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Spin–spin interactions in massive gravity and higher derivative gravity theories

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ABSTRACT

We show that, in the weak field limit, at large separations, in sharp contrast to General Relativity (GR), all massive gravity theories predict distance-dependent spin alignments for spinning objects. For all separations GR requires anti-parallel spin orientations with spins pointing along the line joining the sources. Hence total spin is minimized in GR. On the other hand, while massive gravity at small separations ($m_g r \leqslant 1.62$) gives the same result as GR, for large separations ($m_g r > 1.62$) the spins become parallel to each other and perpendicular to the line joining the objects. Namely, the potential energy is minimized when the total spin is maximized in massive gravity for large separations. We also compute the spin–spin interactions in quadratic gravity theories and find that while at large separations GR result is intact, at small separations, spins become perpendicular to the line joining sources and anti-parallel to each other.

1. Introduction

Consider two widely separated spinning massive objects (for example two galaxies or galaxy clusters) that interact via gravity: What is the minimum energy configuration for their spin orientations, and how does the result depend on whether the graviton is massive or not? In this work we will compute the spin–spin interactions of point-like objects in massive gravity. We will show that introducing a small graviton mass gives the highly unexpected result of changing the spin orientations of sources from the one predicted in GR. Arguably, massive gravity is the most natural modification of GR that has implications in the overall dynamics – accelerated expansion – of the universe and hence a detailed study of gravitomagnetic effects such as the one done in this work is needed.

Before we give a detailed derivation of the results in the next section in D-dimensional spacetimes and higher curvature theories, let us summarize our findings here for the case of $D = 3+1$ for GR and massive gravity. Consider two localized spinning point-like sources described with the components of the energy-momentum tensor

$$T_{00} = m_a \delta^{(3)}(\vec{x} - \vec{x}_a),$$
$$T^i{}_0 = -\frac{1}{2} J_a^k \epsilon^{ikj} \partial_j \delta^{(3)}(\vec{x} - \vec{x}_a), \tag{1}$$

where $a = 1, 2$. Here m_a is the mass and $\vec{J}_a$ is the spin of the particle. Then, working in a flat background, from the tree-level diagram of one graviton exchange, we can calculate the potential energy as

$$U = -\frac{4\pi G}{t} \int d^4x\, d^4x'\, T^{\mu\nu}(x) G_{\mu\nu\alpha\beta}(x, x') T^{\alpha\beta}(x'), \tag{2}$$

where $G_{\mu\nu\alpha\beta}(x, x')$ is the Green's function of the theory at hand and t is a large time that will drop at the end. In GR this computation gives

$$U_{\rm GR} = -\frac{G m_1 m_2}{r} - \frac{G}{r^3}\left[\vec{J}_1 \cdot \vec{J}_2 - 3 \vec{J}_1 \cdot \hat{r}\, \vec{J}_2 \cdot \hat{r}\right], \tag{3}$$

where $\vec{r} = r\hat{r}$ is the distance between the two sources. Spin–spin part can be attractive or repulsive depending on the spin orientations. Maximum value of $\vec{J}_1 \cdot \vec{J}_2 - 3\vec{J}_1 \cdot \hat{r}\, \vec{J}_2 \cdot \hat{r}$, that is the minimum of the potential energy is achieved when $\vec{J}_1$ and $\vec{J}_2$ are anti-parallel and point along $\hat{r}$ as depicted in Fig. 1. That means in GR, for *any given* r, potential energy is minimized for anti-parallel spin orientations, if we neglect the tidal and orbital angular momentum effects. (The computation here is of course not a good approximation for close binary systems, such as two neutron stars *etc.*, but it is a valid approximation for two widely separated galaxies or galaxy clusters.) Let us give the results of the same computation

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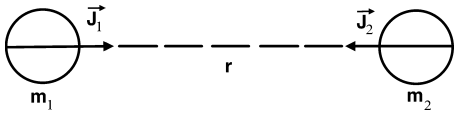

Fig. 1. Minimum energy configuration in GR, as long as weak field limit is applicable.

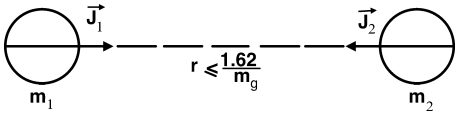

Fig. 2. Minimum energy configuration in massive gravity for $m_g r \leqslant 1.62$.

