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ROUGH SETS ANALYSIS OF DIAGNOSTIC CAPACITY OF VIBROACOUSTIC SYMPTOMS

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Abstract—The paper refers to the problem of diagnostic classification of mechanical objects using vibroacoustic symptoms. A new approach based on the rough sets theory is applied to evaluate the symptoms from the point of view of their diagnostic capacity, i.e., the quality of estimation of a technical state of a mechanical object. The approach enables reduction of the set of symptoms to a minimal subset ensuring a satisfactory estimation. The minimal subset is then used to create a classifier of a technical state. Particular attention is paid to a comparison of different methods of calculation of symptom limit values which divide domains of symptoms into intervals corresponding to classes of technical states. The analysed set of data concerns the technical state of rolling bearings installed in a laboratory stand. They are described by a set of symptoms which result from measurements of noise and vibration of bearing housings. The bearings are in good or bad technical states. The paper presents particular steps of the rough sets methodology and gives, as a final result, a classifier of a technical state of bearings based on a minimal subset of symptoms with the greatest diagnostic capacity.

1. INTRODUCTION

One of the main problems in technical diagnostics is evaluation of a technical state of controlled objects (cf.[1–3]). The process of change of their technical state develops with different speed. An attainment of a critical state can result in several secondary failures. So, in consequence the cost of repair is much higher than it would be if potential dangers were detected earlier and an object was repaired before attaining this critical state.

Impossibility of a direct measurement of the technical state for a working object is the main difficulty in technical diagnostics. In fact, a lot of factors influence the technical state of a working object and many of them are inaccessible for measurement during its work. As a result, evaluation of the technical state can be performed only on a base of an indirect measurement of physical quantities which are changing in accordance with the technical state of an object. These quantities are called *symptoms of the technical state*.

Let us notice that values of symptoms usually change monotonically with deterioration of the technical state (i.e., values of symptoms increase or decrease continuously). On the contrary, the technical state is estimated in a qualitative way, i.e., by considering two-, three- or more classes of technical states [4]. Owing to this fact, it is accepted to trend to determine such values of symptoms which would separate considered classes of technical states. The values are called *symptom limit values* and classes are called conventional classes of the technical state.

Evaluation of the technical state using only one symptom is the most desired in practice. However, very often it is impossible to perform such evaluation in a reliable way. So, instead

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Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX of a single symptom, a subset of symptoms with the best diagnostic capacity is used for this purpose. Chosen symptoms can be used to create a *classifier of the technical state*. The classifier deduces types and degree of failures from the values of symptoms. In simple cases, it enables only gradation of the technical state on a scale from good to bad state. The classifier consists of a chosen subset of symptoms together with their limit values ensuring the best evaluation of the technical state.

According to their construction (mechanical, electrical, etc.) objects are diagnosed by means of different methods enabling recognition of different types of failures. In this paper, we consider methods of technical diagnostics concerning mechanical objects [5].

The general aims of a majority of technical diagnostic investigations are the following:

- evaluation of capacity of particular symptoms to estimation of technical states of considered objects,
- reduction of a set of symptoms to a subset of symptoms ensuring the best evaluation of the technical state; it should be noticed that results of the reduction can be non-unique, i.e., as a result of this reduction one may obtain several possible reduced subsets of symptoms; in such cases, the choice of a specific subset is based on additional criteria, e.g., minimum number of symptoms, cost of measurement,
- creation of a classifier of the technical state basing on a chosen subset of symptoms.

The quality of diagnostic investigations depends on an *a priori* knowledge concerning:

- the kind of technical symptoms which are supposed to carry useful diagnostic information,
- the possibility of satisfactory estimation of symptom limit values which separate conventional classes of the technical state.

In [6] a new method based on the rough sets theory has been applied to analysis of newly proposed aggregated symptom measures in vibroacoustic diagnostics of reducers. In this paper, taking into account encouraging results of this application, we are going to use the rough sets theory to determine the:

- evaluation of different methods of defining symptom limit values, proposed to use in so-called badly conditioned diagnostic tasks [5,6],
- reduction of a set of symptoms to a minimal subset of symptoms ensuring the best evaluation of the technical state,
- creation of classifiers of the technical state using the best method of defining symptom limit values and the chosen subset of symptoms.

The above problems have not been solved in a satisfactory way until now. The first problem, i.e., evaluation of different methods of defining symptom limit values, was so far undertaken in [6] and was limited to analysis of simulation data. The second problem, i.e., a proper choice of symptoms, has a great practical importance because it is closely connected with the cost of diagnostics. However, only few attempts for solving it are known. For example, the algorithm BEDNID, described in [7], is far from solving this problem in a satisfactory way. The research reported in [8] and [9], shows that the rough sets theory provides a good tool for a proper choice of symptoms to create the classifier.

In the present paper, we analyse data concerning a set of identically constructed mechanical objects (i.e., rolling bearings) which are in one of two technical states (good or bad). The considered symptoms are based on noise and vibration characteristics of objects. In addition, two possible scales of noise symptoms are taken into account: logarithmic and linear ones.

In the next section, basic notions of the rough sets theory are recalled. Then, in Section 3, the analysed data set is described together with the methods defining symptom limit values. Results of analysis by means of the rough sets theory are presented in Section 4. Conclusions are drawn in the last section.

2. BASIC CONCEPTS OF THE ROUGH SETS THEORY

In this section only a short reminder of basic notions of the rough sets theory created by Pawlak is given. These concepts should be useful to understand the analysis of the considered diagnostic problem performed in the next sections. More exhaustive presentation of the rough sets theory seems to be unnecessary because of existence of several papers devoted to this theory (cf. e.g., [10–17]). Moreover, some information about applications of this method to medical data analysis and medical diagnostics can be found in [18–22]. The case of medical diagnostics is methodologically similar to the problems of technical diagnostics considered in this paper.

