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Quark confinement from colour confinement

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ABSTRACT

We relate quark confinement, as measured by the Polyakov-loop order parameter, to colour confinement, as described by the Kugo–Ojima/Gribov–Zwanziger scenario. We identify a simple criterion for quark confinement based on the IR behaviour of ghost and gluon propagators, and compute the order-parameter potential from the knowledge of Landau-gauge correlation functions with the aid of the functional RG. Our approach predicts the deconfinement transition in quenched QCD to be of first order for SU(3) and second order for SU(2) – in agreement with general expectations. As an estimate for the critical temperature, we obtain $T_c \simeq 284$ MeV for SU(3).

1. Introduction

Aside of the confinement of quarks, the confinement of gluons is a challenging and unsolved problem. Various scenarios predict the confinement mechanism to be manifest in the infrared domain of gauge-dependent correlation functions. In the Kugo–Ojima [1] and Gribov–Zwanziger scenarios [2] (KOGZ) an infrared enhancement of the ghost and an infrared suppression of the gluon signal confinement. These scenarios have been investigated by a variety of non-perturbative field theoretical tools such as functional methods [3–5] and lattice gauge theory [6]. The results provide strong support for these scenarios even though the infrared enhancement of the ghost is a subject of ongoing debate, for a summary see e.g. [8,9]. This paves the way for a comprehensive understanding of the non-perturbative mechanisms of strongly-coupled gauge systems.

A pressing open question is the relation of colour confinement to quark confinement. Typical quark-confinement criteria based on the Wilson-loop or Polyakov-loop expectation value [10] in quenched QCD have so far remained inaccessible from the pure knowledge of low-order correlation functions of the gauge sector, although evidence for a linearly rising potential between static quarks has been collected within certain approximation schemes, e.g. [11–13].

In this Letter, we propose a method for computing the full Polyakov-loop potential from background-field-dependent Green functions. Our approach relates the order parameter of quark confinement, the expectation value of the Polyakov loop, to the momentum dependence of gauge-dependent Green functions. This leads to a simple confinement criterion in any gauge. The method is explicitly applied in the Landau gauge, where it relates the KOGZ scenario of gluon confinement to quark confinement. We evaluate the effective potential of a purely temporal background field configuration A_0, being directly related to the Polyakov loop variable,

$$L(x) = \frac{1}{N_c}\,\mathrm{tr}\,\mathcal{P}\exp\left(\mathrm{i}g\int_0^\beta dx_0\, A_0(x_0,x)\right), \tag{1}$$

where $\mathcal{P}$ denotes time ordering, and the group trace is taken in the fundamental representation. The negative logarithm of the Polyakov-loop expectation value relates to the free energy of a static fundamental colour source. Moreover, $\langle L\rangle$ measures whether center symmetry is realised by the ensemble under consideration, see e.g. [14]. A center-symmetric confining (disordered) ground state ensures $\langle L\rangle = 0$, whereas deconfinement $\langle L\rangle \neq 0$ signals the ordered phase and center-symmetry breaking.

The order parameter $\langle L[A_0]\rangle$ is conveniently parametrised in the Polyakov gauge: $\partial_0 A_0 = 0$ with A_0 in the Cartan subalgebra. Then, $\langle A_0\rangle$ is sensitive to topological defects related to confinement [15], and also serves as a deconfinement order parameter. More specifically, $\langle L[A_0]\rangle$ is bounded from above by $L[\langle A_0\rangle]$ owing to the Jensen inequality $L[\langle A_0\rangle] = \mathrm{tr}\exp(\mathrm{i}g\beta\langle A_0\rangle)/N_c \geqslant \langle L\rangle$, such that $L[\langle A_0\rangle]$ is nonzero in the center-broken phase. In the center-symmetric phase where the order parameter $\langle L[A_0]\rangle$ vanishes, also the observable $L[\langle A_0\rangle]$ can be shown to be strictly zero [16]. This establishes both $\langle A_0\rangle$ as well as $L[\langle A_0\rangle]$ as a deconfinement order parameter.

In the present work, we compute the effective potential for $\langle A_0\rangle$ from Green functions in the background-field formalism [17] in the Landau–DeWitt gauge by means of the functional RG. These Green functions can be deduced from that in the Landau gauge,

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that is at vanishing background field. Our construction relates gluon confinement encoded in the IR behaviour of Green functions to the potential of the order parameter for quark confinement, and provides a simple confinement criterion.

2. Background-field flows

The effective potential is given by $V(L[A_0]) = \Gamma/\Omega$, where Γ is the effective action taken at the mean field A_0, and Ω is the space–time volume. We evaluate the effective action Γ in the background field approach, where Γ on the one hand depends on the field variable A, being the expectation value of the fluctuating quantum field. On the other hand, a dependence on an auxiliary background field $\bar{A}$ is introduced by gauge-fixing the fluctuating field with respect to the background,

$$D_\mu(\bar{A})(A-\bar{A})_\mu = 0. \tag{2}$$

Implementing this gauge condition at vanishing gauge parameter constitutes the Landau–DeWitt gauge. With the gauge fixing (2), the field dependence of the effective action can be summarised as, $\Gamma = \Gamma[\Phi, \bar{A}]$ with fluctuation fields $\Phi = (A - \bar{A}, C, \bar{C})$ relative to the background. The important connection to the standard effective action depending only on A is established through the identity $\Gamma[A] = \Gamma[0, \bar{A} = A]$, [17].

