THEORETICAL & APPLIED MECHANICS LETTERS **1**, 051008 (2011)

Vibration parameter measurement using the temporal digital hologram sequence and windowed Fourier transform

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(Received 17 July 2011; accepted 30 July 2011; published online 10 September 2011)

Abstract Real time digital recording and numerical reconstruction of a temporal digital hologram sequence have become feasible in recent years. They provide a new measurement method which enjoys the valuable advantages of being full-field, noncontact and high precision. In this paper, a combined method of temporal digital hologram sequence and windowed Fourier transform is proposed to measure the kinematic parameters of random vibration. A series of holograms are recorded by CCD camera and the original phase can be reconstructed by Fresnel reconstruction algorithm. The three-dimensional windowed Fourier transform is used to filter noise in phase and extract the instantaneous kinematic parameters of the specimen, such as the displacement, velocity and acceleration. An experiment is conducted on a chloroprene rubber latex membrane. Results demonstrate that the proposed method determines the vibration parameters precisely and enjoys many merits. © *2011 The Chinese Society of Theoretical and Applied Mechanics.* [doi:10.1063/2.1105108]

Keywords vibration measurement, digital holography, temporal digital hologram sequence, windowed Fourier transform

Optical measurement methods enjoy many unique advantages, such as being full-field, noncontact, nondestructive and high precision, therefore have a wide and particular application prospect. In recent years, due to the demands for higher product quality and reliability in the industrial design and manufacturing, measurement of dynamic behavior such as vibration and transient deformation is of much interest. Many important advances in different optical techniques have been achieved, such as in the moiré fringe method,[1] temporal speckle pattern interferometry,[2,3] photoelasticity,[4] etc. Currently, with the development of digital recording devices, digital holography has attracted many attentions.[5,6] It can be performed in almost real time via the automatic digital recording and numerical reconstruction. Both the amplitude and phase information of the object can be reconstructed by using a single hologram, there does not exist phase ambiguity problem.

Vibration is one of the most important physical quantities which needs to be measured in many engineering fields. Due to the complicated wet chemical processing and difficult repositioning, conventional holographic interferometry is usually confined to the time-average vibration amplitude and mode shape measurement. The digital holographic technique provides a new approach to analyze the vibration in real time.[7] In our previous work, the vibration of a micro-scale cantilever beam was measured by the pre-magnifying digital holography and processed by the two-dimensional windowed Fourier transform (WFT).[8] In this paper, a random vibration of a chloroprene rubber latex membrane was measured. The digital hologram sequence was captured during the course of vibration and the phase difference sequence was calculated via the Fresnel reconstruction algorithm. A three-dimensional windowed Fourier algorithm was used to retrieve the displacement, velocity and acceleration information. Results demonstrate that the proposed combined approach of digital holographic interferometry and windowed Fourier transform is suitable for vibration parameter measurement with its unique and valuable advantages.

In digital holography, a laser light is split into two beams. One beam illuminates the specimen and is diffused to form the object wave $O(x, y)$ in the hologram plane (x, y). The other arrives at the hologram plane directly and generates the reference wave $R(x, y)$. The hologram $H(x, y)$ is the interference of the two waves and can be captured by a CCD sensor, whose the sensitive surface is placed at the hologram plane. A general scheme of digital holography for vibration measurement is shown in Fig. 1. The sensitivity vector $\boldsymbol{S}$ is parallel to $\boldsymbol{e}_i + \boldsymbol{e}_o$, where $\boldsymbol{e}_i$ and $\boldsymbol{e}_o$ are the unit vectors of illumination and observation directions, respectively.

When the hologram is sampled by the CCD sensor, the maximum angle $\theta_{\max}$ between the object and reference waves ought to satisfy the Nyquist sampling theorem expressed as

$$\theta_{\max} \leq 2\arcsin[\lambda/(4\Delta)], \tag{1}$$

where Δ denotes the pixel distance of the CCD sensor and λ denotes the wavelength of the laser. The Nyquist sampling theorem is also ought to be satisfied in the temporal domain. The sampling frequency should be greater than twice the maximum frequency of the signal being sampled.

