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Synchronization stability of general complex dynamical networks with time-varying delays: A piecewise analysis method

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ABSTRACT

The synchronization problem of some general complex dynamical networks with time-varying delays is investigated. Both time-varying delays in the network couplings and time-varying delays in the dynamical nodes are considered. The delays considered in this paper are assumed to vary in an interval, where the lower and upper bounds are known. Based on a piecewise analysis method, the variation interval of the time delay is firstly divided into several subintervals, by checking the variation of the derivative of a Lyapunov function in every subinterval, then the convexity of matrix function method and the free weighting matrix method are fully used in this paper. Some new delay-dependent synchronization stability criteria are derived in the form of linear matrix inequalities. Two numerical examples show that our method can lead to much less conservative results than those in the existing references.

1. Introduction

Complex network models are used to describe various interconnected systems of the real world, such as the world wide web, food webs, electronic power grids, internet etc [1–3], which have become a focal research topic and have drawn much attention from researchers working in different fields, one of the most important reasons is that most practical systems can be modeled by complex dynamical networks. Synchronization motion is one of the most important dynamical properties of complex networks, a lot of work has been done in the literature on synchronization stability analysis for complex networks [4–7].

The characteristic of time-delayed coupling is very common in biological and physical systems etc [8], some of the time delays are trivial and so can be ignored, while some others cannot be ignored, such as in long distance communication, traffic congestions etc. Therefore, time delays should be modeled in order to simulate more realistic networks. Lu and Chen [9] presented a time-varying complex dynamical network model, and investigated its synchronization phenomenon by proving several network synchronization theorems. Li and Chen [10] introduced a complex dynamical network model with coupling delays, and derived some synchronization conditions for both delay-independent and delay-dependent asymptotic stabilities. By utilizing the Lyapunov function method, Cao et al. [4] derived some sufficient conditions for the global exponential synchronization in arrays of coupled delayed neural networks. Li et al. [11] investigated the global synchronization of a class of complex networks with time-varying delays, where delay appeared in the isolated systems but not in the coupling term. Checco et al. [12] studied the synchronization of random networks with given expected degree

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sequences, and showed that random graphs almost surely synchronize. Wang [13] introduced several synchronization criteria for both delay-independent and delay-dependent asymptotic stabilities. Li [14] investigated the synchronization of complex networks with time-varying couplings, the stability criteria were obtained by using Lyapunov–Krasovskii functional method and subspace projection method. Lu [15] studied local and global synchronizations in general complex dynamical networks with delay coupling, some simple criteria of synchronization were given in terms of linear matrix inequalities (LMIs). Dai [16] analyzed the synchronization criteria for complex dynamical networks with neutral-type coupling delay, and the less conservative sufficient conditions were derived in the form of linear matrix inequalities based on the free weighting matrix method. But it is worth noting that most of the existing results on complex networks are concerned with constant delays, little progress has been made towards solving the problem arising from complex networks with time-varying coupling delays. Synchronization stability analysis, as one of the fundamental problems for complex networks, still remains unsolved and challenging, Li [17] introduced the synchronization problem of some general complex dynamical networks with time-varying delays in the network couplings and time-varying delays in the dynamical nodes, but the time-varying delays are required to be differentiable, however, in most cases, these conditions are difficult to be satisfied. Therefore, in this paper we will employ some new techniques such that the above conditions can be removed.

The stability criteria for systems with time delays can be classified into two categories, namely, delay-independent and delay-dependent. Since delay-independent criteria tend to be conservative, especially when the delay is small, considerable attention has been paid to the delay-dependent type. For systems with time-varying delay, fixed model transformations are the main method to deal with delay-dependent stability [18], in which some inequalities such as Park and Moon et al.'s inequalities [19,20], were used to estimate the upper bound of cross-product terms. Recently, in order to reduce the conservatism, the delay-central-point method was proposed [21–24], in the method, employing the central point of variation of the delay, the variation interval of the delay is divided into two subintervals with equal two subintervals respectively. The main advantages of the method [21,22,25,26] are: (1) more information on the variation interval of the delay is employed. (2) the delay-central-point state is introduced.

In this paper, the delay-central-point method will be extended to study the synchronization stability of complex dynamical networks with time-varying coupling delays, By some transformation, the synchronization of the complex networks is transferred equally into the asymptotic stability problem of a group of uncorrelated delay functional differential equations. Firstly, the variation interval of the time delay is divided into several subintervals, by checking the variation of the Lyapunov function for the case when the time delay is in every subinterval some new technique is used in this paper, such as the convexity of matrix inequalities and a piecewise analysis method. The sufficient conditions of delay-dependent synchronization stability are derived in the form of linear matrix inequalities. Numerical examples are given to demonstrate the effectiveness and the advantage of the proposed method.

The rest of this paper is organized as follows. The model of a complex dynamical network with time-varying coupling delays and its synchronization criteria are presented in Section 2. In Section 3, synchronization stability in complex dynamical networks with time-varying delayed nodes are investigated. In Section 4, the effectiveness of the derived results is shown by numerical simulation, and some conclusions and remarks are presented in Section 5.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices, The notation $X > 0$ (respectively, $X < 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, negative definite). For a real matrix B and two real symmetric matrices A and C of appropriate dimensions, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a real symmetric matrix, where $*$ denotes the entries implied by symmetry. R_+ denotes the set of real positive numbers. In this paper, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Complex dynamical network model with coupling delay

Consider delayed complex dynamical networks consisting of N identical nodes, in which each node is an m-dimensional dynamical subsystem

