Effects of prestrain on the ductile-to-brittle transition of ice

Scott A. Snyder*, Erland M. Schulson, Carl E. Renshaw

Ice Research Laboratory, Thayer School of Engineering, Dartmouth College, Hanover, NH 03755, USA

ARTICLE INFO

Article history:
Received 11 August 2015
Received in revised form 25 January 2016
Accepted 25 January 2016
Available online 23 February 2016

Keywords:
Ice
Brittle-to-ductile transition
Creep
Crack propagation
Mechanical behavior

ABSTRACT

The ductile-to-brittle transition was investigated in prestrained columnar ice at ~10 °C. Laboratory-grown specimens of freshwater and saline ice were prestrained under uniaxial across-column compression (to levels from $\varepsilon_p = 0.003$ to $\varepsilon_p = 0.20$, at constant strain rates in the ductile regime) and likewise reloaded (at rates from $1 \times 10^{-8}$ to $3 \times 10^{-2}$ s$^{-1}$). Prestrain caused solid-state recrystallization as well as damage in the form of non-propagating microcracks. The ductile-to-brittle transition strain rate $\varepsilon_{D-B}$ increased by a factor of 3–10 after prestrain of $\varepsilon_p = 0.035$ in both freshwater and saline ice, compared to that of initially undamaged ice of the same type. Additional prestrain had little further effect on $\varepsilon_{D-B}$. The results are interpreted within the framework of a model (proposed by Schulson, 1990, and Renshaw and Schulson, 2001) that predicts the transition strain rate based on the micromechanical boundary between creep and fracture processes. Model parameters primarily affected by prestrain were the power-law creep coefficient $B$ (more so than the creep exponent $n$), Young's modulus $E$ and, by extension, the fracture toughness $K_c$.

* Corresponding author.
E-mail address: scott.snyder@dartmouth.edu (S.A. Snyder).

1 We recognize that at the point of terminal failure material that is initially free from damage possesses microcracks that nucleated during loading. For convenience, we refer to such specimens as “undamaged” or “virgin” to distinguish their initial state from those in which damage has been imparted by specific levels of prestrain prior to testing.

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1. Introduction

The macroscopic behavior of ice is known to be ductile when compressed slowly, but brittle when compressed rapidly [1]. The shift between those two behaviors occurs over a range of up to one order of magnitude in strain rate, $\varepsilon$. Within this range is identified a critical rate of compression, the ductile-to-brittle transition strain rate, $\varepsilon_{D-B}$, which is a function of external conditions (e.g., temperature, pressure) and of internal measures of the ice, including grain size, salinity, as well as damage (for review see Ref. [3]). The influence of damage presents something of a paradox, in that introducing cracks into a body might seem to make the material more brittle, but, in fact, that is not necessarily the case, as will become apparent. Damaged material (containing non-propagating cracks as a result of prior deformation) can behave in a ductile manner under the same loading conditions that cause brittle failure in virgin material.

Previous work has investigated the ductile-to-brittle (D–B) transition in polycrystalline ice that was initially free from damage, for example, as a function of temperature [4], of confinement [5], or of grain size [6]. The mechanical behavior of damaged ice has been a less common subject of inquiry, although precedents include studies of damage in creep of columnar ice [7] and of granular ice [8–10], and in compression of granular ice at constant strain rates [11]. The effect of damage specifically on the D–B transition was explored in moderately prestrained columnar saline ice, in which $\varepsilon_{D-B}$ increased by up to an order of magnitude in strain rate for across-column loading [12]. The level of uniaxial compressive prestrain $\varepsilon_p$ was limited in that study to 0.035, and was imparted at one constant strain rate ($1 \times 10^{-5}$ s$^{-1}$). In the current work we extend the range of prestrain conditions to more fully investigate damage in both saline and freshwater columnar-grained ice that possesses the S2$^c$ growth texture.

The effects on the compressive ductile-to-brittle transition examined herein could be associated with the prestrain of other materials. For example, the strength of highly-confined rock within the earth’s crust can be limited by its plasticity [14], which may depend on prior strain. Compressive prestrain of metals has been
found to decrease ductility upon subsequent tensile loading (e.g., in steel at ambient temperature [15], or in stainless steel in creep [16]). To the authors’ knowledge, however, no similar results to those reported in the present work on ice yet exist for other materials. The only other work on ice in which an effect of prestrain was studied relates to tensile ductility, where compressive prestrain of the magnitude explored in the present work imparted ductile behavior, manifested in elongations of 5% to 10% or greater [17,18].

2. Background

Schulson [1] and Renshaw and Schulson [2] developed a model shown to fit well to data for both the ductile-to-brittle transition and the failure strength of (undamaged) ice and of various types of rock. The pivotal concept expressed by this model is the micro-mechanical competition between two processes: the intensification and the relaxation of internal stresses at crack tips. Crack propagation culminates in brittle fracture, whereas crack blunting via creep culminates in ductility.

The model predicts the ductile-to-brittle transition strain rate for loading under uniaxial compression as

$$\dot{\epsilon}_{\text{D/B}} = \left( \frac{E_0}{E} \right) \left( \frac{n + 1}{n} \right)^2 \left( \frac{3}{4} \right)^{\frac{3}{2}} \left( \frac{B K_0}{n} \right)^{\frac{1}{2}} \left( \frac{1 - \mu}{E} \right)^{\frac{1}{2}} \epsilon_0^n$$

where $E_0$ is the Young’s modulus of undamaged material, $E$ is the effective Young’s modulus (reduced by damage), $n$ and $B$ are the exponent and coefficient, respectively, in the power-law creep relationship ($\dot{\epsilon} = B \sigma^n$ for axial stress $\sigma$), $K_0$ is the fracture toughness, $\mu$ is the coefficient of kinetic friction, and $c$ is the characteristic radius (or half-length) of cracks within the material. All of these parameters can be experimentally determined [3]. Below the threshold $\dot{\epsilon}_{\text{D/B}}$, creep deformation is able to relax internal stresses, concentrated at the crack tips, before they exceed the yield strength of the material; above $\dot{\epsilon}_{\text{D/B}}$, additional cracking leads to brittle failure [2].

The model incorporates the process of frictional sliding that occurs between opposing surfaces of an inclined crack under the action of shear stress. The coefficient of kinetic friction $\mu$ is a function of sliding velocity $v$ and temperature $T$, and values may be obtained from the literature (more below).

For initially undamaged ice under ambient conditions, the creep exponent has been found to have a typical value of $n = 3$, from numerous studies, e.g., freshwater columnar ice [19,20], freshwater granular ice [21], and first-year sea ice [22]. We will show in our results that $n$ remains fairly constant in prestrained ice; at least the evidence is not strong enough to conclude otherwise. Refer to Appendix A for the derivation following Schulson [1] and Renshaw and Schulson [2] of Equation (1), which differs from previous expressions of the model that assumed elastic effects of damage to be negligible and set $E = E_0$.

3. Experimental methods

Freshwater ice and saline ice were both formed in the Ice Research Laboratory at Dartmouth College by unidirectional freezing of filtered ($\leq 20 \mu$m) tap water (or, in the case of saline ice, a 17.5 ± 0.2% (ppt) solution of commercially-available “Instant Ocean” salt mixture), in tanks equilibrated to −4°C. Freezing was controlled by placing a cold plate, chilled using a circulating bath set to −20°C, on the surface of the water or solution, which was seeded with ≤ 4 mm equiaxed ice grains, to produce the S2 (orthotropic columnar) grain structure. The S2 growth texture was verified by the Langway [23] method using thin sections of the as-grown ice [see 24]. Statistics on the mass density, salinity, and grain diameter are listed in Table 1. Blocks of the ice were machined into 152 mm cubes, to a tolerance of 0.076 mm, aligning one edge of the cube parallel to the long axis of the columnar grains, identified as the $x_1$ direction. The ice was machined and tested at $-10^\circ$C.

