrt{\frac{1}{2}\mathrm{tr}(A_1^2)}\right|^{(n-1)}\right]A_1 \quad \text{for Non-Newtonian fluid} \quad n > 1 \tag{4}$$

where A_1, is the rate of deformation tensor, and a, b and n are constants defined differently for different fluids.

$$\begin{aligned} &S + \lambda_1\frac{DS}{Dt} + \frac{\lambda_3}{2}(SA_1 + A_1S) + \frac{\lambda_5}{2}(\mathrm{tr}S)A_1 = \mu\left(A_1 + \lambda_2\frac{DA_1}{Dt} + \lambda_4A_1^2\right) \\ &A_1 = L + L^T \\ &L = \mathrm{grad}\,V \end{aligned} \tag{5a}$$

where A_1, is the first Rivlin–Ericksen tensor, μ, λ_1, λ_2, λ_3, λ_4 and λ_5 are the material constants. The contra variant convected derivative D/Dt is defined by

$$\frac{D(\delta)}{Dt} = \frac{d(\delta)}{dt} - L(\delta) - (\delta)L^T \tag{5b}$$

in which d/dt is the material derivative.

3. First problem

We consider a container having a non-Newtonian fluid in it. A wide moving belt passes through this container and move vertically upward with constant velocity U_0 as shown in Fig. 1. Since the belt moves upward and passes through the fluid, it picks up a film fluid of thickness δ. Due to gravity; the fluid film tends to drain down the belt. For simplicity, some assumptions are made:

(i) The flow is in steady state.
(ii) The flow is laminar and uniform.
(iii) The film fluid thickness δ is uniform.

We choose a xy-coordinate system and position x-axis parallel to the fluid and normal to the belt, y-axis upward along the belt and z-axis normal to the xy-plane. The only velocity component is in the y-direction; therefore,

$$V = (0, v(x), 0) \tag{6}$$

and also the extra stress tensor is function of x only, that is

$$S = S(x) \tag{7}$$

Eq. (6) satisfies the continuity Eq. (1) identically.

3.1. Mathematical formulation

The pressure along x-axis is constant, so using Eqs. (3) and (4) in momentum Eq. (2), then, we obtain

$$-\frac{dP}{dx} = 0 \tag{8}$$

y-momentum:

$$-\frac{dP}{dy} + \rho g + a\frac{d^2v}{dx^2} + b\frac{d}{dx}\left(\frac{dv}{dx}\right)^n = 0 \tag{9}$$

From Eq. (8) we deduce that $p = p(y)$. Thus, we remain with the single equation:

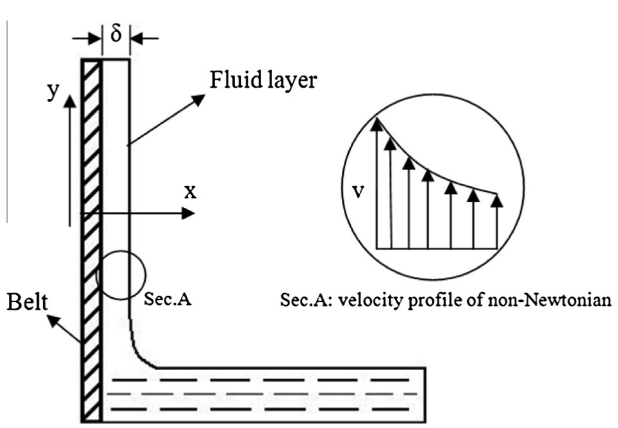

Figure 1 Geometry of the flow of moving belt through a non-Newtonian fluid.

$$a\frac{d^2v}{dx^2} + nb\left(\frac{dv}{dx}\right)^{n-1}\frac{d^2v}{dx^2} + \rho g = \frac{dp}{dy} \quad \text{for non-Newtonian fluid } n > 1 \tag{10}$$

Since the velocity component v in above equation is a function of x while the pressure p is a function of y alone, the two sides of this equation can be equal only if each is constant. As there is no pressure gradient along y-direction, this constant can be taken to be zero and fluid is going upwards; therefore, Eq. (10) becomes

$$a\frac{d^2v}{dx^2} + nb\left(\frac{dv}{dx}\right)^{n-1}\frac{d^2v}{dx^2} - \rho g = 0 \quad \text{for non-Newtonian fluid } n > 1 \tag{11}$$

The boundary conditions will be

$$\begin{cases} x = 0 \to v = U_0 \text{ at the belt} \\ x = \delta \to s_{xy} = 0 \text{ shear stress at free surface} \end{cases} \tag{12}$$

where the shear stress s_{xy} in Eq. (12) for the flow problem under consideration from Eq. (4) is given by

$$s_{xy} = a\frac{dv}{dx} + b\left[\frac{dv}{dx}\right]^n \tag{13}$$

Substituting Eq. (13) in the second boundary condition of Eq. (12), we get

$$\frac{dv}{dx} = 0 \text{ at } x = \delta \tag{14}$$

We obtain the same result in case of a Newtonian fluid.

