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Beam-column resistance interaction criteria for in-plane bending and compression

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Abstract

Present approach to design of steel I-section beam-columns distinguish two design situations of elements subjected to one directional bending and compression in which they are sensitive either to the flexural failure (second order in-plane bending and compression) or to the flexural-torsional failure (second order spatial bending, warping torsion and compression). This paper refers to the former case of member failure and aims in the development of finite element numerical modelling technique that would be useful for the verification of analytical resistance criteria of perfect beam-column elements subjected to different support and loading conditions. Impact of different boundary conditions and nonuniform bending on the member resistance is assessed and concluding remarks in relation to the proposed analytical formulation are drawn.

Keywords: Steel I-section; One directional bending and compression; Perfect beam-column; In-plane failure; Numerical FEM nonlinear stability analysis; Shell modelling; Beam modelling.

1. Introduction

The resistance evaluation of beam-column elements belongs to one of hot topics related to the load bearing capacity of steel structural elements. Different analytical and numerical approaches have been extensively used in the last several decades, first in relation to perfect beam-columns as presented by Chen and Atsuta [1], and more recently in relation to imperfect elements [2]. Formerly and latterly, the finite element method (FEM) is the most
widely used in the form of geometrically and materially nonlinear analysis (GMNA and GMNIA, respectively for perfect and imperfect structural geometry). The results of such analyses are the most important in creating the basic understanding of the behaviour of real beam-column elements and in the development of more sophisticated advanced analyses, the results of which might be directly introduced to new generation of design standards, such as to Eurocode 3 as presented by Simoes da Silva et al. [2]. Present design criteria of I-section beam-column elements in Eurocode 3 distinguish two design situations in which elements subjected to one directional bending and compression are sensitive either to the flexural failure without torsional deformations or to the flexural-torsional failure in which flexural in-plane deformations are associated with torsional deformations. The former case of member failure is relevant for I-section members bent either about minor principal axis or about major principal axis when the beam-column is fully restrained against lateral buckling and lateral-torsional buckling. The latter case of member failure is relevant to I-section beam-columns bent about strong principal axis that are insufficiently restrained against out-of-plane deformations.

This paper refers to beam-column elements fully restrained in the out-of-plane direction that are bent about major principal axis. The aim of this paper is the development of numerical modelling technique that would be useful for the verification of resistance criteria of perfect beam-column elements subjected to different bending conditions and beam-column static schemes. Numerical GMNA results made available through the FEM shell and beam modelling technique of steel I-section perfect beam-columns behaviour are compared with those from the proposed analytical formulation. The effects of different FEM modelling techniques on the plastic zone development and load bearing capacity with regard to elements of different degree of static indeterminacy are examined.

2. Assumptions and fundamental concept of analysis

2.1. Material model and method of stability analysis

The beam-column of HEB 300 wide flange I-section made of S235 steel grade is considered. The member section is of class 1 according to the classification of Eurocode 3 [2]. The material constitutive model of structural steel is the elastic-plastic one. The Huber-Mises yield condition and isotropic hardening are assumed since they are considered as being commonly used for steel. In the elastic range, the material behaviour is described by using the isotropic Hooke’s relationship with Young’s modulus $E$ equal to $210 \text{ N/mm}^2$ and Poisson’s ratio equal to 0.3. In the plastic range, the minimum ductility assumptions of Eurocode 3 [2] are adopted, concerning the ultimate strength $f_u = 1.1 \cdot f_y$ and corresponding ultimate strain $\varepsilon_u = 0.15$, all treated as nominal values.

Let us first take into consideration a perfect member made of adopted I-section and steel grade, being under pure compression, so that the beam-column element becomes a column element. Using the materially nonlinear analysis (MNA), the ultimate load of stocky column (of the slenderness ratio $\lambda_y \leq 0.2$) yields from the characteristic cross-section resistance $N_{c,Rk}$ equal to the squash resistance $N_{p,Rk} = A f_y$ for class 1-3 section members.

Furthermore, the characteristic cross-section resistance is equal to $N_{r,Rk} = N_{eff,Rk} = A_{eff} f_y$ for class 4 section members in compression [2]. Using the linear buckling analysis (LBA), the elastic flexural critical force $N_{cr} = N_{ip}$ of slender members is practically independent from the section class, however it strongly depends upon the column boundary conditions. The following Euler based relationship holds:

$$N_{ip} = \frac{\pi^2 E A}{\lambda_{ip}^2} = \frac{\pi^2 E A}{(L_{ip,cr} / i_{ip})^2} = \frac{\pi^2 E A}{(\mu_{ip} L / i_{ip})^2},$$

where $L_{ip,cr}$ is the buckling length from LBA and $\mu_{ip} = L_{ip,cr} / L$ is the buckling length factor dependent upon the member end boundary conditions.

