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Languages of colonies*

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Abstract

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A colony is a finite set of regular grammars, where each grammar generates a finite language. The component grammars cooperate to derive a common language. In this paper we compare the generative power of colonies with two cooperation strategies and with several types of the selection of the alphabet for the common language. The results give representations of languages of colonies in terms of classes of sequential and parallel languages.

1. Introduction

Colonies are grammatical models of multiagent systems motivated by subsumption architectures [1, 2] and form a special variant of cooperating/distributed grammar systems [3, 6]. The notion of a colony was introduced in [9] as a finite set of regular grammars, where each grammar generates a finite language. The component grammars cooperate to derive a common language. Regular grammars of the colony model agents of the multiagent system and the common language corresponds to the accepted (correct) behaviour of the system. The style of acceptance is expressed by the relation of the alphabet of the common language to the terminal alphabets of the component grammars. The way how the component grammars can take part in the derivation, the derivation mode, corresponds to the strategy under which the

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agents cooperate. More details about acceptance styles can be found in [4]. Variants of cooperation strategies are discussed in [9, 7].

In this paper we study the generative power of colonies with respect to acceptance styles *arb*, *all*, *one*, *ex* and *dist* and with respect to two cooperation strategies, namely to the basic mode of derivation and to the terminal mode of derivation. Acceptance style *arb* means that the terminal set of the colony is an arbitrary subset of terminals of all components. In the case of style *all*, every terminal symbol of the colony is a terminal symbol for all components. If the acceptance style is *one*, then the terminal set of the colony is identical with the terminal set of one of its components and the terminal set of the colony in style *ex* is the union of the terminal sets of the components. Acceptance style *dist* assumes only those terminals of the components which are not nonterminals for any component of the colony.

In the basic mode of derivation a component executes a direct derivation step if it replaces one occurrence of its startsymbol by any of its terminal words. In the terminal mode a component has to replace each occurrence of its startsymbol by one of its terminal words, not necessarily the same one.

The main result of Section 3 is that acceptance styles *all*, *arb*, *one* and *dist* are equally powerful from the generative point of view and the acceptance style *ex* is less powerful than the previously listed ones.

In Section 4 we demonstrate that the above two types of derivation mode differ from each other in the generative power. In the basic mode of derivation the colonies with acceptance style *arb* (or *all* or *one* or *dist*) are as powerful as ε-free context-free grammars. Acceptance style *ex* results in a smaller language class, in a subclass of pure context-free languages. The terminal mode of derivation enhances the generative power. In this case, the colonies with acceptance style *arb* (or *all* or *one* or *dist*) generate all languages those can be obtained by 1-restricted $EPT0L$ systems. Colonies with acceptance style *ex* are of the power of 1-restricted $FPT0L$ systems with nonrecursive tables.

2. Basic definitions and preliminaries

We assume that the reader is familiar with basis of formal language theory. The aim of this section is to recall some types of sequential and parallel grammars and to state some auxiliary relations among corresponding language classes, which we use in the Section 4. For further details and unexplained notions the reader is refered to [8, 12, 13].

For an alphabet Σ we denote by Σ^+ the set of all nonempty words over Σ. The set of all words over Σ, included the empty word ε, is denoted by Σ^*. For a word $w \in \Sigma^*$ we denote by $|w|_a$ the total number of occurrences of symbol $a \in \Sigma$ in w.

A language is an arbitrary set of words. Because colonies do not treate erasing rules, we discuss ε-free languages. For a language L we denote by *alph* L the smallest alphabet Σ such that $L \subseteq \Sigma^*$ holds.

We denote a context-free grammar by $G=(N,T,P,S)$, where N and T are the disjoint finite sets of nonterminals and terminals, respectively, P is the finite set of productions of form $A \to w$, where $A \in N$, $w \in (N \cup T)^*$, and S is the startsymbol.

The language generated by a context-free grammar G is denoted by $L(G)$.

By a regular grammar we mean a context-free grammar $G=(N,T,P,S)$ with productions of form $A \to aB$ and $A \to a$, where A, B are nonterminals and a is a terminal.

By a pure context-free grammar we mean a triple $G=(V,P,\mathscr{S})$, where V is a finite alphabet, $\mathscr{S}$ is a finite set of elements of V^+ and P is a finite set of productions of form $Z \to w$, where $Z \in V$ and $w \in V^+$ hold. (Note that we do not allow erasing rules here.)

The language generated by a pure grammar G is defined by $L(G)=\{y : x \Rightarrow^* y, x \in \mathscr{S}\}$.

A pure context-free grammar $G=(V,P,\mathscr{S})$ is said to be with nonrecursive productions (rules), if P contains no production of type $Z \to xZy$ for $xy \in V^*$. (Derivations of type $Z \Rightarrow u \Rightarrow^+ xZy$ are not forbidden for these grammars.)

We denote the class of languages generated by the context-free grammars (by the pure context-free grammars and by the pure context-free grammars with nonrecursive productions) by $\mathscr{L}(CF)$, ($\mathscr{L}(pCF)$ and $\mathscr{L}(nrpCF)$, respectively).

Proposition 2.1. $\mathscr{L}(nrpCF) \subset \mathscr{L}(pCF) \subset \mathscr{L}(CF)$.