In the present Letter, we identify the background field with the Polyakov loop field, $\bar{A} = A_0$. For evaluating the effective potential $V(L[A_0])$, it suffices to consider A_0 as constant, yielding

$$V_k\big(L[A_0]\big) = \frac{\Gamma_k[0, A_0]}{\Omega}. \tag{3}$$

We compute the effective potential non-perturbatively by means of the functional RG (FRG) for the effective action [18], for reviews see [19,20]. The flow equation for $\Gamma[\Phi, \bar{A}]$ in the background-field approach reads

$$\partial_k \Gamma_k[\Phi, \bar{A}] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2,0)}[\Phi, \bar{A}] + R_k}\,\partial_k R_k, \tag{4}$$

where $\Gamma_k^{(n,m)} = \frac{\delta^n}{\delta\Phi^n}\frac{\delta^m}{\delta\bar{A}^m}\Gamma_k$ [20–22]. The regulator function R_k implements an IR regularisation at $p^2 \simeq k^2$, and the trace Tr sums over momenta, internal indices and species of fields. The flow (4) interpolates between the classical action in the UV and the quantum effective action $\Gamma = \Gamma_{k=0}$ in the IR. For $\Phi = 0$, Eq. (4) entails the flow of $\Gamma_k[A] = \Gamma_k[0, \bar{A} = A]$, and as a specifically interesting case, that of $V_k(L[A_0]) = \Gamma_k[A_0]/\Omega$.

Background-field flows have been applied successfully to non-perturbative analyses of chiral properties in full QCD [23], including quantitative estimates of the critical temperature of the chiral transition from first principles.

The flow (4) is solved utilising optimisation ideas [20,24] that minimise the truncation error. Here, we use a specific optimised regulator [20],

$$R_{\mathrm{opt}} = \big(k^2 - \Gamma_k^{(2)}[0, \bar{A}]\big)\theta\big(k^2 - \Gamma_{k=0}^{(2)}[0, \bar{A}]\big), \tag{5}$$

supplemented by k-dependent fields Φ such that $\Gamma_k^{(2)}[0, \bar{A}] = \Gamma_{k=0}^{(2)}[0, \bar{A}]$ for $\Gamma_{k=0}^{(2)}[0, \bar{A}] > k^2$. With the regulator (5) the flow of the standard effective action $\Gamma_k[A] = \Gamma_k[0, \bar{A} = A]$ is also gauge invariant.

The flow of $\Gamma_k[A]$ can, in principle, be obtained from Eq. (4) by setting $\Phi = 0$ and $\bar{A} = A$. But, this flow is not closed [20,22]: the right-hand side of (4) depends on $\Gamma_k^{(2,0)}[0, A] \neq \Gamma_k^{(2)}[A]$, the flow of which cannot be extracted from $\partial_t \Gamma_k[A]$. This has been neglected in previous non-perturbative applications [25] but turns out to be crucial for confinement. Hence the key input, the two-point function $\Gamma_k^{(2,0)}[0, A]$ in the background field, has to be computed separately.

3. Effective action from Landau-gauge propagators

First, we observe that in the Landau–DeWitt gauge the longitudinal components of Green functions decouple from the transversal dynamics, which further reduces the truncation error, for a detailed discussion see [8]. Moreover, $\Gamma_k^{(2,0)}[0, 0](p^2)$ corresponds to the propagator in the Landau gauge, since the background field gauge with gauge condition (2) reduces to the Landau gauge for vanishing background field. The Landau-gauge propagator has been computed within functional methods, [3,8,26], as well as within lattice gauge theory [6]; for reviews and further literature, see [8, 14,19,20,27].

Recalling the results for Landau-gauge propagators, the gluon propagator can be displayed as

$$\Gamma_A^{(2,0)}[0, 0]\big(p^2\big) = p^2 Z_A\big(p^2\big) P_{\mathrm{T}}(p)\mathbb{1} + p^2\frac{Z_{\mathrm{L}}(p^2)}{\xi} P_{\mathrm{L}}\mathbb{1}, \tag{6}$$

where $\Pi_{\mathrm{L},\mu\nu}(p) = p_\mu p_\nu/p^2$, $P_{\mathrm{T}} = 1 - P_{\mathrm{L}}$, $\mathbb{1}_{ab} = \delta_{ab}$, and ξ denotes the gauge parameter. For the ghost, we have

$$\Gamma_C^{(2,0)}[0, 0]\big(p^2\big) = p^2 Z_C\big(p^2\big)\mathbb{1}. \tag{7}$$

The longitudinal dressing function obeys $Z_{\mathrm{L}} = 1 + \mathcal{O}(\xi)$ and hence drops out of all diagrams beyond one loop in the Landau gauge $\xi = 0$. The dressing functions $Z_{A,C}$ encode the nontrivial behaviour of the full propagators. In the deep infrared, they exhibit the leading momentum behaviour

$$Z_A\big(p^2 \to 0\big) \simeq \big(p^2\big)^{\kappa_A}, \qquad Z_C\big(p^2 \to 0\big) \simeq \big(p^2\big)^{\kappa_C}. \tag{8}$$

In the last years it has become clear that Landau gauge Yang–Mills admits a one-parameter family of infrared solutions consistent with renormalisation group invariance [8]. Despite some formal progress the full understanding of the underlying structure is a subject of current research. Technically, the parameter corresponds to an infrared boundary condition, the value of $Z_C(0)$, and is also relates to $Z_A(p^2 \to 0)$ [8]. This fact is reflected in recent lattice solutions [29] and indications thereof have also been seen in the strong coupling limit [30]. For $Z_C(p^2 \to 0) \to 0$ it can be shown that there is a unique scaling solution, [31,32]. Then the two exponents are related and obey the sum rule

$$0 = \kappa_A + 2\kappa_C + \frac{4-d}{2}, \tag{9}$$

in d-dimensional space–time [4,28,31]. Possible solutions are bound to lie in the range $\kappa_C \in [1/2, 1]$, see [28]. For the truncation used in most DSE and FRG computation, we are led to

$$\kappa_C = 0.595\ldots \quad \text{and} \quad \kappa_A = -1.19\ldots, \tag{10}$$

being the value for the optimised regulator [5]. The regulator dependence in FRG computations leads to a range of $\kappa_C \in [0.539, 0.595]$, see [5]; for a specific flow, see [33]. These results entail the KOGZ confinement scenario: the gluon is infrared screened, whereas the ghost is infrared enhanced with $\kappa_C > 1/2$.