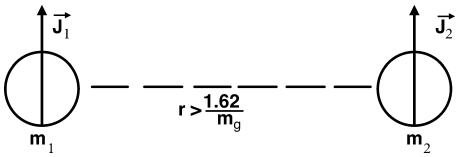

Fig. 3. In massive gravity, at large separations, the potential energy is minimized when the spins are perpendicular to the line joining the sources.

in massive gravity. At this point one might worry about which massive gravity to use. The crucial point is that in the weak field limit around flat space, any viable (non-linear, ghost-free) massive gravity theory reduces to the Fierz–Pauli (FP) theory that describes 5 degrees of freedom. Hence the following computation is a universal, weak field, large distance, prediction of *all* massive gravity theories built to describe 5 degrees of freedom around flat space. The Lagrangian density of the linear massive gravity is

$$\mathcal{L}_{\rm FP} = \frac{1}{16\pi G}\left[R - \frac{m_g^2}{4}\left(h_{\mu\nu}^2 - h^2\right)\right] + \mathcal{L}_{\rm matter}, \tag{4}$$

where m_g is the mass of the graviton, we found that at the lowest order the potential energy is

$$U_{\rm FP} = -\frac{4}{3}Gm_1m_2\frac{e^{-m_g r}}{r} - \frac{Ge^{-m_g r}(1+m_g r+m_g^2r^2)}{r^3} \times\left[\vec{J}_1\cdot\vec{J}_2 - 3\vec{J}_1\cdot\hat{r}\vec{J}_2\cdot\hat{r}\frac{(1+m_g r+\frac{1}{3}m_g^2r^2)}{(1+m_g r+m_g^2r^2)}\right]. \tag{5}$$

It is clear that, in contrast to the GR result, in massive gravity depending on the distance between the sources, spin–spin part of the potential energy is minimized for different spin orientations determined by the maximization of the function (see Appendix A for details)

$$f(\theta,\varphi_1,\varphi_2) = \cos(\theta) - 3\frac{(1+x+\frac{1}{3}x^2)}{(1+x+x^2)}\cos(\varphi_1)\cos(\varphi_2), \tag{6}$$

where $x = m_g r$ and θ is the angle between the spins and φ_i is the angle between $\vec{J}_i$ and $\vec{r}$. Maximization of (6) yields: anti-parallel spins for $x \leqslant \frac{1+\sqrt{5}}{2} \approx 1.62$ as in the case of GR depicted in Fig. 2. On the other hand, for $x > \frac{1+\sqrt{5}}{2} \approx 1.62$, one gets parallel spins which are perpendicular to the line joining the sources as in Fig. 3.

The important conclusion one learns is that while in GR minimal potential energy is realized for minimum total spin at all separations, in massive gravity potential energy is minimized for maximum total spin for $m_g r > 1.62$.[1]

[1] We would like to thank A. Dane whose simulation of the spin–spin interaction led us to realize this point where spins suddenly change orientations. Note that the same point that is the "Golden Number" arises when one considers stable circular orbits in the Newtonian theory with a Yukawa potential. Namely, stable circular orbits exist for $x \leqslant \frac{1+\sqrt{5}}{2}$. We thank F. Öktem for this point.

2. Derivation of the results

To derive the above results and their D-dimensional generalizations in GR, massive gravity and quadratic gravity, it is somewhat more convenient to use the propagator found in [1] to represent (2). In order to avoid repeating the computations of all three theories let us consider the most general theory which includes these theories:

$$S = \int d^Dx\sqrt{-g}\left\{\frac{1}{\kappa}R - \frac{2\Lambda_0}{\kappa} + \alpha R^2 + \beta R_{\mu\nu}^2 + \gamma\left(R_{\mu\nu\sigma\rho}^2 - 4R_{\mu\nu}^2 + R^2\right)\right\} + \int d^Dx\sqrt{-g}\left\{-\frac{m_g^2}{4\kappa}\left(h_{\mu\nu}^2 - h^2\right) + \mathcal{L}_{\rm matter}\right\}. \tag{7}$$

In [1], we computed the scattering amplitude ($A = Ut$) corresponding to a graviton exchange in this theory and presented it with sufficient detail, hence we quote here the result:

$$\begin{aligned} 4A = {} & 2T'_{\mu\nu}\left\{(\beta\bar{\Box}+a)\left(\Delta_L^{(2)} - \frac{4\Lambda}{D-2}\right) + \frac{m_g^2}{\kappa}\right\}^{-1}T^{\mu\nu} \\ & + \frac{2}{D-1}T'\left\{(\beta\bar{\Box}+a)\left(\bar{\Box} + \frac{4\Lambda}{D-2}\right) - \frac{m_g^2}{\kappa}\right\}^{-1}T \\ & - \frac{4\Lambda}{(D-2)(D-1)^2}T'\left\{(\beta\bar{\Box}+a)\left(\bar{\Box} + \frac{4\Lambda}{D-2}\right) - \frac{m_g^2}{\kappa}\right\}^{-1} \\ & \times\left\{\bar{\Box} + \frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T \\ & + \frac{2}{(D-2)(D-1)}T'\left\{\frac{1}{\kappa} + 4\Lambda f - c\bar{\Box} - \frac{m_g^2}{2\kappa\Lambda}(D-1)\right\}^{-1} \\ & \times\left\{\bar{\Box} + \frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T, \end{aligned} \tag{8}$$

where we have dropped the integral signs not to clutter the notation and also to properly account all those theories in the corresponding limits, we have provisionally introduced an effective cosmological constant which is determined via the quadratic equation $\frac{\Lambda-\Lambda_0}{2\kappa} + f\Lambda^2 = 0$. The other parameters that appear above are defined as

$$f \equiv (D\alpha+\beta)\frac{(D-4)}{(D-2)^2} + \gamma\frac{(D-3)(D-4)}{(D-1)(D-2)}, \tag{9}$$

$$a \equiv \frac{1}{\kappa} + \frac{4\Lambda D}{D-2}\alpha + \frac{4\Lambda}{D-1}\beta + \frac{4\Lambda(D-3)(D-4)}{(D-1)(D-2)}\gamma, \tag{10}$$

$$c = \frac{4(D-1)\alpha + D\beta}{D-2}. \tag{11}$$

With all these parameters at hand, one covers all the three theories that we are interested in. For example the result for General Relativity follows from $m_g^2 = \alpha = \beta = \gamma = 0$ which yield $a = \frac{1}{\kappa}$ and $f = c = 0$. For flat backgrounds one has

$$4A = -2\kappa T'_{\mu\nu}\left(\partial^2\right)^{-1}T^{\mu\nu} + \frac{2\kappa}{D-2}T'\left(\partial^2\right)^{-1}T. \tag{12}$$

More explicitly the last equation is

$$4A = -2\kappa\int d^Dx\int d^Dx'\,T_{\mu\nu}(x')G(x,x')T^{\mu\nu}(x) + \frac{2\kappa}{(D-2)}\int d^Dx\int d^Dx'\,T(x')G(x,x')T(x), \tag{13}$$

where the scalar Green's function reads

$$\partial_x^2 G(x,x') = -\delta^D(x,x'),$$

and $\partial_x^2 = -\partial_t^2 + \vec{\nabla}^2$. Of course one must keep in mind that to reach the explicit final result for the potential energies one uses

$$\left(\partial^2\right)^{-1} \equiv G_R(x,x') = \frac{\Gamma(\frac{D-3}{2})}{4\pi^{\frac{D-1}{2}} r^{D-3}} \delta\left[r - (t-t')\right], \tag{14}$$

in the massless case and similarly for the massive case

$$G_R(x,x') = \frac{(\frac{m_g}{r})^{\frac{D-3}{2}}}{(2\pi)^{\frac{D-1}{2}}} K_{\frac{D-3}{2}}(r m_g)\delta\left[r - (t-t')\right], \tag{15}$$

for the retarded Green's functions.

We are now ready to compute the potential energy for the desired theory. We will give two explicit examples below: GR (Einstein's theory) and Fierz–Pauli massive gravity. The analogous computations in the quadratic theory, without an explicit mass term follow similarly.

For massive spinning sources the energy–momentum tensor is given as the D-dimensional generalization of (1)

$$\begin{aligned} T_{00} &= m_a \delta^{(D-1)}(\vec{x} - \vec{x}_a), \qquad T_{ij} = 0, \\ T^i{}_0 &= -\frac{1}{2} J^{klm\cdots} \epsilon^{iklm\cdots j} \partial_j \delta^{(D-1)}(\vec{x} - \vec{x}_a), \end{aligned} \tag{16}$$

where J has $D-3$ and ϵ has $D-1$ indices and $a = 1, 2$. Generators of rotations will be given as $M^{ij} = \int d^{D-1}x\,(x^i T^j{}_0 - x^j T^i{}_0)$, which yields $M^{ij} = \epsilon^{ijk} J^k$ in $D = 3+1$. It is important to note that taking the background to be the Minkowski space, with $\eta_{\mu\nu} = \text{diag}(-,+,\cdots+)$, one has $T = \eta^{\mu\nu} T_{\mu\nu} = -T_{00}$ and hence the trace part does not play a role in the spin–spin interactions.