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t-\tau(t)) \quad (i = 1, 2, \ldots, N) \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots x_{im}(t))^{\mathrm{T}} \in R^m$ is the state vector of the ith node. $f(\cdot) \in R^m$ is a continuously differentiable vector function. The constant $c > 0$ represents the coupling strength. $\Gamma = (\gamma_{ij}) \in R^{m\times m}$ is an inner-coupling matrix, if some pairs (i, j), $1 \le i, j \le m$, with $\gamma_{ij} \ne 0$, which means two coupled nodes are linked through their ith and jth state variables, and $\tau(t)$ is the time-varying delay, which satisfies the assumption that

$$\tau_m \le \tau(t) \le \tau_M \tag{2}$$

where $0 \le \tau_m < \tau_M$, note that τ_m may not be equal to 0. $G = (g_{ij})_{N\times N}$ represents the outer-coupling matrix of the networks, in which g_{ij} is defined as follows: if there exists a connection between node i and node j ($j \ne i$), then $g_{ij} = g_{ji} = 1$, otherwise $g_{ij} = g_{ji} = 0$, ($j \ne i$), and the diagonal elements of matrix G are defined by

$$g_{ii} = -\sum_{j=1, j\ne i}^{N} g_{ij} = -\sum_{j=1, j\ne i}^{N} g_{ji} \quad (i = 1, 2, \ldots N). \tag{3}$$

Definition 1. The delayed dynamical networks (1) are said to achieve asymptotic synchronization if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \quad \text{as } t \to \infty \tag{4}$$

where $s(t)$ is a solution of an isolated node, satisfying $\dot{s}(t) = f(s(t))$.

To obtain the main results, the following lemmas are needed.

Lemma 1 (*[10]*). *Consider the delayed dynamical networks (1), the eigenvalues of the outer-coupling matrix G are denoted by*

$$0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N$$

if the following $N-1$ of m-dimensional time-varying delayed differential equations are asymptotically stable about their zero solution

$$\dot{\eta}_k(t) = J(t)\eta_k(t) + c\lambda_k \Gamma \eta_k(t-\tau(t)) \quad (k = 2, 3, \ldots N) \tag{5}$$

where $J(t)$ is the Jacobian of $f(x(t))$ at $s(t)$, then the synchronized states (4) are asymptotically stable.

Lemma 2 (*Jensen's Inequality [27]*). *Suppose $0 < \tau_m \leq \tau(t) \leq \tau_M$ and $x(t) \in R^n$, for any positive matrix $R \in R^{n\times n}$, then*

$$-(\tau_M - \tau_m)\int_{t-\tau_M}^{t-\tau_m} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s \leq \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_M) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_M) \end{bmatrix}.$$

Lemma 3. *Suppose $\tau(t)$ is defined in Lemma 1, Q_i $(i = 1, 2, 3)$ are some constant matrices with appropriate dimensions, then*

$$Q_1 + (\tau_M - \tau(t))Q_2 + (\tau(t) - \tau_m)Q_3 < 0 \tag{6}$$

if the following inequalities hold

$$Q_1 + (\tau_M - \tau_m)Q_2 < 0 \tag{7}$$

$$Q_1 + (\tau_M - \tau_m)Q_3 < 0. \tag{8}$$

Proof. (I) If $\tau(t) = \tau_m$ or $\tau(t) = \tau_M$, the conclusion is obvious.
(II) If $\tau_m < \tau(t) < \tau_M$
(Sufficiency): define a function as

$$\begin{aligned} f(\tau(t)) &= Q_1 + (\tau_M - \tau(t))Q_2 + (\tau(t) - \tau_m)Q_3 \\ &= \frac{\tau_M - \tau(t)}{\tau_M - \tau_m}[Q_1 + (\tau_M - \tau_m)Q_2] + \frac{\tau(t) - \tau_m}{\tau_M - \tau_m}[Q_1 + (\tau_M - \tau_m)Q_3] \end{aligned} \tag{9}$$

from (7) and (8), we have

$$f(\tau(t)) < 0 \tag{10}$$

which is equal to (6).
(Necessity): Letting $\tau(t) = \tau_m$ and $\tau(t) = \tau_M$ respectively in (6), we can easily obtain (7) and (8). □

Clearly, synchronization of dynamical networks (1) is equivalent to the stability of systems (5) about zero solution, we divide the variation of the delay into two parts with equal length, that is, let $\tau_1 = \frac{1}{2}(\tau_m + \tau_M)$, $\delta = \frac{1}{2}(\tau_M - \tau_m)$, then $\tau_1 = \tau_m + \delta$, $\tau_M = \tau_m + 2\delta$. Define

$$\mathcal{A}_k = \begin{bmatrix} J(t) & 0 & c\lambda_k\Gamma & 0 & 0 \end{bmatrix}$$

$$\xi^{\mathrm{T}}(t) = \begin{bmatrix} \eta_k^{\mathrm{T}}(t) & \eta_k^{\mathrm{T}}(t-\tau_m) & \eta_k^{\mathrm{T}}(t-\tau(t)) & \eta_k^{\mathrm{T}}(t-\tau_1) & \eta_k^{\mathrm{T}}(t-\tau_M) \end{bmatrix}$$

then (5) becomes

$$\dot{\eta}_k(t) = \mathcal{A}_k \xi(t). \tag{11}$$

Based on (11), a sufficient condition for delay-dependent asymptotic stability of system (5) is given as follows.