The first stage of testing involved prestraining the cube-shaped specimens under uniaxial compression at constant strain rate $\dot{\epsilon}_p$ in an across-column direction, identified as $x_2$. Loads were applied to the opposing $x_1$ faces of the specimen by polished brass brush platens fixed to servo-hydraulic controlled actuators. Levels of prestrain were specified from $\dot{\epsilon}_p = 0.003$ to 0.20 to impart permanent deformation to the ice. To avoid collapse of the specimens, the prestrain rate $\dot{\epsilon}_p$ was kept in the ductile regime, either one or two orders of magnitude below the nominal ductile-to-brittle transition strain rate $\dot{\epsilon}_{\text{D/B,0}}$ inherent to undamaged material, for each type of ice (at $-10^\circ$C, $\dot{\epsilon}_{\text{D/B,0}} \approx 1 \times 10^{-5}$ s$^{-1}$ for virgin saline ice, and $\dot{\epsilon}_{\text{D/B,0}} \approx 1 \times 10^{-4}$ s$^{-1}$ for virgin freshwater ice [5,4]). Table 2 lists which type of ice was tested at each prestrain condition.

After being prestrained, each parent specimen was quartered into subspecimens, retaining material along the center planes for thin sections. Before being measured and subsequently reloaded, the subspecimens were machined into rectangular prisms (120 mm × 60 mm × 60 mm, such as those photographed in Fig. 1) with the long dimension running across the columnar grains either parallel ($x_1$) or perpendicular ($x_2$) to the initial prestrain direction.

Porosity was measured before and after prestraining, calculated as $\phi = (\rho_0 - \rho)/\rho_0$, where $\rho$ is the specimen mass density and $\rho_0 = 917.5$ kg m$^{-3}$ is the expected density of pure ice, free of damage, bubbles, salinity, etc., at $-10^\circ$C and ambient pressure [25]. Although bubbles were not visible within the as-grown freshwater ice, its mean mass density (Table 1) was slightly below $\rho_0$, impinging a porosity of $\phi = 0.0024 \pm 0.0014$. In contrast, as-grown saline ice contained visible pores and brine pockets; its mean porosity was $\phi = 0.015 \pm 0.013$ using the same value for $\rho_0$. Dynamic elastic properties (e.g., Young’s modulus, $E$) were determined for undamaged and prestrained ice by measuring ultrasonic transmission velocities. See Snyder et al. [26], Snyder [24] for further details on the ice preparation, prestrain, and measurement procedures.

Finally, the rectangular prisms milled from the pretrained parent specimens were individually reloaded at a constant strain rate $\dot{\epsilon}_t$, ranging from $1 \times 10^{-6}$ to $3 \times 10^{-5}$ s$^{-1}$, compressing uniaxially in the long across-column dimension (either $x_1$ or $x_2$). The loaded faces of the subspecimens were small relative to the bristle ends of the brass brush platens used in prestraining the parent specimens, so solid aluminum platens were used in this step instead. To reduce boundary confinement, a thin (∼0.15 mm) sheet of polyethylene was placed between the subspecimen and each loading platen. Fig. 1 shows subspecimens of saline ice (a, after prestrain $\dot{\epsilon}_p = 0.10$ in this case) and freshwater ice (b, after prestrain $\dot{\epsilon}_p = 0.035$) situated between the platens prior to reloading. The elapsed time between prestraining and reloading was held constant at 24 ± 6 hours. The results obtained following this reloading procedure were similar (as will be shown) to those of lower $\dot{\epsilon}_p$.

<table>
<thead>
<tr>
<th>Ice type</th>
<th>Mass density (kg m$^{-3}$)</th>
<th>Salinity (ppt)</th>
<th>Column diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshwater ice</td>
<td>915.3 ± 1.5</td>
<td>–</td>
<td>5.6 ± 1.9</td>
</tr>
<tr>
<td>Saline ice</td>
<td>903.6 ± 11.0</td>
<td>–</td>
<td>4.4 ± 1.5</td>
</tr>
</tbody>
</table>

Table 1 Measured mass density (at $-10^\circ$C), salinity (of melt), and columnar grain diameter (by linear intercept) of laboratory-produced freshwater ice and saline columnar-grained ice. Values are means ± one standard deviation.
Table 2

Uniaxial compressive prestrain conditions tested for (F) freshwater ice and (S) saline ice at −10 °C.

<table>
<thead>
<tr>
<th>Prestrain rate, ( \dot{\varepsilon} ) (s(^{-1}))</th>
<th>Prestrain level, ( \varepsilon_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 10^{-6} )</td>
<td>0.003 F, 0.035 F, 0.085 F, 0.100 F, 0.15 F</td>
</tr>
<tr>
<td>( 1 \times 10^{-5} )</td>
<td>0.003 F, 0.035 F, 0.085 F, 0.100 F, 0.15 F</td>
</tr>
<tr>
<td>( 1 \times 10^{-4} )</td>
<td>0.003 S, 0.035 S, 0.085 S, 0.100 S</td>
</tr>
</tbody>
</table>

4. Results

4.1. Prestrain effect on microstructure

Two primary microstructural changes occurred as a result of the applied prestrain: damage, in the form of non-propagating cracks, and recrystallization, i.e., the nucleation of refined, equiaxed grains. Damage and recrystallization were quantified by thin-section analysis, described in detail in Ref. [26] along with bulk property measurements. Those findings are summarized here. A representative thin section is shown in Fig. 2 under scattered light (a) to illuminate cracks, and using cross-polarized light (b) to reveal the grain structure. Fig. 3 shows two thin sections for saline ice after prestrain of \( \varepsilon_p = 0.035 \) and \( \varepsilon_p = 0.10 \). Recrystallization was quantified from the thin sections by measuring recrystallized grain diameter \( d_{rx} \) (using the linear intercept method) and area fraction \( f_{rx} \). A weighted average of parent and recrystallized grain diameters \( d_{avg} \) listed in Table 3, characterizes the prestrained microstructure.

Both porosity and elastic compliance increased proportionally with prestrain, up to \( \varepsilon_p = 0.10 \). Beyond this level of prestrain the nature of the damage changed, with cracks opening by several millimeters up to 2 cm in the originally-15 cm parent cube, making it difficult to measure representative bulk properties. The magnitudes of the increases in porosity and in elastic compliance (1/\( E \)) were greater in specimens that had been prestrained at the higher of the two rates tested, closer to the transition strain rate of initially undamaged material (i.e., at \( \dot{\varepsilon} \) one order of magnitude below \( \dot{\varepsilon}_{D/B,O} \)). We also observed, in such specimens of freshwater ice, higher crack densities but fewer recrystallized grains compared to specimens prestrained at a rate two orders of magnitude below \( \dot{\varepsilon}_{D/B,O} \). These observations led us to conclude that cracking plays the dominant role (compared to that of recrystallization) in affecting porosity and Young's modulus [26].

4.2. Ductile-to-brittle transition

When prestrained specimens of both types of ice were reloaded, like the undamaged control specimens, they exhibited a range of ductile to brittle behavior depending upon strain rate. Fig. 4 compares representative stress—strain curves and photographs of saline ice subspecimens upon being reloaded at three different rates after similar prestrain conditions. Macroscopically ductile deformation

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3 One point of difference between this work and that of [12] occurred at the strain rate \( \dot{\varepsilon} = 1 \times 10^{-3} \) s\(^{-1}\), where only 1 out of 3 tests run here produced brittle behavior, as will be defined, compared to 3 out of 3 samples classified as brittle failures in the earlier work. As discussed below, such variability typifies behavior near the transition—and, indeed, it defines the transition.
occurred at the lowest strain rate (Fig. 4a); at higher strain rates, brittle failure occurred by axial splitting or by fracturing along a shear fault.\textsuperscript{4} Splitting tended to occur along planes parallel to the direction of the columnar grains, a characteristic mode of fracturing that supports the wing-crack model\textsuperscript{[1,27]}, evidence for which are the step-like features on the fracture surface (Fig. 4c).

We observed some variability in the fracturing process of the bulk material when specimens were reloaded at strain rates just below that which caused brittle failure, as marked by a sudden drop in load-bearing ability. In some of these cases (as in Fig. 4b), as plastic deformation proceeded, the interaction of fortuitously aligned cracks caused portions of the specimen to collapse. In other cases when loaded at the same rate, the specimen remained intact albeit severely cracked and deformed throughout reloading (up to 0.10 additional strain). Within the vicinity of the ductile-to-brittle transition, the visual appearance of the specimens is therefore inadequate to determine $\varepsilon_D$. A more consistent characterization of ductile versus brittle behavior can be made by examining the stress-strain curves.