Languages a^*S and $\{a^iSb^i\colon i \geq 0\}$ are examples of languages in $\mathscr{L}(pCF)$ but not in $\mathscr{L}(nrpCF)$ and $\{a^ib^i\colon i \geq 1\}$ is in $\mathscr{L}(CF)$ but not in $\mathscr{L}(pCF)$.

Context-free grammars, regular grammars and pure context-free grammars use sequential derivations. We shall use also some types of grammars with parallel derivation.

By an $ET0L$ system we mean an $(n+3)$ tuple $H=(V,T,P_1,\ldots,P_n,S)$, where V is a finite set of symbols, $T \subseteq V$ is a set of terminals, $S \in V$ is the startsymbol and P_i, for every i, $1 \leq i \leq n$, is a finite set of productions of form $Z \to w$, where $Z \in V$, $w \in V^*$. Moreover, every P_i contains at least one production of form $Z \to w$ for each $Z \in V$. The set P_i is called the ith table of H.

A sentential form $x=x_1 \ldots x_m$ with $x_j \in V$, $1 \leq j \leq m$, derives a sentential form $y=y_1 \ldots y_m$ with $y_j \in V^*$, $1 \leq j \leq m$, in $ET0L$ system H directly, denoted by $x \Rightarrow y$, if there is a table P_i, for some i, $1 \leq i \leq n$, such that $x_j \to y_j$ is a production in P_i for each j, $1 \leq j \leq m$.

The language $L(H)$ generated by H is defined by $L(H)=\{w\colon S \Rightarrow^* w, w \in T^*\}$, where $\Rightarrow^*$ denotes the reflexive transitive closure of $\Rightarrow$.

An $ET0L$ system with $V=T$ is a $T0L$ system. We shall use P to distinguish the systems with no erasing rule, i.e. we shall have $PT0L$ systems, $EPT0L$ systems, etc. A $T0L$ system with finite set $\mathscr{S}$ of axioms from V^+ instead of S will be denoted as an $FT0L$ system.

An $ET0L$ system $H=(V,T,P_1,\ldots,P_n,S)$ is said to be 1-restricted $ET0L$ system, abbreviated as $ET0L_{[1]}$ system, if for every P_i, $1 \leq i \leq n$, there exists a symbol Z in V such that if $B \neq Z$ and $B \to w \in P_i$, then $w=B$ holds.

Thus, 1-restricted $ET0L$ systems allow to rewrite by each table at most one symbol into something else than the symbol itself.

We say that an $ET0L_{[1]}$ system $H=(V, T, P_1, \ldots, P_n, S)$ is a system with nonrecursive tables, abbreviated as $nrET0L_{[1]}$ system, if there is no table P_i, $1 \leqslant i \leqslant n$, such that P_i contains a production $Z \to xZy$, where $xy \in V^+$ and, moreover, if $Z \to \alpha$ is in P_i for $\alpha \neq Z$ then $Z \to Z$ is not in P_i.

An $nrET0L_{[1]}$ system with $T=V$ and with a finite set $\mathcal{S}$ of words called axioms (instead of the single startsymbol S) is an $nrFT0L_{[1]}$ system.

Thus, for $\alpha \Rightarrow \beta$ in $nrET0L_{[1]}$ system, either $\beta=\alpha$ or there is exactly one letter in α, say Z, such that $\alpha=\alpha_1 Z\alpha_2 \ldots Z\alpha_{n+1}$ (α_i's do not contain Z) and $\beta=\alpha_1 u_1 \alpha_2 \ldots u_n \alpha_{n+1}$, where u_i's (and β) do not contain Z.

Similarly as for sequential grammars we shall use $\mathcal{L}(X)$ to denote the class of languages generated by L systems in a class X (i.e. $\mathcal{L}(ET0L)$, $\mathcal{L}(ET0L_{[1]})$, $\mathcal{L}(nrET0L_{[1]})$, $\mathcal{L}(FT0L), \ldots$).

Proposition 2.2.

$$\mathcal{L}(CF) \subset \mathcal{L}(nrEPT0L_{[1]}) = \mathcal{L}(EPT0L_{[1]}) \subset \mathcal{L}(EPT0L),$$

$$\mathcal{L}(nrFPT0L_{[1]}) \subset \mathcal{L}(FPT0L_{[1]}) \subset \mathcal{L}(FPT0L).$$

Proof. (a) We show first that $\mathcal{L}(CF) \subset \mathcal{L}(nrEPT0L_{[1]})$. Let L be an arbitrary ε-free context-free language. Without loss of the generality we may assume that L is generated by a context-free grammar $G=(N, T, P, S)$, with all productions of form $A \to BC$, $A \to B$, $A \to a$, where A, B, C are pairwise different nonterminals and a is a terminal. Let $V=N \cup T$ and let us denote by V' the primed version of alphabet V. We construct for L an $nrEPT0L_{[1]}$ system H. Let $P=\{p_1, \ldots, p_n\}$, where $p_i: X \to \alpha$. Let for every i, $1 \leqslant i \leqslant n$, $P_i=\{p_i\} \cup \{X \to X'\} \cup \{z \to z\colon z \in (V \cup V' - \{X\})\}$ and $P_i'=\{X' \to X\} \cup \{z \to z\colon z \in (V \cup V' - \{X'\})\}$. Then, $H=(V \cup V', T, P_1, \ldots, P_n, P_1', \ldots, P_n', S)$ is obviously an $nrEPT0L_{[1]}$ system.