2.1. Introductory Remarks

The data analysed by the rough sets theory concern a set of objects (observations, individuals, states, etc.) described by a set of multi-valued attributes (features, symptoms, variables, etc.). For each pair (object, attribute) there is known a value called descriptor. Objects, attributes and descriptors are three basic components of an information system which can be viewed as a table with rows corresponding to objects and columns corresponding to attributes. Each row of the table contains descriptors representing information about the corresponding object. Moreover, the set of objects is classified into disjoint family of classes.

The observation that we cannot distinguish objects on the basis of imprecise information about them is the starting point of the rough sets approach. In other words, imprecise information causes indiscernibility of objects. Indiscernibility of objects prevents generally their precise classification. Given an equivalence relation viewed as an indiscernibility relation which thus induces an approximation space made of equivalence classes, a *rough set* is a pair of lower and upper approximation of a set in terms of these classes of indiscernible objects. Using lower and upper approximation of a set (or a family of sets—classification) one can define an accuracy and a quality of approximation. These are numbers from interval $[0,1]$ which define how exactly one can describe the examined set (or classification) of objects using available information.

The rough sets approach enables to solve two main problems in the analysis of information systems:

- reduction of all redundant objects and attributes so as to get the minimum subset of attributes ensuring a good approximation of classes and an acceptable quality of classification,
- representation of all important relationships between the most significant attributes and particular classes in a form of a set of decision rules.

2.2. Information System

By an information system we understand the 4-tuple $S = \langle U, Q, V, \rho \rangle$, where U is a finite set of *objects*, Q is a finite set of *attributes*, $V = \bigcup_{q \in Q} V_q$ and V_q is a *domain* of the attribute q, and $\rho : U \times Q \rightarrow V$ is a total function such that $\rho(x,q) \in V_q$ for every $q \in Q$, $x \in U$, called an *information function.*

Let $S = \langle U, Q, V, \rho \rangle$ be an information system and let $P \subseteq Q$ and $x, y \in U$. We say that x and y are *indiscernible* by the set of attributes P in S (denotation $x \tilde{P} y$) iff $\rho(x,q) = \rho(y,q)$ for every $q \in P$. Equivalence classes of relation $\tilde{P}$ are called *P-elementary sets* in S. Q-elementary sets are called *atoms* in S.

The family of all equivalence classes of relation $\tilde{P}$ on U is denoted by $P^{\star}$.

2.3. Approximation of Sets

Let $P \subseteq Q$ and $Y \subseteq U$. The *P-lower approximation* of Y denoted by $\underline{P}Y$ and the *P-upper approximation* of Y denoted by $\overline{P}Y$ are defined as:

$$\underline{P}Y = \bigcup X \ : \ \{X \in P^{\star} \text{ and } X \subseteq Y\}$$
$$\overline{P}Y = \bigcup X \ : \ \{X \in P^{\star} \text{ and } X \cap Y \neq \emptyset\}$$

The P-boundary (doubtful region of classification) is defined as

$$Bn_P(Y) = \overline{P}Y - \underline{P}Y.$$

Set $\underline{P}Y$ is the set of all elements of U which can be certainly classified as elements of Y, employing the set of attributes P; Set $\overline{P}Y$ is the set of elements of U which can be possibly classified as elements of Y, using the set of attributes P. The set $Bn_P(Y)$ is the set of elements which cannot be certainly classified to Y using the set of attributes P.

With every subset $Y \subseteq U$, we can associate an ***accuracy of approximation*** of set Y by P in S, or in short, accuracy of Y, defined as:

$$\alpha_P(Y) = \frac{\text{card}(\underline{P}Y)}{\text{card}(\overline{P}Y)}.$$

2.4. Rough Classification

Let S be an information system, $P \subseteq Q$, and let $\mathbf{X} = \{Y_1, Y_2, \ldots, Y_n\}$ be a ***classification*** of U, i.e., $Y_i \cap Y_j = \emptyset$ for every $i, j \leq n$, $i \neq j$ and $\bigcup_{i=1}^{n} Y_i = U$. Y_i are called ***classes*** of $\mathbf{X}$. By P-lower (P-upper) approximation of $\mathbf{X}$ in S we mean sets $\underline{P}\mathbf{X} = \{\underline{P}Y_1, \underline{P}Y_2, \ldots, \underline{P}Y_n\}$ and $\overline{P}\mathbf{X} = \{\overline{P}Y_1, \overline{P}Y_2, \ldots, \overline{P}Y_n\}$, respectively. The coefficient

$$\gamma_P(\mathbf{X}) = \frac{\sum_{i=1}^{n} \text{card}\,(\underline{P}Y_i)}{\text{card}\,(U)}$$

is called the quality of approximation of classification $\mathbf{X}$ by set of attributes P, or in short, ***quality of classification*** $\mathbf{X}$. It expresses the ratio of all P-correctly classified objects to all objects in the system.

2.5. Reduction of Attributes

We say that the set of attributes $R \subseteq Q$ ***depends*** on the set of attributes $P \subseteq Q$ in S (denotation $P \rightarrow R$) iff $\tilde{P} \subseteq \tilde{R}$. Discovering dependencies between attributes enables the reduction of the set of attributes. Subset $P \subseteq Q$ is ***independent*** in S iff for every $P' \subset P$, $\tilde{P}' \supset \tilde{P}$; otherwise subset $P \subseteq Q$ is ***dependent*** in S. Subset $P \subseteq Q$ is a ***reduct*** of Q in S iff P is the independent set in S and $P^* = Q^*$. If subset $P \subseteq Q$ is the reduct of Q then attributes from subset $Q - P$ are redundant. The quality of classification $\mathbf{X}$ can be also used for a practical detection of redundant attributes because the reduct gives the same quality as the whole set of attributes in the system. Sometimes some attributes can be removed from the reduct without decreasing the quality of classification. The least independent set which ensures the same quality of classification as the reduct is called the ***minimal subset*** in S. Let us notice that an information system may have more than one minimal subset or reduct. Intersection of all minimal sets is called the ***core***. The core is a collection of the most significant attributes for the classification in the system.

