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The electric dipole form factor of the nucleon in chiral perturbation theory to sub-leading order

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ABSTRACT

The electric dipole form factor (EDFF) of the nucleon stemming from the QCD $\bar{\theta}$ term and from the quark color-electric dipole moments is calculated in chiral perturbation theory to sub-leading order. This is the lowest order in which the isoscalar EDFF receives a calculable, non-analytic contribution from the pion cloud. In the case of the $\bar{\theta}$ term, the expected lower bound on the deuteron electric dipole moment is $|d_d| \gtrsim 1.4 \cdot 10^{-4}\bar{\theta}e$ fm. The momentum dependence of the isovector EDFF is proportional to a non-derivative time-reversal-violating pion-nucleon coupling, and the scale for momentum variation—appearing, in particular, in the radius of the form factor—is the pion mass.

The electric dipole form factor (EDFF) completely specifies the parity (P) and time-reversal (T)-violating coupling of a spin 1/2 particle to a single photon [1,2]. At zero momentum, it reduces to the electric dipole moment (EDM), and its radius provides a contribution to the Schiff moment (SM) of a bound state containing the particle [3]. The full momentum dependence of the form factor can be used in lattice simulations to extract the EDM by extrapolation from a finite-momentum calculation [4] (in addition to the required extrapolations in quark masses and volume [5]).

There has been some recent interest [1,2,6–9] in the nucleon EDFF motivated by prospects of experiments that aim to improve the current bound on the neutron EDM, $|d_n| < 2.9 \cdot 10^{-13} e$ fm [10], by nearly two orders of magnitude [11], and to constrain the proton and deuteron EDMs at similar levels [12]. We would like to understand the implications of a possible signal in these measurements to the sources of T violation at the quark level, which include, in order of increasing dimension, the QCD $\bar{\theta}$ term, the quark color-EDM (qCEDM) and EDM, the gluon color-EDM, etc. [13, 14]. Unfortunately, as with other low-energy observables, both the EDM and the SM of hadrons and nuclei are difficult to calculate directly in QCD. However, long-range contributions from pions can, to some extent, be calculated using the low-energy effective field theory of QCD, chiral perturbation theory (ChPT) [15–17]. ChPT affords a systematic expansion of low-energy observables in powers of Q/M_{QCD}, where Q represents low-energy scales such as external momenta and the pion mass m_π, and $M_{QCD} \sim 1$ GeV denotes the characteristic QCD scale. (For introductions, see for example Refs. [18,19].)

In Refs. [1,9] the nucleon EDFF stemming from T-violation sources of effective dimension up to 6 was considered in ChPT to the lowest order where momentum dependence appears. It was argued [9] that the nucleon EDFF partially reflects the source of T violation at the quark level. The various sources differ in particular in the expectation for the behavior of the isoscalar EDFF. For $\bar{\theta}$ and qCEDM, the isoscalar momentum dependence appears only at NLO. The nucleon EDFF from $\bar{\theta}$ was calculated at LO in Ref. [1], generalizing to finite momenta earlier calculations of the EDM [20,21] and SM [3]. At this order, the momentum dependence is isovector and completely due to a T-violating coupling of the pion cloud to the nucleon, with a radius fixed by m_π^2 [3]. In Refs. [2,8] the EDFF calculation was extended to NLO, and corrections found to be significant. For the qCEDM, the nucleon EDFF has been calculated at LO [9], at which order it is identical to that from $\bar{\theta}$. For the other sources of effective dimension 6, the quark EDM and the gluon color-EDM, the nucleon EDFF, including its isoscalar component, has been calculated to

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NNLO [9] and found to be mostly determined by short-distance physics.

Since the proposed deuteron experiment will probe the isoscalar combination of neutron and proton EDMs (in addition to T-violating two-nucleon effects), we present here results for the nucleon EDFF to NLO from both $\bar{\theta}$ and qCEDM, using $SU(2)_L \times SU(2)_R$ heavy-baryon ChPT [16]. For $\bar{\theta}$, we extend the calculation of Ref. [2] and reproduce the EDM results of Ref. [8], the latter obtained from a relativistic version of large-N_c $U(3)_L \times U(3)_R$ ChPT, except for isospin-violating terms neglected in Ref. [8]. At this order, the isoscalar momentum dependence, and so the SM, is entirely due to an isospin-breaking term related to the nucleon mass splitting. As we are going to see, no new undetermined parameters appear, other NLO contributions being given by non-analytic recoil corrections proportional to m_π/m_N, where m_N is the nucleon mass, and by another isospin-breaking term, related to the pion mass splitting. We use the non-analytic contributions to the isoscalar EDFF to provide an estimate of the minimum expected size of the deuteron EDM. The EDFF from the qCEDM depends at NLO on an additional T-violating pion-nucleon coupling, although it is unlikely that the difference could be isolated experimentally.