Illustrating the full range of mechanical behavior under uniaxial compression of ice of both types—freshwater and saline—three-dimensional graphs of $\sigma$-$\varepsilon$ curves at each strain rate $\varepsilon_r$ are depicted in Fig. 5 (for initially undamaged ice) and Fig. 6 through 7 (for prestrained ice, at each prestrain rate $\varepsilon_p$ and level $\varepsilon_p = 0.10$). The areas under the curves are shaded with a semi-transparent color that appears darker where the curves overlap. Some of the curves are overlaid at the same $\varepsilon_r$ values, where multiple tests were run under the same prestrain and reloading conditions. The overlays demonstrate close reproducibility at the lower strain rates where ductile behavior was

\textsuperscript{4} Our use of polyethylene sheets at the ice–platen interfaces likely did not completely eliminate lateral confinement, which even at very small levels can cause shear faulting instead of axial splitting [27].
observed. Some minor variations may be attributed to differences
between individual specimens and to non-uniform damage dis-
tribution. Greater discrepancies appear among replicate tests
at higher strain rates, with notable variation in post-peak-
strain behavior (e.g., see Fig. 6aa at \( \dot{\varepsilon}_r = 3 \times 10^{-4} \text{s}^{-1} \)) at
rates near the ductile-to-brittle transition.

4.2.1. Ductile and brittle behavior quantitatively de
fined

In order to identify the transition quantitatively, we focus on the
features of the \( \sigma - \varepsilon \) curves\(^5\) that uniquely characterize the two be-
haviors. In the tests at lower strain rates which produced macro-
scopically ductile behavior, the stress–strain curve followed a
smooth trajectory over the duration of the test. The uniaxial stress \( \sigma \)
typically reached a peak, \( \sigma_{\text{max}} \), followed by a gradual descent to-
wards an essentially steady-state stress condition at a plateau,
\( \sigma_{\text{low}} \). At very low strain rates, no peak developed in the \( \sigma - \varepsilon \) curve and \( \sigma \)
approached \( \sigma_{\text{low}} \) monotonically. In contrast, both modes of brittle
failure (axial splitting and shear faulting) were marked by a sharp
peak in the stress–strain curve followed by an abrupt drop in stress,
as seen at higher strain rates. To be specific, we consider the drop in
stress \( \Delta \sigma \) that occurs within a certain increment of strain \( \Delta \varepsilon \) after
the peak stress, that is,

\[
\Delta \sigma \triangleq \sigma(\varepsilon_{\text{max}}) + \Delta \varepsilon - \sigma_{\text{max}}
\]

(2)

where \( \varepsilon(\sigma_{\text{max}}) \) is the strain corresponding to the peak stress. Based
on the above observations, we define the macroscopic mechanical
behavior quantitatively as:

Brittle:
\[
-\Delta \sigma > f_c \sigma_{\text{max}},
\]

Ductile:
\[
-\Delta \sigma \leq f_c \sigma_{\text{max}},
\]

taking \( f_c = 0.5 \) and \( \Delta \varepsilon = 0.001 \), where \( f_c \) is a critical fraction of the
peak stress. Results were not particularly sensitive to these arbi-
trarily chosen values for \( f_c \) and \( \Delta \varepsilon \), as borne out by additional
analysis [24]. According to this definition, within the regime
termed as ductile, post-peak weakening to various degrees may
occur over several percent additional shortening. Brittle behavior,
in contrast, is distinguished by sudden and catastrophic\(^6\) loss of
strength, i.e., by material collapse under uniaxial loading. In prac-
tice, the ductile-to-brittle transition is better described as a range
that spans somewhere between the strain rate where every spec-
imen is ductile, to that where only brittle failure occurs, as de
fined by the strictest criteria. Within that range, both brittle and ductile
behavior should be expected. Applying the definition above,
though, allowed us to identify the two behaviors unequivocally.

For reloading in the \( x_1 \) direction, Fig. 8 charts the character
of mechanical behavior—ductile (D) or brittle (B) as de
fined above—for each condition tested, with prestrain \( \varepsilon_p \) on the hori-
zontal axes (n.b., the scale of the horizontal axes is not linear), and
reloading strain rate \( \dot{\varepsilon}_r \) on the vertical axes. Data are shown in the
top panels (a, b) for freshwater ice and in the bottom panels (c, d)
for saline ice. Tests on undamaged specimens are shown at \( \varepsilon_p = 0\)

\(^5\) In this work, compressive stress is taken to be positive.

\(^6\) ‘Catastrophic’ is used here in the mathematical sense to describe an abrupt and
discontinuous process.
and are repeated for reference on both panels for each of the two
types of ice. Note again the inherent difference between
\[ \varepsilon_D = B \]
for freshwater ice (~1 × 10^{-4} s^{-1}) and for saline ice (~1 × 10^{-3} s^{-1}).

Prestrain already began to have an effect at the lowest level
tested (\( \varepsilon_p = 0.003 \)), raising the rate \( \dot{\varepsilon}_r \) at which brittle failure was
consistently observed by a factor of three in both materials (Fig. 8a, b, and c). This effect increased with prestrain, but not indefinitely,
and is somewhat clearer for freshwater ice than for saline ice. At
\( \varepsilon_p = 0.10 \), in most cases, the transition rate \( \dot{\varepsilon}_{D/B} \) had increased by a
factor of three to ten, with apparently little change beyond that. Most of the effect on \( \dot{\varepsilon}_{D/B} \) seems to have occurred around
\( \varepsilon_p = 0.035 \), where extensive damage (microcracks) and some
recrystallization had occurred, with greater proportion of cracks to
recrystallized grains at the higher prestrain rate, based on previous
analysis of freshwater ice [26].

In addition to the tests described above for reloading in the \( \chi_1 \)
direction, parallel to the applied prestrain, other similarly pre-
strained subspecimens were prepared such that their long

Fig. 5. Stress–strain curves by strain rate for undamaged specimens of S2 columnar-grained (a) freshwater and (b) saline ice loaded in uniaxial compression across the columnar grains at –10 °C. Arrows mark the location of the ductile-to-brittle transition.
dimensions were in the $x_2$ direction, perpendicular to prestrain loading. Reloading these specimens in the $x_2$ direction gave similar results for $\dot{\varepsilon}_{D/B}$ as those shown in the $x_1$ chart (Fig. 8) for both types of ice. Thus, in S2 ice prestrained uniaxially across the columns, the transition strain rate appears not to depend on the direction of reloading within the $x_1$-$x_2$ plane.7

Our results were consistent with the work of previous researchers who made preliminary tests on saline ice [12] after imparting some similar prestrain as in the current experiments. Those researchers distinguished ductile from brittle behavior

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7 Differences were detected, however, between the $x_1$ and $x_2$ orientations of prestrained ice with regard to elastic properties [26] and to (tensorial) crack density components, as well as to the shape of the stress–strain curve recorded during reloading in either direction [28]. Discussion of this evidence of strain-induced anisotropy falls beyond the scope of this paper.
primarily on qualitative criteria (i.e., on the shape of the stress–strain curve) but arrived at classifications that match our observations. Our results identifying $\varepsilon_{D/B}$ are fairly insensitive to the specific $\Delta \sigma/\sigma_{\text{max}}$ ratio, defined above, or to the strain increment $\Delta \varepsilon$ over which it is evaluated. $\Delta \sigma$ could be varied by as much as $\pm 10$ percent, or $\Delta \varepsilon$, by a factor of 3, without changing the results.

An alternative criterion for $\varepsilon_{D/B}$ is developed in Snyder [24], using an analysis of strain energy density (by integration under the $\sigma$–$\varepsilon$ curves) and comparing the $\varepsilon_{D/B}$ definition above, based on a post-peak stress drop, with another definition, from rock mechanics, which labels ductile behavior by the retention of load-bearing capacity through axial strains of at least 0.05 [29]. The main point of that analysis is that $\varepsilon_{D/B}$ does not change significantly regardless of which of these criteria is used.

Do the results depend on the prestrain rate $\dot{\varepsilon}_p$? If an effect of prestrain rate $\dot{\varepsilon}_p$ (as opposed to prestrain level $\varepsilon_p$) on $\varepsilon_{D/B}$ exists, it appears to be minor for both freshwater ice and saline ice. The implication, based on the fact that the cracks—to–recrystallized-

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**Fig. 7.** Stress–strain curves by strain rate $\dot{\varepsilon}$ for freshwater ice reloaded under uniaxial compression in $x_1$ after prestrain $\varepsilon_p = 0.10$ in $x_1$ imparted at (a) $\dot{\varepsilon}_p = 1 \times 10^{-5}$s$^{-1}$ or at $\dot{\varepsilon}_p = 1 \times 10^{-4}$s$^{-1}$. Arrows mark the location of the ductile-to-brittle transition.
area ratio is a function of prestrain rate \[ \varepsilon_p \], is that the D–B transition rate is not highly sensitive to the mixture of microcracks and recrystallized grains.