In turn it can be shown that for non-vanishing $Z_C(0)$ the gluon propagator tends to a constant in the infrared, $p^2 Z_A(p^2) \to m^2$, for related work see e.g. [8,34–40]. Note that the gluon propagator then does not correspond to the propagator of a massive physical particle. Instead, we observe clear indications for positivity violation in the numerical solutions for the gluon propagator related to gluon confinement, [8,41]. Still the gluon decouples from the

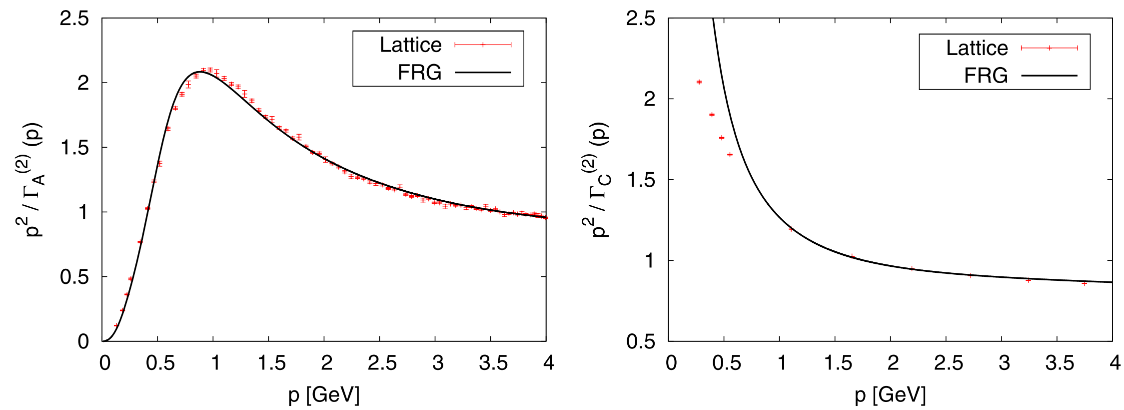

Fig. 1. Momentum dependence of the gluon (left panel) and ghost (right panel) 2-point functions at vanishing temperature. We show the FRG results from Ref. [8] (black solid line) and from lattice simulations from Ref. [6] (red points). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

dynamics as does a massive particle, hence the name decoupling solution. The value of Z_C seems to be bounded by its perturbative value from above, and the gluon mass parameter is bounded from below [8]. The qualitative infrared behaviour is then given by the infrared exponents

$$\kappa_A = -1, \quad \text{and} \quad \kappa_C = 0. \tag{11}$$

We emphasise that even though the infrared exponents for the scaling solution (10) and the decoupling solution (11) are rather different, the propagators do only differ in the deep infrared. It has been suggested in [8] that the infrared boundary condition is directly related to the global part of the gauge fixing, and hence to different resolutions of the Gribov problem. Indeed in [29] the infrared boundary condition has been implemented directly as a global completion of the gauge fixing. Note also, that for Landau gauge Yang–Mills with standard local BRST invariance the requirement of global BRST singles out the scaling solution. The existence of such a formulation on the lattice has been shown recently in [42]. In summary the results are affirmative for the above interpretation and are supported by results in the strong-coupling limit [30] for different implementations of lattice Landau gauge.

In turn, it has been also shown in a series of works that an infrared condition also is present in Landau gauge Yang–Mills with the horizon function, e.g. [35–38]. The latter introduces an explicit (or soft) breaking of BRST invariance as it restricts the functional integral to the first Gribov region. Still this does not fix global gauge degrees of freedom as also the first Gribov region contains infinite many gauge copies. The possibility of a scaling solution in this framework hints at the validity of Zwanziger proposal: full BRST invariance is recovered in the thermodynamic limit if the path integral is restricted to the fundamental modular domain with only one gauge copy.

In summary a consistent picture has emerged with nicely relates all current results. The confirmation of this picture certainly would provide further insight to the confinement mechanism. For the present work, we simply note that the scaling solution is singled out by global BRST invariance which allows the construction of a physical Hilbert space from gauge fixed correlation functions. Nonetheless, the whole one-parameter family provides consistent gauge-fixed correlation functions of Yang–Mills theory and physical observables should be insensitive to the parameter choice. In the present work, we can test this statement.

We proceed by extending the Landau-gauge propagator to that in a given background $\bar{A}$. The Landau-gauge two-point function $\Gamma_k^{(2,0)}[0,0](p^2)$ is, apart from its Lorentz structure provided by the projection operators $P_{\mathrm{T/L}}(p)$, a function of only the momentum squared p^2, cf. Eq. (6). At vanishing temperature, the background field propagator $\Gamma_k^{(2,0)}[0,A]$ can be related to the Landau-gauge propagator in a unique fashion owing to gauge covariance,

$$\begin{aligned}&\big(\Gamma_k^{(2,0)}[0,A]\big)^{ab}_{\mu\nu}\\ &\quad = \big(\Gamma_k^{(2,0)}[0,0]\big(-D^2\big)\big)^{ab}_{\mu\nu} + F^{cd}_{\rho\sigma} f^{abcd}_{\mu\nu\rho\sigma}(D),\end{aligned} \tag{12}$$

with non-singular $f(0)$ in order to ensure the proper limit of a vanishing background. The projection operators $P_{\mathrm{T/L}}$ implicitly contained in $\Gamma_k^{(2,0)}[0,0](-D^2)$ generalise to projectors on transversal and longitudinal spaces respectively with respect to the covariant momentum D, $P_{\mathrm{T/L}} = P_{\mathrm{T/L}}(D)$. The f terms cannot be obtained from the Landau-gauge propagator, but are related to higher Green functions in Landau–DeWitt gauge. However, fortunately they do not play a rôle for our purpose.