For simplicity we focus here on QCD with two light quark flavors u and d, most relevant for low momenta $Q \sim m_\pi$, and consider as explicit degrees of freedom only nucleons, pions, and photons. In the framework of ChPT, the most general effective Lagrangian is built up using QCD symmetries as a guide, in particular the chiral $SU_L(2) \times SU(2)_R \sim SO(4)$ symmetry, which is spontaneously broken down to $SU(2)_{L+R} \sim SO(3)$. A power-counting argument must be used to order interactions according to the expected size of their contributions. In order to fulfill chiral-symmetry requirements, pions couple derivatively in the chiral limit, which brings to amplitudes powers of pion momenta. Chiral-symmetry-breaking terms involve the quark masses m_u and m_d, so they bring into amplitudes powers of the pion mass. Since nucleons are non-relativistic, we remove the large, inert nucleon mass from nucleon fields [16]. This gives one a chiral index (Δ) with which to order terms in the Lagrangian [15,17], i.e. $\mathcal{L} = \sum_\Delta \mathcal{L}^{(\Delta)}$. For strong interactions, the index is given by $\Delta = d + n/2 - 2$, where n is the number of fermion fields and d counts the number of derivatives and powers of the pion mass. Electromagnetic interactions are proportional to the small proton charge $e = \sqrt{4\pi\alpha_{em}}$, and it is convenient to account for factors of e by enlarging the definition of d accordingly. Each interaction is associated with a parameter, or low-energy constant (LEC), which can be estimated using naive dimensional analysis (NDA) [22,14]. In this case, the index Δ tracks the number of inverse powers of $M_{QCD} \sim 2\pi F_\pi \simeq$ 1.2 GeV, with $F_\pi \simeq 186$ MeV the pion decay constant, associated with an interaction. (Note that since NDA associates the LECs of chiral-invariant operators with $g_s/4\pi$, for consistency one should take the strong-interaction coupling $g_s \sim 4\pi$.)

The theory can be enlarged in a straightforward way to include the delta isobar. Note that the delta isobar does not contribute to the nucleon EDFF at the order in which we work. As is the case for the nucleon anapole form factor [23], the structure of the delta interactions that would contribute at NLO vanishes in ChPT. The first non-vanishing delta contribution occurs at a higher order than we are considering here.

The T-conserving terms that we will need consist of the following [19,24,25,2]:

$$\mathcal{L}^{(0)}_{str/em} = \frac{1}{2} D_\mu \boldsymbol{\pi} \cdot D^\mu \boldsymbol{\pi} - \frac{m_\pi^2}{2D} \boldsymbol{\pi}^2 + \bar{N} i v \cdot \mathcal{D} N - \frac{2g_A}{F_\pi} (D_\mu \boldsymbol{\pi}) \cdot \bar{N} \boldsymbol{\tau} S^\mu N \tag{1}$$

and

$$\begin{aligned}\mathcal{L}^{(1)}_{str/em} &= \frac{1}{2m_N} \left\{ -\bar{N} \mathcal{D}_\perp^2 N + \frac{2g_A}{F_\pi} (iv \cdot D\boldsymbol{\pi}) \cdot \bar{N} \boldsymbol{\tau} S \cdot \mathcal{D}_- N \right\} \\ &\quad + \frac{e}{4m_N} \epsilon_{\mu\nu\rho\sigma} \bar{N} \left\{ 1 + \kappa_0 + (1+\kappa_1) \right. \\ &\quad \left. \times \left[\tau_3 - \frac{2}{F_\pi^2 D} \left(\boldsymbol{\pi}^2 \tau_3 - \pi_3 \boldsymbol{\pi} \cdot \boldsymbol{\tau} \right) \right] \right\} v^\mu S^\nu N F^{\rho\sigma} \\ &\quad - \frac{\breve{\delta} m_\pi^2}{2D^2} \left(\boldsymbol{\pi}^2 - \pi_3^2 \right) + \frac{\delta m_N}{2} \bar{N} \left(\tau_3 - \frac{2\pi_3}{F_\pi^2 D} \boldsymbol{\pi} \cdot \boldsymbol{\tau} \right) N. \end{aligned} \tag{2}$$

Here $\boldsymbol{\pi}$ denotes the pion field in a stereographic projection of $SO(4)/SO(3)$, with $D = 1 + \boldsymbol{\pi}^2/F_\pi^2$; $N = (p\ n)^T$ is a heavy-nucleon field of velocity v^μ and spin S^μ ($S^\mu = (0, \vec{\sigma}/2)$ in the nucleon rest frame where $v^\mu = (1, \vec{0})$); and A_μ is the photon field. In addition, $(D_\mu)_{ab} = D^{-1}(\delta_{ab}\partial_\mu - e\epsilon_{3ab} A_\mu)$ is the pion covariant derivative; $\mathcal{D}_\mu = \partial_\mu + i\boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times D_\mu \boldsymbol{\pi})/F_\pi^2 - ieA_\mu(1+\tau_3)/2$ is the nucleon covariant derivative; and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength. The component of $\mathcal{D}^\mu$ perpendicular to v^μ is written

$$\mathcal{D}_\perp^\mu = \mathcal{D}^\mu - v^\mu v \cdot \mathcal{D}, \tag{3}$$

and we use the shorthand notation

$$\mathcal{D}_\pm^\mu \equiv \mathcal{D}^\mu \pm \mathcal{D}^{\dagger\mu}, \qquad \tau_i \mathcal{D}_\pm^\mu \equiv \tau_i \mathcal{D}^\mu \pm \mathcal{D}^{\dagger\mu} \tau_i, \tag{4}$$

where $\mathcal{D}^\dagger$ is defined through $\bar{N}\mathcal{D}^\dagger \equiv \overline{\mathcal{D}N}$.