Thus, the flow of a Sisko fluid on a vertical moving belt is governed by the system:

$$\frac{d^2v}{dx^2} + \frac{nb}{a}\left[\frac{dv}{dx}\right]^{n-1}\frac{d^2v}{dx^2} - \frac{\rho g}{a} = 0 \tag{15}$$

$$\begin{cases} x = 0 \to v = U_0 \\ x = \delta \to \frac{dv}{dx} = 0 \end{cases} \tag{16}$$

To non-dimensionalize Eq. (15) subject to (16), we introduce the dimensionless parameters as follows:

$$v^* = \frac{v}{U_0}, \quad x^* = \frac{x}{\delta}, \quad b^* = \frac{b}{a(\delta/U_0)^{n-1}}, \quad k^* = \frac{\rho g \delta^2}{aU_0}$$

Thus, the dimensionless form of Eq. (15) subject to (16) without '*' is

$$\frac{d^2v}{dx^2} + \frac{nb}{a}\left[\frac{dv}{dx}\right]^{n-1}\frac{d^2v}{dx^2} - \frac{\rho g}{a} = 0 \tag{17}$$

$$\begin{cases} x = 0 \to v = 1 \\ x = 1 \to \frac{dv}{dx} = 0 \end{cases} \tag{18}$$

We give the solution of problem (17) with $n = 3$ under the boundary conditions (18) by variational iteration method (VIM).

4. Second problem

Physics of the problem includes a moving flat plate with constant velocity (i.e. v_0). The flat plate is cooled with a kind of oil through which its properties follow the Sisko fluid model. It is one of the main applications of this problem. In Fig. 2a total outline of the added segment to collect the fluid from the moving plate is shown. The right side of the collector is closed, oil enters to the collector and because of the closed end part of the collector the oil exits from the upper part of the duct. The pressure before duct entrance equals p_0 and in the collector is p_i.

For simplicity, some assumptions are considered:

(i) The flow is in steady state.
(ii) The flow is laminar and uniform.
(iii) The gravity is negligible.

xy-coordinate system is chosen and y-axis is posed parallel to the fluid and normal to the plate. x-axis is toward the plate and also z-axis is normal to the xy-plane. The only velocity component is in the x direction; therefore,

$$V = (v(y), 0, 0) \tag{19}$$

and also the extra stress tensor is function of y only, that is defined as follows:

$$S = S(y) \tag{20}$$

4.1. Mathematical formulation

By entering the velocity field, s becomes

$$S = a\left(\frac{\partial v}{\partial y}\right) + b\left(\frac{\partial v}{\partial y}\right)^n \tag{21}$$

$$\nabla \cdot S = a\left(\frac{\partial^2 v}{\partial y^2}\right) + bn\left(\frac{\partial^2 v}{\partial y^2}\right)\left(\frac{\partial v}{\partial y}\right)^{n-1} \tag{22}$$

$$\nabla \cdot T = \frac{-\partial p}{\partial x} + a\frac{d^2 v}{dy^2} + nb\left[\frac{dv}{dy}\right]^{n-1}\frac{d^2 v}{dy^2} = 0 \tag{23}$$

Acceleration vector is written DV/Dt and it is defined as follows:

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + (\nabla \cdot V)V \tag{24}$$

By substituting Eqs. (23) and (24) into Eq. (2) and considering these assumptions, the final momentum equations are obtained. For further information about this part, one can refer to Refs. [24,25]. However, power law model is considered in these two references but also non-Newtonian fluid is investigated and they are worthy references due to behavior of non-Newtonian fluid for researchers. The pressure along the x axis direction is constant.