We show that $L(G)=L(H)$. We first note that for i, $1 \leqslant i \leqslant n$, the subsequent application of tables P_i and P_i' for a sentential form $v \in V^+$ corresponds to the application of production $p_i: X \to \alpha$ in G for some occurrence of X in v. By this fact, $L(G) \subseteq L(H)$ is obvious. $L(H) \subseteq L(G)$ also holds since P_i' changes only X' to X and, therefore, it can be applied immediately after the occurrence to X' in a sentential form. Therefore, for arbitrary derivation

$$\text{D: } S \Rightarrow \alpha_1 \Rightarrow \alpha_2 \Rightarrow \cdots \Rightarrow \alpha_n$$

$\alpha_n \in T^*$ in H, there is a derivation

$$\text{D}'\text{: } S \Rightarrow \alpha_1' \Rightarrow \alpha_2' \Rightarrow \cdots \Rightarrow \alpha_m'$$

in H such that $\alpha_m'=\alpha_n$ and if $\alpha_j' \Rightarrow \alpha_{j+1}'$ by P_i then exactly one occurrence of X in α_j' is rewritten to α and $\alpha_{j+1}' \Rightarrow \alpha_{j+2}'$ rewrites the occurrences of all X' to X (if there is any). In the case that in D in some derivation step $\alpha_j \Rightarrow \alpha_{j+1}$ P_i is used to rewrite more occurrences of X to α then in D′ we rewrite X's "sequentially" using corresponding number of derivation steps. Thus, terminating derivations in H correspond to terminating derivations in G. This implies that the equality of languages holds and, thus, $\mathscr{L}(CF) \subseteq \mathscr{L}(nrEPT0L_{[1]})$.

The corresponding proper inclusions come out from the fact that $L=\{a^{2^n}: n \geqslant 1\} \notin \mathscr{L}(CF)$ can be generated by the $nrEPT0L_{[1]}$ system $H=(\{A\}, \{a\}, P_1, P_2, A)$, where $P_1=\{A \to aa, a \to a\}$ and $P_2=\{a \to A, A \to A\}$.

(b) To prove $\mathscr{L}(nrEPT0L_{[1]})=\mathscr{L}(EPT0L_{[1]})$ we have to show $\mathscr{L}(EPT0L_{[1]}) \subseteq \mathscr{L}(nrEPT0L_{[1]})$. The reverse inclusion is obvious.

Let $H=(V, T, P_1, \ldots, P_n, S)$ be an $EPT0L_{[1]}$ system with an alphabet $V=\{A_1, \ldots, A_t\}$. Let V' denote the primed version of V. For a word w and for a nonterminal A we denote by w^A the word that is obtained from w by replacing every occurrence of A by a new nonterminal A'.

Let $P_i=\{A \to \alpha_1 | \cdots | \alpha_{n_i}\} \cup \{X \to X\colon X \in V-\{A\}\}$ be a fixed table. We define P_i' by $P_i'=\{A \to \alpha_1^A | \cdots | \alpha_{n_i}^A\} \cup \{X \to X\colon X \in V \cup V'-\{A\}\}$ and for $A \in V$ we put $P_A=\{A' \to A\} \cup \{X \to X\colon X \in (V \cup V'-\{A'\})\}$. Then the system $H'=(V \cup V', T, P_1', \ldots, P_n', P_{A_1}, \ldots, P_{A_t}, S)$ is an $nrEPT0L_{[1]}$ system and $L(H)=L(H')$.

(c) $\mathscr{L}(ET0L_{[1]}) \subset \mathscr{L}(ET0L)$ was proved in [10]. All other inclusions in the Proposition 2.2 are evident. They are proper because $\{a^{2^n}\colon n \geqslant 1\} \in \mathscr{L}(FPT0L_{[1]})-\mathscr{L}(nrFPT0L_{[1]})$ and $\{a^{2^n}b^{2^n}\colon n \geqslant 1\} \in \mathscr{L}(FPT0L)-\mathscr{L}(FPT0L_{[1]})$. □

3. Basic properties of colonies

In this section we turn to special systems of grammars, called colonies. Detailed information on grammar systems can be found in [5, 6]. First we recall the notion of a colony from [9].

Definition 3.1. By a colony we mean an $(n+2)$-tuple $C=(T, R_1, \ldots, R_n, S)$, where

(i) $R_i=(N_i, T_i, P_i, S_i)$, for every i, $1 \leqslant i \leqslant n$, is a regular grammar generating a *finite language*; R_i is called a component of C;

(ii) $S=S_i$ for some i, $1 \leqslant i \leqslant n$; S is called the startsymbol of C;

(iii) $T \subseteq \bigcup_{i=1}^n T_i$ is called the set of terminals of C.

We denote the total alphabet of C by V, i.e. $V=\bigcup_{i=1}^n (T_i \cup N_i)$.

Colonies can generate languages in basic mode (b-mode) of derivation and in terminal mode (t-mode) of derivation.

Definition 3.2. Let $C=(T, R_1, \ldots, R_n, S)$ be a colony and let $x, y \in V^+$, where V is the total alphabet of C.

(i) We say that x derives y in C in basic mode (b-mode) of derivation directly, denoted by $x \overset{b}{\Rightarrow}_C y$, if there is a component R_i of C for some i, $1 \leqslant i \leqslant n$, such that $x=x_1 S_i x_2$ and $y=x_1 w x_2$ hold, where $x_1 x_2 \in V^*$ and $w \in L(R_i)$.

