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Procedia Materials Science 6 (2014) 907 – 918

Procedia Materials Science

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3rd International Conference on Materials Processing and Characterisation (ICMPC 2014)

A Finite Element Approach to Bending, Contact and Fatigue Stress Distribution in Helical Gear Systems

S. Jyothirmai[a], R. Ramesh[b], T. Swarnalatha[c], D. Renuka[d]

Centre for Intelligent Manufacturing Automation (CIMAT),
Department of Mechanical Engineering, MVGR College of Engineering, Vizianagaram, India

Abstract

In the face of extensive research into the theoretical basis and performance characteristics of helical gear design, a complete mathematical description of the relationship between the design parameters and the performance matrices is still to be clearly understood because of the great complexity in their interrelationship. The objective of this work is to conduct a comparative study on helical gear design and its performance based on various performance metrics through finite element as well as analytical approaches. The theoretical analysis for a single helical gear system based on American Gear Manufacturing Association (AGMA) standards has been assessed in Matlab. The effect of major performance metrics of different helical gear tooth systems such as single, herringbone and crossed helical gear are studied through finite element approach (FEA) in ANSYS and compared with theoretical analysis of helical gear pair. Structural, contact and fatigue analysis are also performed in order to investigate the performance metrics of different helical gear systems. The benefit of such a comparison is quickly estimating the stress distribution for a new design variant without carrying out complex theoretical analysis as well as the FEA analysis gives less scope for manual errors while calculating complex formulas related to theoretical analysis of gears. It will significantly reduce processing time as well as enhanced flexibility in the design performance.

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Keywords: tooth bending stress; surface fatigue strength; contact stress; tooth surface strength of gear; herringbone helical gear;

* Corresponding author. Tel.: 09492500137.
E-mail address: jyothirmai.srinathu@gmail.com

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doi:10.1016/j.mspro.2014.07.159

Introduction

The main purpose of gear mechanisms is to transmit rotation and torque between shaft axes. The gear wheel is a machine element that has intrigued many engineers because of numerous technological problems arising in a complete mesh cycle. In order to achieve high load carrying capacity with reduced weight of gear drives but with increased strength in gear transmission, gear design on the basis of tooth stress analysis, tooth modifications and optimum design of gear drives are becoming major research areas. Gears with involute teeth have widely been used in industry because of the low cost of manufacturing. Critical evaluation of helical gear design performance therefore plays a crucial role in estimating the degree of success of such gear systems in terms of stresses and deformation developed in helical gears. Helical gears have more advantages than other gears especially spur gears like it has smoother engagement of teeth, silent in operation, can handle heavy loads and power can be transferred between non parallel shafts, high efficient etc. Due to these advantages it has wide range of applications in high speed high power mechanical systems.

In the evaluation of helical gear designs, certain basic gear design performance metrics such as tooth bending stress, Permissible bending stress, contact stress, bending fatigue strength, allowable surface fatigue stress, tooth surface strength of gear and pinion etc. are to be carefully considered. The effectiveness of the helical gear design can be improved only when all these metrics are controlled properly. Gear designers are constantly looking for ways to improve effectiveness through various techniques. Despite such attempts, the control of all these metrics and achieving the desired performance is a very complicated task. Therefore, there is great need for detailed study of the intricacies of helical gear design especially for different types of gear profiles.

In this paper, an attempt is made to study the performance of a helical gear system for three different types of helical gear systems namely single, herringbone and crossed helical gear system. The objective of this work is to conduct a comparative study on helical gear design and its performance based on various performance metrics through finite element as well as analytical approaches. The theoretical analysis for a single helical gear system based on American Gear Manufacturing Association (AGMA) standards has been assessed in Matlab. The effect of major performance metrics of different helical gear tooth systems such as single, herringbone and crossed helical gear are studied through finite element approach (FEA) in ANSYS and compared with theoretical analysis of helical gear pair. Structural, contact and fatigue analysis are also performed in order to investigate the performance metrics of different helical gear systems. The benefit of such a comparison is quickly estimating the stress distribution for a new design variant without carrying out complex theoretical analysis as well as the FEA analysis gives less scope for manual errors while calculating complex formulas related to theoretical analysis of gears. It will significantly reduce processing time as well as enhanced flexibility in the design performance.