Constant pressure along y-axis is as follows:

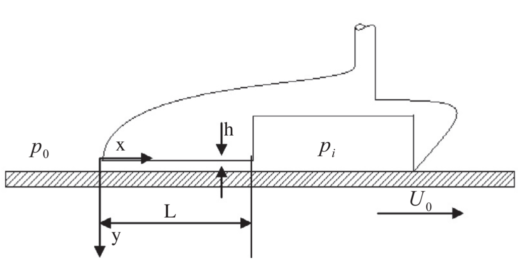

Figure 2 Geometry of the fluid collector over a moving flat plate.

$$\frac{dp}{dy} = 0 \tag{25}$$

x-momentum:

$$a\frac{d^2 v}{dy^2} + nb\left(\frac{dv}{\partial y}\right)^{n-1}\frac{d^2 v}{dy^2} = \frac{dp}{dx} \tag{26}$$

The reason of considering the velocity only as function of y is the change in the partial differential equation to the ordinary differential one. From Eq. (25) it is concluded that the pressure is only the function of x. The right side of Eq. (26) only depends on y and the pressure gradient dp/dx is constant, Therefore:

$$\frac{dp}{dx} = const \tag{27}$$

The proposed condition is implied as follows:

$$\begin{cases} p(0) = p_0 \\ p(L) = p_i \end{cases} \tag{28}$$

Finally, the pressure profile is linear and this profile is obtained as Eq. (29):

$$p(x) = \frac{p_0 - p_i}{L}x + p_0 \tag{29}$$

So, in Eq. (27) the internal pressure of the collector is not known, clearly. Obtaining the internal pressure of the collector is important because it leads to prevent the critical stress in collecting oil segment by limiting the internal pressure. Furthermore, the momentum equation and the related boundary conditions for the flowing non-Newtonian Sisko fluid in accordance with the movement are defined as the following equation:

$$a\frac{d^2 v}{dy^2} + nb\left(\frac{dv}{dy}\right)^{n-1}\frac{d^2 v}{dy^2} - \frac{dp}{dx} = 0 \tag{30}$$

The relevant boundary conditions are as follows:

$$\begin{cases} y = 0 \rightarrow v = 0 \\ y = h \rightarrow v = v_0 \end{cases} \tag{31}$$

To solve Eqs. (30) and (31) the non-dimensional variables and parameters are defined as follows:

$$v^* = \frac{v}{U_0}, \quad y^* = \frac{y}{h}, \quad b^* = \frac{b}{a(h/v_0)^{n-1}}, \quad k^* = \frac{\frac{dp}{dx}h^2}{av_0}$$

Thus, the dimensionless form of Eq. (30) which is subjected to Eq. (31) without '*' is

$$\frac{d^2 v}{dy^2} + nb\left[\frac{dv}{dy}\right]^{n-1}\frac{d^2 v}{dy^2} - k = 0 \tag{32}$$

With the following boundary conditions:

$$\begin{cases} y = 0 \rightarrow v = 0 \\ y = 1 \rightarrow v = 1 \end{cases} \tag{33}$$

Eq. (33) is a second order nonlinear differential equation with two boundary conditions. The solution of Eq. (32) under the above boundary conditions is obtained by VIM. It should be noted that Eq. (32) is considered for integer parameter n to overcome problems due to $dv/dy < 0$ because of the nature of article that investigated the velocity field of gap.

5. Basic concepts of VIM

To illustrate the basic concepts of VIM, we consider the following differential equation:

$$Lu + Nu = h(x) \tag{34}$$

where L, N and $h(x)$ are the linear operator, the nonlinear operator and a heterogeneous term, respectively.

Assuming $u_0(x)$ is the solution of $Lu = 0$ we can write down an expression to correct the value of some special point, for example at $x = 1$:

$$u_{cor}(1) = u_0(1) + \int_0^1 \lambda[Lu_0 + Nu_0 - h]dx \tag{35}$$

where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory, and the second term on the right is called the correction. He [18,19] has modified the above method into an iteration method in the following way:

$$u_{n+1}(x_0) = u_n(x_0) + \int_0^{x_0} \lambda[Lu_n + N\tilde{u}_n - h]dx \tag{36}$$

With $u_0(x)$ as initial approximation with possible unknowns, $\tilde{u}_n$ is considered a restricted variation and is chosen suitably to satisfy the restricted variation conditions, i.e. $\delta\tilde{u}_n = 0$. For arbitrary of u_0, we can rewrite Eq. (36) as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda[Lu_n(\xi) + N\tilde{u}_n(\xi) - h(\xi)]d\xi \quad \text{for non-Newtonian fluid} \quad n \geqslant 0 \tag{37}$$