(ii) We say that x derives y in C in terminal mode (t-mode) of derivation directly, denoted by $x \stackrel{t}{\Rightarrow}_C y$, if there is a component R_i of C for some i, $1 \leqslant i \leqslant n$, such that $x = x_1 S_i x_2 S_i x_3 \ldots x_m S_i x_{m+1}$ and $y = x_1 w_1 x_2 w_2 x_3 \ldots x_m w_m x_{m+1}$, where $x_1 x_2 \ldots x_{m+1} \in (V - \{S_i\})^*$ and $w_j \in L(R_i)$, for each j, $1 \leqslant j \leqslant m$.

The language generated by C in x-mode of derivation for $x \in \{\mathrm{b}, \mathrm{t}\}$ is defined by $L_x(C) = \{w\colon S \stackrel{x}{\Rightarrow}{}_C^* w,\ w \in T^*\}$, where $\stackrel{x}{\Rightarrow}{}_C^*$ denotes the reflexive transitive closure of $\stackrel{x}{\Rightarrow}_C$. If there is no misunderstanding, then subscript C can be omitted.

According to different selections of the terminal set of the colony we can distinguish colonies with different styles of acceptance.

Definition 3.3. We say that colony $C = (T, R_1, \ldots, R_n, S)$ has an acceptance style

(i) *arb* if $T \subseteq \bigcup_{i=1}^n T_i$,
(ii) *one* if $T = T_i$ for some i, $1 \leqslant i \leqslant n$,
(iii) *ex* if $T = \bigcup_{i=1}^n T_i$,
(iv) *all* if $T = \bigcap_{i=1}^n T_i$,
(v) *dist* if $T = (\bigcup_{i=1}^n T_i) - (\bigcup_{i=1}^n N_i)$.

Notation 3.4. For $x \in \{\mathrm{b}, \mathrm{t}\}$ and $f \in \{one, arb, ex, all, dist\}$ the class of languages generated by colonies in x-mode of derivation with acceptance style f is denoted by $\mathscr{L}(Col, x, f)$.

Theorem 3.5. *For* $x \in \{\mathrm{b}, \mathrm{t}\}$

$$\mathscr{L}(Col, x, ex) \subseteq \mathscr{L}(Col, x, arb),$$

$$\mathscr{L}(Col, x, one) = \mathscr{L}(Col, x, all) = \mathscr{L}(Col, x, dist) = \mathscr{L}(Col, x, arb).$$

Proof. Acceptance styles *one*, *ex*, *all* and *dist* are special cases of the style *arb*. So it is sufficient to prove that $\mathscr{L}(Col, x, arb) \subseteq \mathscr{L}(Col, x, f)$ for f being *one* or *all* or *dist*. Let $C = (T, R_1, \ldots, R_n, S)$ with $T = \{a_1, \ldots, a_p\}$ be a colony with acceptance style *arb*. Let $C' = (T, R'_1, \ldots, R'_n, R_{a_1}, \ldots, R_{a_p}, S')$ be a colony, where $R'_i = (N'_i, T \cup T'_i, P'_i, S'_i)$, for $1 \leqslant i \leqslant n$, and N'_i, T'_i, P'_i, S'_i are primed versions of N_i, T_i, P_i, S_i in $R_i = (N_i, T_i, P_i, S_i)$, respectively. Let $R_{a_j} = (\{a'_j\}, T, \{a'_j \to a_j\}, a'_j)$ for $1 \leqslant j \leqslant p$. T in C' obviously fulfils the conditions for any of the acceptance styles *one*, *all* or *dist*. We show that $L_x(C') = L_x(C)$. Inclusion $L_x(C) \subseteq L_x(C')$ holds clearly, because we can simulate every derivation $S \stackrel{x}{\Rightarrow} w_1 \stackrel{x}{\Rightarrow} w_2 \stackrel{x}{\Rightarrow} \cdots \stackrel{x}{\Rightarrow} w_n = w$ in C where $w_j \in V^+, 1 \leqslant j \leqslant n$, $w \in T^+$ by a derivation $S' \stackrel{x}{\Rightarrow} w'_1 \stackrel{x}{\Rightarrow} w'_2 \stackrel{x}{\Rightarrow} \cdots \stackrel{x}{\Rightarrow} w'_n = w' \stackrel{x}{\Rightarrow}{}^* w$ in C', where w'_j is the primed version of w_j, $1 \leqslant j \leqslant n$, and w can be derived from w' using components of $R_{a_1}, \ldots, R_{a_p}$.

The reverse inclusion $L_x(C') \subseteq L_x(C)$ holds, too. Because there is no component that changes any of letters a_j, $1 \leqslant j \leqslant p$, we can reorganize every terminating derivation

$S' \overset{x}{\Rightarrow} z'_1 \overset{x}{\Rightarrow} z'_2 \overset{x}{\Rightarrow} \cdots \overset{x}{\Rightarrow} z'_s = z$ in C', where $z'_j \in (V' \cup T)^+$, $1 \leqslant j \leqslant s$, $z \in T^+$, into a terminating derivation $S' \Rightarrow w'_1 \Rightarrow w'_2 \Rightarrow \cdots \Rightarrow w'_s = z$ with $w'_j \in (V' \cup T)^+$, $1 \leqslant j \leqslant s$, such that for some m, with $1 \leqslant m < s$, it holds that $w'_m = z'$ and we use in the subderivation $w'_m \Rightarrow w'_{m+1} \Rightarrow \cdots \Rightarrow w'_{s-1} \Rightarrow w'_s = z$ only components $R_{a_1}, \ldots, R_{a_p}$.