Nomenclature

TBS	Tooth bending stress
ASFS	Allowable surface fatigue stress
CS	Contact stress
PBS	Permissible bending stress
SFSP	surface fatigue strength of pinion
BFS	Bending fatigue strength

Literature review

Gear analysis is one of the most significant issues in the machine elements theory particularly in the field of gear design and gear manufacturing. Many of the researchers have proposed several concepts for gear design optimization to enhance the performance of gear systems. Cavdar et al. [1] has developed tooth model of involute spur gears with asymmetric teeth to improve the performance of gears such as increasing the load capacity or reducing noise and vibration. In this study, a computer program was developed for asymmetric gears with greater drive side pressure angle than coast side pressure angle to determine bending load carrying capacities and contact conditions of asymmetric gear drives Huang and Liu [2] proposed a dynamic stiffness based method to calculate the dynamic response of a gear tooth subject to meshing force on equations of motion for a Timoshenko beam model. Li [3] has developed a loaded tooth contact analysis program to calculate all of the three-dimensional, thin-rimmed gear structures with all of the gear parameters. Kapelevich [4] has developed a basic geometric theory of the gears with asymmetric teeth profile that allows for an increase in load capacity while reducing weight and dimensions for some types of gears to research and design gears independently from generating rack parameters. It also provides wide variety of solutions for a particular couple of gears that are included in the area of existence. Kahraman and Bajpai [5] has developed a surface wear for helical gear pairs to study the influence of tooth modifications on helical gear wear. The model uses a finite element based gear contact mechanics model to predict the contact pressures at a number of discrete rotational gear positions and a computational procedure for determining relative sliding distances of mating points on each gear for each rotational increment. In this method a simplified design formula was also proposed that links modification parameters directly to initial wear rates. Fong et al. [6] proposed a mathematical model of parametric tooth profile of spur gears where the line of action is given. The line of action usually comprises a simple curve. The proposed mathematical model was aimed at enhancing the freedom of tooth profile design by combining the simple curves into the line of action. The curvature, sliding velocity, contact ratio and the limitation of undercutting can be derived directly from the equation of line of action. Chen and Tsay [7] proposed a mathematical model of the modified helical gear with small number of teeth. This was developed by tooth-profile shifting and basic geometry modification to investigate the condition of tooth undercutting for the involute profile gears using the developed mathematical models. Alipiev [8] conducted research related to the geometric design of spur gear drives of symmetric and asymmetric teeth and proposed realized potential method for geometric design of involute gear drives of symmetric and asymmetric meshing. In addition, for the realization of gear drive potential, the introduction of different parameters exerts a decisive role for the determination of bottom clearances and depths of fillet curves of the rack-cutters. Imrek and Duzcukoglu [9] conducted experimental study on width modification of a spur gear to fix instantaneous pressure changes along single meshing area on the gear profile. In this gear, variable pressure distribution caused by the single and double teeth meshing and the radius of curvature along the active gear profile was approximately kept constant by maintaining a constant ratio of applied load to the tooth width on every point. The amount of wear in the teeth profiles between the modified and unmodified gears was compared. Costopoulos and Spitas [10] proposed several tooth designs alternative to the standard involute for increasing the load carrying capacity of geared power transmissions and to combine the good meshing properties of the driving involute and the increased strength of non-involute curves to provide constant direction of rotation although they can be used in a limited way for reverse rotation.

All of the above works have attempted to enhance effectiveness of gear systems through weight reduction, wear reduction, vibration and noise reduction. Studies have also been performed mostly using involute and asymmetric gear tooth profiles. In addition, most of these works have estimated tooth bending stress and contact stress. The estimation of allowable surface fatigue stress, contact stress, surface fatigue strength, tooth surface strength of gear and pinion and permissible bending stress have not received much attention. The performance of alternative tooth profiles such as circular and cycloidal, generally not in use on account of manufacturing difficulties or reduced strength at root, have also not received much attention.

Modeling of helical gears

1.1. Basis for comparative study

In this work, an attempt has been made to study three different helical gear systems namely single, double and crossed in terms of tooth bending stress and contact stress as studied by most of the researchers as well as other critical stresses such as allowable surface fatigue stress, contact stress, bending fatigue strength, tooth surface strength of gear and pinion. The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate. Two mating helical gears must have equal helix angle but opposite hand. They run smoother and more quietly. They have higher load capacity, are more expensive to manufacture. Helical gears can be used to mesh two shafts that are not parallel and can also be used in a crossed gear mesh connecting two perpendicular shafts. They have longer and strong teeth. They can carry heavy load because of the greater surface contact with the teeth. The efficiency is also reduced because of longer surface contact. The gearing is quieter with less vibration. One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees. An attempt is thus made to identify the best suited tooth helical gear for a given application in terms of all these stresses. This would give a complete picture of the load bearing performance of a given gear. The refined form of the Lewis equation for tooth bending stress is adopted. Relationships for permissible tooth bending stress, tooth surface strength of the pinion and gear, dynamic contact stress, bending fatigue strength, allowable surface fatigue stress as per AGMA standards are adopted. Based on these relationships, the performance metrics were computed for the design specifications mentioned in Table 1.