It is obvious that the successive approximations $u_j, j \geqslant 0$ can be established by determining λ, general Lagrange's multiplier, which can be identified optimally via the variational theory. As mentioned before, the function $\tilde{u}_n$ is a restricted variation which means $\delta\tilde{u}_n = 0$. Therefore, we first determine the Lagrange multiplier λ which will be identified optimally via integration by parts. The successive approximations $u_{n+1}(x), n \geqslant 0$ of the solution $u(x)$ will be readily obtained upon using the obtained optimal Lagrange multiplier and by using admissible function u_0. Once λ is determined, then several approximations $u_j(x), j \geqslant 0$, follow immediately. Consequently, the exact solution may be obtained using following equation:

$$u = \lim_{n\to\infty} u_n \tag{38}$$

6. VIM solution

In order to obtain VIM solution of Eq. (17) with $n = 3$, we construct a correction functional which reads as follows:

$$v_{n+1}(x) = v_n(x) + \int_0^x \lambda\left(\frac{d^2 v_n(\tau)}{d\tau^2} + 3b\left[\frac{d\tilde{v}_n(\tau)}{d\tau}\right]^2 \frac{d^2\tilde{v}_n(\tau)}{d\tau^2} - k\right)d\tau \tag{39}$$

where λ is the general Lagrangian multiplier that is to be determined later and $\tilde{v}_n(\tau)$ is considered as a restricted variation, i.e. $\delta\tilde{v}_n(\tau) = 0$. To find the optimal value of λ:

$$\delta v_{n+1}(x) = \delta v_n(x) + \delta\int_0^x \lambda\left(\frac{d^2 v_n(\tau)}{d\tau^2} + 3b\left[\frac{d\tilde{v}_n(\tau)}{d\tau}\right]^2 \frac{d^2\tilde{v}_n(\tau)}{d\tau^2} - k\right)d\tau \tag{40}$$

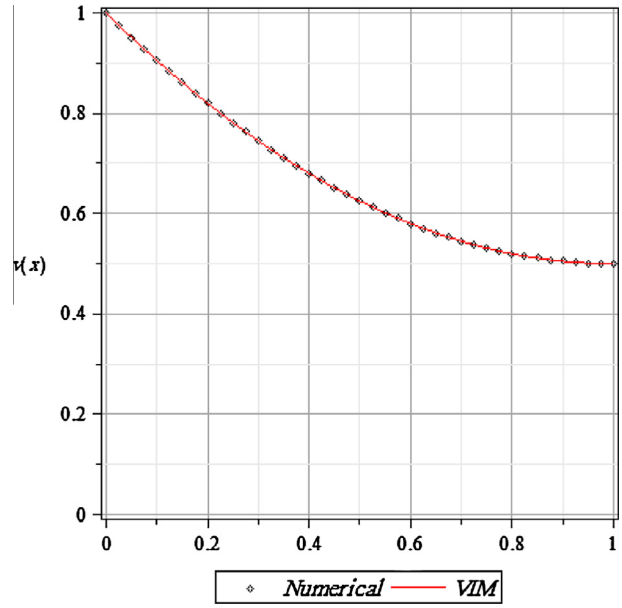

Figure 3 Dimensionless velocity profile of the belt with $b = 0$ and $n = 3$, for fixed value $k = 1$.

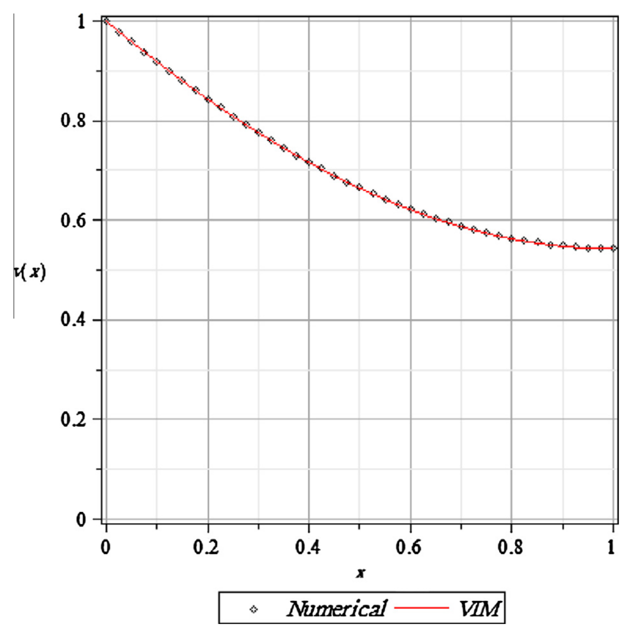

Figure 4 Dimensionless velocity profile of the belt with $b = 0.25$ and $n = 3$, for fixed value $k = 1$.