2.6. Decision Tables

An information system can be seen as a decision table assuming that $Q = C \cup D$ and $C \cap D = \emptyset$, where C are called ***condition attributes***, and D, ***decision attributes***. Decision table $S = \langle U, C \cup D, V, \rho \rangle$ is ***deterministic*** iff $C \rightarrow D$; otherwise it is ***non-deterministic***. The deterministic decision table uniquely describes the decisions to be made when some conditions are satisfied. In the case of a non-deterministic table, decisions are not uniquely determined by the conditions. Instead, a subset of decisions is defined which could be taken under circumstances determined by conditions.

From the decision table a ***decision algorithm*** can be derived. The decision algorithm consists of a set of ***decision rules*** which are logical statements (if ... then ...). A general procedure for the derivation of a decision algorithm from decision tables was presented in [23] or in [15].

Let us notice, however, that the rough sets analysis of information systems gives satisfactory results when domains of attributes are finite sets of rather low cardinality. This requirement is often met naturally when attributes have a qualitative character. If attributes take arbitrary values from given intervals, i.e., have a quantitative character, they can be handled in the analysis after translating of their values into some qualitative terms, e.g., low, medium or high levels. This translation involves a division of the original domain into some subintervals and an assignment

Table 1. Information system S1 (with noise symptoms represented in the logarithmic scale).

No.	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	D
1	0.50	0.79	1.41	0.94	0.79	1.77	79.5	81.5	72.0	72.5	76.5	75.5	0
2	0.89	1.00	1.25	0.94	0.59	1.77	74.0	78.0	75.0	74.5	66.5	78.0	0
3	1.49	1.49	1.25	0.70	0.53	1.33	85.0	88.0	79.5	79.5	74.0	68.5	0
4	1.67	2.51	2.11	1.12	1.25	1.58	93.0	92.0	83.0	72.0	72.0	69.0	0
5	3.16	3.16	2.66	1.41	0.56	1.77	81.5	84.0	81.5	96.5	96.0	91.5	0
6	0.50	0.70	0.79	0.53	0.63	1.18	78.0	83.0	80.5	96.0	94.0	86.5	0
7	0.79	0.89	1.77	0.44	0.70	1.18	80.0	76.0	82.0	78.5	94.5	92.0	0
8	1.41	2.11	2.51	1.58	1.12	3.16	80.0	80.0	79.5	83.5	79.5	94.0	0
9	1.05	0.56	0.56	1.33	1.05	0.39	85.5	79.5	74.5	74.0	77.5	71.0	0
10	0.63	1.12	0.70	0.79	0.70	0.39	78.0	75.0	78.0	77.5	75.0	76.5	0
11	0.63	1.12	0.70	0.79	0.70	0.39	75.5	71.0	82.0	78.5	78.0	78.0	0
12	3.98	2.81	1.33	1.05	1.33	1.41	69.5	75.5	75.0	85.0	77.5	80.0	0
13	2.23	1.58	1.05	1.12	2.11	3.16	69.5	70.0	76.0	80.5	79.5	80.5	0
14	2.23	2.51	1.33	0.63	0.74	0.56	68.0	75.0	69.0	71.5	80.0	85.5	0
15	1.18	0.74	0.44	0.39	0.70	0.66	63.0	63.0	70.0	70.0	64.0	75.5	0
16	1.41	1.41	1.33	1.05	1.77	1.67	60.0	68.0	72.5	79.0	71.0	72.0	0
17	1.88	2.66	1.25	1.58	3.54	1.77	67.5	60.0	79.0	76.0	77.0	74.0	0
18	1.25	1.18	0.50	1.00	2.66	1.05	60.0	60.0	60.0	62.0	60.0	66.5	0
19	1.67	1.41	0.89	1.49	2.98	0.75	67.0	69.0	66.0	60.0	68.0	67.0	0
20	0.39	0.56	0.28	0.14	0.11	0.11	79.5	81.5	72.0	72.5	76.5	75.5	1
21	0.23	0.33	0.29	0.18	0.22	0.26	78.0	75.0	74.5	66.5	78.0	75.5	1
22	0.43	0.21	0.16	0.10	0.35	0.29	79.5	79.5	74.0	68.5	67.0	68.0	1
23	0.28	0.31	0.18	0.10	0.10	0.13	72.0	72.0	69.0	71.5	73.5	76.5	1
24	0.25	0.31	0.31	0.26	0.23	0.33	80.0	78.0	77.0	73.0	83.0	85.5	1
25	0.10	0.10	0.12	0.10	0.15	0.22	72.5	72.0	72.0	70.0	74.0	80.0	1
26	0.23	0.37	0.35	0.22	0.15	0.22	75.5	72.5	74.5	69.5	75.0	78.5	1
27	0.22	0.26	0.20	0.15	0.15	0.20	71.0	73.0	69.0	69.0	76.0	82.0	1
28	0.26	0.44	0.50	0.39	0.37	0.53	71.0	77.0	67.0	66.5	78.0	76.0	1
29	0.23	0.26	0.16	0.10	0.10	0.10	66.0	66.5	68.0	64.0	71.5	71.5	1
30	0.10	0.11	0.10	0.10	0.10	0.10	72.5	72.0	72.0	70.0	74.0	80.0	1
31	0.18	0.23	0.16	0.13	0.11	0.20	75.5	72.5	74.5	69.5	75.0	78.5	1
32	0.12	0.22	0.16	0.10	0.10	0.15	70.5	73.5	69.0	69.0	76.5	82.0	1
33	0.31	0.63	0.22	0.12	0.12	0.17	71.0	77.0	67.0	66.5	78.5	76.0	1
34	0.10	0.14	0.14	0.10	0.10	0.14	67.0	70.5	66.0	63.0	64.5	68.0	1
35	0.33	0.42	0.70	0.35	0.33	0.50	70.0	68.5	75.0	69.5	79.5	80.5	1
36	0.20	0.21	0.35	0.12	0.14	0.14	70.0	68.0	72.0	63.5	76.0	74.0	1
37	0.11	0.15	0.10	0.10	0.10	0.10	65.5	72.0	71.0	67.0	66.0	71.0	1
38	0.15	0.15	0.16	0.15	0.11	0.10	70.0	74.0	70.0	67.0	77.5	82.0	1