The pion-nucleon coupling g_A in Eq. (1) and the anomalous magnetic photon–nucleon couplings κ_0 and κ_1 in Eq. (2) are not determined by symmetry, but are expected to be $\mathcal{O}(1)$, and indeed $g_A = 1.3$, $\kappa_0 = -0.12$, and $\kappa_1 = 3.7$. The pion mass term in Eq. (1) originates in explicit chiral-symmetry breaking by the average quark mass $\bar{m} \equiv (m_d + m_u)/2$; from NDA, $m_\pi^2 = \mathcal{O}(\bar{m} M_{QCD})$. The contribution to the nucleon mass from a similar term, the nucleon sigma term, has been removed by an appropriate definition of the heavy-nucleon field, and the surviving nucleon-pion interactions do not contribute below. The Goldberger–Treiman relation $g_{\pi NN} = 2g_A m_N/F_\pi$ holds in the two lowest orders, and a term in $\mathcal{L}^{(2)}_{str/em}$ provides an $\mathcal{O}(m_\pi^2/M_{QCD}^2)$ correction that accounts [24] for the so-called Goldberger–Treiman discrepancy.

In Eq. (2) we include explicitly the leading isospin-breaking interactions [24] stemming from the quark mass difference, $m_d - m_u \equiv 2\varepsilon\bar{m}$, and from short-range electromagnetic effects. The pion mass splitting is dominated by the electromagnetic contribution $\breve{\delta} m_\pi^2 = \mathcal{O}(\alpha_{em} M_{QCD}^2/4\pi)$; because this is, numerically, of $\mathcal{O}(\varepsilon m_\pi^3/M_{QCD})$ we book this term in $\mathcal{L}^{(1)}_{str/em}$. The quark-mass contribution to the pion splitting is suppressed by a further $\varepsilon m_\pi/M_{QCD}$, one order down in the expansion. Thus, to the accuracy in which we work here, $\breve{\delta} m_\pi^2$ is the observed pion mass splitting, $\breve{\delta} m_\pi^2 = (35.5\ \text{MeV})^2$ [26]. Note that with the way we have written the splitting, in this Letter m_π stands for the neutral pion mass, the charged pion mass squared being $m_{\pi^\pm}^2 = m_\pi^2 + \breve{\delta} m_\pi^2$.

The quark-mass contribution to the nucleon mass difference, δm_N, is expected to be $\mathcal{O}(\varepsilon m_\pi^2/M_{QCD})$ and it is evaluated to be $\delta m_N = 2.26 \pm 0.57 \pm 0.42 \pm 0.10$ MeV from lattice simulations [27], which is in agreement with an extraction from charge-symmetry breaking in the $pn \to d\pi^0$ reaction [28]. This is consistent with the NDA expectation that the corresponding electromagnetic contribution, $\breve{\delta} m_N$, is $\mathcal{O}(\alpha_{em} M_{QCD}/4\pi)$ and thus somewhat smaller; using the Cottingham sum rule [29], $\breve{\delta} m_N = -(0.76 \pm 0.30)$ MeV. In our power counting, $\breve{\delta} m_N$ appears only in $\mathcal{L}^{(2)}_{str/em}$. It gives rise to different multi-pion interactions than does the δm_N term, but these multi-pion interactions do not matter below. Thus, if desired, the dominant effect from $\breve{\delta} m_N$ can be incorporated in our results with the simple substitution $\delta m_N \to \delta m_N + \breve{\delta} m_N$.

We consider two sources of T violation at the scale M_{QCD}: the QCD $\bar\theta$ term and the qCEDM. In terms of a gluon field strength $G^a_{\mu\nu}$ and an appropriate choice [30,31] of quark fields $q = (u\ d)^T$, we can write them as

$$\mathcal{L}_{\not T}^{QCD} = m_\star \bar\theta \bar q i\gamma_5 q - \frac{i}{2}\bar q(\tilde d_0 + \tilde d_3 \tau_3)\sigma^{\mu\nu}\gamma_5 \lambda^a q G^a_{\mu\nu}, \tag{5}$$

where

$$m_\star = \frac{m_u m_d}{m_u + m_d} = \frac{\bar m}{2}\left(1-\varepsilon^2\right) = \mathcal{O}(\bar m) \tag{6}$$

and $\tilde d_0$ ($\tilde d_3$) is the isoscalar (isovector) qCEDM. The first term in Eq. (5) represents the effect of the $\bar\theta$ term under the assumption that $\bar\theta$ is small, as inferred from the bound [10] on the neutron EDM. (For the more general case, see Ref. [31].) The second term in Eq. (5) is a QCD manifestation of sources of T violation at a high scale $M_{\not T}$. At the Standard Model scale it is represented by dimension-6 operators containing the mediator of electroweak-symmetry breaking [13,14], which at lower scales picks up a vacuum expectation value that can be traded for $\bar m$. We write [9]

$$\tilde d_i = \mathcal{O}\left(\frac{4\pi \bar m}{M_{\not T}^2}\tilde\delta\right) \tag{7}$$

in terms of a dimensionless factor $\tilde\delta$. The size of $\tilde\delta$ depends on the exact mechanisms of electroweak and T breaking and on the running to the low energies where non-perturbative QCD effects take over. The minimal assumption is that it is $\mathcal{O}(g_s/4\pi)$, with g_s the strong-coupling constant, but it can be much smaller (when parameters encoding T-violation beyond the Standard Model are small) or much larger (since the first-generation Yukawa couplings are unnaturally small).

The implications of T violation to low-energy observables depend on the way Eq. (5) breaks other QCD symmetries, in particular chiral symmetry [2,31,32]. The $\bar\theta$ term is the fourth component of the same $SO(4)$ vector $P = (\bar q \boldsymbol{\tau} q, \bar q i\gamma_5 q)$ that leads to isospin breaking [1,2,31]. Therefore, it generates EFT interactions that transform as T-violating, fourth components of $SO(4)$ vectors made out of hadronic fields, with coefficients related to those of T-conserving interactions. Similarly, the qCEDM breaks chiral symmetry as a combination of fourth and third components of two other $SO(4)$ vectors [2,9,32]. As in the T-conserving case, we can use NDA to estimate the strength of the effective interactions, and continue to label terms in the effective chiral Lagrangian by the powers of M_{QCD}^{-1}. Details of the construction of the Lagrangian from these terms are discussed in Refs. [31,32].