Table 1. Specifications considered for comparative study.

GEAR PARAMETERS	SPEC.
GEAR RATIO	1.5
FACE WIDTH IN METERS	0.075
TYPE OF GEAR TEETH SYASTEM	20
TORQUE IN NEWTON-METER	132.63
CENTER DISTANCE BETWEEN GEAR AND PINION SHAFT IN METERS	2.5
ANGULAR VELOCITY OF PINION IN RAD/SEC	150.79
MATERIAL FOR GEAR AND PINION	STRUCTURAL
SOURCE OF POWER	UNIFORM
TYPE OF DRIVEN MACHINERY	UNIFORM
TYPE OF LOAD	CONTINOUS
FACTOR OF SAFETY	1.1
POISSONS RATIO	0.3
YOUNG’S MODULUS IN GPA	207
MODULE IN MM	10

3.2 Analysis using Matlab

The analytical analysis for single helical gear system is performed in Matlab. The algorithm for Matlab program has been shown in Fig. 1. Initially all the input parameters such as gear ratio, face width, length between driver and driven shaft, module, torque, speed on pinion has been given. In addition the gear design parameters such as material, Young’s modulus etc has taken as inputs to the program. After giving the input design parameters the performance metrics has been generated.

3.3 Modeling of helical gear systems

Using the specifications listed in Table 1, each of the above tooth geometries were first modelled using Pro/E and then later analysed. Initially, modelling of the gear was carried out in Pro/E. The model was done by sketching the base circle using relations and parameters and after the extrude part is generated the curve is created and the sweep option is performed to obtain the tooth profile. Later on complete gear is generated using pattern feature. In the same way modeling of the pinion was also accomplished. Finally, assembling of both gear and pinion was done to obtain the gear pair. The modeling of single, double and crossed helical gear models in Pro/E is shown in Fig. 2, 3, 4. The meshing of crossed helical gear in Pro/E Fig. 5. Thus the models required for analysis are generated using Pro-e and the inputs required for designing are taken from the Table I. Further on these models are imported to ANSYS workbench and the Structural, fatigue and contact stress analysis is performed which will be illustrates in next section.

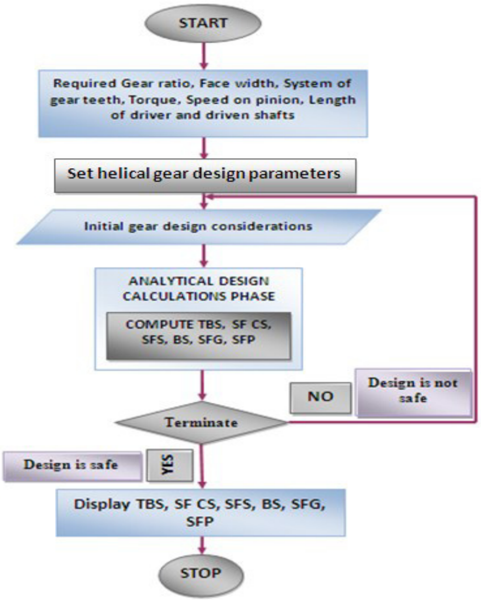

Fig. 1. Algorithm for analytical analysis of helical gear design.