Or

$$\delta v_{n+1}(x) = \delta v_n(x) + \delta\int_0^x \lambda\left(\frac{d^2 v_n(\tau)}{d\tau^2}\right)d\tau \tag{41}$$

Its stationary conditions can be obtained as follows:

$$\ddot{\lambda}(\tau) = 0 \quad 1 - \dot{\lambda}(\tau)\big|_{\tau=x} = 0 \quad \lambda(\tau)\big|_{\tau=x} = 0 \tag{42}$$

The Lagrange multiplier can be identified as same as Eq. (43):

$$\lambda = \tau - x \tag{43}$$

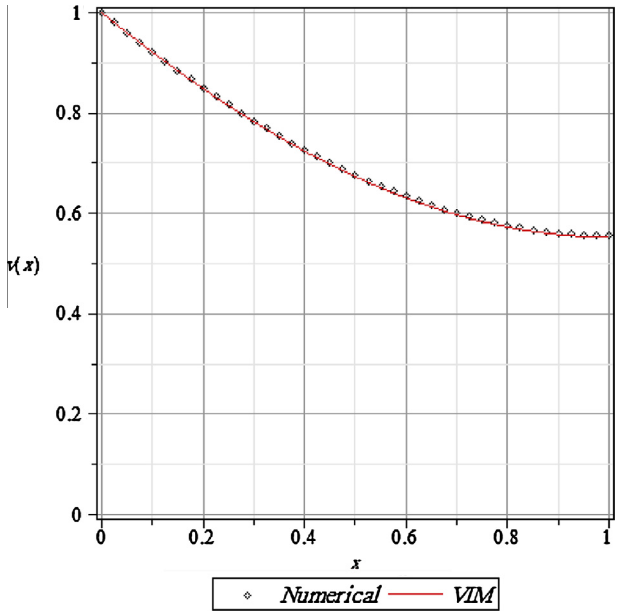

Figure 5 Dimensionless velocity profile of the belt With $b = 0.35$ and $n = 3$, for fixed value $k = 1$.

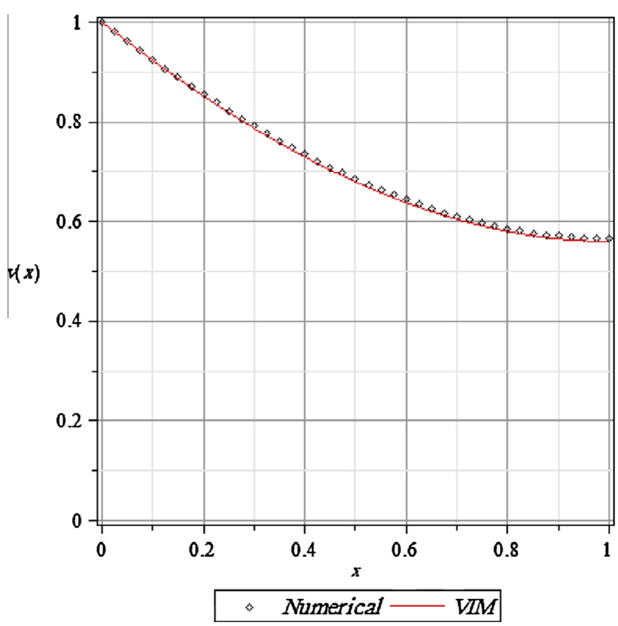

Figure 6 Dimensionless velocity profile of the belt With $b = 0.45$ and $n = 3$, for fixed value $k = 1$.