The phase is reconstructed by the Fresnel reconstruction algorithm.[9] The reference wave $R(x, y)$ serves

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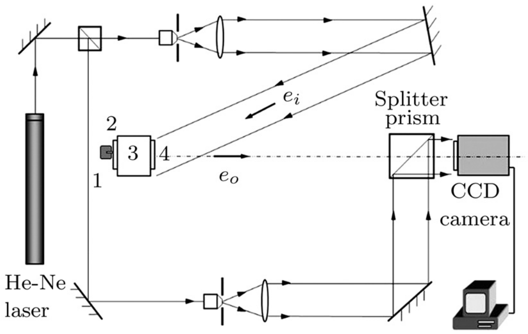


1. PZT stage;
2. Membrane;
3. Airtight hollow cylinder;
4. Membrane awaiting measurement.

Fig. 1. A general scheme of digital holography.

as the reconstruction wave to illuminate the hologram $H(x, y)$. The reconstructed complex wave field $U(x', y')$ at the image plane (x', y') at a distance d from the hologram plane can be expressed as

$$U(x', y') = C \exp\left[\frac{\mathrm{i}\pi}{\lambda d}(x'^2 + y'^2)\right] \cdot FT\left\{H(x, y)R(x, y)\exp\left[\frac{\mathrm{i}\pi}{\lambda d}(x^2 + y^2)\right]\right\}, \quad (2)$$

where FT denotes the two-dimensional Fourier transform, d denotes the reconstruction distance and C is a constant.

While the specimen deforms, a change occurs in the phase of the object wave. If the two holograms are recorded before and after the deformation, the phase difference $\Delta\phi$ between the two states can be calculated by using

$$\Delta\phi = \arg[U_2(m, n)] - \arg[U_1(m, n)]. \quad (3)$$

The phase difference obtained from Eq. (3) is wrapped in a principal value between $-\pi$ to π, so a phase unwrapping processing is needed to get the actual value. The displacement difference D of the specimen is linear to the unwrapped phase difference

$$D = \frac{\lambda \Delta\phi}{2\pi S}. \quad (4)$$

The windowed Fourier transform of a one-dimensional signal $f(x)$ can be written as

$$Sf(u, \xi) = \int_{-\infty}^{\infty} f(x) \exp\left[-\frac{(x - u)^2}{2\sigma^2}\right] \exp(-\mathrm{j}\xi x) \mathrm{d}\, x, \quad (5)$$

where σ is the standard deviation and controls the spatial extension of the Gaussian function, which should be preset appropriately.[10]

Two WFT-based algorithms, windowed Fourier filtering (WFF) and windowed Fourier ridges (WFR), have been developed by Qian.[11] The WFF method processes a fringe pattern by filtering its WFT spectrum. Those spectrum coefficients whose amplitudes are smaller than the preset threshold will be abandoned. Since most noises permeate the spectrum domain with tiny coefficients, a smooth signal can be obtained after an inverse WFT operation. Mathematically, it can be expressed as Eqs. (6) and (7), and the phase $\phi\ (x)$ can be computed by

$$\overline{Sf(u, \xi)} = \begin{cases} Sf(u, \xi), & \text{if} \quad |Sf(u, \xi) \geq Thr|, \\ 0, & \text{if} \quad |Sf(u, \xi) < Thr|, \end{cases} \quad (6)$$

$$\overline{f(x)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{Sf(u, \xi)} \exp\left[-\frac{(x - u)^2}{2\sigma^2}\right] \exp(j\xi x) \mathrm{d}\, u \mathrm{d}\, \xi, \quad (7)$$

$$\phi(x) = \text{angle}[\overline{f(x)}]. \quad (8)$$

The WFR method detects the maxima points of the amplitude spectrum. The frequencies that maximize the normalized amplitude spectrum approximate the local frequencies or alternatively instantaneous frequencies. Mathematically, the phase $\phi\ (x)$ and instantaneous frequency $\phi'\ (x)$ are calculated by

$$\begin{aligned} &\phi(x) = \text{angle}[Sf(u, \xi)]_{\xi = \phi'(u)}, \\ &\phi'(u) = \max\{\text{abs}[Sf(u, \xi)]\}\xi. \end{aligned} \quad (9)$$