This property leads to $L_x(C') \subseteq L_x(C)$ and thus equality $L_x(C) = L_x(C')$ follows. □

Note 3.6. For C' in the previous proof the alphabet of $L(R'_i)$ and $L(R_{a_j})$ is a proper subset of the terminal alphabet of R'_i and of R_{a_j}, respectively. This condition is necessary to prove Theorem 3.5 for acceptance style *all*, otherwise, $\mathscr{L}(Col, x, all)$ is the collection of all ε-free finite languages. For acceptance style *one* and *dist*, Theorem 3.5 remains true even in the case of $T_i = alph\, L(R_i)$. The proof for the case *dist* is straightforward. To prove Theorem 3.5 for style *one*, it is enough to add to colony C one additional component $R_0 = (N_0, T_0 = T, P_0, S)$ such that $L(R_0) \subseteq L_x(C)$ and $alph\, L(R_0) = alph\, L_x(C)$.

Example 3.7. Acceptance style *ex*.

Let $C_{ex} = (\{a, b\}, R_1, R_2, a)$ with $R_1 = (\{a, x\}, \{b\}, \{a \to bx, x \to b\}, a)$ and $R_2 = (\{b\}, \{a\}, \{b \to a\}, b)$. C_{ex} is a colony with acceptance style *ex*.

Let us consider t-mode of derivation. Then every terminating derivation is of the form

$$a \overset{t}{\Rightarrow} bb \overset{t}{\Rightarrow} aa \overset{t}{\Rightarrow} \cdots \overset{t}{\Rightarrow} b^{2^n} \quad \text{or} \quad a \overset{t}{\Rightarrow} bb \overset{t}{\Rightarrow} aa \overset{t}{\Rightarrow} \cdots \overset{t}{\Rightarrow} a^{2^m}.$$

Thus, $L_t(C_{ex}) = \{a^{2^n}: n \geqslant 0\} \cup \{b^{2^n}: n \geqslant 1\}$ and so $L_t(C_{ex}) \notin \mathscr{L}(CF)$.

If C_{ex} uses b-mode of derivation, then terminating derivations are of form

$$a \overset{b}{\Rightarrow} bb \overset{b}{\Rightarrow} \cdots \overset{b}{\Rightarrow} a^{i_1} b^{i_2} a^{i_3} \ldots a^{i_n} b^{i_{n+1}}, \text{ where } \sum_{t=1}^{n+1} i_t \geqslant 2.$$

Thus, $L_b(C_{ex}) = \{a, b\}^+ - \{b\}$ and this language is a regular language.

Example 3.8. Acceptance styles *arb*, *one*, *all*, *dist*.

Let $C = (\{c\}, R_1, R_2, R_3, a)$ with $R_1 = (\{a, x\}, \{b, c\}, \{a \to bx, x \to b\}, a)$, $R_2 = (\{b\}, \{a, c\}, \{b \to a\}, b)$ and $R_3 = (\{b\}, \{c\}, \{b \to c\}, b)$. C is the colony with the arbitrary of the acceptance styles *arb*, *all*, *one* and *dist*. Every terminating derivation in the t-mode in C is of the form

$$a \overset{t}{\Rightarrow} bb \overset{t}{\Rightarrow} aa \overset{t}{\Rightarrow} bbbb \overset{t}{\Rightarrow} \cdots \overset{t}{\Rightarrow} b^{2^n} \overset{t}{\Rightarrow} c^{2^n}.$$

Then $L_t(C) = \{c^{2^n}: n \geqslant 1\}$ and $L_t(C) \notin \mathscr{L}(CF)$.

For the basic mode of derivation we obtain the regular language $L_b(C) = cc^+$.

4. The power of colonies

In this section we determine the generative power of colonies with different modes of derivations and different acceptance styles. We show that colonies with acceptance style *arb* (and therefore also with *one* or *all* or *dist*) in the basic mode of derivation determine ε-free context-free languages, while the acceptance style *ex* in the basic mode results in a less powerful language class, the class of languages determined by sequential forms of grammars with no direct recursive rule. Colonies with acceptance style *arb* (and therefore also *one* or *all* or *dist*) for terminal mode of derivation determine the class of 1-restricted $EPT0L$ languages. In the case of acceptance style *ex* and terminal mode of derivation, we obtain the class of $FPT0L_{[1]}$ languages with nonrecursive tables. These characterizations lead to (Main) Theorem 4.5, is which we present the hierarchy among the language classes of colonies studied in the paper.

We start with the basic mode of derivation.

Theorem 4.1 (Kelemen and Kelemenová [10]). $\mathscr{L}(Col, \mathrm{b}, dist) = \mathscr{L}(CF)$.

Acceptance style *ex* is less powerful.

Theorem 4.2. $\mathscr{L}(Col, \mathrm{b}, ex) = \mathscr{L}(nrpCF)$.

Proof. (a) First we show that for a given colony C_{ex} with acceptance style *ex* there exists a pure context-free grammar G with nonrecursive rules such that $L_{\mathrm{b}}(C_{ex}) = L(G)$ holds.