The relative insensitivity of \( \dot{\varepsilon}_{D/B} \) to the prestrain rate \( \dot{\varepsilon}_p \) is different from the effects of prestrain on elastic properties, which are more pronounced at the higher prestrain rate \[ \varepsilon_p \]. Freshwater ice that was prestrained, for instance, to \( \varepsilon_p = 0.035 \) at \( \dot{\varepsilon}_p = 1 \times 10^{-6} \text{s}^{-1} \) contained modest damage, and recrystallized grains covered roughly one fourth of the area measured in along-column thin sections. Young’s modulus was reduced by only about 5% in both \( x_1 \) and \( x_2 \) directions. The same level of prestrain imparted at the higher rate \( \dot{\varepsilon}_p = 1 \times 10^{-5} \text{s}^{-1} \) produced half as many recrystallized grains but numerous cracks—its Young’s modulus was reduced by ~15% in \( x_1 \) and more than 20% in \( x_2 \). However, \( \dot{\varepsilon}_{D/B} \) was increased by a factor of 3–10 in both cases (Fig. 8a and b). Recrystallization may play only a minor role with respect to elastic properties, but a more significant role in enabling ductility, as manifested by a shift in \( \dot{\varepsilon}_{D/B} \) towards higher values.

**Fig. 8.** Macroscopic mechanical behavior of freshwater ice (a,b) and saline ice (c,d) compressed uniaxially in \( x_p \), parallel to the direction of prestrain \( \varepsilon_p \) (shown on the horizontal axis) at strain rate \( \dot{\varepsilon}_p \) shown on the vertical axis. Symbols indicate ductile (D, \( \Box \)) or brittle (B, \( \bullet \)) behavior, annotated with the number of replicate tests at each condition performed by the present authors, \( n \), and by previous researchers, \( k \) [42,12]. Red curves indicate the predicted strain rates \( \dot{\varepsilon}_{D/B} \) evaluated using Equation (8) (Section 5.3). Note the scale of the horizontal axes is not linear.
5. Discussion

5.1. Possible causes for observed prestrain effects

To account for the observed effect of prestrain on the transition strain rate, we consider three possibilities: that the effect is caused entirely by recrystallization accompanied by grain refinement; that it is caused entirely by cracking and an attendant increase in creep rate (more below); or that it is caused by a combination of both recrystallization and cracking.

The recrystallization–cum–grain refinement explanation implies an effect of the rate of prestrain. At the lower rate, as already noted, recrystallization dominates [26]; correspondingly, the grain size is reduced (see Table 3). If the transition is governed by recrystallization, then the transition strain rate would be expected to increase with decreasing prestrain rate. This follows from earlier work [6] that showed that the transition rate scales as grain size $^{-3/2}$.

The cracking-cum-enhanced creep explanation also implies an effect of prestrain rate, but opposite from the one just noted. Cracks enhance creep through an effect of prestrain rate, but opposite from the one just noted. At the lower rate, as already noted, recrystallization and cracking. Yet, as noted above, it is difficult to detect any significant effect of prestrain rate. The opposing effects of prestrain rate on recrystallization and on cracking appear to lead to an unappreciable overall effect on the transition strain rate. In the relatively narrow range of prestrain rates tested, the evidence fails to single out one or the other feature as the dominant factor, but suggests that the effects of recrystallization complement the influence of damage on the transition strain rate. Often both cracks and recrystallized grains were abundantly present after high levels of prestrain (e.g., Fig. 2), and it was in such specimens that we saw the greatest increase in the ductile-to-brittle transition.

The most likely explanation is therefore the third possibility; namely, that the increase in the transition strain rate with prestrain is probably caused by both recrystallization and cracking.

That said, could changes in the other factors that appear in the model (Equation (1)) contribute to the observed behavior? Cracking can affect factors $K_{ic}$ and $E$, for example; recrystallization can affect $c$ via grain size. We turn now to draw further insights by addressing each of the factors in the model.

5.2. Comparison of the creep–versus–fracture model to experiment

To compare the observed transition strain rates $\dot{\varepsilon}_{D/B}$ (Fig. 8) with those predicted by the model (Equation (1)), the materials parameters ($c$, $E$, $K_{ic}$, $B$, $n$) must be determined. Each of these parameters has been measured independently. Effects of damage on Young’s modulus were demonstrated in recent tests on columnar ice [26], as already noted, that gave values for $E$, as well as data for mean crack lengths $2c$ and crack densities in freshwater ice, measured for various prestrain conditions (listed below in Table 4). Using the dimensionless scalar crack density

$$\rho_c = \sum_i c_i^2 / A$$

where $c_i$ are the lengths of individual crack traces on a two-dimensional thin-section image, and $A$ is the thin-section area, the results [26] showed a relationship between $E$ and $\rho_c$ in close accordance with theory—namely, the prediction of a diminishing degradation of stiffness with increasing damage, based on the non-interacting crack model [30].

Concerning friction and its dependence on velocity and temperature, values for $\mu$ have been derived from double-shear experiments [31,32], for sliding across relatively smooth ($\approx 1 \mu$m) surfaces. The appropriate velocity $v$, for cracks sliding within polycrystalline ice of grain size $d$ loaded under compression at constant strain rate $\dot{\varepsilon}$, is estimated from the relationship [1].

$$v = 2d / N.$$  

where $N$ denotes the number of cracks that slide simultaneously. Assuming for simplicity that $N = 1$, then from the literature cited, under the conditions of the present experiments, the velocity so calculated leads to the range of values $\mu = 0.4$ to 0.6 at $-10\,\text{C}$. Given that the model (Equation (1)) dictates that $\dot{\varepsilon}_{D/B} \propto (1 - \mu)$ for uniaxial compression, the variation noted in $\mu$ leads to a factor of only $\approx 1.5$ variation in transition strain rate, well below the sensitivity of the measured transition strain rate. Thus, in comparing the model and measurements, we use the value of $\mu = 0.5$. The assumption here is that the coefficient of friction for sliding across the faces of a microcrack is essentially the same as for sliding across a smooth interface.

The resistance of a material to crack propagation can be measured by its plane-strain critical stress intensity factor, or fracture toughness, $K_{ic}$. Fracture toughness in freshwater ice was studied by Ref. [33]. Two aspects of their work are relevant here. First, they found that freshwater S2 ice has greater fracture toughness in the across-column plane ($K_{ic} = 120$ kPa m$^{1/2}$ at 4s failure times) than in a plane parallel to the columnar grain axes ($K_{ic} = 87$ kPa m$^{1/2}$ independent of time to failure). Second, they showed that damage affects $K_{ic}$ by investigating prestrained specimens containing various number densities of cracks $n_c$, ranging approximately from 1 cm$^{-2}$ to 7 cm$^{-2}$. As damage increased over this range, the values for $K_{ic}$ decreased by up to 20%, although the functional relationship was not clear due to the scatter in their data. That range of damage corresponds to scalar crack density $\rho_c \leq 0.3$, as calculated by Schulson and Duval [3, p. 203], who speculated that damage-reduced stiffness may be responsible for the observed reduction in $K_{ic}$, which is proportional to the square root of Young’s modulus [33]. We can estimate the damage-reduced fracture toughness as

$$K_{ic} = K_{ic,0} \left( \frac{E}{E_0} \right)^{1/2}$$

using a value of $K_{ic,0} = 120$ kPa m$^{1/2}$ for undamaged ice. The upper limit of $\rho_c$ in those samples (0.3) was similar to the crack density we measured in freshwater ice after compressive prestrain $\rho_p = 0.100$ at $\dot{\varepsilon}_p = 10^{-5}s^{-1}$ (in experiments described below). At that level of prestrain, we also measured a 23% reduction in $E$, $E_0$. By Equation (5), then, we estimate $K_{ic} = 105$ kPa m$^{1/2}$, close to the values obtained by Ref. [33] in comparably damaged ice.