The three-dimensional process can be deduced easily from that in the one-dimensional space due to the separabilities in dimensions of WFT and Gaussian function.[12]

The vibration of a chloroprene rubber latex membrane was measured in this experiment by using the digital holographic scheme shown in Fig. 1. The real holographic scheme in our experiment is shown in Fig. 2. Two circular holes are made on an airtight hollow aluminum cylinder. The membrane awaiting measurement is covered onto one hole and its periphery is fixed to the cylinder by using a strong adhesive. The diameter of the hole is 20 mm. This setup is usually used in bulge test to measure the elastic modulus of film materials. Since the chloroprene rubber is very soft, a PZT stage replaces the ordinary air pressure controller in this test. Another membrane is covered onto the other hole and its periphery is also fixed to the cylinder. While the PZT stage moves randomly, the air pressure in the cylinder changes and the membrane awaiting measurement vibrates, synchronously.

In the experiment, the angle between the illumination and observation directions was about 18° and the laser wavelength is 632.8 nm. The PZT moved randomly, whose displacement variation was shown in Fig. 3(a). During the course of vibration, 420 consecutive holograms in the format of 1 236 pixels × 1 236 pixels were recorded with a rate of 1 frame per second.

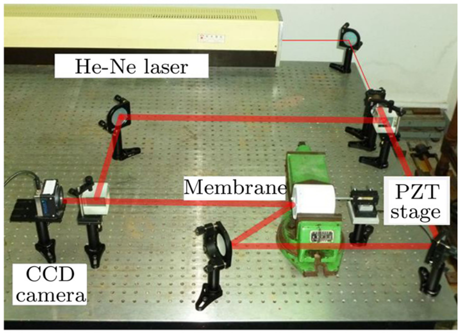


Fig. 2. The real scheme of digital holographic measuring system in our test.

The original wrapped phase ϕ between two holograms recorded at t_1 and t_n was calculated by the Fresnel reconstruction algorithm and an effective area of 240 pixels × 240 pixels in each phase map was selected for analysis, as shown in Fig. 3(b). In the reconstruction plane, one pixel in both the x and y direction is equivalent to a distance of approximately 0.093 mm.

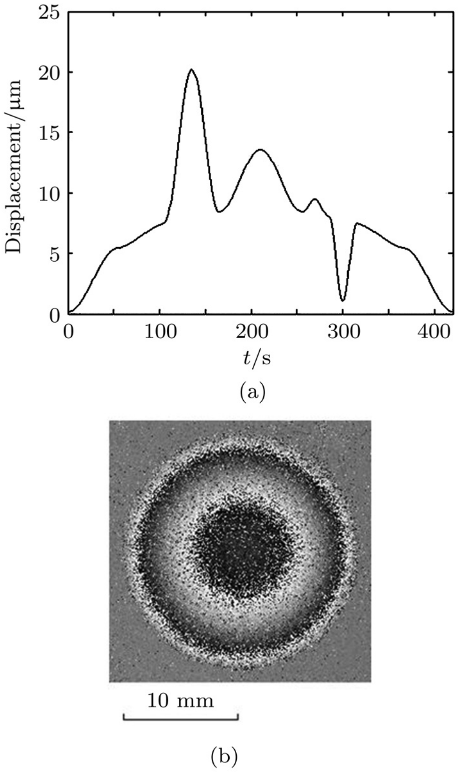


Fig. 3. (a) The motion distribution of the PZT stage. (b) The selected effective area of original phase difference.

