Renormalizability of noncommutative quantum electrodynamics at $\theta^2$ order

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**Abstract**

In $\theta$-expanded approach, we expand the noncommutative quantum electrodynamics action to $\theta^2$ order and calculate the one loop divergent corrections to gauge field propagators. It is shown that the gauge field propagators are one loop renormalizable at $\theta^2$-order with a massive or massless fermion field.

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**1. Introduction**

Field theories on noncommutative space were suggested long time ago by Snyder [1] to find a natural cutoff for the loop integrals in quantum field theories. Recently, due to the development of string theories, noncommutative field theories have been studied extensively [2–5]. The noncommutative space is defined as follows:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu},$$

where $\hat{x}^\mu$ are generators of the noncommutative space and $\theta^{\mu\nu}$ are real antisymmetric constants. From results of noncommutative geometry, the algebra (1) can be represented by the algebra of functions on ordinary space with deformed multiplication, Moyal–Weyl product [6], which is defined as

$$f(x) \star g(x) = \exp\left(\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}\right) f(x) g(y) \bigg|_{y \to x}.$$  

Field theories on noncommutative space can be obtained by replacing ordinary product between fields with the Moyal–Weyl product. Then the noncommutative effects can be explored by the usual perturbative method. One special noncommutative effect is the nonplanar graph which leading the phenomena of UV/IR mixing [7–9]. Noncommutative U(1) and U(N) gauge theories can be discussed in this approach. But for general noncommutative gauge theories, the Moyal–Weyl product makes the noncommutative gauge fields cannot close in gauge algebras. The noncommutative gauge fields should be enveloping-algebra valued [10]. The expansion of noncommutative gauge fields in enveloping algebra can be determined by Seiberg–Witten map [5]. Seiberg and Witten have argued that there is a mapping between noncommutative gauge theories and ordinary gauge theories based on their research in string theory. The noncommutative fields can be expanded by ordinary fields in powers of $\theta^{\mu\nu}$. From the Seiberg–Witten map, one can obtain a $\theta$-expanded action and consider the noncommutative effects order by order in $\theta$. In this $\theta$-expanded approach, all gauge theories can be discussed. Obviously, one can study the renormalizability of noncommutative gauge theories in this approach.

Renormalizability of noncommutative gauge theories in $\theta$-expanded approach have been discussed in many papers [11–14]. It is shown that the pure noncommutative U(1) gauge propagator is one loop renormalizable to all orders in $\theta$ [11]. For pure noncommutative SU(N) theory, the gauge propagators are renormalizable at the first order in $\theta$ [14]. But when matter fields appear, the renormalizability of the gauge theories become complicated problems, which are still not completely solved. In noncommutative U(1) Higgs–Kibble model, the gauge sector is one loop renormalizable at first order in $\theta$ no matter whether the gauge symmetry is spontaneously broken or not [16], but the whole $\theta$-order renor-
malizability is spoiled by the matter sector. There is a similar result for fermion fields. In noncommutative quantum electrodynamics (QED), the gauge sector is one loop renormalizable at first order in $\theta$ [12,15], and a four fermions term can not be renormalizable by field redefinitions.

As it is argued in [16], the renormalizability of gauge sector is very surprising when matter fields appear. If the gauge sector can be renormalized to all order in $\theta$, we may expect that there are new symmetries in these noncommutative gauge theories, which make the gauge sector be renormalizable. Then we can modify the matter sector according these new symmetries so that the whole theory becomes renormalizable. But in order to achieve these, we should first prove that the gauge sector is renormalizable to all orders in $\theta$ when matter fields exist. In this direction, the one-loop UV-divergent matter contributions to gauge sectors have been computed in Ref. [17] using path integral method for noncommutative fields. In this Letter, using the $\theta$-expanded approach, we expand the noncommutative QED action to $\theta^2$-order by Seiberg–Witten map, and calculate all the divergent fermionic contributions to gauge field propagators up to $\theta^2$-order. It is shown that these divergent terms can be renormalized by field redefinitions. When one only considers contributions from action of $\theta^2$-order, there are non-renormalizable gauge field divergent terms involving fermion mass [12]. Because the divergent gauge field contributions to the gauge field propagators are renormalizable [11], we can conclude that the gauge field propagators are one loop renormalizable to $\theta^2$-order in noncommutative QED. We have more confidence in the renormalizability of the gauge propagators to all orders in $\theta$ in noncommutative QED. In Section 2, we give the $\theta^2$-order noncommutative QED action using the Seiberg–Witten map. In Section 3, we calculate the divergent contributions to the gauge field propagator and Section 4 is for the conclusion.