The creep parameters, $B$ and $n$, were derived using the peak stress $\sigma_{max}$ on the $\sigma$–$\varepsilon$ curves (e.g., Fig. 6) generated at different strain rates $\dot{\varepsilon}_i$ for various levels and rates of prestrain. Plotting $\sigma_{max}$ against strain rate $\dot{\varepsilon}_i$ on a log–log scale reveals near log-linear trends in the ductile regime, shown in Fig. 9. We make the conceptual approximation that the point of peak stress in a constant strain rate test can be considered to correspond to the minimum strain rate $\dot{\varepsilon}_{min}$ in a creep test, as demonstrated by Mellor and Cole [34]. The correspondence between $\sigma_{max}$ and $\dot{\varepsilon}_{min}$ has been described in terms of the “beginning of significant structural damage” and damage-induced deformation signified by the onset of failure that occurs at the minimum strain rate $\dot{\varepsilon}_{min}$ [35]. Thus we evoke the power law creep equation, $\dot{\varepsilon}_i = B\sigma_{max}^n$, and rewrite it as
\[ \log \sigma_{\text{max}} = -m \log B + m \log \dot{\varepsilon}_t, \]

where \( m = 1/n \) and \( B \) is a constant.

Fitting a least squares linear regression on the log \( \sigma_{\text{max}}-\log \dot{\varepsilon}_t \) data from specimens displaying ductile behavior provides the slope, \( m \), and intercept, \(-m \log B\). Note that \( \sigma_{\text{max}} \) did not generally occur at the same level of strain; \( \varepsilon(\sigma_{\text{max}}) \) tended to shift towards...
higher levels as $\varepsilon_t$ decreased, until the rate of deformation was low enough that no peak in stress developed at all, e.g., in freshwater ice reloaded at $\varepsilon_t < 1 \times 10^{-5}$ s$^{-1}$. The regression included the data only where a clear peak stress was observed. For $\varepsilon_t = 1 \times 10^{-4}$ s$^{-1}$, the peak stresses occurred on average at inelastic strains of 0.0035 $\pm$ 0.0010 in freshwater ice and 0.0065 $\pm$ 0.0033 in saline ice.

Different values for the parameters $B$ and $n$ might be determined if a stress other than $\sigma_{\text{max}}$ were used in Equation (6), such as the stress corresponding to a certain arbitrary level of strain. However, it seemed more meaningful to us to use the peak stress because the peak carries some natural significance relating to the strength of the material (and to the minimum strain rate).

The regressions based on Equation (6) were performed on grouped data for undamaged and prestrained specimens at each prestrain rate. Statistical uncertainty is shown by shaded areas about the fit lines on the graphs (e.g., Fig. 9). These graphs show that the trend lines for the prestrained cases are shifted noticeably toward higher strain rates, compared to the trend lines for undamaged ice. The slopes appear fairly similar, however, for all cases regardless of damage, which implies that prestrain has little effect on $n$. Whereas the graphs in Fig. 9 combine data for prestrain cases ($\varepsilon_p = 0.035$ and 0.10) to fit the trend lines on each panel, we also performed separate regressions for the two levels of $\varepsilon_p$ to derive the creep parameters listed in Table 4. Given the error in fitting the regression lines, we have only weak evidence that the $n$ value may increase marginally with prestrain. The (unit log) intercepts that determine the $B$ values, however, were clearly affected by prestrain, with a greater effect observed at the higher prestrain rate, for both materials.

It is noteworthy that uniaxial prestrain affects the $B$ parameter but not so much the $n$ value. Others have also found this to be true in creep tests of granular ice damaged by uniaxial compression [8,10]. So, along with $E$ and $K_0$, the $B$ value appears to be a key parameter in the model with regard to damage. Before discussing this parameter further, let us now look to see how well the model predicts the ductile-to-brittle transition in prestrained columnar ice.

The observed and predicted values for transition strain rate are compared in Fig. 10 for both freshwater and saline ice reloaded in the $x_1$ direction. For each material and prestrain rate, a vertical line connects the highest strain rate at which ductile behavior was consistently observed (open symbols) to the lowest strain rate at which brittle behavior was observed (filled symbols). Thus the transition is shown to occur over a range of values, as discussed above.

The increase in the ductile-to-brittle transition with prestrain appears somewhat stronger for freshwater ice than for saline ice, a point noted with Fig. 8, Fig. 10 also shows that the model slightly over-predicts the transitions for saline ice. Considering that the model is based on physics of deformation at crack tips, we tentatively attribute this discrepancy for saline ice to its relatively lower propensity to cracking, as observed in the bulk prestrained material (although as yet unable to be quantified).

Some observations can be made by comparing Tables 3 and 4 pertaining to the influence of cracking versus recrystallization. Table 3 reveals that $d_{\text{cr}}$ appears to decrease with increasing pre-

### Table 4

<table>
<thead>
<tr>
<th>Prestrain rate</th>
<th>Prestrain level</th>
<th>Porosity</th>
<th>Power-law creep coefficient</th>
<th>Creep exponent</th>
<th>Coefficient of kinetic friction</th>
<th>Young's modulus</th>
<th>Fracture toughness</th>
<th>Crack length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_p$</td>
<td>$\varepsilon_p$</td>
<td>$\phi$</td>
<td>$\log B$</td>
<td>$B$</td>
<td>$n$</td>
<td>$\mu$</td>
<td>$E$</td>
<td>$K_0$</td>
</tr>
<tr>
<td>Freshwater ice</td>
<td>0</td>
<td>$0.002 \pm 0.001$</td>
<td>$-6.32$</td>
<td>$4.8 \times 10^{-7}$</td>
<td>3.1</td>
<td>0.5</td>
<td>9.52</td>
<td>120</td>
</tr>
<tr>
<td>$1 \times 10^{-5}$</td>
<td>0.010</td>
<td>$0.009 \pm 0.002$</td>
<td>$-5.54$</td>
<td>$2.9 \times 10^{-6}$</td>
<td>3.3</td>
<td>0.5</td>
<td>9.09</td>
<td>117</td>
</tr>
<tr>
<td>Saline ice</td>
<td>0</td>
<td>$0.015 \pm 0.003$</td>
<td>$-5.26$</td>
<td>$5.5 \times 10^{-6}$</td>
<td>3.0</td>
<td>0.5</td>
<td>8.44</td>
<td>113</td>
</tr>
<tr>
<td>$1 \times 10^{-5}$</td>
<td>0.015</td>
<td>$0.024 \pm 0.009$</td>
<td>$-5.26$</td>
<td>$5.5 \times 10^{-6}$</td>
<td>3.2</td>
<td>0.5</td>
<td>8.14</td>
<td>111</td>
</tr>
</tbody>
</table>

$^8$ Separate linear regressions for both types of ice give $\Delta(\varepsilon_{\text{max}})/\Delta(\log \varepsilon_t) = -0.002$ over the range of $\varepsilon_t > 1 \times 10^{-5}$ s$^{-1}$ tested, with a p-value of 0.0002 for freshwater ice, but 0.06 for saline ice, indicating similar trends between the two materials despite greater variability inherent in saline ice.
observation.) In the meantime, the level of prestrain has a stronger impact on $B$ for freshwater ice than for saline ice, which is consistent with the fact that the model over-predicts the transition for saline ice. Future work to analyze the difference in behavior between the two materials could shed light on the underlying physics.

Experimental uncertainty also affects the parameters (Table 4) entered into the model (Equation (1)). The estimated crack half-length $c$ is a factor that contributes significantly to error in the predicted transition strain rates plotted on the horizontal axes (Fig. 10). Direct measurement of crack traces in thin sections showed $c$ to be approximately log-normally distributed with high variance owing to the attenuated grain geometries [26]. However, for the purposes of predicting the $D$-$B$ transition, it is not clear that a mean crack length is appropriate to use as the representative value. Instead, we follow the reasoning in which the characteristic crack length is set by the grain diameter [3], using $2c = d$ to calculate the predicted strain rates $r_{DB}$. As $r_{DB} \propto d^{n/2}$, for $n = 3$, a factor of 2 uncertainty in crack length translates to an error of $\pm 0.5$ on the logarithmic scale. Given this uncertainty, the model matches the experimental data for freshwater and saline ice quite closely, especially up to moderate levels of prestrain ($\epsilon_p \leq 0.035$).

Note that none of the parameters in the model directly represent the effects of recrystallization that we have observed to occur during compressive prestrain. The experimental evidence supports the use of the model where compression causes recrystallization accompanied by cracks on the scale of parent grain diameters. The application of the model to predict prestrain effects has not yet been tested where the dominant microstructural change is expected to be recrystallization with negligible cracking, such as for prestrain rates lower than two orders of magnitude below $r_{DB0}$; the transition strain rate for virgin material. It is expected that, to the extent to which grain size governs characteristic crack half-length $c$, the model could account, via that parameter, for grain reduction effects attendant to recrystallization.