A phase unwrapping algorithm was applied to each original phase map. Then the unwrapped phase values in the form of a three-dimensional matrix were packed to the exponential form exp(jϕ) and processed by the three-dimensional WFT. Since the phase was rewrapped after the windowed Fourier processing, it was unwrapped again. Two-dimensional WFF generates a series of clear phase maps in spatial domain, but the phase distribution along the time axis may not be so smooth. While a filtering operation along the time axis is performed, some disturbances may occur in the spatial phase distribution. The three-dimensional processing eliminates the noise in both the spatial and temporal domains, so the results are more precise.

Figure 4 shows two original wrapped phase maps (at $t = 1$ and 150 s) in the left column, wrapped phase maps after windowed Fourier processing in the middle column, and unwrapped phase maps in the right column. According to the experimental arrangement, a phase change of 1 rad is equivalent to a displacement of approximately 51 nm. The displacement distribution of the membrane at each time instant and the temporal displacement variation on each pixel can be obtained. Figure 5(a) shows the whole-field displacement distribution at $t = 150$ s and Fig. 5(b) plots the displacement variation along the time axis on the randomly selected pixel (76, 55) and the central pixel (120, 120).

In the mean time, the instantaneous velocity and acceleration can be obtained by detecting the WFR along the time axis. The instantaneous velocity and acceleration distribution along the time axis on pixels (76, 55) and (120, 120) is shown in Figs. 6(a) and 6(b), respectively.

The digital holographic interferometry combined with three-dimensional WFT is proposed for vibration measurement. It is a full-field, high precision and non-contact measurement method. The real time transformation and automatization of digital hologram recording has been realized. The unambiguous phase information can be obtained from a single hologram with no request of phase shifting or carrier. Compared with other optical interferometry methods, the speed of image acquisition can be improved or the measurement range of vibration frequency and amplitude can be extended. The three-dimensional WFT is selected to improve the phase precision and retrieve the phase derivative value.

In this paper, the experiment was conducted on a chloroprene rubber latex membrane. Holograms were captured during the random vibration course of the membrane. The important vibration parameters, such as displacement, velocity and acceleration, were extracted successfully by the three-dimensional windowed Fourier processing. The proposed measurement method can be used for random vibration, whose maximum frequency is less than half the sampling frequency. It is suggested that the sampling frequency is greater than this critical value to improve the precision. Different schemes of digital holography can be employed in the practical application for different objects, such as the pre-imaging digital holography for a big object and digital holographic microscopy for a micro-scale object, etc. For a low frequency vibration measurement, the phase shifting technique or the synthetic aperture method can be introduced to improve the resolution of digital holography.