At finite temperature, the Polyakov loop L is a further invariant, and the 00 component of the gluon two-point function (12) receives further contributions proportional to derivatives of L. For constant fields A_0, we arrive at

$$\big(\Gamma_k^{(2,0)}[0,A_0]\big)^{ab}_{\mu\nu} = \big(\Gamma_k^{(2,0)}[0,0]\big(-D^2\big)\big)^{ab}_{\mu\nu} + L\text{-terms}, \tag{13}$$

as the f term in (12) vanishes: $F(A_0) = 0$. In this Letter, we take only the explicit T dependence due to Matsubara frequencies into account and drop any implicit T dependence: first, this amounts to dropping the L contribution in (13). This term is related to the second derivative of the effective potential $V_k^{(2)}$ via Nielsen identities [20,22], and can indeed be estimated by $V_k^{(2)}$. Its influence on the confinement-deconfinement phase transition temperature is parametrically suppressed, and can be neglected for a first estimate of the critical temperature T_c. Second, this amounts to using the zero-temperature propagators. First results indeed indicate that transversal and longitudinal gluon and ghost propagators are little modified [43–45] for higher Matsubara frequencies $2\pi T n$ for $n > 2, 3$. The biggest change appears in the gluon propagator longitudinal with respect to the heat bath that develops some enhancement compared to the transversal counterpart. The inclusion of the full temperature dependence is necessary for an accurate determination of, e.g., the critical exponents or the equation of state (see, e.g., [46]). This will be subject of a forthcoming paper.

4. A simple order–disorder confinement criterion

The preceding analysis gives rise to a simple confinement criterion which relates the IR behaviour of gluon and ghost 2-point

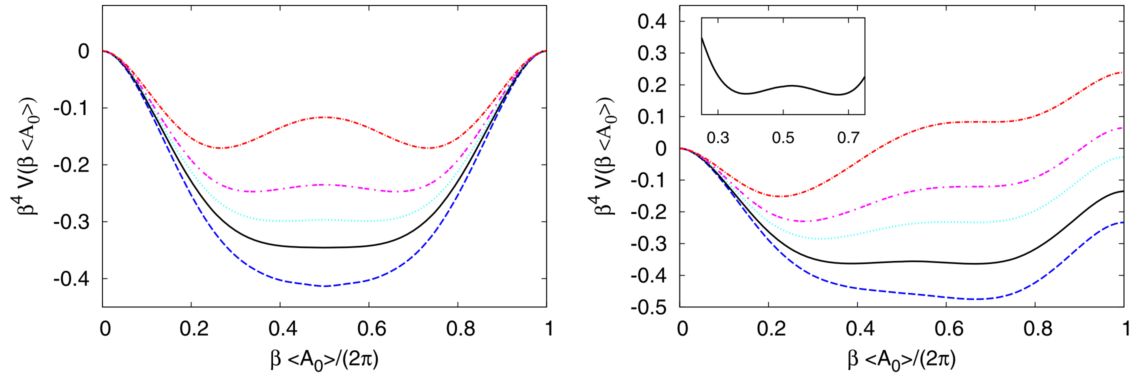

Fig. 2. Order-parameter potential for SU(2) (left panel) and SU(3) (right panel) for various temperatures. For SU(2) we show the potential for $T = 260, 266, 270, 275, 285$ MeV (from bottom to top). We find $T_c \approx 266$ MeV for SU(2). In case of SU(3), the relevant minima occur in the A_0^8 direction in the Cartan subalgebra. A slice of the potential in this direction is shown for $T = 285, 289.5, 295, 300, 310$ MeV (from bottom to top). A magnified view on the potential at the phase transition is shown in the inlay, revealing the 1st-order nature of the phase transition with two equivalent minima at $T_c \approx 289.5$ MeV.

functions to the deconfinement order parameter. Integrating the flow (4), we obtain

$$\Gamma[A] = \frac{1}{2}\,\mathrm{Tr}\ln \Gamma^{(2,0)}[0, A] + \mathcal{O}\big(\partial_t \Gamma_k^{(2,0)}\big) + \text{c.t.}, \tag{14}$$

where the counterterms (c.t.) denote the appropriate UV initial conditions of the flow, and the $O(\partial_t \Gamma_k^{(2,0)})$ terms correspond to integrated RG improvement terms. The first term is explicitly regulator-independent, and so is the improvement term. This can be used to show within the specific choice (5) that the improvement term is subdominant for the following analytic argument, which is confirmed by the full numerical solution.

The effective action in (14) involves the Laplacian $-D^2$ for vanishing field strength. In the constant A_0 background, we use the parametrisation $gA_0^a = 2\pi T\phi^a$, where ϕ^a is a vector in the Cartan subalgebra. The spectrum of the Laplacian then reads

$$\mathrm{spec}\big\{-D^2[A_0]\big\} = \vec{p}^{\,2} + (2\pi T)^2\big(n - \nu_\ell|\phi|\big)^2, \tag{15}$$

where the ν_ℓ denote the $N_c^2 - 1$ eigenvalues of the hermitian colour matrix $T^a\phi^a/|\phi|$, $(T^a)^{bc} = -\mathrm{i} f^{abc}$ being the generators of the adjoint representation. From Eq. (15), it is clear that ϕ is a compact variable.