of qualitative codes to these subintervals. Definition of boundary values of the subintervals can influence considerably the quality of classification; it should take into account experience, habits and conventions used by the experts and, possibly, an error of measurement (cf. [18,20,21]). In technical diagnostics attributes are symptoms of the technical state. They are translated into qualitative attributes using symptom limit values which divide an original domain of a symptom into subintervals corresponding to conventional classes of the technical state.

3. THE PROBLEM DEFINITION

The analysed data set is composed of observations collected during a laboratory experiment with a set of 38 rolling bearing. The set of examined bearings is divided into two subsets. The first one consists of 19 bearings which were recognized to be good ones. At the end of their production, they were checked by a product quality control which proved that they were made according to a technical documentation. Other 19 bearings had different elements artificially damaged (rolling elements or one of the bearing races).

The investigated bearings were successively assembled on a laboratory stand. Measurements of a chosen set of vibration and noise symptoms were collected in conditions of the simulated working loads.

Vibration and noise level of bearing housing were taken as supposed symptoms of the technical state. In each case, measuring quantities were obtained as a result of band filtering of a signal for 6 different frequency bands. The filters were 1/3-octave filters with standard middle frequencies from range: 800–2500 Hz.

The data collected from the measurements set up the information system which is presented in Table 1. It contains information about 38 bearings described by means of 12 symptoms $s_1 - s_{12}$. Symptoms $s_1 - s_6$ are measured as accelerations of vibration [m/s^2] while symptoms $s_7 - s_{12}$ correspond to levels of noise in decibels [dB]. The information about each object is additionally extended by the two-valued decision attribute (denoted by D). This attribute characterises the real technical state of a bearing (0—bearings in a good technical state, 1—bearings in bad state). The information system presented in Table 1 will be denoted by **S1**.

The symptom limit values can be defined in different ways (cf.[6,24–29]). We shall use four methods described below:

A. the C-method:

$$b = \bar{s} + \sigma\sqrt{\frac{P_g}{2A}} \tag{1}$$

where:

$\bar{s}$ - mean value of a symptom, calculated as:

$$\bar{s} = \sum_{i=1}^{M} \frac{S_i}{M} \tag{2}$$

M - number of measurements of a symptom (number of observations);
S_i - result of measurement of a symptom,
σ - standard deviation of a symptom calculated as:

$$\sigma = \frac{\sqrt{\sum_{i=1}^{M} (S_i - \bar{s})^2}}{M} \tag{3}$$

P - the fiability index of an object (a ratio of the work time to the work time increased by the repair time),
A - the permissible probability of superfluous repairs performed in order to avoid breakdown;

B. the P-method:

$$b = \left(1 - \gamma^{-1}\right)\, \bar{s}\sqrt[\gamma]{\frac{P_g}{A}} \tag{4}$$

where:

γ - Pareto's shape coefficient calculated as:

$$\gamma = 1 + \sqrt{1 + \left(\frac{\bar{s}}{\sigma}\right)^2} \tag{5}$$

C. the W-method:

$$b = s_{\min} + (\bar{s} - s_{\min})\, \Gamma^{-1}\, \left(1 + k^{-1}\right) \sqrt[k]{\ln\left(\frac{P_g}{A}\right)} \tag{6}$$

Table 2. Symptom limit values for system S1.

Symptom	Method	symptom limit values			
		$b1$	$b2$	$b3$	b
s_1	L	0.22	0.34	0.46	0.58
	C	1.57	2.27	2.97	3.36
	W	1.34	1.78	2.21	2.64
	P	0.60	0.98	1.35	1.73
s_2	L	0.20	0.30	0.40	0.50
	C	1.60	2.26	2.93	3.60
	W	1.21	1.54	1.88	2.22
	P	0.61	1.01	1.42	1.82
s_3	L	0.18	0.26	0.34	0.42
	C	1.28	1.81	2.34	2.87
	W	0.96	1.23	1.50	1.77
	P	0.50	0.83	1.15	1.47
s_4	L	0.15	0.20	0.25	0.30
	C	0.96	1.35	1.73	2.12
	W	0.70	0.89	1.09	1.29
	P	0.39	0.64	0.88	1.12
s_5	L	0.21	0.32	0.43	0.54
	C	1.39	2.05	2.70	3.36
	W	1.35	1.86	2.36	2.86
	P	0.53	0.85	1.16	1.48
s_6	L	0.20	0.30	0.40	0.50
	C	1.42	2.05	2.69	3.32
	W	1.21	1.59	1.98	2.37
	P	0.54	0.89	1.23	1.58
s_7	L	79.60	99.20	118.80	138.40
	C	78.94	84.43	89.91	95.40
	W	21.14	35.16	49.18	63.20
	P	81.40	83.20	85.00	86.81
s_8	L	71.00	82.00	93.00	104.00
	C	79.56	84.81	90.07	95.32
	W	20.86	34.89	48.93	62.96
	P	75.03	79.06	83.09	87.12
s_9	L	75.10	90.20	105.30	120.40
	C	77.45	81.52	85.59	89.66
	W	19.44	33.60	47.76	61.92
	P	77.17	79.25	81.32	83.40
s_{10}	L	71.40	82.80	94.20	105.60
	C	78.97	85.23	91.49	97.75
	W	22.13	36.14	50.15	64.15
	P	75.51	79.62	83.73	87.83
s_{11}	L	76.90	93.80	110.70	127.60
	C	81.67	87.53	93.40	99.26
	W	21.60	35.59	49.59	63.58
	P	80.19	83.47	86.76	90.05
s_{12}	L	76.00	85.50	95.00	104.50
	C	82.73	88.03	93.33	98.63
	W	22.50	38.12	53.74	69.36
	P	79.59	83.19	86.78	90.37