A simplified approach to the ultimate strength evaluation of class 1-2 section columns uses MNA and LBA that entirely neglect the effect of coupling of prebuckling and postbuckling deformation states. Failure modes of inelastic column are therefore treated as independent ones and the nominal buckling strength can be evaluated as $N_{b,ip,Rk} = \min(N_{c,Rk} ; N_{ip})$. Thus, the buckling reduction factor is calculated as:
\[ \chi_{ip} = \frac{N_{b,ip,Rk}}{N_{c,Rk}} = \min \left( 1; \frac{N_{ip}}{N_{c,Rk}} \right) = \min \left( 1; \frac{1}{\lambda_{ip}} \right), \]

where the relative principal axis flexural slenderness \( \lambda_{ip} = \sqrt{N_{c,Rk}/N_{ip}} \).

In stability analysis with nonlinear geometry and nonlinear relationship between stress and strain adopted in this paper, the full coupling between the prebuckling and postbuckling deformation states is considered. To solve nonlinear boundary value problems of buckling, ABAQUS program is used, in which the theory of moderately large deformation (large displacement and moderate rotation approach) is implemented and available through the option NLGEOM [3,4].

2.2. Considered member slenderness, support conditions and load cases

The beam-column of HEB 300 wide flange I-section made of S235 steel grade is considered with different lengths corresponding to four different values of the relative major principal axis flexural slenderness \( \lambda_y \), namely, 0.5; 1.0; 1.5 and 2.0. Three different support conditions are considered for the study carried out in this paper, namely, the statically determinate ones (Fig. 1a, c) and the statically indeterminate to one degree (Fig. 1b).

![Diagrams of support conditions and load cases](image)

Fig. 1. Considered support conditions and load cases of the beam-column HEB300; (a) H-H support conditions; (b) F-H support conditions; (c) F-FE support conditions.
The statically determinate, simply supported beam-column is loaded with the $P_{x,d}$ force and the bending moment of the maximum value $M_{y,d} = M_{y,d,max}$ applied to the hinged roller support. Three load cases of the moment $M_{y,d,min}$ application at the immovable hinged support are considered hereafter. They are indicated by the factor $\psi = M_{y,d,min} / M_{y,d,max}$, namely, (+1.0); 0.0 and (-1.0). The first one, denoted by SM is referred to the symmetrical internal first order moment diagram $M_{y,Ed}'$ corresponding to the constant value of $M_{y,d}$, however the second one, denoted by TM – to the triangular diagram of the first order moment, $M_{y,Ed}' = (3xM_{y,d}) / L$ and the third one, denoted by AM – to the antisymmetric internal moment diagram $M_{y,Ed}' = (2x / L - 1)M_{y,d}$. For all these three load cases of the simply supported beam-column, the critical force is calculated for the flexural buckling length $L_{y,cr}$ equal to the system length $L$, i.e. for the flexural buckling length factor $\mu_y = L_{y,cr} / L = 1.0$.

The statically determinate, cantilever element is referred to the load case of a $P_{x,d}$ force and the bending moment $M_{y,d}$ applied to the free end. This case corresponds to the first order internal moment being constant throughout the member. The critical force is calculated for the flexural buckling length $L_{y,cr}$ equal to the doubled system length $L$, i.e. for the flexural buckling length factor $\mu_y = L_{y,cr} / L = 2.0$.

The statically indeterminate element is referred to the load case of a $P_{x,d}$ force and the bending moment $M_{y,d}$ applied to the hinged roller support. This case corresponds to the linearly changing first order internal moment with the factor $\psi = -0.5$; i.e. $M_{y,Ed}' = 0.5(3x / L - 1)M_{y,d}$ . The critical force is calculated for the flexural bucking length $L_{y,cr}$ equal to approximately 0.7 of the system length $L$, i.e. for the flexural bucking length factor $\mu_y = L_{y,cr} / L = 0.7$.

It is worthy to note that because of the geometrically nonlinear analysis, the equilibrium conditions are set in the member deflected configuration, i.e. the $P-$δ and P-Δ effects are automatically considered in the analysis. The former effect is relevant to nonsway members (see Fig. 1a, b) while both effects need to be considered for sway members (see Fig. 1c). Because of the mentioned above effects, the second order internal moment $M_{y,Ed}''$ differs from that of the first order one, and as a result its maximum value $M_{y,Ed,max}$ is not at the same section along the member length as identified by $M_{y,Ed,max}'$ for the first order internal moment diagram. This is clearly shown in Fig. 1 for all the considered beam-column support conditions and load cases.