2. $\theta^2$-order action

The QED action on noncommutative space is

$$S = -\frac{1}{4} \int d^4x \hat{F}_{\mu\nu} \cdot \hat{F}^{\mu\nu} + \int d^4x \hat{\psi} \cdot (i\hat{D} - m) \hat{\psi},$$

where $\hat{\psi}$ is Moyal–Weyl star product. The NC covariant derivative $D_{\mu} \hat{\psi}$ and the noncommutative U(1) gauge field strength $F_{\mu\nu}$ are defined as

$$\hat{D}_{\mu} \hat{\psi} = \partial_{\mu} \hat{\psi} - i \hat{A}_{\mu} \hat{\psi},$$

$$\hat{F}_{\mu\nu} = \partial_{\mu} \hat{A}_{\nu} - \partial_{\nu} \hat{A}_{\mu} - i[A_{\mu}, A_{\nu}].$$

According to Seiberg–Witten map, one can express the NC fields $\hat{\psi}, \hat{A}_{\mu}$ using ordinary QED fields $\psi, A_{\mu}$. In order to get the $\theta^2$-order action, one should express the NC fields to $\theta^2$-order, which are given by

$$\hat{A}_{\mu} = a_{\mu} + \frac{1}{2} \theta^{\mu\nu} (a_{\mu} a_{\nu} - a_{\nu} a_{\mu} a_{\alpha} a_{\beta}) + \frac{1}{2} \theta^{\mu\nu} \theta^{\rho\lambda} (f_{\rho\lambda} a_{\alpha} a_{\mu} - a_{\alpha} a_{\rho} a_{\lambda} a_{\mu}),$$

$$\hat{F}_{\mu\nu} = -\frac{1}{2} \theta^{\mu\nu} a_{\alpha} a_{\beta} + \frac{1}{16} \theta^{\mu\nu} \theta^{\alpha\beta} (\partial_{\mu} a_{\alpha} a_{\beta} a_{\gamma} - 2 i \partial_{\mu} a_{\alpha} a_{\beta} a_{\gamma} + 4 a_{\alpha} a_{\beta} a_{\gamma} - 2 a_{\alpha} a_{\beta} a_{\gamma} + 2 a_{\beta} a_{\gamma} - 2 a_{\gamma} a_{\beta} a_{\gamma}),$$

where $f_{\mu\nu} = \partial_{\mu} a_{\nu} - \partial_{\nu} a_{\mu}$, $a_{\alpha} a_{\beta}$ is the usual U(1) gauge field strength and $D_{\mu} \psi = \partial_{\mu} \psi - i a_{\mu} \psi$ is the usual covariant derivative of fermion field $\psi$.

Then plugging the above results (6) and (7) into (3), one can get the action to $\theta^2$ order as follows:

$$S = S_0 + S_1 + S_{2,a} + S_{2,\psi},$$

where

$$S_0 = \int d^4x \left[ \bar{\psi} i D^\alpha D_\alpha \psi - m \bar{\psi} - \frac{1}{2} f_{\mu\nu} f^{\mu\nu} \right],$$

$$S_1 = -\frac{1}{2} \theta^{\mu\nu} \left( f_{\alpha\mu} \bar{\psi} i \gamma^\rho D_\rho \psi + f_{\alpha\mu} f_{\beta\nu} \gamma^{\alpha\beta} \right)$$