The relevant terms here are the pion-nucleon interactions

$$\mathcal{L}_{\not T}^{(n)} = -\frac{1}{F_\pi D}\bar N(\bar g_0 \boldsymbol{\pi}\cdot\boldsymbol{\tau} + \bar g_1 \pi_3)N \tag{8}$$

and

$$\mathcal{L}_{\not T}^{(n+1)} = \frac{2}{F_\pi^2 D}(D_\mu \boldsymbol{\pi})\cdot \bar N(\bar h_0 \boldsymbol{\pi} + \bar h_1 \pi_3 \boldsymbol{\tau})S^\mu N + \frac{\bar h_2}{F_\pi D}\left(\delta_{i3} - \frac{2\pi_i \pi_3}{F_\pi^2 D}\right)\bar N(\boldsymbol{\tau}\times v\cdot D\boldsymbol{\pi})_i N, \tag{9}$$

and the short-range contributions to the nucleon EDM,

$$\mathcal{L}_{\not T}^{(n+2)} + \mathcal{L}_{\not T}^{(n+3)} = 2\bar N\left\{\left(1 - \frac{2\boldsymbol{\pi}^2}{F_\pi^2 D}\right)\left[\bar d_0 + \bar d_1'\left(\tau_3 - \frac{2}{F_\pi^2 D}(\boldsymbol{\pi}^2\tau_3 - \pi_3 \boldsymbol{\pi}\cdot\boldsymbol{\tau})\right)\right] + \bar d_1\left(\tau_3 - \frac{2\pi_3}{F_\pi^2 D}\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right)\right\} S^\mu\left(v^\nu + \frac{i\mathcal{D}_{\perp-}^\nu}{2m_N}\right)NF_{\mu\nu}. \tag{10}$$

Here $\bar g_i$, $\bar h_i$, $\bar d_i$, and $\bar d_1'$ are parameters of sizes

$$\bar g_0 = \mathcal{O}\left(\bar\theta \frac{m_\pi^2}{M_{QCD}}, \tilde\delta \frac{m_\pi^2 M_{QCD}}{M_{\not T}^2}\right), \qquad \bar g_1 = \mathcal{O}\left(\tilde\delta\frac{m_\pi^2 M_{QCD}}{M_{\not T}^2}\right), \tag{11}$$

$$\bar h_0 = \mathcal{O}\left(\bar\theta\frac{m_\pi^2}{M_{QCD}^2}, \tilde\delta\frac{m_\pi^2}{M_{\not T}^2}\right), \qquad \bar h_{1,2} = \mathcal{O}\left(\tilde\delta\frac{m_\pi^2}{M_{\not T}^2}\right), \tag{12}$$

and

$$\bar d_{0,1}, \bar d_1' = \mathcal{O}\left(e\bar\theta\frac{m_\pi^2}{M_{QCD}^3}, e\tilde\delta\frac{m_\pi^2}{M_{\not T}^2 M_{QCD}}\right). \tag{13}$$

The isoscalar ($\bar d_0$) and isovector ($\bar d_1$, $\bar d_1'$) contributions to the nucleon EDM occur for both T-violation sources. Direct short-range contributions to the momentum dependence of the EDFF first appear in $\mathcal{L}_{\not T}^{(n+4)}$, being further suppressed by $\mathcal{O}(Q/M_{QCD})$. For $\bar\theta$, $n = 1$ and among the T-violating πN interactions only the $I = 0$ interactions with coefficients $\bar g_0$ and $\bar h_0$ appear [1]. Because of the link with isospin violation [31],

$$\bar g_0 = \frac{m_\star \delta m_N}{\varepsilon \bar m}\bar\theta \simeq \frac{\delta m_N}{2\varepsilon}\bar\theta. \tag{14}$$

A similar connection exists between $\bar h_0$ and the leading isospin breaking in the pion-nucleon vertex. For qCEDM, $n = -1$ and all πN terms should be included. Since in this case there is no analogous link to T-conserving quantities, one cannot do better than the NDA estimates (11) and (12) without lattice or dynamical-model input.

The T-violating current-current nucleon-electron interaction is of the form

$$iT = -ie\bar e\left(l'\right)\gamma^\mu e(l) D_{\mu\nu}(q)\,\bar N\left(p'\right) J^\nu_{ed}(q,K) N(p), \tag{15}$$

where $e(l)$ ($N(p)$) is an electron (nucleon) spinor with momentum l (p) and $D_{\mu\nu}(q) = -i(\eta_{\mu\nu}/q^2 + \cdots)$ is the photon propagator with $q^2 = (p-p')^2 \equiv -Q^2 < 0$. The nucleon electric dipole current J^μ_{ed} can be expressed in terms of $q = p - p'$ and $K = (p+p')/2$ as an expansion in powers of Q/m_N that reads [1,31,9]

$$J^\mu_{ed}(q,K) = 2\left(F_0\left(Q^2\right) + F_1\left(Q^2\right)\tau_3\right)\left[S^\mu v\cdot q - S\cdot q v^\mu + \frac{1}{m_N}\left(S^\mu q\cdot K - S\cdot q K^\mu\right) + \ldots\right], \tag{16}$$

where $F_0(Q^2)$ ($F_1(Q^2)$) is the isoscalar (isovector) EDFF of the nucleon. We will write

$$F_i\left(Q^2\right) = d_i - S_i' Q^2 + H_i\left(Q^2\right), \tag{17}$$

where d_i is the EDM, S_i' the SM, and $H_i(Q^2)$ accounts for the remaining Q^2 dependence.