At high temperature, $2\pi T \gg \Lambda_{\mathrm{QCD}}$, the effective potential is dominated by the perturbative regime, and the background-covariant inverse propagators of both gluons and ghosts are approximately given by their tree-level values $\Gamma^{(2),\mathrm{tree}}(-D^2) = -D^2$. The perturbative limit of the effective potential V in $d > 2$ is given by the well-known Weiss potential [47],

$$\begin{aligned} V^{\mathrm{UV}}(\phi^a) &= \left\{\frac{d-1}{2} + \frac{1}{2} - 1\right\}\frac{1}{\Omega}\,\mathrm{Tr}\ln\big(-D^2[A_0]\big) \\ &= -\frac{(d-2)\Gamma(d/2)}{\pi^{d/2}}T^d \sum_{l=1}^{N_c^2-1}\sum_{n=1}^{\infty}\frac{\cos 2\pi n\nu_\ell|\phi|}{n^d}, \end{aligned} \tag{16}$$

where the terms in curly brackets in the first row denote the contributions from transversal gluons, longitudinal gluons and ghosts, respectively. In the second row, we have dropped a T- and field-independent constant. The Weiss potential exhibits maxima at the center-symmetric points where $L[\langle A_0\rangle] = 0$, implying that the perturbative ground state is not confining, $\langle L\rangle \neq 0$.

Now, we perform the same analysis at low temperature $2\pi T \ll \Lambda_{\mathrm{QCD}}$. The series in (16) converges rather rapidly due to the $1/n^d$ suppression of higher terms. Hence, the effective potential $V(\phi^a)$ is dominantly induced by fluctuations with momenta near the temperature scale $p^2 \sim (2\pi T)^2$. This does not change qualitatively in the presence of a non-trivial momentum dependence of the propagators. We conclude that only the first 10–20 Matsubara frequencies play a rôle. Moreover, changing the propagator for the first two or three Matsubara frequencies, even though their weight is higher, only gives rise to minimal changes in the potential. This fully justifies the zero-temperature estimate on the propagators.

With the parametrisation (6), (7), the dressing functions $Z_A(p^2)$, $Z_C(p^2)$ in the KOGZ scenario are characterised by the power-law behaviour (8) in the deep IR, $p^2 \ll \Lambda_{\mathrm{QCD}}^2$. For low enough temperature, the spectral window $-D^2[A_0] \simeq (2\pi T)^2$ is in this asymptotic regime, and thus the effective potential arises dominantly from fluctuations in the deep IR,

$$\begin{aligned} V^{\mathrm{IR}}(\phi^a) &= \left\{\frac{d-1}{2}(1+\kappa_A) + \frac{1}{2} - (1+\kappa_C)\right\} \\ &\quad\times \frac{1}{\Omega}\,\mathrm{Tr}\ln\big(-D^2[A_0]\big) \\ &= \left\{1 + \frac{(d-1)\kappa_A - 2\kappa_C}{d-2}\right\}V^{\mathrm{UV}}(\phi^a). \end{aligned} \tag{17}$$

If the anomalous dimensions are such that the expression in curly brackets becomes negative, the effective potential is reversed and the confining center-symmetric points become order-parameter minima.

We conclude that the effective action (17) predicts a center-symmetric quark-confining ground state if

$$f(\kappa_A, \kappa_C; d) = d - 2 + (d-1)\kappa_A - 2\kappa_c < 0. \tag{18}$$

Provided that the $\mathcal{O}(\partial_t \Gamma_k(2,0))$ terms in Eq. (14) remain subdominant, this equation provides a simple, necessary and sufficient criterion for quark confinement in Yang–Mills theory: if Eq. (18) is satisfied the order parameter for quark confinement vanishes, $\langle L[A_0]\rangle = 0$. It is satisfied for the whole one-parameter family of infrared solutions of Landau-gauge Yang–Mills theory. For the scaling solution with the sum rule (9), we are led to

$$\kappa \equiv \kappa_C > \frac{d-3}{4} \tag{19}$$

which is satisfied for the numerical values for the scaling exponents κ_d in $d = 2, 3, 4$, see [4,28]. Specifically in $d = 4$, we have Eq. (10), and hence

$$f(-2\kappa_C, \kappa_C; 4) = -2.76\ldots. \tag{20}$$

For the decoupling solution (11), we are led to

$$f(-1, 0; d) = -1. \tag{21}$$

Both values imply confinement, and hence the whole one parameter family of solutions is confining. Note that this is to be expected as corresponding propagators can be obtained within lattice simulations with different gauge fixings.

The above confinement criterion has to be compared to the Kugo–Ojima criterion for colour confinement $\kappa > 0$ and the Zwanziger horizon condition for the ghost $\kappa > 0$ and for the gluon $\kappa > 1/2$ in $d = 4$. The Kugo–Ojima criterion and the Zwanziger horizon condition are necessary but not sufficient for confinement. Indeed for $0 < \kappa < 1/4$ in four dimensions, we observe that the Kugo–Ojima criterion is satisfied but does not lead to confinement according to the present confinement criterion (19). We would also like to emphasise that, in effective theories for QCD, Eq. (18) only serves as a necessary condition. It only restricts the propagators, and other Green functions in effective theories might violate related constraints.