3. Spin–spin interaction in General Relativity

For massless gravity in D dimensions, we have

$$\begin{aligned} 4A = &-2\kappa T'_{00} \left\{\frac{1}{\partial^2}\right\} T^{00} + \frac{2\kappa}{(D-2)} T' \left\{\frac{1}{\partial^2}\right\} T \\ &- 4\kappa T'_{0i} \left\{\frac{1}{\partial^2}\right\} T^{0i}. \end{aligned} \tag{17}$$

The first two terms give $4t$ times the usual Newtonian potential energy which we need not depict here. The last term, which is the relevant part for spin–spin interactions, reads

$$\begin{aligned} &-4\kappa T'_{0i} \left\{\frac{1}{\partial^2}\right\} T^{0i} \\ &= 2\kappa J_1^{a_1 a_2 \ldots a_{D-3}} \epsilon^{i a_1 a_2 \ldots a_{D-3} n} \partial'_n \delta^{(D-1)}(\vec{x}' - \vec{x}_1) \\ &\quad \times \left\{\frac{1}{\partial^2}\right\} \left(\frac{1}{2}\right) J_2^{b_1 b_2 \ldots b_{D-3}} \epsilon^{i b_1 b_2 \ldots b_{D-3} m} \partial_m \delta^{(D-1)}(\vec{x} - \vec{x}_2) \\ &= \kappa J_1^{a_1 a_2 \ldots a_{D-3}} J_2^{b_1 b_2 \ldots b_{D-3}} \epsilon^{i a_1 a_2 \ldots a_{D-3} n} \epsilon^{i b_1 b_2 \ldots b_{D-3} m} \\ &\quad \times \partial'_n \delta^{(D-1)}(\vec{x}' - \vec{x}_1) \left\{\frac{1}{\partial^2}\right\} \partial_m \delta^{(D-1)}(\vec{x} - \vec{x}_2). \end{aligned} \tag{18}$$

This expression looks somewhat cumbersome, to understand the crux of the computation, let us carry it out more explicitly in $D = 3+1$ dimensions.

$$\begin{aligned} &-4\kappa T'_{0i} \left\{\frac{1}{\partial^2}\right\} T^{0i} \\ &= 4\kappa \int d^4x' \int d^4x \frac{1}{4} J_1^k \epsilon^{ikj} \partial_j \delta^{(3)}(\vec{x} - \vec{x}_1) \\ &\quad \times \frac{1}{4\pi |\vec{x} - \vec{x}'|} \delta\left(|\vec{x} - \vec{x}'| - (t - t')\right) \\ &\quad \times J_2^l \epsilon^{ilm} \partial'_m \delta^{(3)}(\vec{x}' - \vec{x}_2). \end{aligned} \tag{19}$$

Carrying out the time integrals and performing integration by parts, one gets

$$\begin{aligned} &-4\kappa T'_{0i} \left\{\frac{1}{\partial^2}\right\} T^{0i} \\ &= \frac{t\kappa}{4\pi} \left(\delta^{ij} \vec{J}_1 \cdot \vec{J}_2 - J_1^i J_2^j\right) \frac{\partial}{\partial x_1^i} \frac{\partial}{\partial x_2^j} \frac{1}{|\vec{x}_1 - \vec{x}_2|}. \end{aligned} \tag{20}$$

Since the sources do not coincide, $\vec{x}_1 \neq \vec{x}_2$, one has

$$\frac{\partial}{\partial x_1^i} \frac{\partial}{\partial x_2^j} \frac{1}{|\vec{x}_1 - \vec{x}_2|} = \frac{1}{r^3}\left(\delta^{ij} - 3\hat{r}^i \hat{r}^j\right), \tag{21}$$

and therefore spin–spin interaction potential energy of GR is found (3).

In D-dimensions contractions of the ϵ tensor only change the relative coefficients of the two terms in the spin–spin part. To obtain the generic result, one way is to do the computation in several other dimensions and find the formula or one can use the contractions of the ϵ tensor. We have done both ways, the result is

$$\begin{aligned} &-4\kappa T'_{0i} \left\{\frac{1}{\partial^2}\right\} T^{0i} \\ &= (D-3)!\kappa \left\{ J_1^{a_1 a_2 \ldots a_{D-3}} J_2^{a_1 a_2 \ldots a_{D-3}} \partial_m \partial'_m \right. \\ &\quad \left. - (D-3) J_1^{a_1 a_2 \ldots a_{D-4} m} J_2^{a_1 a_2 \ldots a_{D-4} n} \partial_m \partial'_n \right\} \\ &\quad \times \left\{\frac{1}{\partial^2}\right\} \delta^{(D-1)}(\vec{x}' - \vec{x}_1) \delta^{(D-1)}(\vec{x} - \vec{x}_2). \end{aligned} \tag{22}$$