$$+ \frac{1}{2} f_{\mu\nu} \left( \bar{\psi} i \gamma^\alpha D_\alpha \psi - m \bar{\psi} - \frac{1}{2} f_{\alpha\beta} f^{\alpha\beta} \right),$$

$$S_{2,a} = \frac{1}{4} \theta^{\mu\nu} \theta^{\alpha\beta} \int d^4x \left[ 2 f_{\mu\nu} f_{\beta\rho} f_{\alpha\xi} f^{\alpha\beta} f_{\xi\rho} + f_{\kappa\lambda} f_{\beta\mu} f_{\alpha\nu} f^{\alpha\beta} + \frac{1}{4} f_{\mu\nu} f_{\kappa\lambda} f^{\mu\nu} f_{\kappa\lambda} - \frac{1}{8} f_{\mu\nu} f_{\kappa\lambda} f_{\xi\rho} f^{\mu\nu} f_{\kappa\lambda} f^{\xi\rho} \right],$$

$$S_{2,\psi} = \frac{1}{16} \theta^{\mu\nu} \theta^{\alpha\beta} \left[ f_{\alpha\beta} f_{\mu\nu} \bar{\psi} i \gamma^\alpha D_\alpha \psi + \frac{1}{2} f_{\mu\nu} f_{\kappa\lambda} \bar{\psi} i \gamma^\kappa D_\kappa \psi + 2 f_{\mu\nu} f_{\alpha\xi} \bar{\psi} i \gamma^\alpha D_\xi \psi + 2 f_{\mu\nu} f_{\alpha\xi} \bar{\psi} i \gamma^\alpha D_\xi \psi \right. \left. + 3 f_{\mu\nu} f_{\alpha\xi} \bar{\psi} i \gamma^\alpha D_\xi \psi + 3 f_{\mu\nu} f_{\alpha\xi} \bar{\psi} i \gamma^\alpha D_\xi \psi \right]$$

$$- m \left( \frac{1}{2} f_{\mu\nu} f_{\kappa\lambda} - f_{\mu\nu} f_{\kappa\lambda} \right) \bar{\psi} \psi + m f_{\kappa\lambda} \bar{\psi} i D_{\kappa} \psi \right].$$

3. One-loop divergent terms

In this section, we calculate the divergent contributions to the U(1) gauge field propagator at one-loop level. These contributions come from two sectors, the gauge sector and fermionic field sector. Because the gauge sector is renormalizable, we just consider the fermionic contributions. To carry out the calculation, we consider terms in the action (8) quadratic in $\psi$. These terms can be written as

$$S^{(2)} = \bar{\psi} (B^{(0,0)} + B^{(0,1)} + B^{(1,1)} + B^{(1,2)})$$

$$+ B^{(2,1)} + B^{(2,2)} + B^{(2,3)} \psi,$$

where $B^{(m,n)}$ are terms containing $m$ factors of $\theta$ and $n$ factors of $a_{\mu}.$

From the action (8), we can obtain

$$B^{(0,0)} = i \theta - m,$$

$$B^{(0,1)} = \theta,$$

$$B^{(1,1)} = -\frac{1}{2} \theta^{\mu\nu} \left( i \gamma^\alpha f_{\alpha\mu} \partial_\nu + \frac{1}{2} f_{\mu\nu} \bar{\theta} - \frac{m}{2} f_{\mu\nu} \right),$$

$$B^{(1,2)} = -\frac{1}{2} \theta^{\mu\nu} \left( \gamma^\alpha f_{\alpha\mu} a_\nu + \frac{1}{2} f_{\mu\nu} \theta \right),$$

$$B^{(2,1)} = \frac{1}{16} \theta^{\mu\nu} \theta^{\alpha\beta} (2 \gamma^\alpha \partial_{\mu} f_{\alpha\beta} \partial_\nu + 2 i f_{\mu\nu} f_{\kappa\lambda} \partial_\kappa \partial_\lambda)$$