5. Results for the phase transition

In contradistinction to the simple confinement criterion put forward above, the physics of the confinement–deconfinement phase transition, e.g., the transition temperature and the order of the phase transition, is determined by the dynamics of the system and not by its IR asymptotics. Indeed, we find that fluctuations in the non-perturbative mid-momentum regime induce the center-symmetric minimum of the A_0 potential long before the propagators acquire their deep IR scaling form (8). As only the deep infrared is sensitive to the infrared boundary condition the critical temperature is insensitive to this choice which is confirmed in the explicit computation.

The results presented below are achieved by numerically integrating the flow equation (4) in order to obtain the potential for an A_0 background. The present truncation is optimised by using Landau-gauge propagators and RG improvement terms at zero temperature computed from the FRG for different infrared boundary conditions. It is also compared to results obtained by using fits to Landau-gauge propagators as measured by lattice gauge theory [7] and the RG improvement computed in [8]. For our numerical study of the order-parameter potential we have suitably amended the lattice propagators by the perturbative behaviour in the UV and the corresponding power laws (8) in the IR. In Fig. 1 we show the gluon and ghost propagators as obtained from FRG computations [8] and lattice simulations [7]. There is an impressive agreement of the results for the ghost and gluon propagators for momenta larger than about $p \gtrsim 700$ MeV which holds for the whole one parameter family of solutions including the scaling one. The results for the ghost dressing from the scaling solution of the FRG and lattice simulations start deviating for $p \lesssim 700$ MeV whereas the scaling solution for the gluon starts deviating for even lower momenta. Since the lowest non-vanishing Matsubara mode is associated with momenta at about $|p| \sim 2\pi T_c \sim 1700$ MeV, the differences in the IR are hardly probed in the present study of the deconfinement phase transition. This is confirmed by the explicit computation. In the vacuum limit, $T \to 0$, the picture arising from the preceding simple confinement criterion is confirmed: a sufficient amount of gluon screening with or without an IR enhancement of the ghost creates a center-disordered ground state with quark confinement.

The confinement–deconfinement transition is taking place in the mid-momentum regime that interpolates between the perturbative regime and the IR asymptotics. The effective potentials for SU(2) and SU(3) for various temperature values near the phase transition are displayed in Fig. 2. For SU(3) (right panel), the slice of the potential in A_0^8 direction is depicted where the relevant minima for the phase transition occur. Reading off $\langle A_0 \rangle$ from the minimum of the potential at a given temperature, we can determine $L[\langle A_0 \rangle]$ which is plotted in Fig. 3. For SU(2) (blue/dashed line), the phase transition is of second order. For SU(3) (black/solid line), we clearly observe a first-order phase transition at a critical temperature of $T_c \simeq 284 \pm 10$ MeV with a lattice string tension $\sqrt{\sigma} = 440$ MeV, that is $T_c/\sqrt{\sigma} = 0.646 \pm 0.023$. The error relates to the uncertainties of the fits for the lattice propagators which exceed the estimate on the systematic error in the FRG computation. The result compares favourably both qualitatively and quantitatively with lattice simulations, see e.g. [7,48]. Also, our result for $L[\langle A_0 \rangle]$ in the deconfined phase is higher than the lattice measurement of the Polyakov-loop expectation value $\langle L \rangle$ in agreement with the Jensen inequality $L[\langle A_0 \rangle] > \langle L \rangle$. Note however that this statement has to be taken with care as the lattice result involves a non-trivial renormalisation factor which is absent in the definition of $L[\langle A_0 \rangle]$. Indeed, $L[\langle A_0 \rangle] \leqslant 1$ whereas the renormalised Polyakov loop $\langle L \rangle_{\text{ren}}$ necessarily exceeds unity for some temperature range as can be deduced from perturbation theory.

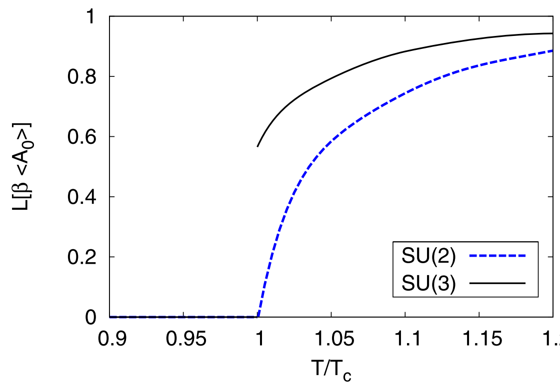

Fig. 3. Polyakov loop for the A_0 expectation value $L[\beta\langle A_0 \rangle]$ for SU(2) (blue/dashed line) and SU(3) (black/solid line). The phase transition is of second order for SU(2) and of first order for SU(3). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

As discussed above, corrections to our estimate arise from finite-T modifications of the propagators as well as from order-parameter fluctuations; the latter are more pronounced for SU(2) owing to the second-order nature of the transition. As expected, the critical temperature is not sensitive to the one-parameter family of solutions, it is only sensitive to the mid-momentum regime at about 1 GeV. Indeed, this also explains the fact that the gluon mass parameter is restricted from below: small gluon mass parameters would also trigger changes in the mid-momentum regime and almost certainly change physical quantities such as the critical temperature.

In summary, we have established a simple confinement criterion that relates quark confinement to the infrared behaviour of ghost and gluon Green functions. This confinement criterion is applicable in arbitrary gauges. Our full numerical analysis of the IR dynamics predicts a second-order phase transition for SU(2) and a first-order phase transition for SU(3), the critical temperature of which is in quantitative agreement with lattice results. The related Polyakov loop potential also plays an important rôle for full QCD computations with dynamical quarks within functional methods, for first results on the QCD phase diagram see [49].

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References

[1] T. Kugo, I. Ojima, Prog. Theor. Phys. Suppl. 66 (1979) 1;
T. Kugo, I. Ojima, Prog. Theor. Phys. 71 (1984) 1121, Erratum;
T. Kugo, arXiv:hep-th/9511033.