Finally the spin–spin interaction in D-dimensional massless gravity reads

$$\begin{aligned} U_{\text{GR}} = &-\frac{G_D (D-2)!(D-3)^2}{2r^{D-1}} \\ &\times \left(J_1 \cdot J_2 - (D-1)(J_1 \cdot \hat{r})(J_2 \cdot \hat{r})\right), \end{aligned} \tag{23}$$

where the D-dimensional Newton's constant is

$$G_D = \frac{\kappa \Gamma(\frac{D-3}{2})}{8(D-2)\pi^{\frac{D-1}{2}}},$$

which gives $\kappa = 16\pi G$ in $D = 3+1$. Here we defined the scalar products between the anti-symmetric objects as

$$\begin{aligned} J_1 \cdot J_2 &\equiv J^{a_1 a_2 \ldots a_{D-3}} J^{a_1 a_2 \ldots a_{D-3}}, \\ (J_1 \cdot \hat{r})(J_2 \cdot \hat{r}) &\equiv J^{a_1 a_2 \ldots a_{D-3}} \hat{r}_{D-3} J^{a_1 a_2 \ldots a_{D-3}} \hat{r}_{D-3}. \end{aligned} \tag{24}$$

4. Scattering in massive D-dimensional gravity

Let us now do the same computation in the linearized massive gravity. The relevant scattering amplitude is

$$\begin{aligned} 4A = &-2\kappa T'_{00} \left\{\partial^2 - m_g^2\right\}^{-1} T^{00} \\ &+ \frac{2\kappa}{(D-1)} T' \left\{\partial^2 - m_g^2\right\}^{-1} T - 4\kappa T'_{0i} \left\{\partial^2 - m_g^2\right\}^{-1} T^{0i} \\ = &-2\kappa \left(\frac{D-2}{D-1}\right) m_1 m_2 \frac{1}{(2\pi)^{\frac{D-1}{2}}} \frac{1}{r^{\frac{D-3}{2}}} \\ &\times \left[\left(\frac{1}{m_g^2}\right)^{\frac{3-D}{4}} K_{\frac{D-3}{2}}(r m_g)\right] \end{aligned}$$

$$+\kappa(D-3)!\Big[J_1^{a_1a_2\ldots a_{D-3}}J_2^{a_1a_2\ldots a_{D-3}}\partial_m\partial'_m - (D-3)J^{a_1a_2\ldots a_{D-4}m}J^{a_1a_2\ldots a_{D-4}n}\partial_m\partial'_n\Big] \times\frac{1}{(2\pi)^{\frac{D-1}{2}}}\frac{1}{r^{\frac{D-3}{2}}}\left[\left(\frac{1}{m_g^2}\right)^{\frac{3-D}{4}}K_{\frac{D-3}{2}}(rm_g)\right]. \tag{25}$$

Just like the massless case that we studied in detail in the previous section, one performs partial integrations and carries out the integrals to get

$$U_{\rm FP}=-\frac{\kappa(D-2)m_1m_2\,m_g^{\frac{D-3}{2}}}{2(D-1)(2\pi)^{\frac{D-1}{2}}r^{\frac{D-3}{2}}}K_{\frac{D-3}{2}}(rm_g) +\frac{\kappa(D-3)!\,m_g^{\frac{D+1}{2}}}{4(2\pi)^{\frac{D-1}{2}}r^{\frac{D-3}{2}}}K_{\frac{D+1}{2}}(rm_g) \times\left[J_1\cdot J_2\left(\frac{2K_{\frac{D-1}{2}}(rm_g)}{rm_gK_{\frac{D+1}{2}}(rm_g)}-1\right) +(D-3)(J_1\cdot\hat{r})(J_2\cdot\hat{r})\right]. \tag{26}$$

For $D=3+1$ (26) gives (5).