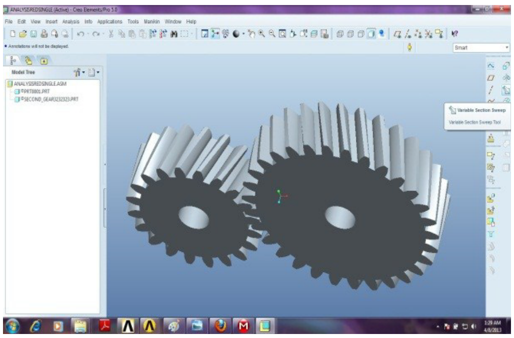

Fig. 2. Modeling of single helical gear model in Pro/E

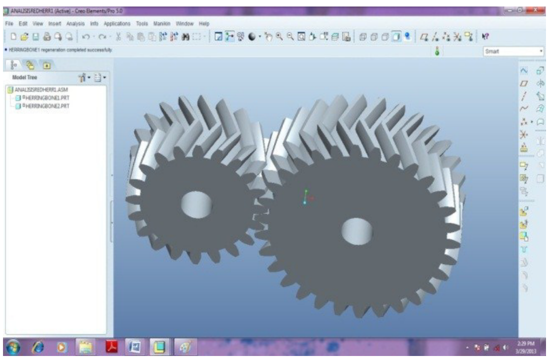

Fig. 3. Modeling of double helical gear model in Pro/E

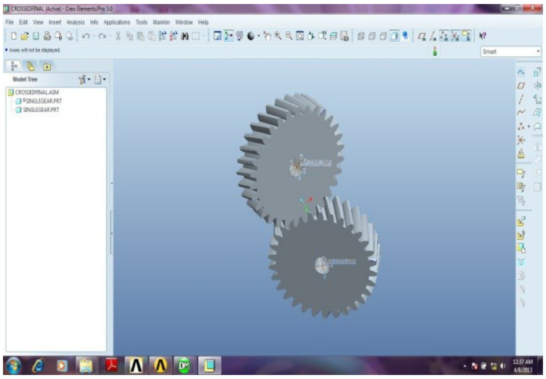

Fig. 4. Modeling of crossed helical gear model in Pro/E

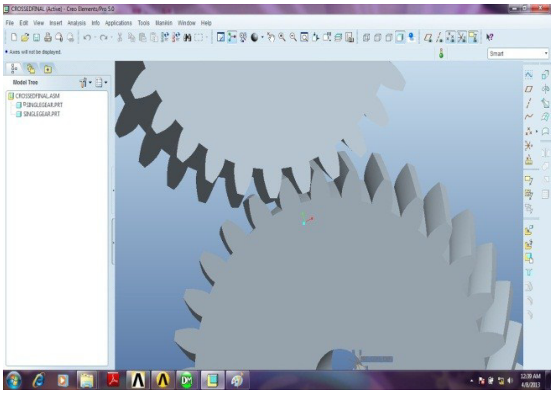

Fig. 5. Mating of crossed helical gear in Pro/E

2. Finite element analysis of helical gears

It deals with the development of finite element analysis that has been implemented for various gear systems that were developed in the previous chapter. The main objective of developing finite element analysis was in order to estimate bending, fatigue and contact stress distribution in the pinion and gear. Finite element analysis of the developed helical gear pair was executed in ANSYS. The first step is to perform structural analysis in order to calculate tooth bending stress and permissible bending stress, bending fatigue strength of pinion. The second step in the finite element analysis approach is to perform contact stress analysis in order to calculate contact stress. The final step involved is to perform fatigue stress analysis in order to calculate allowable surface fatigue stress, surface fatigue strength of pinion. Each of these steps was executed and is described below.

The structural analysis of the helical gear train was performed in six stages namely input of engineering data, definition of geometry, development of model, setup and generation of solution and results. Structural steel was used in this problem having material properties of elastic modulus 207 GPa and Poisson’s ratio 0.3. After input of these data, the model created in Pro/E was imported. After the model was imported, meshing operation was performed on the model to divide the model into several elements or nodes. The type of node element considered was tetrahedron and the torque, angular velocity of required range as specified in Table I were applied on the helical gear pair entities after the meshing operation. Two coordinate systems were taken for helical gear pair one is global coordinate system for gear and another is normal coordinate system for pinion. Torque was applied on the pinion by considering normal coordinate system means torque will be applied on pinion about pinion central axis and angular velocity of pinion is considered by considering the coordinating system for pinion about pinion central axis. After completion of pre-processing steps post processing steps were accomplished in ANSYS. In order to execute this several tools were imported such as fatigue tool, contact tool etc. In addition vonmises stresses, principal stresses were also given for analysis in order to calculate the performance metrics of helical gear pair. Based on these input details, the solution was generated by ANSYS. This structural analysis was executed for all the three helical gears listed earlier. The tooth bending stress distribution for the various helical gears are in Fig. 6. (for single helical gear), Fig. 7. (for double helical gear).