[2] V.N. Gribov, Nucl. Phys. B 139 (1978) 1;
D. Zwanziger, Nucl. Phys. B 412 (1994) 657.

[3] L. von Smekal, R. Alkofer, A. Hauck, Phys. Rev. Lett. 79 (1997) 3591, arXiv:hep-ph/9705242.

[4] D. Zwanziger, Phys. Rev. D 65 (2002) 094039.

[5] J.M. Pawlowski, D.F. Litim, S. Nedelko, L. von Smekal, Phys. Rev. Lett. 93 (2004) 152002, arXiv:hep-th/0312324.

[6] F.D.R. Bonnet, P.O. Bowman, D.B. Leinweber, A.G. Williams, Phys. Rev. D 62 (2000) 051501, arXiv:hep-lat/0002020;
J. Gattnar, K. Langfeld, H. Reinhardt, Phys. Rev. Lett. 93 (2004) 061601, arXiv:hep-lat/0403011;
A. Cucchieri, T. Mendes, O. Oliveira, P.J. Silva, Phys. Rev. D 76 (2007) 114507, arXiv:0705.3367 [hep-lat];
A. Cucchieri, T. Mendes, PoS LATTICE (2007) 297, arXiv:0710.0412 [hep-lat];
A. Sternbeck, L. von Smekal, D.B. Leinweber, A.G. Williams, PoS LATTICE (2007) 340, arXiv:0710.1982 [hep-lat];
I.L. Bogolubsky, E.M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, PoS LATTICE (2007) 290, arXiv:0710.1968 [hep-lat].

[7] A. Sternbeck, E.M. Ilgenfritz, M. Muller-Preussker, A. Schiller, I.L. Bogolubsky, PoS LAT2006 (2006) 076, arXiv:hep-lat/0610053.

[8] C.S. Fischer, A. Maas, J.M. Pawlowski, Ann. Phys. 324 (2009) 2408, arXiv:0810.1987 [hep-ph].

[9] L. von Smekal, arXiv:0812.0654 [hep-th].

[10] A.M. Polyakov, Phys. Lett. B 72 (1978) 477;
L. Susskind, Phys. Rev. D 20 (1979) 2610.

[11] D. Zwanziger, Nucl. Phys. B 518 (1998) 237.

[12] D. Epple, H. Reinhardt, W. Schleifenbaum, Phys. Rev. D 75 (2007) 045011, arXiv:hep-th/0612241;
H. Reinhardt, D. Epple, Phys. Rev. D 76 (2007) 065015, arXiv:0706.0175 [hep-th].

[13] R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, Mod. Phys. Lett. A 23 (2008) 1105, arXiv:hep-ph/0607293.

[14] B. Svetitsky, Phys. Rep. 132 (1986) 1;
J. Greensite, Prog. Part. Nucl. Phys. 51 (2003) 1.

[15] H. Reinhardt, Nucl. Phys. B 503 (1997) 505;
C. Ford, U.G. Mitreuter, T. Tok, A. Wipf, J.M. Pawlowski, Ann. Phys. 269 (1998) 26, arXiv:hep-th/9802191;
O. Jahn, F. Lenz, Phys. Rev. D 58 (1998) 085006, arXiv:hep-th/9803177.

[16] F. Marhauser, J.M. Pawlowski, arXiv:0812.1144 [hep-ph].

[17] L.F. Abbott, Nucl. Phys. B 185 (1981) 189.

[18] C. Wetterich, Phys. Lett. B 301 (1993) 90.

[19] D.F. Litim, J.M. Pawlowski, arXiv:hep-th/9901063;
J. Berges, N. Tetradis, C. Wetterich, Phys. Rep. 363 (2002) 223, arXiv:hep-ph/0005122;
J. Polonyi, Central Eur. J. Phys. 1 (2003) 1, arXiv:hep-th/0110026;
H. Gies, arXiv:hep-ph/0611146;
B.J. Schaefer, J. Wambach, Phys. Part. Nucl. 39 (2008) 1025, arXiv:hep-ph/0611191.

[20] J.M. Pawlowski, Ann. Phys. 322 (2007) 2831, arXiv:hep-th/0512261.

[21] M. Reuter, C. Wetterich, Nucl. Phys. B 417 (1994) 181;
M. Reuter, C. Wetterich, Phys. Rev. D 56 (1997) 7893;
F. Freire, D.F. Litim, J.M. Pawlowski, Phys. Lett. B 495 (2000) 256, arXiv:hep-th/0009110;
H. Gies, Phys. Rev. D 66 (2002) 025006.

[22] J.M. Pawlowski, Int. J. Mod. Phys. A 16 (2001) 2105;
D.F. Litim, J.M. Pawlowski, Phys. Rev. D 66 (2002) 025030;
D.F. Litim, J.M. Pawlowski, JHEP 0209 (2002) 049;
D.F. Litim, J.M. Pawlowski, Phys. Lett. B 546 (2002) 279.

[23] J.M. Pawlowski, Phys. Rev. D 58 (1998) 045011;
H. Gies, C. Wetterich, Phys. Rev. D 69 (2004) 025001, arXiv:hep-th/0209183;
J. Braun, H. Gies, Phys. Lett. B 645 (2007) 53, arXiv:hep-ph/0512085;
J. Braun, H. Gies, JHEP 0606 (2006) 024.

[24] D.F. Litim, Phys. Lett. B 486 (2000) 92;
D.F. Litim, Phys. Rev. D 64 (2001) 105007.