5. Quadratic gravity

With the tools at our hand we can extend the above results to D-dimensional quadratic gravity without a Fierz–Pauli mass term with the Lagrangian density

$$\mathcal{L}=\frac{1}{\kappa}R+\alpha R^2+\beta R_{\mu\nu}^2+\gamma\big(R_{\mu\nu\sigma\rho}^2-4R_{\mu\nu}^2+R^2\big).$$

The amplitude can be written as

$$U_{\rm quad}\times t=-\frac{\kappa}{2}T'_{\mu\nu}\big(\partial^2\big)^{-1}T^{\mu\nu}+\frac{\kappa T'(\partial^2)^{-1}T}{2(D-2)} +\frac{\kappa}{2}T'_{\mu\nu}\big(\partial^2-m_\beta^2\big)^{-1}T^{\mu\nu}-\frac{\kappa T'(\partial^2-m_\beta^2)^{-1}T}{2(D-1)} -\frac{\kappa T'(\partial^2-m_c^2)^{-1}T}{2(D-2)(D-1)}, \tag{27}$$

where $m_\beta^2=-\frac{1}{\kappa\beta}$ and $m_c^2=\frac{1}{\kappa(4\alpha(D-1)+D\beta)}$. All the terms in the above expression have been computed above: The first line is pure GR, the second and third lines come from the quadratic terms in the Lagrangian. The fourth and the fifth terms do not contribute to the spin–spin interactions, the third term gives a negative contribution to the spin–spin interaction in comparison with the GR result. The full expression is somewhat cumbersome to depict, $D=2+1$ case was given in [2], here let us write down the $D=3+1$ result.

Let $U_{\rm quad}\equiv U_{\rm GR}+U_2$, then the contribution coming from the quadratic part reads

$$U_2=\frac{Gm_1m_2}{r}\left(\frac{4}{3}e^{-m_\beta r}-\frac{1}{3}e^{-m_c r}\right)+\frac{Ge^{-m_\beta r}(1+m_\beta r+m_\beta^2r^2)}{r^3} \times\left[\vec{J}_1\cdot\vec{J}_2-3\vec{J}_1\cdot\hat{r}\,\vec{J}_2\cdot\hat{r}\,\frac{(1+m_\beta r+\frac{1}{3}m_\beta^2r^2)}{(1+m_\beta r+m_\beta^2r^2)}\right]. \tag{28}$$

At long distances, GR part dominates and hence spins are anti-parallel to each other and point along $\hat{r}$. In short distances quadratic part dominates and spin–spin interaction part is just like the one in massive gravity but with an overall negative sign. Therefore, for quadratic gravity, at short distances spins are anti-parallel to each other but they are perpendicular to $\hat{r}$ (Fig. 4).

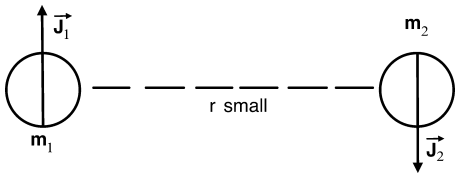

Fig. 4. Minimum energy configuration in quadratic gravity for small separations.

6. Conclusions and discussions

In this work, we have initiated a study of (linear) gravitomagnetic effects in the Fierz–Pauli massive gravity which is the unique linearized massive spin-2 theory describing 5 degrees of freedom in $(3+1)$-dimensional flat backgrounds. (Non-linear extensions of the FP theory such as the dGRT theory [3] or its extensions which are free of the Boulware–Deser ghost [4], though they still could be acausal [5], yield exactly the same prediction as the FP theory at large distances in the weak field limit.)

For two point-like spinning sources that interact gravitationally, potential energy is minimized for anti-parallel spin orientations pointing in the direction of the vector between the sources in General Relativity at any distance where the linear approximation is valid. On the other hand for massive gravity, potential energy is minimized when the spins point away from the line joining the sources at large separations. Hence the total spin of the system is non-zero even for equal magnitude spins and point perpendicular to the axis joining the sources.

A word about the mass–mass term in (5) is needed: At large distances it is in the desired Yukawa like form which is one of the main motivations of studying massive gravity theories, since it can replace all or part of the dark energy needed to explain the accelerated expansion of the Universe. As is clear that term also has the undesired vDVZ discontinuity [7,8] in the vanishing graviton mass limit. Therefore (5) cannot be applied to the scales where Newtonian (or Einsteinian) gravity is well-tested. For such scales non-linear effects, such as the Vainshtein mechanism [9] or non-linear extensions of massive gravity, such as [3], should come into play to correctly reproduce the observations within massive gravity. The relevant scales that appear depend on the specific non-linear extension of massive gravity. See [6] for an extensive review of massive gravity theories and how Vainshtein radius, below which non-linear theories must be used, can be set to the size of the solar system or the size of the galaxy. The point of view in this current work is that at sufficiently large separations where massive gravity is expected to deviate from GR, (5) describes the lowest order potential energy for all viable massive gravity theories that reduce to the Fierz–Pauli theory at the linear level. [Of course, there may not exist a viable non-linear massive gravity theory free of the ghost, acausality, strong coupling and vDVZ problems, but at this stage, there is still hope.]