The form factor itself can be expanded in powers of Q/M_{QCD}. The leading-order (LO) contributions to the current, which are $\mathcal{O}(e\bar g_0 Q/(2\pi F_\pi)^2)$, have been calculated in Refs. [1,9]. They include the unknown short-range contributions in $\mathcal{L}_{\not T}^{(n+2)}$ (10), and loop diagrams made out of T-violating interactions in $\mathcal{L}_{\not T}^{(n)}$ (8) and T-conserving interactions in $\mathcal{L}_{str/em}^{(0)}$ (1). Here we focus on the next-to-leading order (NLO), that is, terms of relative $\mathcal{O}(Q/M_{QCD})$. They are made of diagrams with one insertion of interactions in $\mathcal{L}_{\not T}^{(n+3)}$ (10), $\mathcal{L}_{\not T}^{(n+1)}$ (9), or $\mathcal{L}_{str/em}^{(1)}$ (2). There are no new, unknown short-range parameters appearing at tree level, the recoil corrections in $\mathcal{L}_{\not T}^{(n+3)}$ (10) simply ensuring—together with those in $\mathcal{L}_{str/em}^{(1)}$ (2)—the form (16) of the current. The loop diagrams contributing to the nucleon EDFF in NLO are shown in Figs. 1 and 2,

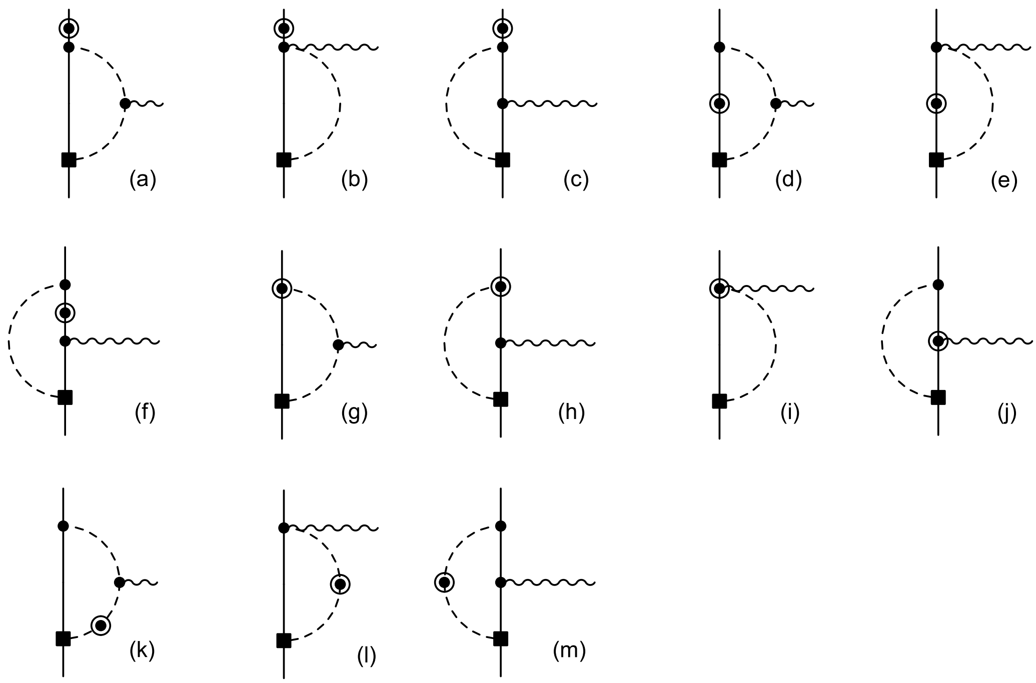

Fig. 1. One-loop diagrams contributing to the nucleon electric dipole form factor in sub-leading order coming from one insertion of an $\mathcal{L}^{(1)}_{str/em}$ operator. Solid, dashed and wavy lines represent nucleons, pions and (virtual) photons, respectively; single filled circles stand for interactions from $\mathcal{L}^{(0)}_{str/em}$ while double circles for interactions from $\mathcal{L}^{(1)}_{str/em}$; squares represent the T-violating vertices from $\mathcal{L}^{(n)}_{\not{T}}$. For simplicity only one possible ordering is shown here.

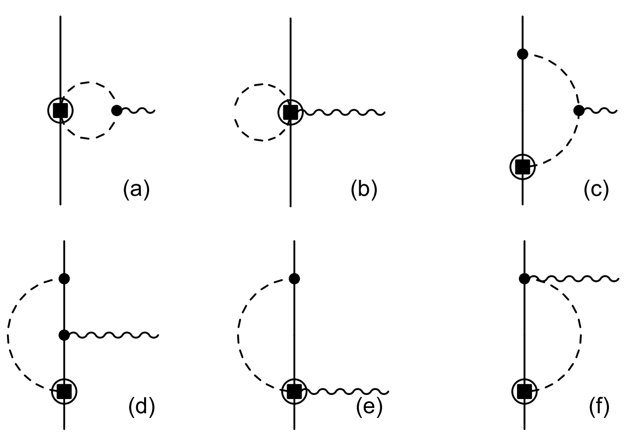

Fig. 2. Diagrams contributing to the nucleon electric dipole form factor in sub-leading order coming from one insertion of the T-violating vertex from $\mathcal{L}^{(n+1)}_{\not{T}}$, represented by a circled square. Other symbols are as in Fig. 1. For simplicity only one possible ordering is shown here.

classified according to the combination of couplings that they contain. All other contributions to the EDFF are formally of higher order: they come from more powers of momenta in diagrams with the same number of loops, or from extra loops.