As a result, the following variational iteration formula can be obtained:

$$v_{n+1}(x) = v_n(x) + \int_0^x (\tau - x)\left(\frac{d^2 v_n(\tau)}{d\tau^2} + 3b\left[\frac{d\tilde{v}_n(\tau)}{d\tau}\right]^2 \frac{d^2\tilde{v}_n(\tau)}{d\tau^2} - k\right) d\tau \tag{44}$$

Now we must start with an arbitrary initial approximation. Therefore, we begin with Eq. (45):

$$v_0(x) = cx + d \tag{45}$$

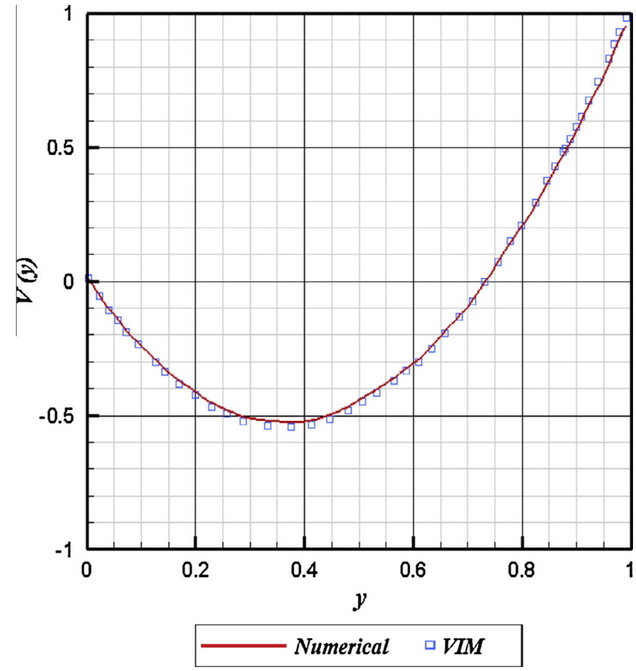

Figure 7 Comparison of VIM velocity field V and Numerical solution with $b = 0.02$, $n = 1$ and $k = 8$.

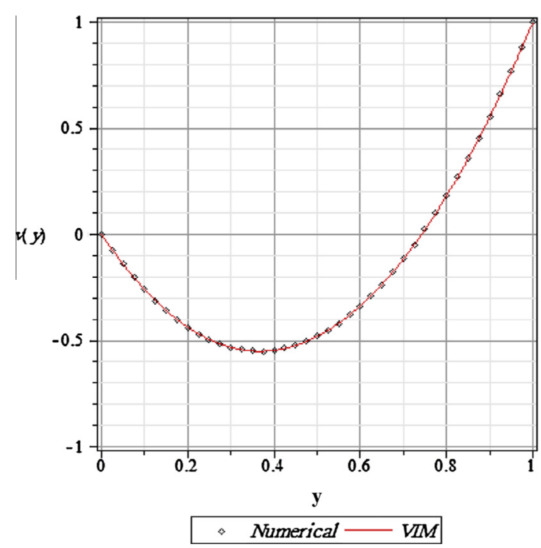

Figure 8 Dimensionless velocity profile With $b = 0.01$, $n = 2$ and $k = 8$.

where c and d are unknown constants to be further determined. By the above variational formula (44), we obtain the following first-order approximate solution:

$$v_1(x) = v_0(x) + \int_0^x (\tau - x)\left(\frac{d^2 v_0(\tau)}{d\tau^2} + 3b\left[\frac{d\tilde{v}_0(\tau)}{d\tau}\right]^2 \frac{d^2\tilde{v}_0(\tau)}{d\tau^2} - k\right) d\tau \tag{46}$$

Substituting Eq. (45) into Eq. (46), we have

$$v_1(x) = cx + d + \frac{1}{2}kx^2 \tag{47}$$

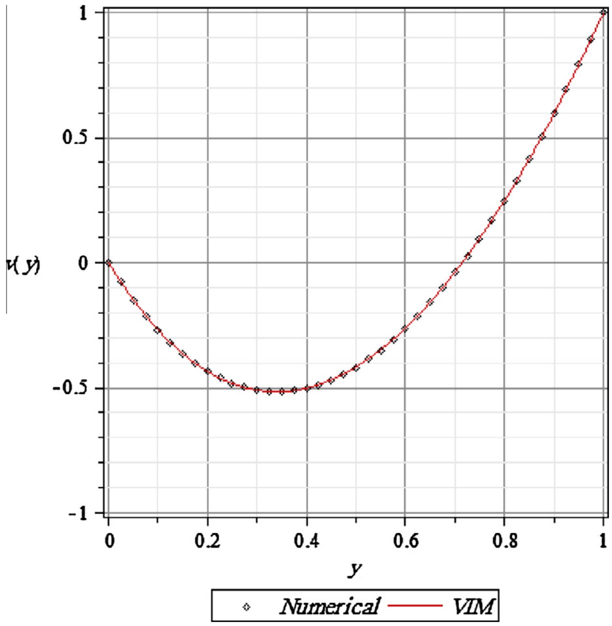

Figure 9 Dimensionless velocity profile with $b = 0.05$, $n = 2$ and $k = 8$.