$$+ 2 f_{\mu\nu} f_{\alpha\xi} \gamma^\alpha \partial_\xi \psi + 2 f_{\mu\nu} f_{\alpha\xi} \gamma^\alpha \partial_\xi \psi + 3 f_{\mu\nu} f_{\alpha\xi} \gamma^\alpha \partial_\xi \psi + 3 f_{\mu\nu} f_{\alpha\xi} \gamma^\alpha \partial_\xi \psi$$

$$- m \left( \frac{1}{2} f_{\mu\nu} f_{\kappa\lambda} - f_{\mu\nu} f_{\kappa\lambda} \right) \bar{\psi} \psi + m f_{\kappa\lambda} \bar{\psi} i D_{\kappa} \psi \right].$$
\[
+ 6i f_{a\mu} f_{\nu k} \gamma^\alpha \partial_\nu - 2i \partial_\mu \partial_\nu a_\nu \gamma^\alpha (\partial_\nu a_\alpha + a_\alpha \partial_\nu + a_\nu a_\alpha ) \\
+ m \partial_\mu f_{\kappa \lambda}, a_\nu \bigg],
\]
\[B^{(2,3)} = \frac{1}{16} i \alpha^2 \Gamma \left( f_{\mu \lambda} f_{\nu k} \gamma^\rho + \frac{1}{2} f_{\mu \nu} f_{k \lambda} \gamma^\rho a_\lambda + 2 f_{\mu \nu} f_{k \lambda} \gamma^\rho a_\lambda \\
+ 6 f_{a \mu} f_{\nu k} \gamma^\alpha a_\nu - 2 \partial_\mu f_{\kappa \lambda} \gamma^\alpha a_\nu a_\nu \\
- 2 \partial_\nu \partial_\alpha a_\alpha a_\mu \bigg). \tag{19}\]

Integrating over \( \psi, \), we can obtain the contributions to the effective action \( I^{(1)} \):
\[
e^{i I^{(1)}} = \int D\psi D\bar{\psi} \exp \left( i \int d^4 x \bar{\psi} \sum_{m,n} B^{(m,n)} \psi \right) \\
= \det \left( \sum_{m,n} B^{(m,n)} \right). \tag{21}\]

Then the effective action can be written as
\[
i I^{(1)} = \text{ln} \det (B^{(0,0)} + B^{(0,1)} + B^{(1,1)} + B^{(1,2)}) \\
+ B^{(2,1)} + B^{(2,2)} + B^{(2,3)}) \\
= \text{Tr} \ln (B^{(0,0)} + B^{(0,1)} + B^{(1,1)} + B^{(1,2)}) \\
+ B^{(2,1)} + B^{(2,2)} + B^{(2,3)}) \\
= \text{Tr} \ln (I - \text{im} \Box^{-1} \phi + B^{(0,1)} + B^{(1,1)} + B^{(1,2)}) \\
+ B^{(2,1)} + B^{(2,2)} + B^{(2,3)}) \phi) + \text{Tr} \ln (i - \text{im} \phi) \tag{22}\]

where \( \Box = -\partial^2 \). The last two terms are infinite constants and can be ignored in the effective action. The contribution term to the effective action is the first term in the final equation. We can calculate it by power expansion
\[
\text{Tr} \ln (I - \text{im} \Box^{-1} \phi + B^{(0,1)} + B^{(1,1)} + B^{(1,2)}) \\
+ B^{(2,1)} + B^{(2,2)} + B^{(2,3)}) \phi) = \text{Tr} \left( \sum_{n=1}^{\infty} \frac{-1)^{n+1}}{n} \left( \text{im} \Box + \sum_{j,k} \Box^{-1} B^{(j,k)} \phi \right)^n \right) \tag{23}\]