Assume that $C_{ex} = (T, R_1, \ldots, R_n, S)$ is the colony with $T = \bigcup_{i=1}^{n} T_i$ for $R_i = (N_i, T_i, P_i, S_i)$, $1 \leqslant i \leqslant n$. Let us define $P = \bigcup_{i=1}^{n} \{S_i \to w: w \in L(R_i)\}$, $V = \bigcup_{i=1}^{n} alph\ L(R_i)$, and $\mathscr{S} = \{S\}$ for $S \in T$ and $\mathscr{S} = \{s: s \in L(R_i)$ for all i such that $S_i = S\}$, otherwise.

The pure context-free grammar $G = (V, P, \mathscr{S})$ has nonrecursive rules and it generates the same language as C_{ex} does. This follows from the fact that $T = \bigcup_{i=1}^{n} T_i$ for C_{ex} and every component of the colony derives a terminal word over its own alphabet, so for every terminating derivation $S \overset{\mathrm{b}}{\Rightarrow} w_1 \overset{\mathrm{b}}{\Rightarrow} w_2 \overset{\mathrm{b}}{\Rightarrow} \cdots \overset{\mathrm{b}}{\Rightarrow} w_n = w$ in C_{ex}, where $w \in T^+$, it holds that strings $w_1, w_2, \ldots, w_{n-1}$ are in T^+, too.

Moreover for each derivation in C_{ex} of type as above there is a corresponding derivation $s \Rightarrow^* w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n = w$ in G and vice versa. Therefore, $\mathscr{L}(Col, \mathrm{b}, ex) \subseteq \mathscr{L}(nrpCF)$.

(b) We continue by proving that for a pure context-free grammar G with nonrecursive rules there is a colony C_{ex} with acceptance style *ex* such that $L_{\mathrm{b}}(C_{ex}) = L(G)$ holds.

Assume that $G = (V, P, \mathscr{S})$ is the given pure context-free grammar with nonrecursive rules and $\mathscr{S} = \{s_1, \ldots, s_r\}$. Let G have n rules. We define $C_{ex} = (T, R_1, \ldots, R_{n+1}, S)$, where S is a new starting symbol and $T = V$. Further, for every rule p: $A \to a_1 \ldots a_t$ in P there is a component R_p in C_{ex} with rules $A \to a_1 X_1, X_1 \to a_2 X_2, \ldots, X_{t-2}$

$\to a_{t-1}X_{t-1}, X_{t-1} \to a_t$, where X's are new pairwise different nonterminals different for each rule. Finally, let the set of productions of the $(n+1)$st component P_{n+1} contain for every $s_i = s_{i,1} \dots s_{i,n_i} \in \mathscr{S}$, where $s_{i,j} \in V$, $1 \leqslant i \leqslant r$, $1 \leqslant j \leqslant n_i$, productions $S \to s_{i,1} Y_1$, $Y_1 \to s_{i,2} Y_2, \dots, Y_{n_i} \to s_{i,n_i}$, where the Y's are pairwise different new nonterminals for different i's and j's where $1 \leqslant i \leqslant r$, $1 \leqslant j \leqslant n_i$.

Evidently, $L(G) = L_b(C_{ex})$, because each derivation $s_i \Rightarrow w_i \Rightarrow \cdots \Rightarrow w_n$ in G can be simulated in C_{ex} by the corresponding derivation $S \overset{b}{\Rightarrow} s_i \overset{b}{\Rightarrow} w_1 \overset{b}{\Rightarrow} \cdots \overset{b}{\Rightarrow} w_n$, and vice versa. Therefore, $\mathscr{L}(nrpCF) \subseteq \mathscr{L}(Col, \mathrm{b}, ex)$. Summarizing parts (a) and (b) of the proof we get the result.

In the following we study terminal mode of the derivation in colonies.

Theorem 4.3. $\mathscr{L}(Col, \mathrm{t}, arb) = \mathscr{L}(EPT0L_{[1]})$

Proof. First we prove that for a colony C with acceptance style *arb* there exists an 1-restricted $EPT0L$ system H such that $L_t(C) = L(H)$ holds.

Assume that $C = (T, R_1, \dots, R_n, S)$ is given with $R_i = (N_i, T_i, P_i, S_i)$, $1 \leqslant i \leqslant n$. We determine the $EPT0L_{[1]}$ system $H = (N, T, P_1, \dots, P_n, S)$ as follows: S and T are that of the colony, $V = \bigcup_{i=1}^{n} (T_i \cup S_i)$ and $P_i = \{x \to x\colon x \in (V - \{S_i\})\} \cup \{S_i \to w\colon w \in L(R_i)\}$.

By the definition of the t-mode derivation, if component R_i, for some i, $1 \leqslant i \leqslant n$, executes a t-mode derivation for a sentential form $x = x_1 \dots x_q$ with letters x_j, $1 \leqslant j \leqslant q$, then we obtain a sentential form $y = y_1 \dots y_q$, where $y_k = x_k$ if $x_k \neq S_i$ and $y_k \in L(R_i)$ if $x_k = S_i$ for $1 \leqslant k \leqslant q$. By the definition of 1-restricted $EPT0L$ systems, the above derivation corresponds to the application of a table of $EPT0L_{[1]}$ system H.