Table 3. Information system S2 (with noise symptoms represented in the linear scale).

No.	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	D
1	0.50	0.79	1.41	0.94	0.79	1.77	188.8	237.7	79.6	84.3	133.6	119.1	0
2	0.89	1.00	1.25	0.94	0.59	1.77	100.2	158.8	112.4	106.1	42.2	158.8	0
3	1.49	1.49	1.25	0.70	0.53	1.33	355.6	502.3	188.8	188.8	100.2	53.2	0
4	1.67	2.51	2.11	1.12	1.25	1.58	893.3	796.2	282.5	79.6	79.6	56.3	0
5	3.16	3.16	2.66	1.41	0.56	1.77	237.7	316.9	237.7	1336.6	1261.9	751.6	0
6	0.50	0.70	0.79	0.53	0.63	1.18	158.8	282.5	211.8	1261.9	1002.3	422.6	0
7	0.79	0.89	1.77	0.44	0.70	1.18	200.0	126.1	251.7	168.2	1061.7	796.2	0
8	1.41	2.11	2.51	1.58	1.12	3.16	200.0	200.0	188.8	299.2	188.8	1002.3	0
9	1.05	0.56	0.56	1.33	1.05	0.39	376.7	188.8	106.1	100.2	149.9	70.9	0
10	0.63	1.12	0.70	0.79	0.70	0.39	158.8	112.4	158.8	149.9	112.4	133.6	0
11	0.63	1.12	0.70	0.79	0.70	0.39	119.1	70.9	251.7	168.2	158.8	158.8	0
12	3.98	2.81	1.33	1.05	1.33	1.41	59.7	119.1	112.4	355.6	149.9	200.0	0
13	2.23	1.58	1.05	1.12	2.11	3.16	59.7	63.2	126.1	211.8	188.8	211.8	0
14	2.23	2.51	1.33	0.63	0.74	0.56	50.2	112.4	56.3	75.1	200.0	376.7	0
15	1.18	0.74	0.44	0.39	0.70	0.66	28.2	28.2	63.2	63.2	31.6	119.1	0
16	1.41	1.41	1.33	1.05	1.77	1.67	20.0	50.2	84.3	178.2	70.9	79.6	0
17	1.88	2.66	1.25	1.58	3.54	1.77	47.4	20.0	178.2	126.1	141.5	100.2	0
18	1.25	1.18	0.50	1.00	2.66	1.05	20.0	20.0	20.0	25.1	20.0	42.2	0
19	1.67	1.41	0.89	1.49	2.98	0.75	44.7	56.3	39.9	20.0	50.2	44.7	0
20	0.39	0.56	0.28	0.14	0.11	0.11	188.8	237.7	79.6	84.3	133.6	119.1	1
21	0.23	0.33	0.29	0.18	0.22	0.26	158.8	112.4	106.1	42.2	158.8	119.1	1
22	0.43	0.21	0.16	0.10	0.35	0.29	188.8	188.8	100.2	53.2	44.7	50.2	1
23	0.28	0.31	0.18	0.10	0.10	0.13	79.6	79.6	56.3	75.1	94.6	133.6	1
24	0.25	0.31	0.31	0.26	0.23	0.33	200.0	158.8	141.5	89.3	282.5	376.7	1
25	0.10	0.10	0.12	0.10	0.15	0.22	84.3	79.6	79.6	63.2	100.2	200.0	1
26	0.23	0.37	0.35	0.22	0.15	0.22	119.1	84.3	106.1	59.7	112.4	168.2	1
27	0.22	0.26	0.20	0.15	0.15	0.20	70.9	89.3	56.3	56.3	126.1	251.7	1
28	0.26	0.44	0.50	0.39	0.37	0.53	70.9	141.5	44.7	42.2	158.8	126.1	1
29	0.23	0.26	0.16	0.10	0.10	0.10	39.9	42.2	50.2	31.6	75.1	75.1	1
30	0.10	0.11	0.10	0.10	0.10	0.10	84.3	79.6	79.6	63.2	100.2	200.0	1
31	0.18	0.23	0.16	0.13	0.11	0.20	119.1	84.3	106.1	59.7	112.4	168.2	1
32	0.12	0.22	0.16	0.10	0.10	0.15	66.9	94.6	56.3	56.3	133.6	251.7	1
33	0.31	0.63	0.22	0.12	0.12	0.17	70.9	141.5	44.7	42.2	168.2	126.1	1
34	0.10	0.14	0.14	0.10	0.10	0.14	44.7	66.9	39.9	28.2	33.5	50.2	1
35	0.33	0.42	0.70	0.35	0.33	0.50	63.2	53.2	112.4	59.7	188.8	211.8	1
36	0.20	0.21	0.35	0.12	0.14	0.14	63.2	50.2	79.6	29.9	126.1	100.2	1
37	0.11	0.15	0.10	0.10	0.10	0.10	37.6	79.6	70.9	44.7	39.9	70.9	1
38	0.15	0.15	0.16	0.15	0.11	0.10	63.2	100.2	63.2	44.7	149.9	251.7	1

where:

s_{min} - minimal observed value of a symptom,

k - Weibul's shape coefficient calculated as:

$$k = \frac{\bar{s} - s_{min}}{\sigma} \tag{7}$$

$\Gamma(n)$ - the gamma function of the (n) argument,

D. the L-method:

$$b = 4\, s_M - 3\, s_{min} \tag{8}$$

where: s_M is the mode value of an empirical distribution of results (of observations).