3. Analytical formulation for codification purposes

Since in design practice the structural analysis is nowadays most likely to be performed using geometrically nonlinear elastic analysis (GNA), local design criteria of beam-column elements should involve second-order stress resultants. It allows to develop a simple analytical ultimate limit state criterion used hereafter that can be written down as follows:

$$\frac{m_{y,ip}''}{m_{N,ip}} = 1$$  

(3)

where $m_{y,ip}'' = M_{y,Ed,max}'' / M_{c,Rk}$ is the normalized second-order moment and $m_{N,ip} = M_{N,ip,Rk} / M_{c,Rk}$ is the normalized member in-plane moment resistance reduced in the presence of compressive axial force and buckling. For the member resistance of class 1 and 2 section beam-columns bent about the major principal axis $y$, the normalized moment resistance $m_{N,ip} = m_{N,y}$ may be written down as follows:

- when $0.5a \leq n_{h,y} \leq 1$, $a = 1 - \frac{2ht_j}{A} \leq 0.5$ :

$$m_{N,y} = 1 - \chi_y \frac{n_{h,y} - 0.5a}{1 - 0.5a}$$  

(4)
- and when \( 0 \leq n_{h,z} \leq 0.5 \alpha \) then \( m_{N,z} = 1 \).

For the member resistance of beam-column elements of class 1 and 2 sections, bent about the minor principal axis \( z \), the normalized moment \( m_{N,ip} = m_{N,z} \). For elements bent in the curvature corresponding to the lowest buckling mode, the normalized moment \( m_{N,ip} = m_{N,z} \) may be written down as follows:

- when \( a \leq n_{h,z} \leq 1 \):

\[
m_{N,z} = 1 - \left( \frac{n_{h,z} - a}{1 - a} \right)^2
\]

- and when \( 0 \leq n_{h,z} \leq a \) then \( m_{N,z} = 1 \).

For class 3 and 4 section members (for \( ip = y, z \)):

\[
m_{N,ip} = 1 - n_{h,ip} \chi_{ip}.
\]

In the above relationships, and in the following, the indicator \( n_{h,y} = \frac{N_{\max,Ed}}{N_{h,y,Rk}} \) where \( N_{\max,Ed} \) is the axial force corresponding to the ultimate limit state of the beam-column subjected to combined loading of \( P_{x,d} \) and \( M_{y,d} \). It is worthy of noticing that for the buckling reduction factor \( \chi_{ip} \) equal to unity, Eq. (3) is the ultimate limit state criterion for the cross-section resistance verification that is based on the reduced cross-section moment resistance of the beam-column element (see criteria adopted in Eurocode 3 \([2, 5]\)). For all boundary and loading conditions, an eventually required correction factor needs to be introduced through the processes of verification and calibration.

4. Resistance of beam-column elements from numerical simulations

In the following, the resistance of perfect beam-column elements bent about the major principal axis \( y \) is considered for which the results of FEM numerical simulations in reference to different finite element (FE) models, namely, shell and beam models (denoted FE BM and FE SM, respectively) and ABAQUS software are presented.

4.1. Beam strength under in-plane moment and column flexural buckling strength

Let us first present the results for two extreme cases of beam-column loading conditions, namely, \( M_{x,d} \) and \( P_{x,d} \) acting alone. For class 1-2 section beam-columns bent without any axial compressive force, the beam ultimate limit state corresponds to the full yielding state of the most stressed section, i.e. \( M_{ip,Ed} = M_{c,ip,Rk} \) where \( M_{c,ip,Rk} = M_{pl,ip,Rd} \). For beam-columns subjected to pure compression without any bending moment, the column ultimate limit state corresponds to buckling and it is dependent upon the boundary conditions of considered element (see Fig. 1a, b, c), i.e. \( N_{Ed} = N_{h,ip,Rk} \) where \( N_{h,ip,Rk} = X_{ip} N_{pl,ip,Rd} \). The results of the latter case are presented in Fig. 2 graphically in the form of relationship of the buckling reduction factor \( \chi_y \) where \( \chi_y = \frac{N_{h,y,Rk}}{N_{pl,Rk}} \) and the relative member slenderness \( \lambda_y \), in discrete points of 0.5; 1.0; 1.5 and 2.0 (in Fig. 2a for FE BM - the finite element beam model and in Fig. 2b for FE SM - the finite element shell model). The solid lines represent the linear buckling solution (Euler's hyperbola) and the point results are those obtained numerically using nonlinear stability theory and the option NLGEOM in ABAQUS \([3, 4]\). Square-shaped points represent the results for H-H boundary conditions, triangle-shaped – the results for F-H boundary conditions and finally circular-shaped – the results for F-FE boundary conditions.
From presented results it is clear that for columns of the section class that is insensitive to local buckling, as for the considered HEB 300 section, there is a negligible difference between the results obtained with use of different finite element modelling techniques. The most important conclusion is that for slender columns (of the slenderness ratio greater than or equal to 1.5) the numerical results correspond fully to those evaluated from linear stability theory (from LBA) and for stocky columns (of the slenderness ratio below 0.5) – to the squash load (from MNA). Members of the slenderness ratio the value of which is around unity seem to be the most sensitive to structural anisotropy resulting from coupling of twofold nonlinear effects, namely material nonlinearity (plasticity) and geometric nonlinearity (large prebuckling displacements) on the equilibrium state. These effects are so important that they cannot be omitted in the slenderness range corresponding to a discontinuity of failure mechanisms evaluated from simple analyses of MNA and LBA. Furthermore, the above identified range of the slenderness ratio claims to be the most sensitive to the influence of imperfections that reduce the resistance from the level associated with perfect columns to that corresponding to real columns [6]. The degree of resistance reduction is dependent upon the technological processes of member production and assembling that are associated with different imperfections (see [2] for details in reference to steelwork Eurocodes).