Theorem 1. *System (5) is asymptotically stable, for given constant $\tau_M > \tau_m \geq 0$, if there exist matrices $P_k > 0$, $Q_{ki} > 0$, $R_{ki} > 0$ and N_k, M_k, T_k and $S_k (i = 1, 2, 3; k = 2, 3, \ldots, N)$ with appropriate dimensions such that the following matrix inequality holds*

$$\begin{bmatrix} \Pi_{ki} + \Sigma_{k11}^{i} + (\Sigma_{k11}^{i})^{\mathrm{T}} & * & * \\ \Sigma_{k21} & \Sigma_{k22} & * \\ \Sigma_{k31}^{ij} & 0 & \Sigma_{k33}^{i} \end{bmatrix} < 0 \quad (i, j = 1, 2; k = 2, \ldots, N) \tag{12}$$

where

$$\Pi_{k1}=\begin{bmatrix} H_k & * & * & * & * \\ R_{k1} & -R_{k1}-Q_{k1} & * & * & * \\ c\lambda_k\Gamma^{\mathrm{T}}P_k & 0 & 0 & * & * \\ 0 & 0 & 0 & -Q_{k2}-\dfrac{R_{k3}}{\delta} & * \\ 0 & 0 & 0 & \dfrac{R_{k3}}{\delta} & -\dfrac{R_{k3}}{\delta}-Q_{k3} \end{bmatrix}$$

$$\Pi_{k2}=\begin{bmatrix} H_k & * & * & * & * \\ R_{k1} & -R_{k1}-Q_{k1}-\dfrac{R_{k2}}{\delta} & * & * & * \\ c\lambda_k\Gamma^{\mathrm{T}}P_k & 0 & 0 & * & * \\ 0 & \dfrac{R_{k2}}{\delta} & 0 & -Q_{k2}-\dfrac{R_{k2}}{\delta} & * \\ 0 & 0 & 0 & 0 & -Q_{k3} \end{bmatrix}$$

$$\Sigma_{k11}^{1}=\begin{bmatrix}0 & N_k & M_k-N_k & -M_k & 0\end{bmatrix}$$

$$\Sigma_{k11}^{2}=\begin{bmatrix}0 & 0 & S_k-T_k & T_k & -S_k\end{bmatrix}$$

$$\Sigma_{k31}^{11}=\sqrt{\delta}N_k^{\mathrm{T}}\qquad \Sigma_{k31}^{12}=\sqrt{\delta}M_k^{\mathrm{T}}\qquad \Sigma_{k31}^{21}=\sqrt{\delta}T_k^{\mathrm{T}}\qquad \Sigma_{k31}^{22}=\sqrt{\delta}S_k^{\mathrm{T}}$$

$$\Sigma_{k33}^{1}=-R_{k2}\qquad \Sigma_{k33}^{2}=-R_{k3}$$

$$\Sigma_{k21}^{\mathrm{T}}=\begin{bmatrix}\tau_m\mathcal{A}_k^{\mathrm{T}}R_{k1} & \sqrt{\delta}\mathcal{A}_k^{\mathrm{T}}R_{k2} & \sqrt{\delta}\mathcal{A}_k^{\mathrm{T}}R_{k3}\end{bmatrix}$$

$$\Sigma_{k22}=\mathrm{diag}\{-R_{k1}\quad -R_{k2}\quad -R_{k3}\}$$

$$H_k=P_kJ(t)+J^{\mathrm{T}}(t)P_k+Q_{k1}+Q_{k2}+Q_{k3}-R_{k1}$$

$$\delta=\frac{1}{2}(\tau_M-\tau_m).$$

Proof. Construct a Lyapunov–Krasovskii functional candidate as

$$\begin{aligned} V(\eta_{kt}) = {} & \eta_k^{\mathrm{T}}(t)P_k\eta_k(t)+\int_{t-\tau_m}^{t}\eta_k^{\mathrm{T}}(v)Q_{k1}\eta_k(v)\mathrm{d}v+\int_{t-\tau_1}^{t}\eta_k^{\mathrm{T}}(v)Q_{k2}\eta_k(v)\mathrm{d}v+\int_{t-\tau_M}^{t}\eta_k^{\mathrm{T}}(v)Q_{k3}\eta_k(v)\mathrm{d}v \\ & +\tau_m\int_{t-\tau_m}^{t}\int_{s}^{t}\dot\eta_k^{\mathrm{T}}(v)R_{k1}\dot\eta_k(v)dvds+\int_{t-\tau_1}^{t-\tau_m}\int_{s}^{t}\dot\eta_k^{\mathrm{T}}(v)R_{k2}\dot\eta_k(v)\mathrm{d}v\mathrm{d}s+\int_{t-\tau_M}^{t-\tau_1}\int_{s}^{t}\dot\eta_k^{\mathrm{T}}(v)R_{k3}\dot\eta_k(v)\mathrm{d}v\mathrm{d}s \end{aligned} \tag{13}$$

where $P_k>0$, $Q_{ki}>0$, $R_{ki}>0$ ($i=1,2,3$; $k=2,3,\ldots,N$).

Calculating the derivative of $V(\eta_{kt})$ leads to the following equality

$$\begin{aligned} \dot V(\eta_{kt}) = {} & 2\dot\eta_k^{\mathrm{T}}(t)P_k\eta_k(t)+\eta_k^{\mathrm{T}}(t)(Q_{k1}+Q_{k2}+Q_{k3})\eta_k(t)-\eta_k^{\mathrm{T}}(t-\tau_m)Q_{k1}\eta_k(t-\tau_m)-\eta_k^{\mathrm{T}}(t-\tau_1)Q_{k2}\eta_k(t-\tau_1) \\ & -\eta_k^{\mathrm{T}}(t-\tau_M)Q_{k3}\eta_k(t-\tau_M)+\dot\eta_k^{\mathrm{T}}(t)(\tau_m^2R_{k1}+\delta R_{k2}+\delta R_{k3})\dot\eta_k(t)-\tau_m\int_{t-\tau_m}^{t}\dot\eta_k^{\mathrm{T}}(v)R_{k1}\dot\eta_k(v)\mathrm{d}v \\ & -\int_{t-\tau_1}^{t-\tau_m}\dot\eta_k^{\mathrm{T}}(v)R_{k2}\dot\eta_k(v)\mathrm{d}v-\int_{t-\tau_M}^{t-\tau_1}\dot\eta_k^{\mathrm{T}}(v)R_{k3}\dot\eta_k(v)\mathrm{d}v \end{aligned} \tag{14}$$