Table 4. Symptom limit values for system S2.

Symptom	Method	symptom limit values			
		$b1$	$b2$	$b3$	b
s_1	L	0.22	0.34	0.46	0.58
	C	1.57	2.27	2.97	3.36
	W	1.34	1.78	2.21	2.64
	P	0.60	0.98	1.35	1.73
s_2	L	0.20	0.30	0.40	0.50
	C	1.60	2.26	2.93	3.60
	W	1.21	1.54	1.88	2.22
	P	0.61	1.01	1.42	1.82
s_3	L	0.18	0.26	0.34	0.42
	C	1.28	1.81	2.34	2.87
	W	0.96	1.23	1.50	1.77
	P	0.50	0.83	1.15	1.47
s_4	L	0.15	0.20	0.25	0.30
	C	0.96	1.35	1.73	2.12
	W	0.70	0.89	1.09	1.29
	P	0.39	0.64	0.88	1.12
s_5	L	0.21	0.32	0.43	0.54
	C	1.39	2.05	2.70	3.36
	W	1.35	1.86	2.36	2.86
	P	0.53	0.85	1.16	1.48
s_6	L	0.20	0.30	0.40	0.50
	C	1.42	2.05	2.69	3.32
	W	1.21	1.59	1.98	2.37
	P	0.54	0.89	1.23	1.58
s_7	L	47.30	74.60	101.90	129.20
	C	252.41	369.69	486.98	604.26
	W	238.39	324.58	410.76	496.94
	P	103.60	159.90	216.19	272.49
s_8	L	92.80	165.60	238.40	311.20
	C	253.91	364.98	476.05	587.12
	W	211.54	278.94	346.34	413.74
	P	140.52	188.25	235.97	283.70
s_9	L	77.50	135.00	192.50	250.00
	C	163.45	215.71	267.97	320.23
	W	86.25	104.68	123.12	141.55
	P	108.23	138.95	169.68	200.40
s_{10}	L	61.20	102.40	143.60	184.80
	C	376.27	593.96	811.65	1029.35
	W	660.21	1037.91	1415.60	1793.30
	P	129.37	197.54	265.71	333.87
s_{11}	L	136.40	252.80	369.20	485.60
	C	411.16	625.32	839.48	1053.65
	W	491.99	706.05	920.11	1134.17
	P	204.12	271.85	339.57	407.30
s_{12}	L	70.70	121.40	172.10	222.80
	C	373.96	538.71	703.46	868.20
	W	307.63	401.46	495.29	589.12
	P	157.10	243.50	329.90	416.31

The value of b, calculated according to formulae (1), (4), (6) and (8) is treated as a threshold value (i.e., an "alarm" value) which separates good and bad technical states. Three additional limit values $b1$, $b2$ and $b3$ are uniformly distributed over the range of symptom variability and are interpreted as "alert" values of symptom [24]. They are defined as follows:

$$b1 = s^* + 0.25 \star (b - s^*) \tag{9A}$$

$$b2 = s^* + 0.50 \star (b - s^*) \tag{9B}$$

$$b3 = s^* + 0.75 \star (b - s^*) \tag{9C}$$

where:

- in the case of the C-, P- and W-methods:

$$s^* = \bar{s}$$

- in the case of the L-method:

$$s^* = s_M.$$

Table 2 shows a list of the limit values for the data included in the Table 1.

It should be noticed, however that the L-method of defining symptom limit values is based on the mode value of the probability density function of a symptom and requires that the skewness of the distribution of observations is greater than a certain value. Let us notice that the noise symptoms were measured in a decibel scale. The logarithm operation leads to normalization of the shape of the distribution of observation and as consequence, the skewness of this distribution decreases. So, we think that the presentation of measurements of noise signals in a decibel scale may be disadvantageous. For this reason, we decided to make transformation of results and to present them in a linear scale [mP].

Table 3 presents the set the transformed values of the noise symptom measurements. The information system containing the transformed values is called information system **S2**. Table 4 shows the limit values for the symptoms from Table 3. These were calculated using the four methods as in the case of information system **S1**. Both information systems **S1** and **S2**, will be examined next in the same way. The difference of results shows the possible influence of noise measurement scales (linear or logarithmic) on the quality of diagnosis.

4. AN ANALYSIS OF INFORMATION SYSTEMS S1 AND S2 USING THE ROUGH SETS THEORY

The information system **S1** was analysed first. In this information systems results of measurements of noise were presented in a logarithmic scale [dB]. Table 5 shows accuracies of approximations of each particular class and quality of classification (i.e., classification of the rolling bearings from the viewpoint of the technical state) for all considered definitions of symptom limit values.

Then, the information system **S2** was analysed. In this system, results of measurements of noise symptoms were presented in a linear scale [mP]. Accuracies of approximations and quality of classification are presented in Table 6.

Let us notice that according to the criterion of the quality of classification, the ranking of the methods of defining limit values is the same for information systems **S1** and **S2**. For both information systems, the highest value of quality of classification (equal to 1.0) is obtained for methods L, W and P. However, the L-method gives higher number of atoms than other methods. So, this method enables better differentiation of considered objects using their available description. Most of the created atoms consist of one object only. In system **S1**, the P-method is the second best from the viewpoint of the number of atoms and the W-method is the third one. In system **S2**, the ranking of methods is just opposite. Anyway, in both information systems the C-method is according to the number of atoms the worst. It gives unsatisfactory quality of classification (less then 0.55) and multiobject-atoms consisting of objects belonging to different classes of technical states.