The equality $L_t(C) = L(H)$ is obvious. Therefore $\mathscr{L}(Col, \mathrm{t}, arb) \subseteq \mathscr{L}(EPT0L_{[1]})$.

Following Proposition 2.2 it remains to prove that for every 1-restricted $nrEPT0L$ system H there exists a colony C with the acceptance style *arb* such that $L_t(C) = L(H)$ holds.

Assume that $H = (V, T, P_1, \dots, P_n, S)$ is a given $nrEPT0L_{[1]}$ system and $P_i = \{A_i \to \alpha\colon A_i \in V, \alpha \in V^+, \alpha_{|A_i|} = 0\} \cup \{x \to x\colon x \in (V - \{A_i\})\}$. We associate to every production $p\colon A_i \to x_1 \dots x_n \in P_i$, where $n \geqslant 2$, a set of productions $\{A_i \to x_1 X'_2, X'_2 \to x_2 X'_3, \dots, X'_n \to x_n\}$, where $X'_2, \dots, X'_n$, are new symbols introduced to p. Let the sets of new symbols, introduced to such productions, be pairwise disjoint. All productions of form $A_i \to x \in P_i$, where $x \in V - \{A_i\}$, remain unchanged. Let us assume that the new symbols, being introduced to tables of H, are pairwise different. Let us denote by P'_i the set of all productions determined in the above way by all productions of P_i.

We define the colony $C = (T, R'_1, \dots, R'_n, S)$ as follows. T and S are the same as in H and $R'_i = (\{A_i\} \cup N'_i, V - \{A_i\}, P'_i, A_i)$, $1 \leqslant i \leqslant n$, where N'_i denotes the set of all new symbols introduced to table P_i in the above way. Since H is propagating, nonrecursive and 1-restricted, and the new symbols are pairwise different, the above-determined structure C is a well-defined colony of acceptance style *arb*. It is clear that R_i' is a regular grammar and it generates $L(R_i')=\{\alpha\colon A_i\to\alpha\in P_i\}$.

We show that $L(H)=L_t(C)$. By the definition of the t-mode of derivation, the application of component R_i' of C for some i, $1\leqslant i\leqslant n$, for some sentential form w corresponds to the application of table P_i of H and, reversely, every application of table P_j, $1\leqslant j\leqslant n$, corresponds a t-mode derivation of component R_j. Thus, $L(H)=L_t(C)$ and $\mathscr{L}(EPT0L_{[1]})\subseteq\mathscr{L}(Col, t, arb)$.

Hence we have the result. □

Theorem 4.4. $\mathscr{L}(Col, t, ex)=\mathscr{L}(nrFPT0L_{[1]})$

Proof. (a) First we prove that for every colony C_{ex} with the acceptance style *ex* there is an $nrFPT0L_{[1]}$ system H such that $L_t(C_{ex})=L(H)$ holds. Assume that $C_{ex}=(T, R_1,\ldots,R_n, S)$ is a given colony with $T=\bigcup_{i=1}^n T_i$, where T_i is the terminal alphabet of R_i for $1\leqslant i\leqslant n$. To determine the $nrFPT0L_{[1]}$ system we put $V=\bigcup_{i=1}^n$ *alph* $L(R_i)$, $P_i=\{S_i\to w\colon w\in L(R_i)\}\cup\{x\to x\colon x\in(V-\{S_i\})\}$, and $\mathscr{S}=\{S\}$ if $S\in T$ in C_{ex} and $\mathscr{S}=\{s\colon s\in L(R_i)$ for all i such that $S_i=S\}$, otherwise. Evidently, $H=(V, P_1,\ldots,P_n,\mathscr{S})$ is an $FT0L_{[1]}$ system. From the properties of colonies it follows that H is also propagating and nonrecursive. For H the equality $L(H)=L_t(C_{ex})$ comes out from the definition of $\mathscr{S}$ and from the fact that to the derivation step $x\overset{t}{\Rightarrow}y$ in C_{ex} which uses the component R_i corresponds in H to the derivation step $x\Rightarrow y$ using the table P_i, and vice versa. So $\mathscr{L}(Col, \mathrm{t}, ex)\subseteq\mathscr{L}(nrFPT0L_{[1]})$.

(b) Assume that $H=(V, P_1,\ldots,P_n,\mathscr{S})$ is a given $nrFPT0L_{[1]}$ system with $\mathscr{S}=\{s_1,\ldots,s_r\}$. We define a colony $C_{ex}=(T, R_1,\ldots,R_{n+1}, S)$ as follows: Let S be a new starting symbol and let $T=V$. The table $P_i=\{A_i\to\alpha_1|\cdots|\alpha_k\}\cup\{x\to x\colon x\in(V-\{A_i\})\}$ determines the set P_i' of productions of the component R_i in the following way. Assume $\alpha_t=a_{t,1}\ldots a_{t,j_t}$ for $1\leqslant t\leqslant k$. Then $P_i'=\bigcup_{t=1}^k\{A_i\to a_{t,1}X_{t,1}, X_{t,1}\to a_{t,2}X_{t,2},\ldots,X_{t,j_t-2}\to a_{t,j_t-1}X_{t,j_t-1}, X_{t,j_t-1}\to a_{t,j_t}\}$. We construct P_{n+1}', the set of rules of the $(n+1)$st component R_{n+1}, as follows: for every $s_i=s_{i,1}\ldots s_{i,n_i}\in\mathscr{S}$, where $s_{i,j}\in V$, $1\leqslant i\leqslant r$, $1\leqslant j\leqslant n_i$, P_{n+1} contains productions $S\to s_{i,1}Y_1$, $Y_1\to s_{i,2}Y_2,\ldots$, $Y_{n_i}\to s_{i,n_i}$, where the Y's are pairwise different new nonterminals for different i's and j's, where $1\leqslant i\leqslant r$, $1\leqslant j\leqslant n_i$. Evidently, the above-defined C_{ex} is a colony, since H is nonrecursive and propagating. $L_t(C_{ex})=L(H)$, since for $w\in V^+$ we have $w\Rightarrow w'$ in H if and only if $w\overset{t}{\Rightarrow}w'$ in C_{ex}. So we have $\mathscr{L}(nrFT0L_{[1]})\subseteq\mathscr{L}(Col, \mathrm{t}, ex)$. Hence we have the result. □