using Lemma 2

$$-\tau_m\int_{t-\tau_m}^{t}\dot\eta_k^{\mathrm{T}}(v)R_{k1}\dot\eta_k(v)\mathrm{d}v\le\begin{bmatrix}\eta_k(t)\\ \eta_k(t-\tau_m)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-R_{k1} & R_{k1}\\ R_{k1} & -R_{k1}\end{bmatrix}\begin{bmatrix}\eta_k(t)\\ \eta_k(t-\tau_m)\end{bmatrix} \tag{15}$$

then

$$\dot V(\eta_{kt})\le\xi^{\mathrm{T}}(t)\Pi_k\xi(t)+\dot\eta_k^{\mathrm{T}}(t)(\tau_m^2R_{k1}+\delta R_{k2}+\delta R_{k3})\dot\eta_k(t)-\int_{t-\tau_1}^{t-\tau_m}\dot\eta_k^{\mathrm{T}}(v)R_{k2}\dot\eta_k(v)\mathrm{d}v-\int_{t-\tau_M}^{t-\tau_1}\dot\eta_k^{\mathrm{T}}(v)R_{k3}\dot\eta_k(v)\mathrm{d}v \tag{16}$$

where

$$\Pi_k=\begin{bmatrix} H_k & * & * & * & * \\ R_{k1} & -R_{k1}-Q_{k1} & * & * & * \\ c\lambda_k\Gamma^{\mathrm{T}}P_k & 0 & 0 & * & * \\ 0 & 0 & 0 & -Q_{k2} & * \\ 0 & 0 & 0 & 0 & -Q_{k3} \end{bmatrix}. \tag{17}$$

It is noted that, for any $t \in R_+$, $\tau(t) \in [\tau_m, \tau_1]$ or $\tau(t) \in (\tau_1, \tau_M]$, define two sets

$$\Omega_1 = \{t : \tau(t) \in [\tau_m, \tau_1]\} \qquad \Omega_2 = \{t : \tau(t) \in (\tau_1, \tau_M]\} \tag{18}$$

in the following, we will discuss the variation of $\dot V(\eta_{kt})$ for two cases, that is $t \in \Omega_1$ or $t \in \Omega_2$

Case 1. For $t \in \Omega_1$

$$-\int_{t-\tau_M}^{t-\tau_1} \dot\eta_k^{\mathrm T}(v)R_{k3}\dot\eta_k(v)\mathrm{d}v \le -\frac{1}{\delta}\begin{bmatrix}\eta_k(t-\tau_1)\\ \eta_k(t-\tau_M)\end{bmatrix}^{\mathrm T}\begin{bmatrix}-R_{k3} & R_{k3}\\ R_{k3} & -R_{k3}\end{bmatrix}\begin{bmatrix}\eta_k(t-\tau_1)\\ \eta_k(t-\tau_M)\end{bmatrix}. \tag{19}$$

Employing the free matrix method, we have

$$2\xi^{\mathrm T}(t)N_k\left[\eta_k(t-\tau_m) - \eta_k(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_m}\dot\eta_k(v)\mathrm{d}v\right] = 0 \tag{20}$$

$$2\xi^{\mathrm T}(t)M_k\left[\eta_k(t-\tau(t)) - \eta_k(t-\tau_1) - \int_{t-\tau_M}^{t-\tau(t)}\dot\eta_k(v)\mathrm{d}v\right] = 0 \tag{21}$$

where

$$N_k^{\mathrm T} = \begin{bmatrix}N_{k1}^{\mathrm T} & N_{k2}^{\mathrm T} & N_{k3}^{\mathrm T} & N_{k4}^{\mathrm T} & N_{k5}^{\mathrm T}\end{bmatrix}$$
$$M_k^{\mathrm T} = \begin{bmatrix}M_{k1}^{\mathrm T} & M_{k2}^{\mathrm T} & M_{k3}^{\mathrm T} & M_{k4}^{\mathrm T} & M_{k5}^{\mathrm T}\end{bmatrix}$$

there exists $R_{k2} > 0$, such that

$$-2\xi^{\mathrm T}(t)N_k\int_{t-\tau(t)}^{t-\tau_m}\dot\eta_k(v)\mathrm{d}v \le (\tau(t)-\tau_m)\xi^{\mathrm T}(t)N_kR_{k2}^{-1}N_k^{\mathrm T}\xi(t) + \int_{t-\tau(t)}^{t-\tau_m}\dot\eta_k^{\mathrm T}(v)R_{k2}\dot\eta_k(v)\mathrm{d}v \tag{22}$$

$$-2\xi^{\mathrm T}(t)M_k\int_{t-\tau_1}^{t-\tau(t)}\dot\eta_k(v)\mathrm{d}v \le (\tau_1-\tau(t))\xi^{\mathrm T}(t)M_kR_{k2}^{-1}M_k^{\mathrm T}\xi(t) + \int_{t-\tau_1}^{t-\tau(t)}\dot\eta_k^{\mathrm T}(v)R_{k2}\dot\eta_k(v)\mathrm{d}v. \tag{23}$$