The NLO diagrams of Fig. 1 are built from the leading interactions in Eqs. (1) and (8), plus one insertion of an operator from Eq. (2). Diagrams 1(a), (b), (c) represent a correction to the external energies,

$$v \cdot q = -\frac{q \cdot K}{m_N}, \tag{18}$$

$$v \cdot K = -\frac{1}{2m_N}\left(K^2 + \frac{q^2}{4}\right) \mp \frac{\delta m_N}{2}, \tag{19}$$

of a proton ($-$ sign) or neutron ($+$ sign) in LO diagrams. (In the remaining NLO diagrams, we set the right-hand side of these equations to zero.) Analogous insertions in the nucleon propagator are represented by Diagrams 1(d), (e), (f). Diagrams 1(g), (h), (i) originate in the recoil correction in pion emission/absorption, while Diagram 1(j) arises from the magnetic photon–nucleon interaction. Diagrams 1(k), (l), (m) represent an insertion of the pion mass splitting in pion propagation. These one-loop diagrams contribute to the current at order $\mathcal{O}(e\bar{g}_i Q^2/(2\pi F_\pi)^2 m_N)$.

The NLO diagrams in Fig. 2 are built from the leading interactions in Eq. (1) with one insertion of an operator from Eq. (9). Diagrams 2(a), (b) stem from the sub-leading pion-nucleon couplings $\bar{h}_{0,1}$, and Diagrams 2(c), (d), (e), (f) from the sub-leading coupling $\bar{h}_2$, present only for qCEDM. These one-loop diagrams contribute to the current at order $\mathcal{O}(e\bar{h}_i Q^2/(2\pi F_\pi)^2)$, which is precisely the same order as the diagrams in Fig. 1.

The diagrams in Figs. 1 and 2 can be evaluated in a straightforward way. We use regularization in d spacetime dimensions, and define

$$L \equiv \frac{2}{4-d} - \gamma_E + \ln 4\pi, \tag{20}$$

where $\gamma_E = 0.557\ldots$ is the Euler constant. The LO loop contributions depend on a renormalization scale μ but this dependence is compensated for by the contribution from the short-range interactions in Eq. (10). The NLO diagrams are finite in this regularization scheme.

Most of the diagrams actually vanish when the on-shell conditions (18) and (19) are consistently enforced. Diagrams (a), (b) in Fig. 2 vanish due to isospin. Since Diagrams 2(c), (d), (e), (f) vanish too, the EDFF to this order depends only on the leading T-violating parameters $\bar{g}_{0,1}$ through Fig. 1. Diagram 1(j) vanishes due to its spin structure and therefore the EDFF does not depend on the anomalous magnetic moments, either. Diagram 1(h) gives both isoscalar and isovector contributions. The remaining non-vanishing diagrams are 1(a), (d), (k). Neglecting T-conserving isospin violation, these diagrams give purely isovector results. In the case of $\bar{\theta}$, the results are proportional to $eg_A\bar{g}_0/(2\pi F_\pi)^2$, as in LO [1], times the recoil suppression factor m_π/m_N. For qCEDM, there is an additional momentum-independent contribution proportional to $\bar{g}_1$.

We have checked each of the isospin-breaking contributions in two ways. The contributions from $\breve{\delta}m_\pi^2$ come through Diagrams 1(k), (l), (m). Because the LO EDFF originates entirely in charged-pion diagrams, these contributions can be obtained alternatively by evaluating the LO EDFF with $m_\pi^2+\breve{\delta}m_\pi^2$, then expanding in powers of $\breve{\delta}m_\pi^2/m_\pi^2$:

$$F_1(Q^2)\big|_{m_\pi^2+\breve{\delta}m_\pi^2} = F_1(Q^2)\big|_{m_\pi^2} + \breve{\delta}m_\pi^2 \frac{\partial F_1(Q^2)}{\partial m_\pi^2}\bigg|_{m_\pi^2} + \cdots. \quad (21)$$

The resulting EDFF is thus isovector. Including the nucleon mass difference δm_N, Diagrams 1(a), (d) generate an additional isoscalar contribution. As a check, we have performed the field redefinition of Ref. [33], which amounts here to adding to Eq. (2)

$$\Delta\mathcal{L}^{(1)}_{str/em} = -\frac{\delta m_N}{2}\bar{N}\tau_3 N - \delta m_N(\boldsymbol{\pi}\times v\cdot D\boldsymbol{\pi})_3. \quad (22)$$

The first term eliminates δm_N from the internal nucleon lines and from the asymptotic states (and thus from Eq. (19)), but the second term generates extra contributions $\propto \delta m_N$ in Diagrams 1(k), (l), (m) and a new isospin-breaking photon-pion coupling, which appears in a diagram with the same topology as Diagram 1(d). The same final result is obtained.