To examine the bending fatigue strength in gear pair, the maximum principal stress at the root on the tensile side of the tooth [11] was used for evaluating the tooth bending strength of a gear and pinion. Surface fatigue strength of the tooth profile is calculated by multiplying allowable surface fatigue stress with factor of safety. The numerical solutions are compared with that of the analytical analysis for single helical gear. Similarly fatigue stress analysis of remaining helical gears has been accomplished in ANSYS as shown in Fig. 8. (for single helical gear), Fig. 9. (for double helical gear). The solution is generated automatically by ANSYS. To examine the contact stresses in the gear pair, the helical gear train with two-dimensional contact developed in Pro/E was analyzed in ANSYS as shown in Fig. 10. The numerical solutions obtained in ANSYS were compared with that of the Hertz theory contact stress through analytical analysis for single helical gear.

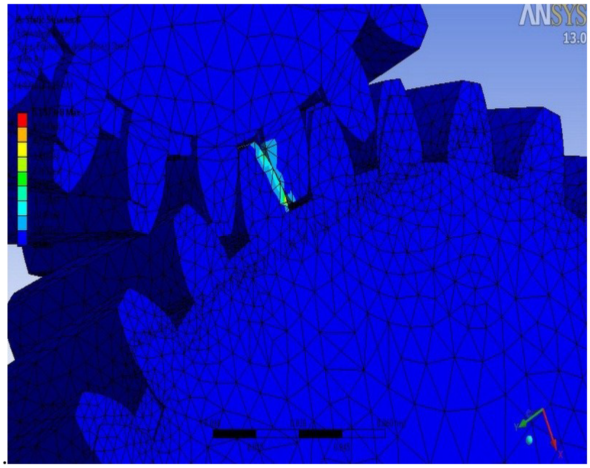

Fig. 6. Tooth bending stress distribution for single helical gear

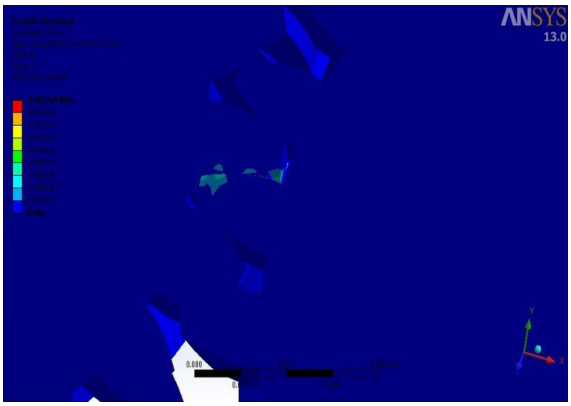

Fig. 7. Tooth bending stress distribution for herringbone helical gear.