[25] J. Braun, H. Gies, H.J. Pirner, AIP Conf. Proc. 775 (2005) 162, arXiv:hep-ph/0610341.

[26] C.S. Fischer, R. Alkofer, Phys. Lett. B 536 (2002) 177, arXiv:hep-ph/0202202;
J.M. Pawlowski, in preparation.

[27] R. Alkofer, L. von Smekal, Phys. Rep. 353 (2001) 281, arXiv:hep-ph/0007355;
C.S. Fischer, J. Phys. G 32 (2006) R253.

[28] C. Lerche, L. von Smekal, Phys. Rev. D 65 (2002) 125006, arXiv:hep-ph/0202194.

[29] A. Maas, arXiv:0907.5185 [hep-lat].

[30] A. Sternbeck, L. von Smekal, arXiv:0811.4300 [hep-lat];
A. Sternbeck, L. von Smekal, PoS LATTICE2008 (2008) 267, arXiv:0810.3765 [hep-lat].

[31] C.S. Fischer, J.M. Pawlowski, Phys. Rev. D 75 (2007) 025012, arXiv:hep-th/0609009;
C.S. Fischer, J.M. Pawlowski, Phys. Rev. D 80 (2009) 025023.

[32] R. Alkofer, M.Q. Huber, K. Schwenzer, arXiv:0801.2762 [hep-th].

[33] C.S. Fischer, H. Gies, JHEP 0410 (2004) 048, arXiv:hep-ph/0408089.

[34] J.M. Cornwall, Phys. Rev. D 26 (1982) 1453.

[35] D. Zwanziger, arXiv:0904.2380 [hep-th].

[36] D. Dudal, J.A. Gracey, S.P. Sorella, N. Vandersickel, H. Verschelde, Phys. Rev. D 78 (2008) 065047, arXiv:0806.4348 [hep-th];
S.P. Sorella, Phys. Rev. D 80 (2009) 025013, arXiv:0905.1010 [hep-th];
D. Dudal, N. Vandersickel, H. Verschelde, S.P. Sorella, arXiv:0911.0082 [hep-th].

[37] M.Q. Huber, R. Alkofer, S.P. Sorella, arXiv:0910.5604 [hep-th].

[38] K.I. Kondo, Phys. Lett. B 678 (2009) 322, arXiv:0904.4897 [hep-th];
K.I. Kondo, arXiv:0909.4866 [hep-th].

[39] Ph. Boucaud, Th. Bruntjen, J.P. Leroy, A. Le Yaouanc, A.Y. Lokhov, J. Micheli, O. Pene, J. Rodriguez-Quintero, JHEP 0606 (2006) 001, arXiv:hep-ph/0604056;
Ph. Boucaud, J.P. Leroy, A.L. Yaouanc, J. Micheli, O. Pene, J. Rodriguez-Quintero, JHEP 0806 (2008) 012, arXiv:0801.2721 [hep-ph];
Ph. Boucaud, J.P. Leroy, A.L. Yaouanc, J. Micheli, O. Pene, J. Rodriguez-Quintero, JHEP 0806 (2008) 099, arXiv:0803.2161 [hep-ph].

[40] A.C. Aguilar, J. Papavassiliou, Phys. Rev. D 77 (2008) 125022, arXiv:0712.0780 [hep-ph];
D. Binosi, J. Papavassiliou, Phys. Rev. D 77 (2008) 061702, arXiv:0712.2707 [hep-ph];
A.C. Aguilar, D. Binosi, J. Papavassiliou, Phys. Rev. D 78 (2008) 025010, arXiv:0802.1870 [hep-ph].

[41] A. Cucchieri, T. Mendes, A.R. Taurines, Phys. Rev. D 71 (2005) 051902, arXiv:hep-lat/0406020.

[42] L. von Smekal, D. Mehta, A. Sternbeck, A.G. Williams, PoS LAT2007 (2007) 382, arXiv:0710.2410 [hep-lat];
L. von Smekal, A. Jorkowski, D. Mehta, A. Sternbeck, PoS CONFINEMENT8 (2008) 048, arXiv:0812.2992 [hep-th].

[43] B. Gruter, R. Alkofer, A. Maas, J. Wambach, Eur. Phys. J. C 42 (2005) 109, arXiv:hep-ph/0408282.

[44] A. Cucchieri, A. Maas, T. Mendes, Phys. Rev. D 75 (2007) 076003, arXiv:hep-lat/0702022.

[45] A. Maas, arXiv:0911.0348 [hep-lat].

[46] M.N. Chernodub, V.I. Zakharov, Phys. Rev. Lett. 100 (2008) 222001, arXiv:hep-ph/0703167.

[47] N. Weiss, Phys. Rev. D 24 (1981) 475;
P.N. Meisinger, M.C. Ogilvie, Phys. Lett. B 407 (1997) 297, arXiv:hep-lat/9703009;
M. Engelhardt, H. Reinhardt, Phys. Lett. B 430 (1998) 161, arXiv:hep-th/9709115;
H. Gies, Phys. Rev. D 63 (2001) 025013.

[48] J. Fingberg, U.M. Heller, F. Karsch, Nucl. Phys. B 392 (1993) 493, arXiv:hep-lat/9208012;
O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, Phys. Lett. B 543 (2002) 41, arXiv:hep-lat/0207002;
B. Lucini, M. Teper, U. Wenger, JHEP 0502 (2005) 033, arXiv:hep-lat/0502003;
C. Wozar, T. Kaestner, A. Wipf, T. Heinzl, Phys. Rev. D 76 (2007) 085004, arXiv:0704.2570 [hep-lat].

[49] J. Braun, L.M. Haas, F. Marhauser, J.M. Pawlowski, arXiv:0908.0008 [hep-ph].