The diagrams in Fig. 1 contribute to both isoscalar and isovector EDMs. Taking the NLO contributions together with the LO from Refs. [1,9], we have

$$d_0 = \bar{d}_0 + \frac{eg_A\bar{g}_0}{(2\pi F_\pi)^2}\pi\left[\frac{3m_\pi}{4m_N}\left(1+\frac{\bar{g}_1}{3\bar{g}_0}\right) - \frac{\delta m_N}{m_\pi}\right], \quad (23)$$

$$d_1 = \bar{d}_1 + \bar{d}_1' + \frac{eg_A\bar{g}_0}{(2\pi F_\pi)^2}\left[L - \ln\frac{m_\pi^2}{\mu^2} + \frac{5\pi}{4}\frac{m_\pi}{m_N}\left(1+\frac{\bar{g}_1}{5\bar{g}_0}\right) - \frac{\breve{\delta}m_\pi^2}{m_\pi^2}\right]. \quad (24)$$

The LO piece in Eq. (24), which depends on $\bar{g}_0$ and is non-analytic in m_π^2, is, with the use of the Goldberger–Treiman relation, the result of Ref. [20], which holds also for the qCEDM [9]. The short-range isovector combination $\bar{d}_1+\bar{d}_1'$ absorbs the divergence and μ dependence of the LO loop. The short- and long-range contributions to the EDM are in general of the same size, but a cancellation is unlikely due to the non-analytic dependence on m_π of the pion contribution. The isoscalar parameter $\bar{d}_0$ is not needed for renormalization at this order, but there is no apparent reason to assume its size to be much smaller than NDA either.

At NLO, the EDM receives finite non-analytic corrections, which depend also on $\bar{g}_1$ for qCEDM. From Eqs. (23) and (24) we see that, as usual in baryon ChPT, the NLO contributions are enhanced by π over NDA. However, the other dimensionless factors are not large enough to overcome the m_π/m_N suppression. Setting μ to m_N as a representative value for the size of d_1 [20], the NLO term in Eq. (24) (Eq. (23)) is about 15% (10%) of the leading non-analytic term in Eq. (24), indicating good convergence of the chiral expansion. The isovector character of the LO non-analytic terms is approximately preserved at NLO. Isospin-breaking contributions, although formally NLO, are pretty small, amounting to 15–20% of the total NLO contribution.

In the case of $\bar{\theta}$ we can use Eq. (14) and expect

$$|d_n| = |d_0 - d_1| \gtrsim \frac{eg_A}{(2\pi F_\pi)^2}\frac{\delta m_N}{2\varepsilon}\left[\ln\frac{m_N^2}{m_\pi^2} + \frac{\pi}{2}\frac{m_\pi}{m_N} - \frac{\breve{\delta}m_\pi^2}{m_\pi^2} + \pi\frac{\delta m_N}{m_\pi}\right]\bar{\theta} \simeq (1.99+0.12-0.04+0.03)\cdot 10^{-3}\bar{\theta}e\ \text{fm} \quad (25)$$

for the neutron EDM and

$$|d_p| = |d_0 + d_1| \gtrsim \frac{eg_A}{(2\pi F_\pi)^2}\frac{\delta m_N}{2\varepsilon}\left[\ln\frac{m_N^2}{m_\pi^2} + 2\pi\frac{m_\pi}{m_N} - \frac{\breve{\delta}m_\pi^2}{m_\pi^2} - \pi\frac{\delta m_N}{m_\pi}\right]\bar{\theta} \simeq (1.99+0.46-0.04-0.03)\cdot 10^{-3}\bar{\theta}e\ \text{fm} \quad (26)$$

for the proton EDM, using the lattice QCD value $\delta m_N/2\varepsilon = 2.8$ MeV [27]. Non-analytic NLO corrections are therefore somewhat larger for the proton EDM, but this difference is unlikely to be significant in light of our ignorance about the size of short-range contributions.

The non-analytic terms in Eq. (23) represent a lower-bound estimate for the size of the nucleon isoscalar EDM, as the short-range contribution $\bar{d}_0$ is nominally of lower order. The expected lower bound on the nucleon isoscalar EDM might have implications for proposed experiments on EDMs of light nuclei. In these cases, there will be additional many-nucleon contributions, but the average of the one-nucleon contributions still provides an estimate of the order of magnitude of the expected nuclear EDM. For the deuteron, the average one-nucleon contribution is exactly d_0 and, in the case of $\bar{\theta}$, we expect for the deuteron EDM

$$|d_d| \gtrsim \frac{eg_A}{(2\pi F_\pi)^2}\frac{\delta m_N}{2\varepsilon}\pi\left[\frac{3m_\pi}{4m_N} - \frac{\delta m_N}{m_\pi}\right]\bar{\theta} \simeq (1.7-0.3)\cdot 10^{-4}\bar{\theta}e\ \text{fm}. \quad (27)$$

Therefore, if there are no cancellations, a deuteron EDM signal from $\bar{\theta}$ is expected to be larger than about 10% of the neutron EDM signal.

Note that short- and long-range physics cannot be separated with a measurement of the neutron and proton EDMs alone. On the other hand, the momentum dependence of the EDFF is completely determined, to the order we are working, by long-range contributions generated by $\bar{g}_0$. It is therefore the same for $\bar{\theta}$ and qCEDM. It turns out that the isoscalar form factor receives momentum dependence only from isospin-breaking terms, while there is a non-vanishing correction to the isovector momentum dependence also from isospin-conserving terms.