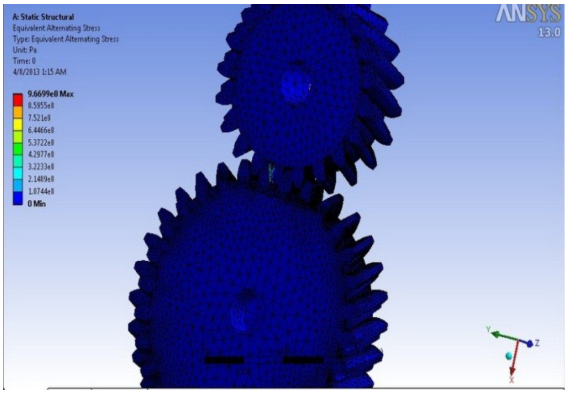

Fig. 8. Fatigue Stress distribution for single helical gear

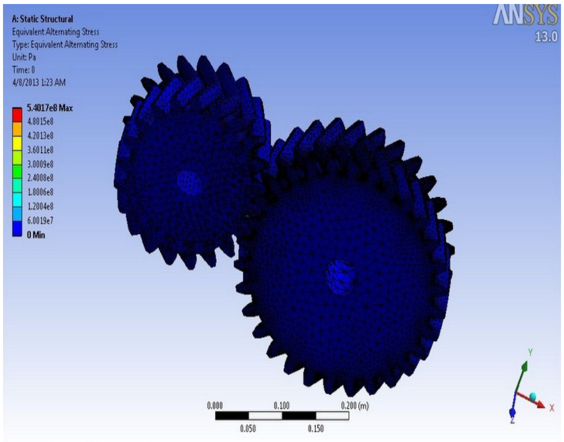

Fig. 9. Fatigue stress distribution for herringbone gears

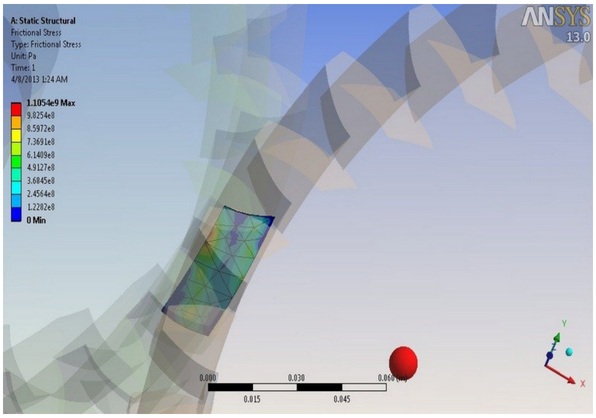

Fig. 10. Contact stress analysis for double helical gear

Similarly contact stress analysis was also carried out for the remaining helical gears in ANSYS and compared with the analytical results. It can be seen from all of these that the maximum tooth bending stress was obtained at the tensile side of tooth of gear. In addition it can also be seen from the figures that the stress distribution is maximum at the contact side and minimum stress distribution obtained at the flank of gear and pinion. The comparison of various performance metrics for different tooth profiles will be illustrated in the next section.

3. Comparative study of helical gear systems

In this section, comparative study of gear teeth performance with different helical gear systems**.** Theoretical analysis has been performed to the single helical gear system using Matlab and at the same time FEA analysis was performed by initially creating a model in Pro/E and importing this file in ANSYS. Now in order to justify our FEA analysis we need to compare the results that are obtained through analytical analysis with that of FEA analysis for single helical gear system. From the study it is observed that the results related to analytical and FEA analysis was closer in case of single helical gear. Hence the FEA analysis for the rest of helical gear systems has been executed and illustrated in Table 2. and Fig. 11.

Table 2. Comparison of various gear design metrics for different helical gear systems in MPa.

	SINGLE		DOUBLE	CROSSED
	AA	FEA	FEA	FEA
TBS	6.787	6.65	15.8	25.14
BFS	185.28	189.10	198	342.5
SFSP	698.49	709.126	625.5	720.147
ASFS	634.99	634.99	634.99	634.99
CS	183.65	180.02	203.45	130.63
PBS	168.432	168.432	168.432	168.432

It is observed that the predicted values from FEA are close to the values obtained through the analytical analysis for single helical gear system. In the case of tooth bending stress, it can be observed from Figure 18 that the FEA values and the values obtained from analytical analysis and are fairly close and the error is about 2%. Out of the 4 performance metrics of the helical gear model, three performance metrics predicted by the FEA show an error less than 1.5% in comparison with the analytical analysis results and for the other performance metrics the FEA show an error less than 2% in comparison with the analytical analysis results. It is observed from the table that the performance metrics like allowable surface fatigue stress and the permissible bending stress are constant for all the helical gear systems. The design for safety is predicted by comparing the tooth bending stress with that of the permissible bending stress and in the same way the allowable surface fatigue stress is also got more than the contact stress values for various helical gear systems. Here the tooth bending strength for any gear system must be less than that of the permissible bending strength so that the factor of safety lies between 1.1 to 1.5.

And similarly the contact stress of the gear system must be less than the allowable surface fatigue stress of the gear system. From the above Table II we can observe that all the above values obtained satisfy the above two conditions and hence the design is safe. And finally it can be concluded thus that the developed FEA model is an accurate representation of the stress distribution pattern. Figure. 12 shows the variation of tooth bending stress for different helical gear systems. It is found from the graph it is observed that the values of tooth bending stress vary over a wide range and it shows that the application of these three helical gear systems is not the same. The crossed helical gear system got more TBS value hence it is clear that the mating of two opposite gear teeth has highest TBS value.Fig. 13. shows the variation of bending fatigue strength for different helical gear systems. It is found from the graph that the bending fatigue strength is more in case of crossed helical gears since the mating of gear tooth are in opposite and accurate direction. Figure. 14 shows the variation of surface fatigue strength of pinion for different helical gear systems. It is found from that the fatigue strength of crossed helical gears are more than other helical gear systems. Since the mating of gear tooth are in opposite and accurate direction and possess non-intersecting, non-parallel shaft axis and different helix angles. Figure. 15 shows the variation of contact stress for different helical gear systems. It is found from the graph that the crossed helical gears has high bending and fatigue strength the stress values corresponding to these type of gears will be less compared other helical gear systems. It can also be seen from the charts that the values corresponding to allowable surface fatigue stress and permissible bending stress are within a range of 634 MPa all helical gears and between 168 MPa for all helical gear for the same loading condition the values of crossed helical gear are much lower that is between 25-130 Mpa in case of stresses. In the same way the strength is ranging from 342-720Mpa.This shows that all the three gear systems are to be used at different loading conditions and also that crossed helical gears gives optimum results for the conditions that we have considered.