Adding (20) and (21) to the right of (16) and substituting (19), (22) and (23) into (16), we have

$$\begin{aligned}\dot V(\eta_{kt}) \le\ & \xi^{\mathrm T}(t)[\Pi_{k1} + \mathcal{A}_k^{\mathrm T}(\tau_m^2R_{k1} + \delta R_{k2} + \delta R_{k3})\mathcal{A}_k + \Sigma_{k11}^1 + (\Sigma_{k11}^1)^{\mathrm T}]\xi(t) + (\tau(t)-\tau_m)\xi^{\mathrm T}(t)N_kR_{k2}^{-1}N_k^{\mathrm T}\xi(t)\\ & + (\tau_1-\tau(t))\xi^{\mathrm T}(t)M_kR_{k2}^{-1}M_k^{\mathrm T}\xi(t)\end{aligned} \tag{24}$$

from (12), when $i = 1, j = 1$ and $j = 2$, by the Schur complement, we have

$$\begin{aligned}&\Pi_{k1} + \mathcal{A}_k^{\mathrm T}(\tau_m^2R_{k1} + \delta R_{k2} + \delta R_{k3})\mathcal{A}_k + \Sigma_{k11}^1 + (\Sigma_{k11}^1)^{\mathrm T} + (\tau_1-\tau_m)M_kR_{k2}^{-1}M_k^{\mathrm T} < 0\\ &\Pi_{k1} + \mathcal{A}_k^{\mathrm T}(\tau_m^2R_{k1} + \delta R_{k2} + \delta R_{k3})\mathcal{A}_k + \Sigma_{k11}^1 + (\Sigma_{k11}^1)^{\mathrm T} + (\tau_1-\tau_m)N_kR_{k2}^{-1}N_k^{\mathrm T} < 0\end{aligned} \tag{25}$$

using Lemma 3, we have

$$\dot V(\eta_{kt}) < 0. \tag{26}$$

Case 2. For $t \in \Omega_2$

$$-\int_{t-\tau_1}^{t-\tau_M} \dot\eta_k^{\mathrm T}(v)R_{k2}\dot\eta_k(v)\mathrm{d}v \le -\frac{1}{\delta}\begin{bmatrix}\eta_k(t-\tau_M)\\ \eta_k(t-\tau_1)\end{bmatrix}^{\mathrm T}\begin{bmatrix}-R_{k2} & R_{k2}\\ R_{k2} & -R_{k2}\end{bmatrix}\begin{bmatrix}\eta_k(t-\tau_M)\\ \eta_k(t-\tau_1)\end{bmatrix}. \tag{27}$$

Employing the free matrix method, we have

$$2\xi^{\mathrm T}(t)T_k\left[\eta_k(t-\tau_1) - \eta_k(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1}\dot\eta_k(v)\mathrm{d}v\right] = 0 \tag{28}$$

$$2\xi^{\mathrm T}(t)S_k\left[\eta_k(t-\tau(t)) - \eta_k(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot\eta_k(v)\mathrm{d}v\right] = 0 \tag{29}$$

where

$$T_k^{\mathrm T} = \begin{bmatrix}T_{k1}^{\mathrm T} & T_{k2}^{\mathrm T} & T_{k3}^{\mathrm T} & T_{k4}^{\mathrm T} & T_{k5}^{\mathrm T}\end{bmatrix}$$
$$S_k^{\mathrm T} = \begin{bmatrix}S_{k1}^{\mathrm T} & S_{k2}^{\mathrm T} & S_{k3}^{\mathrm T} & S_{k4}^{\mathrm T} & S_{k5}^{\mathrm T}\end{bmatrix}$$

there exists $R_{k3} > 0$, such that

$$-2\xi^{\mathrm{T}}(t)T_k \int_{t-\tau(t)}^{t-\tau_1} \dot{\eta}_k(v)\mathrm{d}v \le (\tau(t)-\tau_m)\xi^{\mathrm{T}}(t)T_k R_{k3}^{-1} T_k^{\mathrm{T}}\xi(t) + \int_{t-\tau(t)}^{t-\tau_1} \dot{\eta}_k^{\mathrm{T}}(v) R_{k3}\dot{\eta}_k(v)\mathrm{d}v \tag{30}$$

$$-2\xi^{\mathrm{T}}(t)S_k \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}_k(v)\mathrm{d}v \le (\tau_M-\tau(t))\xi^{\mathrm{T}}(t)S_k R_{k3}^{-1} S_k^{\mathrm{T}}\xi(t) + \int_{t-\tau_M}^{t-\tau(t)} \dot{\eta}_k^{\mathrm{T}}(v) R_{k3}\dot{\eta}_k(v)\mathrm{d}v. \tag{31}$$

Adding (28) and (29) to the right of (16) and substituting (27), (30) and (31) into (16), we have

$$\begin{aligned}\dot{V}(\eta_{kt}) \le\ & \xi^{\mathrm{T}}(t)[\Pi_{k2} + \mathscr{A}_k^{\mathrm{T}}(\tau_m^2 R_{k1} + \delta R_{k2} + \delta R_{k3})\mathscr{A}_k + \Sigma_{k11}^2 + (\Sigma_{k11}^2)^{\mathrm{T}}]\xi(t) + (\tau(t)-\tau_1)\xi^{\mathrm{T}}(t)T_k R_{k3}^{-1}T_k^{\mathrm{T}}\xi(t)\\ & + (\tau_M-\tau(t))\xi^{\mathrm{T}}(t)S_k R_{k3}^{-1}S_k^{\mathrm{T}}\xi(t)\end{aligned} \tag{32}$$

from (12), when $i = 2, j = 1$ and $j = 2$, by the Schur complement, we have

$$\begin{aligned}&\Pi_{k2} + \mathscr{A}_k^{\mathrm{T}}(\tau_m^2 R_{k1} + \delta R_{k2} + \delta R_{k3})\mathscr{A}_k + \Sigma_{k11}^2 + (\Sigma_{k11}^2)^{\mathrm{T}} + (\tau_M-\tau_1)T_k R_{k3}^{-1}T_k^{\mathrm{T}} < 0\\ &\Pi_{k2} + \mathscr{A}_k^{\mathrm{T}}(\tau_m^2 R_{k1} + \delta R_{k2} + \delta R_{k3})\mathscr{A}_k + \Sigma_{k11}^2 + (\Sigma_{k11}^2)^{\mathrm{T}} + (\tau_M-\tau_1)S_k R_{k3}^{-1}S_k^{\mathrm{T}} < 0\end{aligned} \tag{33}$$

using Lemma 3, we have

$$\dot{V}(\eta_{kt}) < 0. \tag{34}$$

From the above discussion, we can see that for all $t \in R_+$, (13) with $i = 1, 2, j = 1$ and $j = 2$, by the Lyapunov stability theory, we know that the systems (5) are asymptotically stable, according to Lemma 1, the asymptotic synchronization defined in (4) is achieved, the proof is completed. □