Using the above equation, we can calculate the divergent contributions to the gauge field propagators. For \( n = 1, \\text{Tr} \left( \Box^{-1} \phi^{(2,2)} \phi \right) = 0 \). The nonvanishing terms from the \( n = 2 \) case are:
\[
\text{Tr} \left( \Box^{-1} B^{(1,1)} \phi \Box^{-1} B^{(0,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \tilde{a}_\mu (-k) \tilde{a}_\nu (k) \left( \frac{4}{3} k_\mu k_\nu - k_\nu k_\mu \right) \tag{24}\]
\[
\text{Tr} \left( \Box^{-1} B^{(1,1)} \phi \Box^{-1} B^{(1,1,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{25}\]
\[
\text{Tr} \left( \Box^{-1} B^{(0,1)} \phi \Box^{-1} B^{(2,1)} \phi \right) + \text{Tr} \left( \Box^{-1} B^{(2,1)} \phi \Box^{-1} B^{(0,1)} \phi \right) \phi = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{26}\]

For \( n = 3 \), the divergent terms are as follows:
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(0,1)} \phi \Box^{-1} B^{(2,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{27}\]
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(1,1)} \phi \Box^{-1} B^{(1,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{28}\]
where \( [ABC]_A \) means the sum of all inequivalent arrangements of \( ABC \). For \( n = 4 \) case, the divergent contributions are
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(0,1)} \phi \Box^{-1} B^{(2,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{29}\]
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(1,1)} \phi \Box^{-1} B^{(2,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{30}\]
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(2,1)} \phi \Box^{-1} B^{(0,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{31}\]
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(2,1)} \phi \Box^{-1} B^{(1,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{32}\]
\[
\text{Tr} \left( \text{im} \Box^{-1} \phi \Box^{-1} B^{(2,1)} \phi \Box^{-1} B^{(2,1)} \phi \right) = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \gamma^\rho \phi \gamma^\lambda \tilde{a}_\nu (-k) \tilde{a}_\nu (k) \left( \frac{1}{2} m^2 k^2 k_\mu k_\nu \right) \tag{33}\]
For $n = 5$, we can obtain the nonvanishing contributions as follows:

$$\text{Tr} \left[ (-im)^{-1} \gamma^4 \left( -\Gamma^{-1}(2,1;i\theta) \right) \right]_A = \int \frac{d^4k}{(2\pi)^4} \partial_\mu \Gamma^{-1}(\theta) \vec{a}_\nu(k) \left( \frac{5}{2} m_4 k_\mu k_\nu \right). \quad (34)$$

$$\text{Tr} \left[ (-im)^{-1} \gamma^4 \left( -\Gamma^{-1}(B,1;i\theta) \right) \right]_A = \int \frac{d^4k}{(2\pi)^4} \partial_\mu \Gamma^{-1}(\theta) \vec{a}_\nu(k) \left( \frac{25}{2} m_4 k_\mu k_\nu \right). \quad (35)$$

For $n = 6$, the divergent contributions are

$$\text{Tr} \left[ (-im)^{-1} \gamma^4 \left( -\Gamma^{-1}(B,1;i\theta) \right)^2 \right]_A = \int \frac{d^4k}{(2\pi)^4} \partial_\mu \Gamma^{-1}(\theta) \vec{a}_\nu(k) \left( \frac{9}{2} m_4 k_\mu k_\nu \right). \quad (36)$$

$$\text{Tr} \left[ (-im)^{-1} \gamma^4 \left( -\Gamma^{-1}(B,1;i\theta) \right)^2 \right]_A = \int \frac{d^4k}{(2\pi)^4} \partial_\mu \Gamma^{-1}(\theta) \vec{a}_\nu(k) \left( \frac{3}{2} m_4 k_\mu k_\nu \right). \quad (37)$$

For $n = 7, 8, \ldots$, there is no divergent contribution. Sum up all the divergent contributions (24)–(37), we can obtain

$$i\Gamma^{(1)} = \int \frac{d^4k}{(2\pi)^4} \left[ \vec{a}_\mu(-k) \vec{a}_\nu(k) \frac{4}{3} \left( k^2 g_{\mu\nu} - k_\mu k_\nu \right) + \theta^{\mu\nu} \Gamma^{\kappa\lambda} \left( \frac{1}{60} \vec{a}_\nu(-k) \vec{a}_\lambda(k) k^4 k_\mu k_\kappa \right) \right]. \quad (38)$$