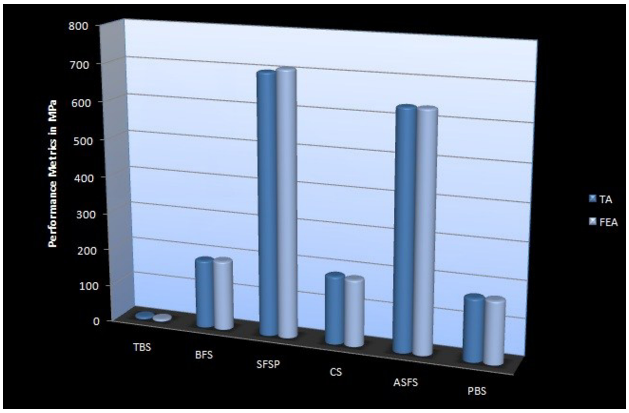

Fig.11. Variation of different performance metrics for different analysis

Hence, it can be concluded that the single and herringbone helical gears are useful where there are heavy loads high rotational speeds because the stress induced in them are very large. But at the same time the surface fatigue strength, Surface fatigue strength of gear and pinion are large for crossed helical gear which permits its use in larger speed reduction at low speeds. On the other hand the single helical gear fails at the strength criteria because of the axial thrust acting on it in single direction. And also as the stresses are much lower in the present scenario hence these gears can be used at larger speeds and also at pitch line velocity greater than 25 m/sec.

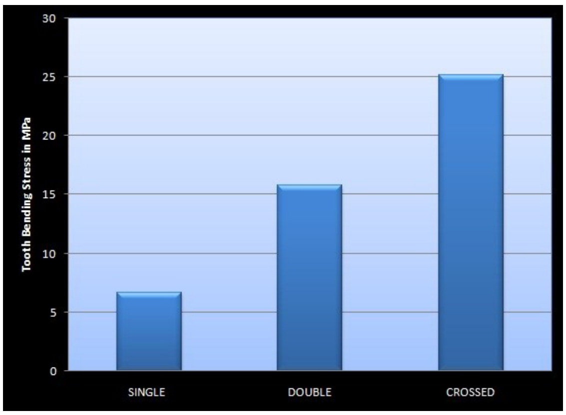

Fig.12. Variation of tooth bending stress in MPa for different helical gears

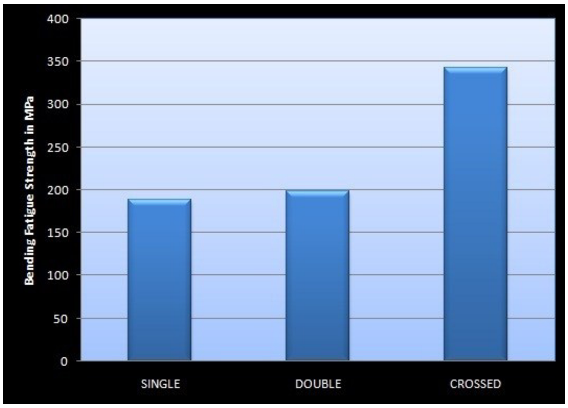

Fig. 13. Variation of bending fatigue strength in MPa for different helical gears.

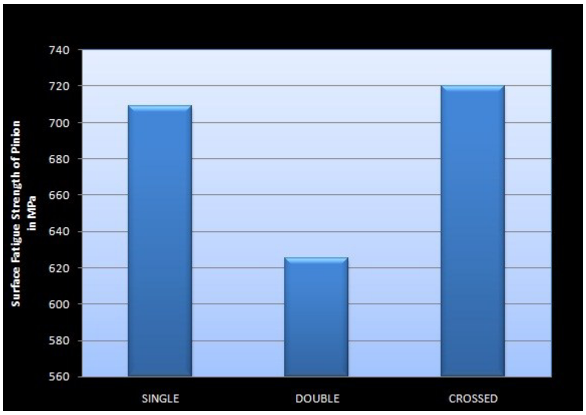

Fig. 14. Variation of surface fatigue strength of pinion in MPa for different helical gears