### 5.3. Discussion of prestrain effects on strain rate

Why does the $B$ value in particular, as opposed to $n$, exhibit sensitivity to prestrain? Fig. 11 illustrates the effect of damage on the creep coefficient $B$ in freshwater ice. Specifically, using values listed in Table 4 obtained from the present experiments, the ratio of $\Delta B/B_0$, where $\Delta B = B - B_0$, is plotted as a function of dimensionless scalar crack density $\rho_c$ (values from Ref. [26]). With results from two prestrain levels at two prestrain rates, the sample size is small but is fit well by a power function of $\rho_c$ (solid line, Fig. 11):

$$\Delta B/B_0 = C_6 \rho_c^\beta$$

with $C_6 = (52.5 \pm 1.1)$ and $\beta = (0.67 \pm 0.05)$ as constant parameters $\pm$ standard error terms as determined by linear regression on the data transformed to log--log scale. The power function with exponent $\beta\leq1$ implies a natural saturation effect.

A similar relationship between $\Delta B/B_0$ and damage in granular ice was found by Refs. [10], using a different damage parameter, $\rho'$, based on the square of the sum of crack lengths in a representative area, rather than on the sum of squares used in $\rho_c$. Despite using a different definition of crack density, the trend in that data had the same form as Equation (7) relating $\Delta B/B_0$ to $\rho'$, but with constants $C_6 = 2.38 \times 10^{-2}$ and $\beta = 0.74 \pm 0.12$ [10]. To explain why the values of the coefficient $C_6$ differ by a factor of over 2000, while the exponents $\beta$ had similar values, we note that the material in that study was granular ice of comparatively fine grain size (1.5 mm). Thus the crack population was likely to have been significantly larger, albeit with a shorter average crack length. Even very many short cracks do not contribute much in the sum of squares in $\rho_c$, however they would count significantly in the square of the sum in $\rho'$. Further analysis [24] bears out that the similarities outweigh the differences among the results of these investigations. Apart from the scaling factor, both results of the present work and of [10] indicate an effect on $B$ that scales with damage to a power $\beta$ between 0.5 and 0.8.

The work by Ref. [10] (which, again, showed no change in creep exponent $n$) provided a number of insights: 1) enhanced creep rate due to damage was observed even at low stress levels, as in our reloading cases at low strain rates where $\sigma_{max}$ remained small ($\leq 1$ MPa). 2) Neither the reduction in elastic stiffness nor cata-elastic flow (the frictional deformation of localized fragmented material) accounts for creep-rate enhancement. 3) The increase in creep rate, specifically in the $B$ value, results from stress concentrations created by the presence of cracks, and these stress concentrations in turn cause intense, localized viscoplastic deformation [10].

The last point, regarding crack-induced concentrations in the internal stress field, introduces an explanation for the somewhat counter-intuitive damage effects we have observed. The first part of our work [26] verified in columnar ice that elastic compliance systemically increases with damage in accordance with non-interacting crack models [30]. These models—it must be emphasized—do not claim that crack interactions are insignificant, but rather that the stress-amplifying and stress-shielding effects of different crack-interactions tend to cancel each other in their effects on elastic properties. Even though their net effect on elastic properties may be negligible, crack interactions “may produce a strong impact on SIFs” (stress intensity factors) [emphasis original] [30]. The physical meaning of this plays out in the resulting deformation under increasing loads (as in constant strain-rate tests, and as in many practical scenarios). Whereas the elastic component of strain depends linearly on stress, the inelastic components are non-linear ($\epsilon_p \propto \sigma^n$). As damage and crack interaction increases, the stress intensification...
effects overwhelm elastic effects, with the result manifested in higher $B$ values and higher ductile-to-brittle transition rates.

The role of fracture toughness on the transition strain rate is also non-linear ($\dot{\varepsilon}_{D/B} \propto K_{IC}^2$) and, although we did not measure this material property directly, it appears to be reduced by damage, i.e., decreases with increasing strain, but only modestly so ($K_{IC} \propto \epsilon^{1/2}$) [33]. An increased propensity for fracture propagation in damaged material, therefore, can counteract increased viscoplasticity to some extent, which perhaps explains why the shifts in $\dot{\varepsilon}_{D/B}$ were limited to about one order of magnitude in strain rate.

5.4. Consolidation of analysis

Through derivations described in Appendix B, we can consolidate the various independently-measured parameters on which the transition strain rate depends to express it as a function of prestrain:

$$\dot{\varepsilon}_{D/B} = \left( \frac{16K_{IC}^2}{\sqrt{\pi(1-\mu)c^3/2}} \right) E_0 \left[ 1 + \left( 7.41 + 1290 \left( \frac{\partial \phi}{\partial \epsilon_p} \right)^{0.67} \right) \right] \sqrt{1 + \frac{1}{E_0} \left( \frac{\partial \phi}{\partial \epsilon_p} \frac{\partial \phi}{\partial \epsilon_p} \right)},$$

(Equation 8)

which generates the curves (in red) indicating the predicted D–B transition on the charts of observed ductile (D) and brittle (B) behavior (Fig. 8). Comparison of these predictions with the experimental data further illustrates the accuracy of the model, which is in excellent agreement with the observed transition, especially up to prestrain levels of $\epsilon_p = 0.10$, in both types of ice, albeit with the slight over-prediction for saline ice discussed earlier.

5.5. Final comments

Clearly, there are multiple factors at play, but their effects are not unbounded, as indicated by the lack of change in $\dot{\varepsilon}_{D/B}$ beyond moderate levels of prestrain. The increase in transition strain rate is limited because the microstructural changes resulting from prestrain that cause $\dot{\varepsilon}_{D/B}$ to shift cannot proliferate indefinitely. For example, the extent of recrystallization, shown to increase with prestrain [26], is bounded by the volume of the specimen. Of course, even if the fraction of recrystallized grains were to completely saturate to 1, the discontinuous process of recrystallization theoretically could still continue under continued deformation, nucleating successive generations of yet newer grains. However, let us suppose some of the enhancement in ductility observed in prestrained ice is due to reorientation by dynamic recrystallization, localized where existing grains were less favorably oriented for basal slip [36], and thus to the development of a macroscopic crystallographic texture more favorable for plastic flow. If this is the case, it stands to reason that further ductility should be increasingly difficult to realize as more of the material becomes favorably oriented relative to the applied state of stress.

Likewise, damage evolves in a non-linear way, dependent on applied stresses as well as on strain-rate: microcracks nucleate at particular sites that may be random but not arbitrary, such as grain-boundary triple points in columnar ice [37]. There is a finite number of such sites in a given volume, so it is not surprising that the nature of damage should begin to change, as we described earlier, for example, with notable opening of cracks occurring at prestrain levels $\epsilon_p \geq 0.10$. Another example of damage evolution is the eventual crushing or comminution of material that enables cataclastic flow, which other investigators have reported remains negligible at lower levels of prestrain (before the onset of tertiary creep, i.e., beyond the regime where $n \approx 3$) and therefore does not contribute significantly to the observed increase in $B$ [10]. The point of these comments is to suggest a physical hypothesis, as yet not rigorously tested, to explain why the prestrain effects on the D–B transition seem to level off around $\epsilon_p \approx 0.10$.

The experiments described herein involved specific conditions of prestrain applied under uniaxial compression. Other prestrain conditions of engineering relevance, such as biaxial compression, may change the character of imparted damage—or suppress it altogether—and possibly result in different effects.

The model has the capacity to accommodate various effects of cracking and of recrystallization. If damage occurs during prestrain, the onset of cracking precedes recrystallization [26] and the characteristic crack length $c$ is governed by the original, i.e., parent, grain size. Effects of cracks on $B$ and $E$ dominate, and the model ignores recrystallization. The D–B transition strain rate increases even though $c$ remains constant; $\dot{\varepsilon}_{D/B} \propto B/E$ (Equation 5).

On the other hand, if no damage occurs during prestrain (e.g., at lower rates of prestrain than those tested), recrystallization alone should not affect $B$ or $E$. However, $c$ might decrease because it represents cracks that have yet to occur, but which upon reloading would nucleate within the new microstructure. In the extreme case, completely recrystallized ice (free of cracks) should behave the same as virgin granular ice. The D–B transition strain rate increases as grain size (and thus $c$) decreases [6]; $\dot{\varepsilon}_{D/B} \propto c^{-3/2}$ (Equation 1).