Remark 1. The differentiability of the delay functions $\tau(t)$ is removed in Theorem 1, hence the proposed results are less conservative and restrictive than [17]. Moreover, the lower bound of the delay $\tau_1 \neq 0$ is considered in this paper.

Remark 2. To further reduce the conservatism, we may divide the variation interval of the delay into l ($l \ge 3$) parts with equal length. Defining

$$\tau_i = \tau_m + \frac{i(\tau_M-\tau_m)}{l} \quad (i = 1, 2, \ldots, l)$$

then $[\tau_m, \tau_M] = [\tau_m, \tau_1] \cup \cup_{i=1}^{l-1} (\tau_i, \tau_{i+1}]$, we may use a similar method in Theorem 1, and construct a Lyapunov function $V(x)$, then by checking the variation of derivative of $V(x)$ for the case when $\tau(t) \in [\tau_m, \tau_1]$ or $\tau(t) \in (\tau_i, \tau_{i+1}]$, $i = 1, 2, \ldots, l-1$, respectively, we can derive some delay-dependent conditions which can guarantee $\dot{V}(x) < 0$, it can also be easily extended by the proposed method in Theorem 1. For the brevity of the analysis, we omit it here.

3. Complex dynamical network model with delayed nodes

In this section, we will study the synchronization of dynamical node delayed complex networks. Unless otherwise defined, we endorse the same notations used in the above section.

Consider general complex networks consisting of N delayed dynamical nodes. Each node of the networks is an m-dimensional dynamical system with time-varying delay, which is described by

$$\dot{x}_i(t) = f(x_i(t), x_i(t-\tau(t))) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t) \quad (i = 1, 2, \ldots, N) \tag{35}$$

where $f(\cdot) \in R^m$ is a continuously differentiable vector function, the time-varying delay $\tau(t)$ satisfies (2). The dynamical node delayed networks (35) are said to be achieve asymptotic synchronization if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \quad \text{as } t \to \infty \tag{36}$$

where $s(t)$ is a solution of an isolated node, satisfying $\dot{s}(t) = f(s(t), s(t-\tau(t)))$.

Lemma 4 (*[17]*). *Consider the dynamical node delayed networks (35), let $0 = \lambda_1 > \lambda_2 \ge \lambda_3 \ge \cdots, \ge \lambda_N$ denote the eigenvalues of the outer-coupling matrix G. If the following $N-1$ linear time-varying delayed differential equations are asymptotically stable about their zero solution*

$$\dot{\eta}_k(t) = (J_1(t) + c\lambda_k\Gamma)\eta_k(t) + J_2(t)\eta_k(t-\tau(t)) \quad (k = 2, 3, \ldots N) \tag{37}$$

where $J_1(t)$ is the Jacobian of $f(x(t), x(t-\tau(t)))$ at $s(t)$, and $J_2(t)$ is the Jacobian of $f(x(t), x(t-\tau(t)))$ at $s(t-\tau(t))$. Then the synchronization states (36) are asymptotically stable.

Clearly, synchronization of dynamical networks (35) is equivalent to the stability of systems (37) about zero solution. By similar method in Theorem 1, we divide the variation interval of the delay into two parts with equal length, that is, let $\tau_1 = \frac{1}{2}(\tau_m + \tau_M)$, $\delta = \frac{1}{2}(\tau_M - \tau_m)$, then $\tau_1 = \tau_m + \delta$, $\tau_M = \tau_m + 2\delta$. Define

$$\bar{\mathcal{A}}_k = \begin{bmatrix} J_1(t) + c\lambda_k \Gamma & 0 & J_2(t) & 0 & 0 \end{bmatrix}$$

$$\xi^{\mathrm{T}}(t) = \begin{bmatrix} \eta_k^{\mathrm{T}}(t) & \eta_k^{\mathrm{T}}(t-\tau_m) & \eta_k^{\mathrm{T}}(t-\tau(t)) & \eta_k^{\mathrm{T}}(t-\tau_1) & \eta_k^{\mathrm{T}}(t-\tau_M) \end{bmatrix}$$

then (37) becomes

$$\dot{\eta}_k(t) = \bar{\mathcal{A}}_k \xi(t). \tag{38}$$

Based on (38), a sufficient condition for delay-dependent asymptotical stability of system (37) is given in the following theorem.