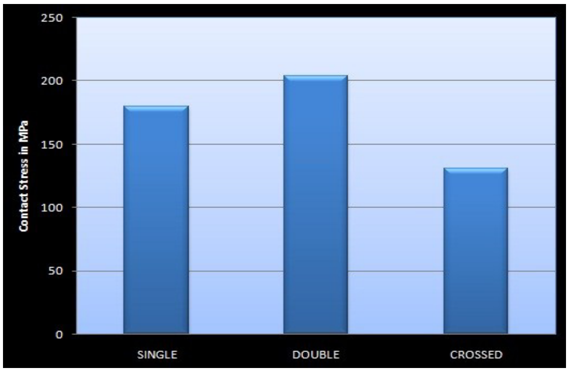

Fig. 15. Variation of contact stress in MPa for different helical gears

In this work, an attempt has been made to compare the performance of various helical gear systems for a given set of specification through an analytical approach based on AGMA standards as well as a finite element analysis approach. Four different helical gear systems namely single, herringbone and crossed helical gear systems were evaluated. The developed FEA model was validated against the analytical approach and was found to be very close. Further stress analysis was carried out using FEA. It was found that the overall performance of crossed helical gear was found to be the best in terms of stress as well as tooth strength at low speeds and low loads whereas herringbone and single helical gear systems are employed for optimum values of speeds and loads. The low stresses observed in case of single helical gear makes its use in case of high speeds and heavy loads.

4. Conclusion

In this paper, an attempt has been made to compare the performance of various helical gear systems for a given set of specification through an analytical approach based on AGMA standards as well as a finite element analysis approach. Three different helical gear systems namely single, herringbone, crossed helical gear systemswere evaluated. The developed FEA model was validated against the analytical approach and was found to be very close. Further stress analysis was carried out using FEA. The developed FEA model was validated against the analytical approach and was found to be very close. Further stress analysis was carried out using FEA. It was found that the overall performance of crossed helical gear was found to be the best in terms of stress as well as tooth strength at low speeds and low loads whereas herringbone and single helical gear systems are employed for optimum values of speeds and loads. The low stresses observed in case of single helical gear makes its use in case of high speeds and heavy loads.

References

Alipiev, O., 2011. Geometric Design of Involute Spur Gear Drivesw Symmetric and Asymmetric Teeth using the Realized Potential Method. Mechanism and Machine Theory 46,10–32.

Cavdar, K., Karpa, F., Babalik, F.C., 2005. Computer Aided Analysis of Bending Strength of Involute Spur Gears with Asymmetric Profile. Journal of Mechanical Design 127(3), 477-484.

Costopoulos, Th., Spitas, V., 2009. Reduction of Gear Fillet Stresses by using One-Sided Involute Asymmetric Teeth. Mechanism and Machine Theory 44, 1524–1534.

Chen, C. F., Tsay, C. B., 2005. Tooth Profile Design for The Manufacture of Helical Gear Sets with Small Numbers of Teeth. International Journal of Machine Tools & Manufacture 45, 1531–1541.

Fong, Z. H., Chiang, T. W., Tsay, C. W., 2002. Mathematical Model for Parametric Tooth Profile of Spur Gear using Line of Action. Mathematical and Computer Modeling 36, 603-614.

Gurumani, R., Shanmugam, S., 2011. Modeling and Contact Analysis of Crowned Spur Gear Teeth. Engineering Mechanics 18(1), 65-78.

Huang, K. J., Liu, T. S., 2000. Dynamic Analysis of a Spur Gear by the Dynamic Stiffness Method. Journal of Sound and Vibration 234(2), 311-329.

Imrek, H., Duzcukoglu, H., 2007. Relation Between Wear and Tooth Width Modification in Spur Gears. Wear 262, 390–394.

Li, S. T., 2002. Gear Contact Model and Loaded Tooth Contact Analysis of a Three-Dimensional Thin-Rimmed Gear. Journal of Mechanical Design 124(3), 511-517.

Kapelevich, A., 2000.Geometry and Design of Involute Spur Gears with Asymmetric Teeth. Mechanism and Machine Theory 35, 117-130.

Kahraman, A., Bajpai, P., 2005. Influence of Tooth Profile Deviations on Helical Gear Wear. Journal of Mechanical Design 127, 656-663.