Although these experiments were performed on columnar ice, none of the processes discussed above should differ significantly in granular versus columnar ice. Therefore, we expect granular ice to exhibit the same effects of prestrain on the D–B transition. Given the apparent universality of the model [2], now extended to prestrain conditions, the behavior seen here might be expected to occur in rocks and other minerals, as well.

6. Conclusions

The following conclusions regarding the effect of prestrain on the ductile-to-brittle transition of columnar ice at $-10^\circ C$ can be drawn from these experiments:

(i) Prestrain causes a reproducible increase by a factor of 3–10 in the D–B transition strain rate, $\dot{\varepsilon}_{D/B}$.

(ii) The effect on $\dot{\varepsilon}_{D/B}$ is similar for prestrain rates one and two orders of magnitude below the inherent transition strain rate $\dot{\varepsilon}_{D/B}$ of undamaged material.

(iii) The transition rate changes only up to a certain level of prestrain, $\dot{\varepsilon}_{D/B}$ remains fairly constant for prestrain levels beyond about 0.035.

(iv) Shifts in the ductile-to-brittle transition were observed in both freshwater ice and saline ice, indicating that prestrain effects are not isolated to one material.

(v) The creep-fracture model [12] accurately predicts the transition strain rate $\dot{\varepsilon}_{D/B}$ in prestrained ice of both types, especially up to moderate levels of prestrain ($\epsilon_p < 0.035$).

(vi) The model is an instrumental device, which gave quantitative insights into the interrelated materials parameters (the most important being the creep coefficient $B$ and Young’s modulus $E$) that are affected by prestrain.
Appendix A. Model derivation

Schulson and Duval [3] present the predicted ductile-to-brittle transition strain rate under compression as

\[ \dot{\varepsilon}_{\text{D/B}} = -\frac{(n + 1)^2 E_{\text{B}} B K_n^m}{n \sqrt{\pi} (1 - R) - \mu (1 + R) c^{n+2}} \]  

(A.1)

Defined as the ratio between the least (\(\sigma_3\)) and the greatest (\(\sigma_1\)) principal (compressive) stresses, the confinement parameter

\[ R \equiv \frac{\sigma_3}{\sigma_1} \]  

(A.2)

affects the frictional resistance to sliding. When

\[ R < (1 - \mu)/(1 + \mu) \quad \text{for} \quad \mu < 1, \]

sliding can occur along a primary crack inclined to the direction of \(\sigma_1\). Under higher confinement, sliding is suppressed [2]. By arresting frictional mechanisms in a wing-crack or comb-crack process [38,39], confinement can also suppress crack propagation. However, the role of confinement is beyond the scope of the present work, which involves only uniaxial loading scenarios (\(R = 0\)).

To be more specific, two confinement ratios can be defined. With respect to columnar ice having the \(x_3\) direction in a Cartesian coordinate system, \(R_{21} \equiv \sigma_{22}/\sigma_{11}\) is the ratio of the minor across-column normal stress to the major across-column normal stress, and \(R_{31} \equiv \sigma_{33}/\sigma_{11}\) is the ratio of the along-column normal stress to the major across-column normal stress. For uniaxial loading, \(R = R_{21} = R_{31} = 1\).

Equation (A.1) is derived by estimating the radius \(r_c\) of the creep deformation zone around a crack tip (subject to Mode I opening) using the model of Riedel and Rice [40].

\[ r_c = \frac{1}{2 \pi} \left( \frac{K_1}{E_0} \right)^2 \frac{(n + 1)^2 E_{\text{B}} B K_n^m}{2 n \sigma_0 (n + 1)} F_{cr} \]  

(A.3)

incorporating the stress intensity factor \(K_1\), the Young’s modulus of undamaged material \(E_0\), creep parameters \(B\) and \(n\), the time of loading \(t\), an angular function \(F_{cr} \approx 1\), and a (stress and strain) field amplitude factor \(\sigma_0 \approx 1\).

Creep behavior is modeled by a power-law relationship, \(\dot{\varepsilon} = \dot{\varepsilon} \text{a}^n\), where \(\dot{\varepsilon}\) denotes the (minimum) creep rate. This power law is consistent with empirical findings at relevant levels of stress, although other relationships have been proposed, such as a hyperbolic sine function [21], to describe creep across a broader range of stress levels [see also 41]. The familiar power-law above finds widespread use by virtue of balancing mathematical simplicity with empirical validity.

Schulson [1] approximates the loading time in Equation (A.3) as

\[ t \approx K_1/K_4 \]

and factors the time derivative of the stress intensity factor as

\[ K_4 = \left( \frac{\partial K_1}{\partial \varepsilon} \right) \text{a} \left( \frac{\partial \varepsilon}{\partial t} \right) \]  

(A.4)

The first partial derivative relates the change in stress intensity factor to the change in applied stress, which, in the scenario in which secondary cracks initiate from frictional sliding of primary cracks, can be written [2].

\[ \frac{\partial K_1}{\partial \sigma} = \frac{\sqrt{\pi} \varepsilon}{2} \left[ (1 - R) - \mu (1 + R) \right] \]  

(A.5)

where \(\mu\) is the coefficient of kinetic friction and \(R = \sigma_3/\sigma_1\) is the confinement ratio between the least and the greatest principal stresses, respectively. The second factor in Equation (A.4) is an effective (sustainable to damage) Young’s modulus \(E = \partial \varepsilon/\partial \sigma\), and the third factor is the applied strain rate \(\dot{\varepsilon} = \partial \varepsilon/\partial t\).

In the remaining steps of the derivations by Schulson [1] and Renshaw and Schulson [2], the effect of damage on Young’s modulus was neglected, that is, it was assumed \(E \approx E_0\). We are now in a position to remove that assumption, so we proceed by rewriting Equation (A.3) with the approximation of \(t\) without canceling \(E\):

\[ r_c = \frac{1}{2 \pi \varepsilon_{cr}} \left( \frac{K_1}{E_0} \right)^2 \frac{(n + 1)^2 E_{\text{B}} B K_n^m}{2 n \sqrt{\pi} (1 - R) - \mu (1 + R) c^{n+2}} \]  

(A.6)

Solving for strain rate,

\[ \dot{\varepsilon} = \left( \frac{1}{2 \pi r_c} \right) \frac{\varepsilon_{cr}}{\varepsilon} \left( \frac{E_0}{E} \right) \frac{(n + 1)^2 B K_n^m}{\sqrt{\pi} (1 - R) - \mu (1 + R) c^{n+2}} \]  

(A.7)

We can now resume following [1], who substitutes \(K_0\) for \(K_1\) at the ductile-to-brittle transition, i.e., the point at which secondary crack initiation occurs. This crack initiation is triggered where the stress near the tip of a sliding crack segment exceeds the yield stress of the material, in a region of stress concentration of radius \(r_c\). The estimation of \(r_c\) is explained further in Refs. [2], relating it to the primary crack size as

\[ r_c = \frac{c}{6 \pi} \]  

(A.8)

The transition to brittle behavior occurs where the size of the zone of stress concentration \(r_c\) will have just exceeded the size of the zone of creep deformation \(r_{cr}\). Thus \(r_{cr} = r_c\) marks the transition point [3]. Substituting \(2 \pi r_{cr} = c/3\) in Equation (A.7), we obtain the transition strain rate

\[ \dot{\varepsilon}_{\text{D/B}} = \frac{3}{c} \left( \frac{E_0}{E} \right) \frac{(n + 1)^2 B K_n^m}{\sqrt{\pi} (1 - R) - \mu (1 + R) c^{n+2}} \]  

(A.9)

which is simply Equation (A.1) modified by a damage factor.

\[ \dot{\varepsilon}_{\text{D/B}} = \left( \frac{E_0}{E} \right) \dot{\varepsilon}_{\text{D/B}} \]  

(A.10)

In this project we have dealt with uniaxial compression, \(R = 0\). Eliminating confinement,
\[
\dot{\varepsilon}_{D/B} = \left( \frac{E_0}{E} \right) \frac{(n + 1)^2(3)^{3/2} B K_{lc}^n}{\pi (1 - \mu) c^{3/2}}.
\]  
(A.11)

With zero confinement and using \( n = 3 \), Equation (A.11) then simplifies to
\[
\dot{\varepsilon}_{D/B} = \left( \frac{E_0}{E} \right) \frac{16 B K_{lc}^3}{\sqrt{\pi (1 - \mu) c^{3/2}}} \quad \text{for } \mu < 1.
\]  
(A.12)

Appendix B. Towards a single-variable expression of the transition strain rate

In Section 5.2, we showed that the creep-vs.-fracture model reasonably accurately predicts the D–B transition strain rates, compared against our experimental observations, in both freshwater ice and saline ice that was prestrained and reloaded in uniaxial compression in the \( x_1 \) direction (Fig. 10). In each case, the predicted transition strain rate \( \dot{\varepsilon}_{D/B} \) was calculated using Equation (1) with parameters obtained independently for the prestrain conditions tested. The transition strain rate was effectively a function of multiple unknowns:

\[
\dot{\varepsilon}_{D/B} = \dot{\varepsilon}_{D/B}(E, c, B, n, K_{lc}, \mu).
\]  
(B.1)

It could be useful instead to have a formula for \( \dot{\varepsilon}_{D/B} \) expressed more directly in terms of prestrain. The reasoning that follows aims to develop such a formula.