In the next step of the analysis we checked the quality of classification (i.e., evaluation of technical state) using single symptoms. Results for both information systems and for all considered methods L, C, P and W are given in Table 7.

Table 5. Accuracies of approximations and quality of classification for information system S1.

	Methods			
	L	C	W	P
Number of atoms	35	22	22	26
Class 0				
Lower approximation	19	18	19	19
Upper approximation	19	35	19	19
Accuracy of approx.	1.0	0.51	1.0	1.0
Class 1				
Lower approximation	19	3	19	19
Upper approximation	19	20	19	19
Accuracy of approx.	1.0	0.15	1.0	1.0
Accuracy of classification	1.0	0.38	1.0	1.0
Quality of classification	1.0	0.55	1.0	1.0

Table 6. Accuracies of approximations and quality of classification for information system S2.

	Methods			
	L	C	W	P
Number of atoms	37	19	32	23
Class 0				
Lower approximation	19	17	19	19
Upper approximation	19	37	19	19
Accuracy of approx.	1.0	0.46	1.0	1.0
Class 1				
Lower approximation	19	1	19	19
Upper approximation	19	21	19	19
Accuracy of approx.	1.0	0.05	1.0	1.0
Accuracy of classification	1.0	0.31	1.0	1.0
Quality of classification	1.0	0.47	1.0	1.0

Table 7. Quality of classification using single symptoms.

infor. system		Symptoms											
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
S1	L	1.0	0.45	0.45	0.45	1.0	0.42	0.0	0.11	0.0	0.11	0.08	0.11
	C	0.18	0.16	0.21	0.26	0.13	0.21	0.08	0.05	0.24	0.18	0.11	0.08
	P	0.32	0.29	0.32	0.37	0.24	0.34	0.11	0.08	0.21	0.26	0.11	0.16
	W	0.45	0.47	0.32	0.47	1.0	0.34	0.03	0.0	0.0	0.16	0.0	0.0
S2	L	1.0	0.45	0.45	0.45	1.0	0.42	0.0	0.11	0.13	0.32	0.11	0.0
	C	0.18	0.16	0.21	0.26	0.13	0.21	0.08	0.11	0.21	0.05	0.08	0.08
	P	0.32	0.29	0.32	0.37	0.24	0.34	0.11	0.11	0.26	0.13	0.11	0.11
	W	0.45	0.47	0.32	0.47	1.0	0.34	0.08	0.11	0.11	0.29	0.11	0.18

These results demonstrate that single symptoms ensuring satisfactory quality of classification (equal to 1) are the following: symptoms s_1 and s_5 for the L-method or for a symptom s_5 for the W-method. For other single symptoms, the quality of classification is much lower. Moreover the comparison of results obtained for noise and vibration symptoms shows that vibration symptoms are considerably better.

Then, we searched for minimal subsets of symptoms for both information systems and all methods of defining symptom limit values. Let us remark that this step of analysis is very important because it enables reducing a number of symptoms. Results obtained for methods L and W, P are presented below. We do not give results for the C-method because they are inadmissible from the viewpoint of the quality of classification. Results obtained for information system S1 are the following:

- the L-method
  - the core is empty,
  - 17 minimal subsets:

$$\{s_3, s_7, s_8, s_9, s_{11}, s_{12}\}, \{s_4, s_7, s_8, s_9, s_{11}, s_{12}\},$$
$$\{s_6, s_8, s_{10}, s_{11}, s_{12}\}, \{s_6, s_7, s_{10}, s_{11}, s_{12}\},$$
$$\{s_6, s_8, s_9, s_{11}, s_{12}\}, \{s_6, s_7, s_8, s_{10}, s_{12}\},$$
$$\{s_6, s_8, s_9, s_{10}, s_{11}\}, \{s_6, s_7, s_9, s_{10}, s_{11}\},$$
$$\{s_4, s_8, s_{10}, s_{12}\}, \{s_3, s_8, s_{10}, s_{12}\},$$
$$\{s_3, s_{10}, s_{11}\}, \{s_4, s_{10}, s_{11}\},$$
$$\{s_2, s_6\}, \{s_2, s_4\}, \{s_2, s_3\}, \{s_5\}, \{s_1\},$$

- the W-method
  - the core is empty,
  - 31 minimal subsets: the minimal subsets have the following structure: one is a singleton (symptom s_1); 5 subsets consist of two elements, 15 subsets of three elements, 7 subsets of four elements and other subsets of five elements,
- the P-method
  - the core is symptom s_1,
  - 10 minimal subsets: four of them are composed of three elements and other subsets of four elements.

Results obtained for information system S2 are the following:

- the L-method
  - the core is empty,
  - 18 minimal subsets:

$$\{s_6, s_8, s_9, s_{10}\}, \{s_4, s_8, s_9, s_{10}\}, \{s_3, s_8, s_9, s_{10}\},$$
$$\{s_6, s_7, s_9, s_{10}\}, \{s_6, s_7, s_8, s_{10}\},$$
$$\{s_6, s_{10}, s_{12}\}, \{s_4, s_{10}, s_{12}\}, \{s_3, s_{10}, s_{12}\}, \{s_6, s_{10}, s_{11}\},$$
$$\{s_4, s_{10}, s_{11}\}, \{s_3, s_{10}, s_{11}\}, \{s_4, s_7, s_{10}\}, \{s_3, s_7, s_{10}\},$$
$$\{s_2, s_6\}, \{s_2, s_4\}, \{s_2, s_3\}, \{s_5\}, \{s_1\}.$$

- the W-methods
  - the core is empty,
  - 7 minimal subsets: minimal subsets have the following structure: one is a singleton (symptom s_1); 4 subsets consist of two elements and 2 subsets of three elements,
- the P-method
  - the core is symptom s_1,
  - 21 minimal subsets: four of them are composed of three elements, five of four elements and other subsets of five elements.