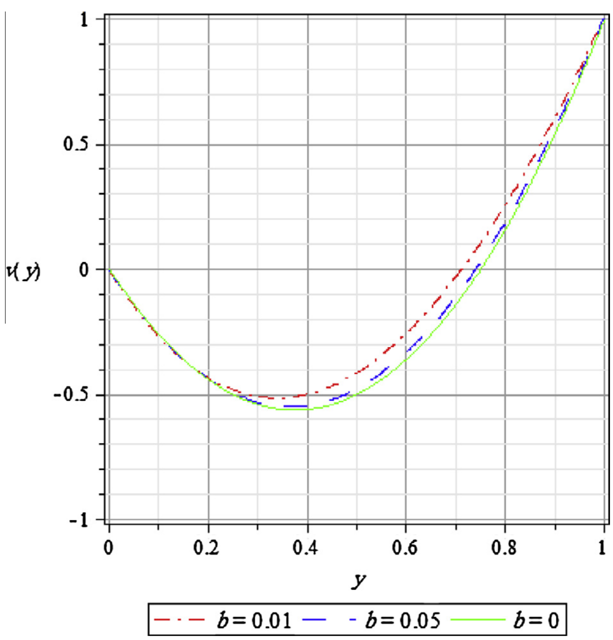

Figure 10 Comparison of dimensionless velocity profile With $n = 2$ and $k = 8$, for different values of b.

Incorporating the boundary conditions, Eq. (18), into $v_1(x)$ we can determine the values of the unknown constants as follows:

$$c = -k,\ d = 1 \tag{48}$$

Therefore, we obtain the first-order approximate solution, which reads as follows:

$$v_1(x) = \frac{1}{2}kx^2 - kx + 1 \tag{49}$$

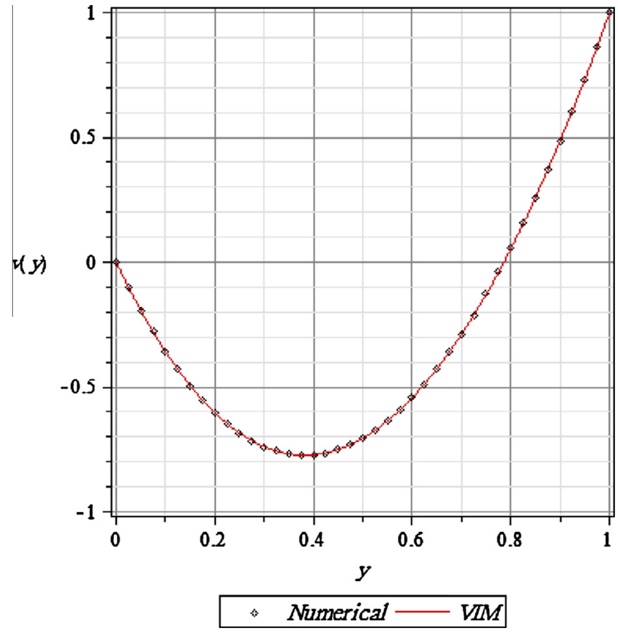

Figure 11 Dimensionless velocity profile with $b = 0.02$, $n = 2$ and $k = 10$.

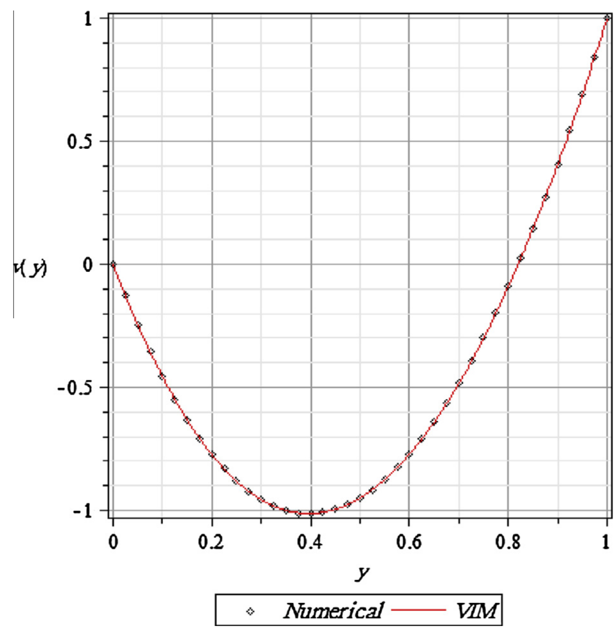

Figure 12 Dimensionless velocity profile with $b = 0.02$, $n = 2$ and $k = 12$.