Among the parameters with the greatest influence in Equation (B.1) was \( c \), the crack half-length. In Section 5.2, we argued that the critical crack length \( 2c \) should not be strongly affected by prestrain, so we set it equal to the average parent grain diameter \( d \) and held it constant in the model. The equivalence between crack length and grain size also supported our argument that the coefficient of kinetic friction \( \mu \) likewise remains unaffected by prestrain. The creep exponent \( n \) was determined to remain fairly constant (\( n = 3 \)) as well, for the range of prestrain (\( \epsilon_p < 0.10 \)) that we examined (see Fig. 9).

We start by recalling Equation (1), which was derived for \( n = 3, \)
\[
\dot{\varepsilon}_{D/B} = \left( \frac{E_0}{E} \right) \frac{16 B K_{lc}^3}{\sqrt{\pi (1 - \mu) c^{3/2}}} \quad \text{for } \mu < 1.
\]

Now consider the fracture toughness, \( K_{lc} \), which is raised to the \( n \)th power in the model and so influences the transition strain rate in a non-linear way. We did not measure \( K_{lc} \) but we can assume that whatever effect prestrain has on it is accounted for by the proportionality (Equation (5)) between \( K_{lc} \) and the square root of Young’s modulus \( E \) [33]. Recall Equation (5), which we can write as

\[
K_{lc} = K_{lc,0} \left( \frac{E}{E_0} \right)^{1/2},
\]  
(B.2)

in which the terms with subscript ‘0’ are constants representing properties of initially undamaged material, either freshwater ice or saline ice. Raising the fracture toughness to the 3rd (i.e., \( n \))th power as in Equation (1), we get
\[
K_{lc}^3 = K_{lc,0}^3 \left( \frac{E}{E_0} \right)^{3/2}.
\]  
(B.3)

This, combined with Young’s modulus factor in the model (Equation (1)), gives
\[
\left( \frac{E_0}{E} \right) K_{lc}^2 = K_{lc,0}^2 \left( \frac{E}{E_0} \right)^{3/2} \left( \frac{E}{E_0} \right)^{-1} = K_{lc,0}^2 \left( \frac{E}{E_0} \right)^{1/2} = K_{lc,0}^2 \sqrt{\frac{E}{E_0}}
\]  
(B.4)

Substituting for the terms on the left hand side above, Equation (1) becomes
\[
\dot{\varepsilon}_{D/B} = \left( \frac{16 K_{lc,0}^3}{\sqrt{\pi (1 - \mu) c^{3/2}}} \right) B \left( \frac{E}{E_0} \right) \sqrt{\frac{E}{E_0}},
\]  
(B.5)

with the expression inside the parentheses comprising only constant terms (assuming the friction coefficient \( \mu \) and crack half-length \( c \) are constants).

We have reduced to 2 from 6 the number of variables in Equation (B.1), such that
\[
\dot{\varepsilon}_{D/B} = \dot{\varepsilon}_{D/B}(E, B).
\]  
(B.6)

Young’s modulus \( E \) and the power-law creep coefficient \( B \) emerge as the key parameters of influence on \( \dot{\varepsilon}_{D/B} \) that are both significantly affected by prestrain. We are now able to express both \( E \) and \( B \) in more fundamental terms based on our analysis of the experimental results.

In Refs. [26], data from ultrasonic measurements of \( E \) were fit with a linear function of porosity \( \phi \), which can be written as
\[
\frac{E}{E_0} = 1 + \frac{1}{E_0} \frac{\partial E}{\partial \phi} \phi,
\]  
(B.7)

In Refs. [26], values of \( B \) were fit by a power-law function of dimensionless scalar crack density \( \rho_c \) expressed in terms of the ratio of \( \Delta B/B_0 \) by Equation (7):
\[
\frac{\Delta B}{B_0} = C \phi^\beta.
\]  
(B.8)

The dimensionless crack density itself was shown by Ref. [26] to relate in a linear way to the change in porosity \( \Delta \phi = \phi - \phi_0 \):
\[
\rho_c = 0.020 + 3.5(\phi - \phi_0) \quad \text{for } 0 < \phi < 1.
\]  
(B.9)

Using this relationship to substitute for the crack density in Equation (B.8), with the values \( C = 52.5 \) and \( \beta = 0.67 \) determined by regression (Fig. 11), leads to
\[
B = B_0 \left( 1 + 52.5(0.020 + 3.5(\phi - \phi_0))^{0.67} \right) = B_0 \left( 1 + [7.41 + 1290(\phi - \phi_0)]^{0.67} \right).
\]  
(B.10)

Inserting Equations B.7 and B.10 into Equation B.5, we have
\[
\dot{\varepsilon}_{D/B} = \left( \frac{16 K_{lc,0}^3}{\sqrt{\pi (1 - \mu) c^{3/2}}} \right) B_0 \left( 1 + [7.41 + 1290(\phi - \phi_0)]^{0.67} \right) \left( 1 + \frac{1}{E_0} \frac{\partial E}{\partial \phi} \phi \right),
\]  
(B.11)

in which, if we assume that the critical crack length \( 2c \) remains constant (and equal to the grain diameter \( d \)) and that the friction coefficient is also constant (\( \mu = 0.5 \)), porosity \( \phi \) is the only variable dependent on prestrain. Porosity is a function not only of prestrain level \( \epsilon_p \), but also of prestrain rate \( \dot{\epsilon}_p \).
\[ \phi = \phi (\varepsilon_p, \dot{\varepsilon}_p), \]  
\text{for both freshwater ice and saline ice, when } E \text{ is measured in the } x_1 \text{ direction [26].}

To express \( \dot{\varepsilon}_{D/B} \) as a function of prestrain \( \varepsilon_p \), we replace \( \phi \) using Equation (B.13) in Equation (B.11) to get

\[ \dot{\varepsilon}_{D/B} = \left( \frac{16K^3}{\sqrt{\pi}(1-\mu)^{3/2}} \right) B_0 \left( 1 + \left[ 7.41 + 1290 \left( \frac{\partial \phi}{\partial \varepsilon_p} \varepsilon_p \right)^{0.67} \right] \right) \right) \sqrt{1 + \frac{\partial E}{\partial \phi} \frac{\partial \phi}{\partial \varepsilon_p} + \frac{\partial E}{\partial \varepsilon_p} \varepsilon_p}. \]  
\text{Fig. B.1 plots this prediction for } \dot{\varepsilon}_{D/B} \text{ using Equation (B.14) for } \partial E/\partial \phi \text{ and values for } \partial \phi/\partial \varepsilon_p \text{ from Ref. [26] for the respective prestrain rates } \dot{\varepsilon}_p \text{ indicated in the legend, for both freshwater and saline ice. The graph shows that the model does predict, in both materials, a greater change in the } D-B \text{ transition for the higher prestrain rates. However, the difference due to prestrain rate is smaller than what we were able to detect with the set of strain rates } \dot{\varepsilon}_p \text{ that were tested.}

It should be noted that the domain of prestrain in which this model is valid does not extend indefinitely. At higher levels of prestrain \( (\varepsilon_p > 0.10) \) Equation (B.15) predicts the transition strain rate to reach a maximum near \( \phi = 0.15 \) and then to begin decreasing, until Young’s modulus \( E \) reaches negative values, which is a non-physical scenario. The linear relationship given by Equation B.7 is an oversimplification for \( E \), but it is adequate for prestrain in the range of practical interest.

References