Theorem 2. *System (37) is asymptotically stable, for given constant $\tau_M > \tau_m \geq 0$, if there exist matrices $P_k > 0$, $Q_{ki} > 0$, $R_{ki} > 0$ and N_k, M_k, T_k and $S_k (i = 1, 2, 3; k = 2, 3, \ldots, N)$ with appropriate dimensions such that the following matrix inequality holds*

$$\begin{bmatrix} \bar{\Pi}_{ki} + \Sigma_{k11}^i + (\Sigma_{k11}^i)^{\mathrm{T}} & * & * \\ \bar{\Sigma}_{k21} & \Sigma_{k22} & * \\ \Sigma_{k31}^{ij} & 0 & \Sigma_{k33}^i \end{bmatrix} < 0 \quad (i, j = 1, 2; k = 2, 3, \ldots, N) \tag{39}$$

where

$$\bar{\Pi}_{k1} = \begin{bmatrix} \bar{H}_k & * & * & * & * \\ R_{k1} & -R_{k1} - Q_{k1} & * & * & * \\ J_2^{\mathrm{T}}(t)P_k & 0 & 0 & * & * \\ 0 & 0 & 0 & -Q_{k2} - \dfrac{R_{k3}}{\delta} & * \\ 0 & 0 & 0 & \dfrac{R_{k3}}{\delta} & -\dfrac{R_{k3}}{\delta} - Q_{k3} \end{bmatrix}$$

$$\bar{\Pi}_{k2} = \begin{bmatrix} \bar{H}_k & * & * & * & * \\ R_{k1} & -R_{k1} - Q_{k1} - \dfrac{R_{k2}}{\delta} & * & * & * \\ J_2^{\mathrm{T}}(t)P_k & 0 & 0 & * & * \\ 0 & \dfrac{R_{k2}}{\delta} & 0 & -Q_{k2} - \dfrac{R_{k2}}{\delta} & * \\ 0 & 0 & 0 & 0 & -Q_{k3} \end{bmatrix}$$

$$\Sigma_{k11}^1 = \begin{bmatrix} 0 & N_k & M_k - N_k & -M_k & 0 \end{bmatrix}$$

$$\Sigma_{k11}^2 = \begin{bmatrix} 0 & 0 & S_k - T_k & T_k & -S_k \end{bmatrix}$$

$$\Sigma_{k31}^{11} = \sqrt{\delta} N_k^{\mathrm{T}} \qquad \Sigma_{k31}^{12} = \sqrt{\delta} M_k^{\mathrm{T}} \qquad \Sigma_{k31}^{21} = \sqrt{\delta} T_k^{\mathrm{T}} \qquad \Sigma_{k31}^{22} = \sqrt{\delta} S_k^{\mathrm{T}}$$

$$\Sigma_{k33}^1 = -R_{k2} \quad \Sigma_{k33}^2 = -R_{k3}$$

$$\bar{\Sigma}_{k21}^{\mathrm{T}} = \begin{bmatrix} \tau_m \bar{\mathcal{A}}_k^{\mathrm{T}} R_{k1} & \sqrt{\delta} \bar{\mathcal{A}}_k^{\mathrm{T}} R_{k2} & \sqrt{\delta} \bar{\mathcal{A}}_k^{\mathrm{T}} R_{k3} \end{bmatrix}$$

$$\Sigma_{k22} = \mathrm{diag}\{-R_{k1} \ \ -R_{k2} \ \ -R_{k3}\}$$

$$\bar{H}_k = P_k(J_1(t) + c\lambda_k \Gamma) + (J_1(t) + c\lambda_k \Gamma)^{\mathrm{T}} P_k + Q_{k1} + Q_{k2} + Q_{k3} - R_{k1}$$

$$\delta = \frac{1}{2}(\tau_M - \tau_m).$$

Proof. The proof is similar to that of Theorem 1, we omit it here. □

4. Numerical examples

In this section, we use two examples to illustrate the results derived in this work. The above synchronization conditions can be applied to networks with different topologies and different sizes. In order to illustrate the main results, we consider a lower-dimensional network model.

Example 1. Consider 5-node complex dynamical networks, in which each node is a simple three-dimensional linear system[28].

$$\begin{cases} \dot{x}_{i1}(t) = -x_{i1}(t) \\ \dot{x}_{i2}(t) = -2x_{i2}(t) \\ \dot{x}_{i3}(t) = -3x_{i3}(t) \end{cases} \tag{40}$$

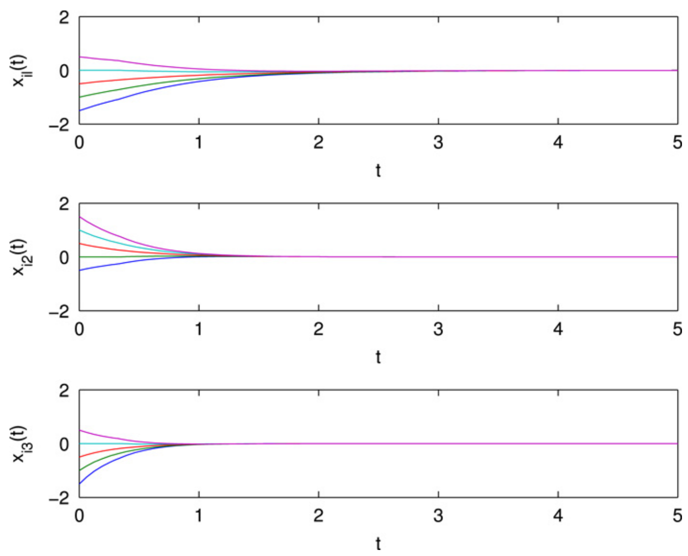

Fig. 1. State response $x_i(t)(i = 1, 2, \ldots, 5)$ for the coupling delayed networks when $c = 0.5$, $\tau(t) = \frac{|\sin(t)|+1}{4}$.

which is asymptotically stable at the equilibrium point $s(t) = (0, 0, 0)^{\mathrm{T}}$, and its Jacobin matrix is $J(t) = \mathrm{diag}\{-1, -2, -3\}$. For simplicity, we suppose that the inner-coupling matrix is $\Gamma = \mathrm{diag}\{1, 1, 1\}$, and the coupling matrix is

$$G = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}. \tag{41}$$

Obviously, G is an irreducible symmetric matrix. Therefore, if the delayed equations (5) are asymptotically stable, then the synchronized state is asymptotically stable. Using Theorem 1, we can obtain Table 1 by LMI toolbox of MATLAB, which lists the maximum allowable bounds for different c and τ_m. Firstly, it is found that the maximum delay bounds for $\tau_m = 0$ are less conservative than that obtained in[17], Secondly, [17] only discussed this for $\tau_m = 0$, Theorem 1 in this paper also considers $\tau_m \neq 0$. Moreover the differentiability of the time delay $\tau(t)$ is removed here. Fig. 1 shows the state response $x_{i1}(t), x_{i2}(t), x_{i3}(t)(i = 1, 2, \ldots, 5)$ for $c = 0.5$, $\tau(t) = \frac{|\sin(t)|+1}{4}$. We see that the state converges to zero under the above conditions.