One could argue that compared to the Newtonian potential energy between the sources, the spin–spin potential energy is rather small and does not contribute much to the overall force. While this is correct, the overall force is not the relevant issue here: spin–spin force is quite distinct from the mass–mass force. The former is the sole force that determines the spin orientations.The situation is similar to the magnetic force in electrodynamics: While the magnetic force between two slowly moving charges with magnetic dipole moments (spins) is much smaller compared to the Coulomb force, it has a distinct effect on the charges. In fact interacting magnetic-dipole moments of charged particles give rise to ferromagnetic effects. In the context of massive gravity, a similar situation arises: spin–spin interaction of galaxies give rise to an overall spin of the system. Of course to derive observable consequences

from our calculations above, one must carry out an N-body simulation of galaxies. The situation is actually quite similar to the Heisenberg model of three-dimensional spins. It is an open question to see if massive gravity could explain the observations of [10,11] who found that galaxies in a region have a non-zero total spin which cannot be easily explained by GR.

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Appendix A. Finding the spin-orientations in GR and in massive gravity

Here let us derive the minimum energy configuration for the spins in both GR and massive gravity. The relevant part to be maximized in the potential energy is

$$h \equiv \vec{J}_1 \cdot \vec{J}_2 - f(x)\, \vec{J}_1 \cdot \hat{r}\, \vec{J}_2 \cdot \hat{r}, \tag{29}$$

where $x = m_g r$ and $f(x) = 3$ for GR and the general form of it is

$$f(x) = \frac{3(1 + x + \frac{1}{3}x^2)}{(1 + x + x^2)}. \tag{30}$$

Note that for massive gravity $f(x) \in [3, 1)$.

In spherical coordinates let us choose the plane of $\vec{J}_1$ and $\hat{r}$ as the xy-plane, and choose the direction of $\hat{r}$ as the x-axis. Therefore, $\vec{J}_1$ and $\vec{J}_2$ have the following components in this coordinate system

$$\vec{J}_1 = J_1(\cos\varphi_1\, \hat{i} + \sin\varphi_1\, \hat{j}), \tag{31}$$

and

$$\vec{J}_2 = J_2(\cos\varphi_2 \sin\theta_2\, \hat{i} + \sin\varphi_2 \sin\theta_2\, \hat{j} + \cos\theta_2\, \hat{k}). \tag{32}$$

Then, the relevant scalar products read

$$\vec{J}_1 \cdot \hat{r} = J_1 \cos\varphi_1, \qquad \vec{J}_2 \cdot \hat{r} = J_2 \cos\varphi_2 \sin\theta_2, \tag{33}$$

$$\vec{J}_1 \cdot \vec{J}_2 = J_1 J_2(\cos\varphi_1 \cos\varphi_2 \sin\theta_2 + \sin\varphi_1 \sin\varphi_2 \sin\theta_2). \tag{34}$$

Then (29) becomes

$$h = J_1 J_2\big[\cos\varphi_1 \cos\varphi_2 \sin\theta_2 (1 - f) + \sin\varphi_1 \sin\varphi_2 \sin\theta_2\big], \tag{35}$$

where we wrote $f(x) = f$. From (35) we see that $\vec{J}_1$ and $\vec{J}_2$ must be on the same plane, which follows from $\frac{\partial h}{\partial \theta_2} = 0$, $\theta_2 = \pm\frac{\pi}{2}$. When these are put into (35) we see that h becomes a maximum for $\theta_2 = \frac{\pi}{2}$ and a minimum for $\theta_2 = -\frac{\pi}{2}$. Since we want it to be a maximum (to get the minimum of the potential energy) we choose $\frac{\pi}{2}$. Then

$$h = J_1 J_2\big[\cos\varphi_1 \cos\varphi_2 (1 - f) + \sin\varphi_1 \sin\varphi_2\big], \tag{36}$$

and extremization with respect to two angles yield

$$\frac{\partial h}{\partial \varphi_1} = -\sin\varphi_1 \cos\varphi_2 (1 - f) + \cos\varphi_1 \sin\varphi_2 = 0, \tag{37}$$

$$\frac{\partial h}{\partial \varphi_2} = -\cos\varphi_1 \sin\varphi_2 (1 - f) + \sin\varphi_1 \cos\varphi_2 = 0. \tag{38}$$

From now on the discussion bifurcates whether f is 1 or not.