Summarizing Theorems 4.1–4.4 we obtain the following hierarchy.

Theorem 4.5. *Let* $f\in\{one, arb, all, dist\}$. *Then*

(a) $\mathscr{L}(Col, \mathrm{b}, ex)\subset\mathscr{L}(Col, \mathrm{b}, f)\subset\mathscr{L}(Col, \mathrm{t}, f)$

(b) $\mathscr{L}(Col, \mathrm{t}, ex)\subset\mathscr{L}(Col, \mathrm{t}, f)$

(c) *Families* $\mathscr{L}(Col, \mathrm{b}, f)$ and $\mathscr{L}(Col, \mathrm{t}, ex)$ *are incomparable.*

(d) *Families* $\mathscr{L}(Col, \mathrm{b}, ex)$ and $\mathscr{L}(Col, \mathrm{t}, ex)$ *are incomparable.*

Proof. (a) We have

$$\mathscr{L}(Col, \mathrm{b}, ex) = \mathscr{L}(nrpCF) \subset \mathscr{L}(CF) = \mathscr{L}(Col, \mathrm{b}, f) \subset \mathscr{L}(EPT0L_{[1]})$$
$$= \mathscr{L}(Col, \mathrm{t}, f)$$

by Theorem 4.2, Proposition 2.1, Theorems 4.1 and 3.5, Proposition 2.2 and Theorem 4.3.

(b) $\mathscr{L}(FPT0L_{[1]}) \subseteq \mathscr{L}(EPT0L_{[1]})$ is evident. (One can add a new table, which rewrites a new startsymbol to the original axioms.) So

$$\mathscr{L}(Col, t, ex) = \mathscr{L}(nrFPT0L_{[1]}) \subset \mathscr{L}(FPT0L_{[1]}) \subseteq \mathscr{L}(EPT0L_{[1]})$$
$$= \mathscr{L}(Col, \mathrm{t}, f)$$

by Theorem 4.4, Proposition 2.2 and Theorem 4.3.

(c) Note that in a component of a colony a letter cannot be both terminal and nonterminal symbol. Therefore, a colony over one letter alphabet, say $\{a\}$, with the acceptance style ex degenerates. Its derivations consist of at most one step, rewriting the axiom S into a word over $\{a\}$, i.e. such a colony produces finite language only. So $\{a\}^+ \in \mathscr{L}(Col, \mathrm{b}, f) - \mathscr{L}(Col, \mathrm{t}, ex)$.

$\{a^{2^n}, b^{2^n}\colon n \geq 1\}$ is in $\mathscr{L}(Col, \mathrm{t}, ex)$ but not in $\mathscr{L}(Col, \mathrm{b}, f) = \mathscr{L}(CF)$. See Example 3.1.

Consequently, $\mathscr{L}(Col, \mathrm{b}, f)$ and $\mathscr{L}(Col, \mathrm{t}, ex)$ are incomparable.

(d) $\{a^{2^n}, b^{2^n}\colon n \geq 1\}$ is in $\mathscr{L}(Col, \mathrm{t}, ex)$ according to Example 3.7 and it is not in $\mathscr{L}(Col, \mathrm{b}, ex) \subset \mathscr{L}(CF)$.

$L_0 = \{a, b\}^+ - \{b\}$ is in $\mathscr{L}(Col, \mathrm{b}, ex)$ according to Example 3.7. We shall prove that L_0 is not in $\mathscr{L}(Col, \mathrm{t}, ex)$. Assume we have a colony $C = (T, R_1, \ldots, R_n, S)$ with $T = \{a, b\}$ being the union of terminal alphabets of $R_1, \ldots, R_n$ and such that $L_\mathrm{t}(C) = L_0$. Then only the symbols S, a and b can be startsymbols of components of C. If S is the startsymbol of R_i, then $L(R_i)$ is a finite subset of $(a \cup b)^+$. If a is the startsymbol of R_i, then a is a nonterminal of R_i and $L(R_i) \subset bb^+$ and, analogously, if b is the startsymbol of R_i, then $L(R_i) \subset a^+$. Therefore, only finite many words in $L_\mathrm{t}(C)$ contain both occurrences of a and b, hence $L_0 \neq L_\mathrm{t}(C)$.

Consequently, $\mathscr{L}(Col, \mathrm{t}, ex)$ and $\mathscr{L}(Col, \mathrm{b}, ex)$ are incomparable.

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