Example 2. Consider complex dynamical networks with 5 nodes, where each node is the following dynamical delayed system

$$\begin{cases} \dot{x}_{i1}(t) = -2x_{i1}(t) - x_{i1}(t - \tau(t)) \\ \dot{x}_{i2}(t) = 0.9x_{i2}(t) + x_{i1}(t - \tau(t)) - x_{i2}(t - \tau(t)) \\ \dot{x}_{i3}(t) = -0.1x_{i3}(t) - x_{i3}(t - \tau(t)) \end{cases} \tag{42}$$

which is asymptotically stable at $s(t) = 0$, with the Jacobian given by

$$J_1(t) = \mathrm{diag}\{-2, 0.9, -0.1\}$$
$$J_2(t) = \mathrm{diag}\{-1, 0, 0; 1, -1, 0; 0, 0, -1\}.$$

Assume that the network coupling is similar to that in Example 1. The upper bounds on the time delay obtained from Theorem 2 are listed in Table 2 by LMI toolbox of MATLAB. It is found that the maximum delay bounds for $\tau_m = 0$ are less conservative than that obtained in [17], moreover; [17] only discussed this for $\tau_m = 0$. Theorem 2 in this paper also considers $\tau_m \neq 0$. Fig. 2 shows the state response $x_{i1}(t), x_{i2}(t), x_{i3}(t)(i = 1, 2, \ldots, 5)$ for $c = 0.5$, $\tau(t) = \frac{|\sin(t)|+1}{4}$. We see that the state also converges to zero under the above conditions.

Remark 3. Since the time-varying delay functions in two examples are not continuously differentiable, moreover lower bound $\tau_m = \frac{1}{4} \neq 0$, the synchronization conditions derived in [17] cannot be applied to these examples.

Remark 4. From Table 1, maximum delay bounds τ_M increase with the coupled strength c increasing for given τ_m. But maximum delay bounds τ_M decrease with the coupled strength c increasing for given τ_m from Table 2. The change direction

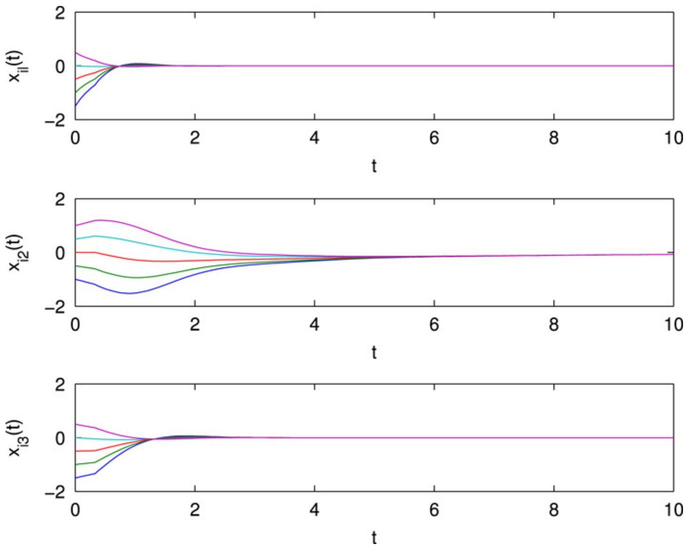

Fig. 2. State response $x_i(t)(i = 1, 2, \ldots, 5)$ for the dynamical node delayed networks when $c = 0.5$, $\tau(t) = \frac{|\sin(t)|+1}{4}$.

Table 1
Maximum allowable τ_2 for different c and τ_m.

		c			
τ_m		0.3	0.4	0.5	0.6
0	Kun Li [17]	0.960	0.710	0.562	0.464
	Theorem 1	1.345	0.950	0.731	0.587
	Over [17]	40.10%	33.80%	30.07%	26.51%
0.1	Theorem 1	1.354	0.951	0.731	0.587
0.5	Theorem 1	1.389	0.967	0.740	0.605

Table 2
Maximum allowable τ_2 for different c and τ_m.

		c			
τ_m		0.3	0.4	0.5	0.6
0	Kun Li [17]	1.115	1.147	1.179	1.209
	Theorem 2	1.723	2.020	2.310	2.486
	Over [17]	54.53%	76.11%	95.93%	105.62%
0.1	Theorem 2	1.723	2.036	2.325	2.501
0.5	Theorem 2	1.725	2.090	2.434	2.724

of upper bounds τ_M is different when delays appear in the network coupling and in the dynamical nodes respectively. A interesting problem will be considered in the future to investigate the maximum delay bounds, when delays appear in the network coupling and in the dynamical nodes at one time.

5. Conclusions

In this paper, the synchronization problem has been investigated for some complex dynamical networks with time-varying delays. Both time-varying delays in network couplings and time-varying delays in dynamical nodes have been considered, the delays considered in this paper are assumed to vary in an interval, where the lower and upper bounds are known. Based on piecewise analysis method and Lyapunov function method, some new delay-dependent synchronization stability criteria are derived by a set of linear matrix inequalities. Numerical examples show that the derived criteria can lead to less conservative results than those obtained based on the existing methods.

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