Let us notice that in most cases the number of minimal subsets is rather high, cores are empty or one-element. These results seem to be typical for data sets in which some attributes are mutually interchangeable. This case occurs in the analysed problem because the measurements of vibroacoustic symptoms concern similar quantities.

Using any of minimal subsets one can create a classifier of the technical state. One of them should be chosen to be a base of evaluation of the technical state. As a great number of minimal subsets was obtained, we decided to use other criteria than the quality of classification alone. The minimum cardinality of a minimal subset was chosen as a secondary criterion (in practice it may be interesting to take into account also such criteria as facility, cost and time of measurement). It can be noticed that the L-method gives minimal subsets composed of one element only, i.e., $\{s_1\}$ and $\{s_5\}$. Similarly the W-method gives $\{s_5\}$. So, these subsets can be used to create classifiers in first order.

Then, we can see two-element and three-element subsets. Two-element subsets are the following:

$$\begin{aligned} \mathbf{S1} - L &: \{s_2, s_3\},\ \{s_2, s_4\},\ \{s_2, s_6\}, \\ \mathbf{S1} - W &: \{s_2, s_7\},\ \{s_1, s_6\},\ \{s_1, s_2\},\ \{s_2, s_4\},\ \{s_1, s_4\}, \\ \mathbf{S2} - L &: \{s_2, s_3\},\ \{s_2, s_4\},\ \{s_2, s_6\}, \\ \mathbf{S2} - W &: \{s_1, s_6\},\ \{s_2, s_4\},\ \{s_1, s_2\}. \end{aligned}$$

Let us notice that the P-method does not give any two-element subsets. Taking into consideration the structure of three-element subsets it can be noticed that vibration symptoms dominate over noise ones. For instance, for system **S1** and the W-method one can generate 15 three-element subsets; they all consist of two vibration symptoms and one noise one. A similar result was noticed for the same W-method and system **S2**.

When the choice of one minimal subset to be a classifier of the technical state is done, one can derive a decision algorithm from the reduced information system. It consists of decision rules which determine the assignment of an object to the real technical state basing on values of symptoms belonging to the chosen minimal subset.

The two examples of decision algorithms are given below. The first algorithm is created using one-symptom minimal subset $\{s_1\}$ and the second one is built using two symptom minimal subset $\{s_2, s_4\}$.

EXAMPLE 1. System **S1**, the L-method and the classifier $\{s_1\}$:

if ($s_1 = 5$) then (class= 1), the rule confirmed by 2 objects;
if ($s_1 = 4$) then (class= 1), the rule confirmed by 17 objects;
if ($s_1 = 3$) then (class= 0), the rule confirmed by 2 objects;
if ($s_1 = 2$) then (class= 0), the rule confirmed by 9 objects;
if ($s_1 = 1$) then (class= 0), the rule confirmed by 8 objects;

where:
(class 1) - denotes bad technical state and
(class 0) - denotes good technical state.

EXAMPLE 2. System **S2**, the L-method, classifier $\{s_2, s_4\}$. The decision algorithm is presented graphically in Figure 1.

5. CONCLUSIONS

The analysis of the diagnostic problem by means of the rough sets theory leads to the following conclusions:

A) The four considered methods of defining symptom limit values give different quality of classification of the rolling bearings from the viewpoint of the technical state. For both information systems **S1** and **S2**, the C-method led to considerably worse results than L-, W- and P-methods.
B) The definition of the noise scale (i.e., logarithmic or linear) does not influence significantly the quality of classification of the rolling bearings.
C) A significant superiority of the vibration symptoms was noticed over the noise symptoms for both systems **S1** and **S2**. It can result from influence of a reflexion of acoustic field and acoustic property of the laboratory room.
D) A majority of possible classifiers of the technical state (minimal subsets) are composed of one or two symptoms. The results point out that the choice of vibration measurement bands has been done correctly from the viewpoint of diagnostic information.

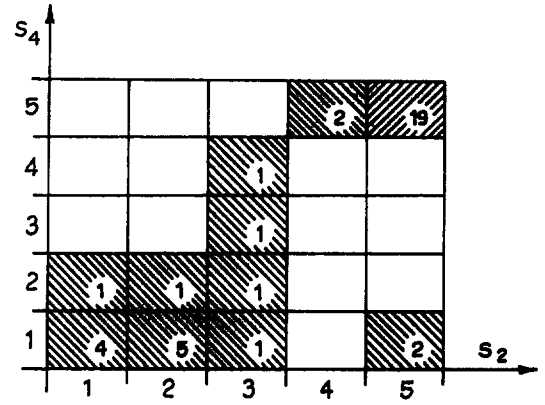

Figure 1. Graphical representation of the decision algorithm for system S2, classifier $\{s_2, s_4\}$ and the L-method;
//// - denotes bad technical state (class 1)
\\\\ - denotes good technical state (class 0)
(numbers in boxes get information about the number of objects which match the given combination of values of symptoms s_4 and s_2).

The three best methods of defining the symptom limit values (L, W, P) can be ranked as follows from the viewpoint of the minimal cardinality of the obtained minimal subset:

L - two singletons $(\{s_1\}, \{s_5\})$,
W - one singleton $(\{s_5\})$,
P - no singletons.

The above results have been obtained using the data collected in a laboratory, so in such conditions the widely-understood measurement noise should be relatively weak. Therefore, it is desirable to verify the obtained results for a similar set of objects working in a real environment. The research which lead to such verification is presently in progress [30].

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