Let us first take $f = 1$ then (37) and (38) become

$$\cos\varphi_1 \sin\varphi_2 = 0, \tag{39}$$

$$\sin\varphi_1 \cos\varphi_2 = 0. \tag{40}$$

From (39) $\varphi_1 = \frac{\pi}{2}$ or $\varphi_2 = 0$ and from (40) $\varphi_1 = 0$ or $\varphi_2 = \frac{\pi}{2}$. Therefore we have two solutions that are

$$\varphi_1 = \varphi_2 = 0,$$
$$\varphi_1 = \varphi_2 = \frac{\pi}{2}. \tag{41}$$

Putting (41) into (35) we get

$$h(\varphi_1 = 0,\ \varphi_2 = 0) = 0, \tag{42}$$

$$h\left(\varphi_1 = \frac{\pi}{2},\ \varphi_2 = \frac{\pi}{2}\right) = J_1 J_2, \tag{43}$$

where (43) gives the minimum potential energy. Both spins point in the same direction and they are perpendicular to $\vec{r}$ joining the sources.

Let us continue our discussion with $f \neq 1$: We plug (37) into (38) to get

$$\big[(1 - f)^2 - 1\big] \sin\varphi_1 \cos\varphi_2 = 0. \tag{44}$$

There are again two cases which must be analyzed separately. One is

$$(1 - f)^2 - 1 = 0 \quad \Rightarrow \quad f(f - 2) = 0. \tag{45}$$

For this case f can be either 0 or 2. We know that f is in between $[3, 1)$. Then f cannot be 0. Therefore, it must be 2. If $f \neq 2$ then $\sin\varphi_1 \cos\varphi_2 = 0$ which is the second case. Before going into the details of the second option, let us exhaust the first one:

$$f = 2 \Rightarrow \frac{3(1 + x + \frac{1}{3}x^2)}{(1 + x + x^2)} = 2,$$
$$x^2 - x - 1 = 0, \tag{46}$$

whose physical relation is

$$x = \frac{1 + \sqrt{5}}{2} \cong 1.62.$$

Note that at this point,

$$h = -J_1 J_2 \cos(\varphi_1 + \varphi_2), \tag{47}$$

which is maximized for $\varphi_1 + \varphi_2 = \pi$, that is the same as the GR case. Lets look at the $f \neq 2$ case. For this case

$$\sin\varphi_1 \cos\varphi_2 = 0.$$

Then we have two possibilities that are $\varphi_1 = 0$ or π and φ_2 is arbitrary or $\varphi_2 = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ and φ_1 is arbitrary. Put $\varphi_1 = 0$ or $\varphi_1 = \pi$ (both will give the same result) into (36)

$$h = J_1 J_2 (1 - f) \cos\varphi_2, \tag{48}$$

taking the derivative of (48) with respect to φ_2 to find the maximum value of h. Then,

$$\frac{\partial h}{\partial \varphi_2} = -J_1 J_2 (1 - f) \sin\varphi_2 = 0,$$

which is solved for $\varphi_2 = 0, \pi$. If we put these results separately into (36) we get

$$h(\varphi_1 = 0, \varphi_2 = 0) = J_1 J_2 (1 - f) < 0, \tag{49}$$

$$h(\varphi_1 = 0, \varphi_2 = \pi) = -J_1 J_2 (1 - f) > 0. \tag{50}$$

Therefore, h is maximum for (50). The second possibility is $\varphi_2 = \frac{\pi}{2}$. Again (36) becomes for this choice as follows:

$$h = J_1 J_2 \sin\varphi_1, \tag{51}$$

note that there is no f dependence. The maximization condition of (51) are

$$\frac{\partial h}{\partial \varphi_1} = J_1 J_2 \cos\varphi_1 = 0,$$

$\varphi_1 = \frac{\pi}{2}, \frac{3\pi}{2}$. Putting these into (36) we get

$$h\left(\varphi_1 = \frac{\pi}{2}, \varphi_2 = \frac{\pi}{2}\right) = J_1 J_2, \tag{52}$$

$$h\left(\varphi_1 = \frac{3\pi}{2}, \varphi_2 = \frac{\pi}{2}\right) = -J_1 J_2. \tag{53}$$

For this case, h is always a maximum for (52). Here note that when $f < 2$ (50) becomes smaller than (52). Then for $f < 2$ the spins point in the same direction and are perpendicular to the line joining them. On the other hand, for $f > 2$ (50) is larger than (52) so the spins are anti-parallel and point along the line joining them. Namely for massive gravity the spins flip at $m_g r \approx 1.62$.

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