The variation of the form factor with Q^2 can be characterized at very small momenta by the electromagnetic contribution to the nucleon SM, the leading and sub-leading contributions of which we find to be

$$S_0' = -\frac{eg_A\bar{g}_0}{6(2\pi F_\pi)^2 m_\pi^2}\frac{\pi}{2}\frac{\delta m_N}{m_\pi}, \quad (28)$$

$$S_1' = \frac{eg_A\bar{g}_0}{6(2\pi F_\pi)^2 m_\pi^2}\left[1 - \frac{5\pi}{4}\frac{m_\pi}{m_N} - \frac{\breve{\delta}m_\pi^2}{m_\pi^2}\right]. \quad (29)$$

The LO, isovector term is the result of Refs. [3,1]. While the EDM vanishes in the chiral limit, the isovector SM is finite. The NLO correction, which agrees with the $\bar{\theta}$ result of Ref. [8] when T-conserving isospin violation is neglected, vanishes in the chiral limit but gives a relatively large correction to the isovector SM of about 60%, due to the numerical factor $5\pi/4$. Again, the isospin-breaking corrections are relatively small, and, as a consequence, at NLO the SM remains mostly isovector.

To this order, the SM is entirely given, apart for $\bar{g}_0$, by quantities that can be determined from other processes. In the case of $\bar{\theta}$, we can again use Eq. (14) to estimate

$$S_0' = -\frac{eg_A}{12(2\pi F_\pi)^2}\frac{\pi(\delta m_N)^2}{2\varepsilon m_\pi^3}\bar{\theta} \simeq -5.0\cdot 10^{-6}\bar{\theta}e\ \text{fm}^3, \quad (30)$$

$$S_1' = \frac{eg_A}{12(2\pi F_\pi)^2}\frac{\delta m_N}{\varepsilon m_\pi^2}\left[1 - \frac{5\pi}{4}\frac{m_\pi}{m_N} - \frac{\breve{\delta} m_\pi^2}{m_\pi^2}\right]\bar{\theta} \simeq 6.8\cdot 10^{-5}\bar{\theta} e\ \mathrm{fm}^3, \tag{31}$$

where again we used the lattice-QCD value [27] for $\delta m_N/2\varepsilon$. From these results we can straightforwardly obtain the SM for the proton and the neutron. Although we could again use the isoscalar component as an estimate for a lower bound on the deuteron SM, there could be potentially significant contributions from the deuteron binding momentum.

The full momentum dependence of the EDFF is given in addition by the functions $H_i(Q^2)$ introduced in Eq. (17),

$$H_0(Q^2) = -\frac{4eg_A\bar{g}_0}{15(2\pi F_\pi)^2}\frac{3\pi}{4}\frac{\delta m_N}{m_\pi}h_0^{(1)}\left(\frac{Q^2}{4m_\pi^2}\right), \tag{32}$$

$$H_1(Q^2) = \frac{4eg_A\bar{g}_0}{15(2\pi F_\pi)^2}\left[h_1^{(0)}\left(\frac{Q^2}{4m_\pi^2}\right) - \frac{7\pi}{8}\frac{m_\pi}{m_N}h_1^{(1)}\left(\frac{Q^2}{4m_\pi^2}\right) - \frac{2\breve{\delta} m_\pi^2}{m_\pi^2}\breve{h}_1^{(1)}\left(\frac{Q^2}{4m_\pi^2}\right)\right]. \tag{33}$$

Here, the LO term,

$$h_1^{(0)}(x) = -\frac{15}{4}\left[\sqrt{1+\frac{1}{x}}\ln\left(\frac{\sqrt{1+1/x}+1}{\sqrt{1+1/x}-1}\right) - 2\left(1+\frac{x}{3}\right)\right], \tag{34}$$

is the one calculated in Refs. [1,9], while we now obtain the NLO isovector functions

$$h_1^{(1)}(x) = -\frac{1}{7}\left[3(1+2x)h_0^{(1)}(x) - 10x^2\right], \tag{35}$$

and

$$\breve{h}_1^{(1)}(x) = -\frac{1}{4(1+x)}\left(h_1^{(0)}(x) - 5x^2\right), \tag{36}$$

and the NLO isoscalar function

$$h_0^{(1)}(x) = 5\left(\frac{1}{\sqrt{x}}\arctan\sqrt{x} - 1 + \frac{x}{3}\right). \tag{37}$$

In compliance with the definition of H_i, the four functions behave as $h_i^{(n)}(x) = x^2 + \mathcal{O}(x^3)$ for $x \ll 1$.

As in lowest order, the momentum dependence is fixed by the pion cloud. Thus the scale for momentum variation is determined by $2m_\pi$. As for the SM, NLO corrections can be significant, but the isospin-breaking contributions are small. Both the SM and the functions $H_{0,1}(Q^2)$ are testable predictions of ChPT. Unfortunately, since the full momentum dependence of the EDFF will not be measured anytime soon, this observation carries no practical implications for the next generation of EDM experiments.

In summary, we have calculated the nucleon electric dipole form factor due to the $\bar{\theta}$ term and to the quark color-electric dipole moment in sub-leading order in ChPT, including isospin-breaking effects. The chiral expansion seems to be converging, although NLO corrections are enhanced by extra factors of π. Under the assumption that higher-order results are not afflicted by anomalously-large dimensionless factors, the relative error of our results at momentum Q should be $\sim (Q/M_{QCD})^2$. The NLO isospin-breaking contributions are relatively small and could be overcome by isospin-conserving contributions at NNLO. We have shown that at NLO the EDFF includes both isoscalar and isovector components, with a Q^2 dependence determined by non-derivative T-violating pion-nucleon couplings and the pion mass. The isoscalar momentum dependence is entirely due to the nucleon mass splitting. We have provided a lower-bound estimate for the isoscalar nucleon EDM, expected to set also the minimum size of the deuteron EDM. A full calculation of the latter in ChPT can now be performed.

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