We can rewrite $i\Gamma^{(1)}$ in coordinate space

$$i\Gamma^{(1)} = \int \frac{d^4k}{(2\pi)^4} \left[ \frac{2}{3} k^\mu k^\nu f_{\mu\nu} \Gamma^{\kappa\lambda} \left( \frac{1}{240} f_{\mu\nu} \square^2 k_\lambda \right) + \frac{1}{48} \partial_\mu f_{\alpha\beta} \Gamma^{\kappa\lambda} \left( \square^2 f_{\alpha\beta} \right) \right]. \quad (39)$$

As we have already mentioned, we want to investigate the one-loop renormalizability of noncommutative QED at $\theta^2$-order. One main point in this investigation is to make use of the freedom in Seiberg–Witten map, which has been discussed in [11]. The freedom of Seiberg–Witten map allows field redefinitions. If $\hat{A}_\mu^{(n)}$ is a solution of Seiberg–Witten map at $\theta^\alpha$-order, then a gauge covariant term $\hat{A}_\mu^{(n)}$ of exactly $\theta^\alpha$-order can be added to $\hat{A}_\mu^{(n)}$ to form another solution

$$\hat{A}_\mu^{(n)} = \hat{A}_\mu^{(n)} + \hat{A}_\mu^{(n)}.$$  

These field redefinitions yield additional terms $\Delta S_n$ to the original action which can be used to absorb divergences. $\Delta S_n$ is of the form

$$\Delta S_n = \int d^4x \partial_\mu f^{\mu\nu} \hat{A}_\nu^{(n)}. \quad (41)$$

At $\theta^2$-order, there are four additional terms to the original action. In momentum space, these additional terms can be expressed as

$$\int \frac{d^4k}{(2\pi)^4} \partial_\mu \hat{a}_\nu(k) \Gamma^{\kappa\lambda} \left( k^2 g_{\mu\nu} - k_\mu k_\nu \right) k^4 a_\nu.$$  

$$\int \frac{d^4k}{(2\pi)^4} \partial_\mu \hat{a}_\nu(k) \Gamma^{\kappa\lambda} k_\nu a_\mu k_\kappa.$$  

$$\int \frac{d^4k}{(2\pi)^4} \partial_\mu \hat{a}_\nu(k) \Gamma^{\kappa\lambda} k_\nu a_\mu k_\kappa.$$  

$$\int \frac{d^4k}{(2\pi)^4} \partial_\mu \hat{a}_\nu(k) \Gamma^{\kappa\lambda} k_\nu a_\mu k_\kappa.$$  

It is easy to check that the $\theta^2$-order divergent contributions in (39) are of the forms (43) and (44). So the gauge field propagators are one-loop renormalizable at $\theta^2$-order in noncommutative QED with massive matter fields.

4. Conclusion

In this Letter, we investigate the renormalizability of gauge field propagators at $\theta^2$ order in noncommutative QED. We expand the noncommutative QED action to $\theta^2$ order. Using the background field method, we calculate the one loop divergent contributions of the fermion fields to the U(1) gauge field propagators.

We have proved that the gauge propagators are renormalizable at $\theta^2$ order no matter whether the fermions are massive or massless. If we only consider contributions from action of $\theta$-order as that in Ref. [12], there are non-renormalizable gauge field divergent terms of $\theta^2$-order when the fermion is massive.

From this result, we have more confidence in extrapolating that the gauge propagators are one loop renormalizable to all order in $\theta$ in noncommutative QED. In order to prove renormalizability at $\theta^n$ order, we should expand the action to $\theta^n$ and calculate effective action to $\theta^n$ order, of course, this is not an easy work.

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References