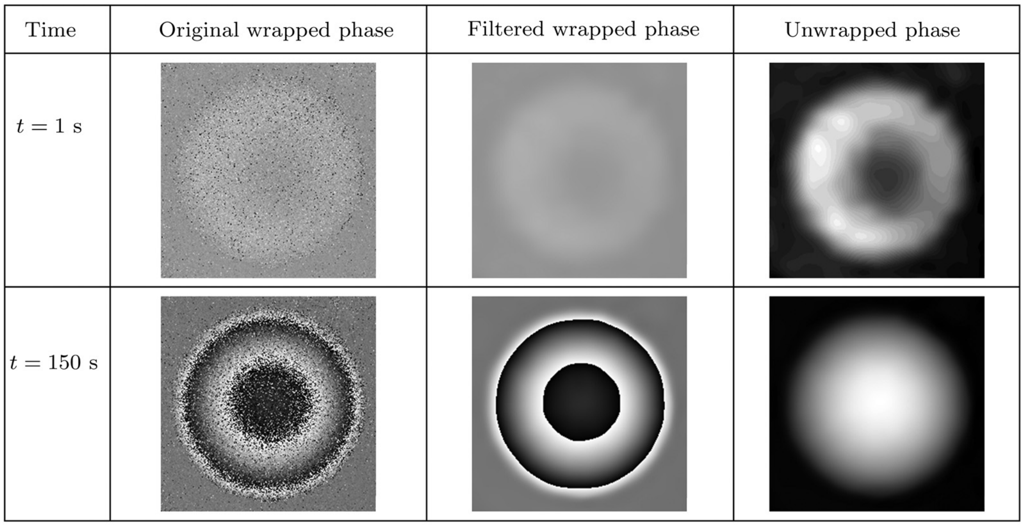
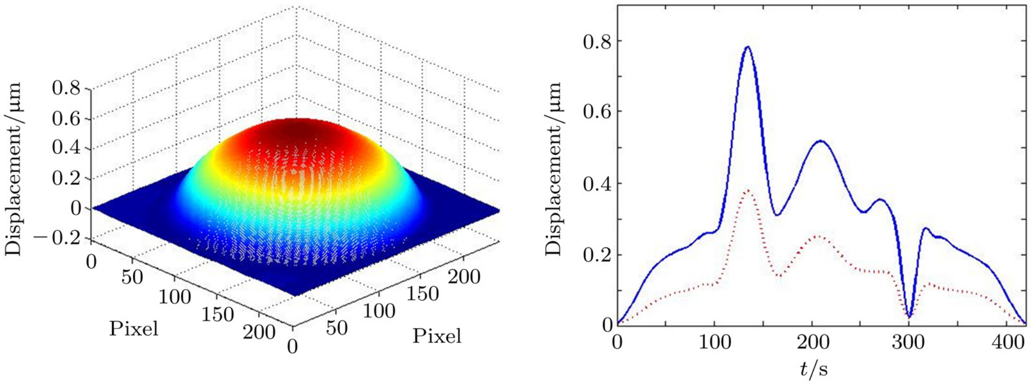
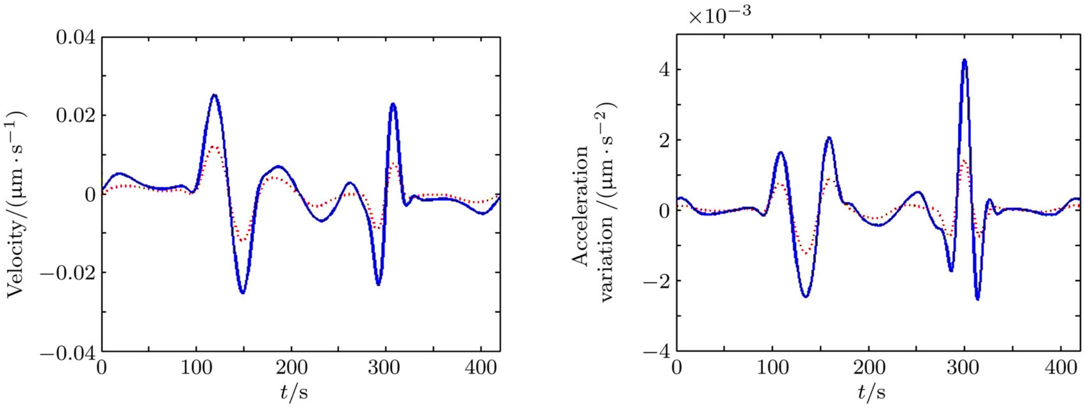


Fig. 4. The phase difference maps between and after the filtering and unwrapping processing.



(a) (b)

Fig. 5. (a) The displacement distribution of the membrane at t = 150 s. (b) The displacement variation along the time axis on the pixels (76, 55) (broken line) and (120,120) (solid line).



(a) (b)

Fig. 6. (a) The velocity variation. (b) The acceleration variation along the time axis on the pixels (76, 55) (broken line) and (120,120) (solid line).

This work was supported by the National Natural Science Foundation of China (10772171 and 10732080), and the National Basic Research Program of China (2007CB936803).

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