Using the variational formula (44) and first-order approximate solution (47), we can obtain $v_2(x)$:

$$v_2(x) = cx + d + \frac{1}{2}kx^2 - \frac{3}{2}bc^2kx^2 - k^2bcx^3 - \frac{1}{4}\mathrm{b}k^3x^4 \tag{50}$$

In the same way, $v_3(x), v_4(x), \ldots$ can be obtained.

Therefore, we are able to give an approximate solution of the considered problem.

To give an approximation solution for the second problem with $n = 2$, we use the same process.

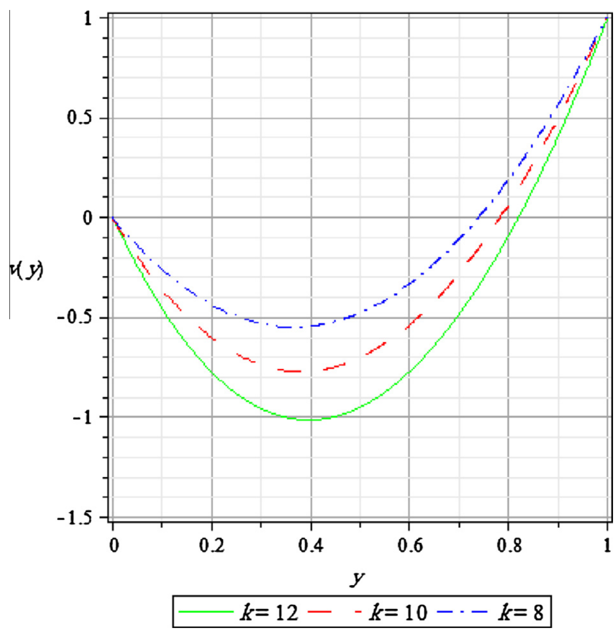

Figure 13 Comparison of dimensionless velocity profile With $b = 0.02$ and $n = 2$ for different values of k.

7. Results and discussion

For the first problem we consider a container having a non-Newtonian fluid in it. A wide moving belt passes through this container and moves vertically upward with constant velocity U_0 as shown in Fig. 1. Since the belt moves upward and passes through the fluid, it picks up a film fluid of thickness δ. Due to gravity, the fluid film tends to drain down the belt. The graphical representation Figs. 3–6 of the velocity v against the horizontal distance x shows that the velocity increases as the non-Newtonian effect increases from $b = 0$ (Newtonian case) to $b = 0.45$, when $n = 3$. Physics of the second problem includes a moving flat plate with constant velocity (i.e. v_0). The flat plate is cooled with a kind of oil through which its properties follow the Sisko fluid model. In case of $n = 1$, the behavior of the fluids changes to Newtonian fluid. Fig. 7 shows the comparable results of Newtonian fluid by variational iteration method. In Figs. 8, 9, 11 and 12 the velocity profile of the Sisko fluid with $n = 2$ in duct is plotted in accordance with duct width in different values of b and k, respectively. Comparison of velocity profile with $n = 2$ and $k = 8$, for different values of b has been shown in Fig. 10. It is observed that in case of larger amount of b the results separate from Newtonian fluid and non-Newtonian fluid. The velocity profile with $b = 0.02$ and $n = 2$, for different values of k has been shown in Fig. 13. Figs. 3–6, 8, 9, 11 and 12 show the comparison of analytical results of velocity with numerical results. It can be found that our analytical results of velocity fit well with numerical results.

8. Conclusion

In the present work, we study two problems of non-Newtonian fluid, namely a Sisko fluid. For the first problem we consider a thin film of fluid on a vertically moving belt and in the second one a fluid flow in collector. We apply the variational iteration method to obtain the velocity profile. The validity of our solutions is compared by the numerical results. In first problem For Sisko fluid when $n = 3$, it is observed that as the non-Newtonian effect is increased the velocity increases. In second problem we obtain the pressure gradient by applying this method (VIM) for fluid flow in collector. Predicting the pressure can help to analyze the extra stresses in the collector.

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