4.2. Beam-column strength for different support conditions and loading cases

The following figures present the results of numerical simulations, illustrating graphically the relationship between the normalized second-order moment $M_{\text{y,Ed,max}}/M_{p,y,Rd}$ and the maximum axial force corresponding to the ultimate limit state normalized with reference to the buckling strength $n_{h,y} = N_{\text{max,Ed}}/N_{h,y,Rd}$. Three different loading cases are considered for simply supported beam-columns. Fig. 3 presents the results for two single curvature loading cases, namely, in Fig. 3a for SM – the results for the symmetric first order bending moment diagram, while in Fig. 3b for TM – the results for the nonuniform, first order triangular bending moment diagram.

In Fig. 3a, the results of application of two finite element models, namely, FE BM given by solid line and FE SM given by dashed line are presented. In the application of FE BM, the second order moment $M_{y,Ed,max}$ was obtained directly from the finite element analysis (FEA). In the application of FE SM, the deflection of middle length point of the member $x$ axis was monitored throughout the whole incremental analysis in order to get the member maximum transverse displacement, responsible for the P-δ effect. The maximum second order moment was then calculated with use of a simple set of equilibrium equations established for the beam-column half-length part. Comparison of results corresponding to FEA using FE BM and FE SM simulations brings us to the conclusion that differences are negligible from the engineering point of view. One may suggest that the beam finite element model may be efficiently used for the evaluation of beam-column resistances in case of sections not sensitive to the local buckling phenomenon. Fig. 3b shows therefore only the results based on FE BM simulations.

Fig. 4 presents the results for the other two boundary cases of beam-column elements in which the first buckling mode corresponds to the deformation state under applied moment. The results for the beam-column with F-H
boundary conditions and the nonuniform first order bending moment diagram are given in Fig. 4a. The results are based on the FE BM simulations only. Fig. 4b presents the results for F-FE boundary conditions when using two finite element models, namely, FE BM given by solid line and FE SM given by dashed line. The similar conclusions can be drawn as presented earlier for the boundary and loading conditions shown in Fig. 3a.

![Fig. 3. Results of beam-column strength simulations for HEB 300 section for two loading cases; (a) SM - two equal end moments; (b) TM - one end moment.](image)

![Fig. 4. Results of beam-column strength simulations for HEB 300 section under end moment; (a) F-H end conditions; (b) F-FE end conditions.](image)

![Fig. 5. Results of beam-column strength simulations for HEB 300 section for H-H support conditions and two antisymmetric end moment.](image)

![Fig. 6. Verification of analytical formula for beam-column resistance evaluation.](image)

Fig. 5 presents the results for a double curvature loading case, namely, for the beam-column with boundary conditions corresponding to the simply supported element and for the AM loading case (antisymmetric nonuniform
bending moment diagram with the deformation shape corresponding to the second lowest buckling mode, instead of that corresponding to the critical state). Only FE BM simulations are presented in this case.

For the beam-column relative slenderness ratio less than approximately 2.0 (definitely below 1.5 where the structural anisotropy does not play an important role in case of design based on the bifurcation load corresponding to the antisymmetric buckling mode), the slenderness dependent resistance curves \( m_{N,y} \) start from \( m_{N,y} = 0.0 \), regardless the value of slenderness ratio. This is in contrast to the other loading and boundary conditions in which the resistance curves start from \( m_{N,y} = 1 - \chi_y \) with the buckling reduction factor corresponding to the critical load (see Figs. 3 and 4). This is indicative to the range of slenderness in which the elastic buckling according to the lowest mode is not possible. The slenderness ratio \( \lambda_y = 2.0 \) corresponds in simply supported beam-columns under antisymmetric bending moment diagram to the equalization of the squash load and the elastic bifurcation load corresponding to the antisymmetric buckling mode. One could expect in this situation that the moment resistance \( m_{N,y} = 1 - \chi_y \) obtained numerically for \( n_{by} = 1.0 \) would be of the similar value like in all other cases considered in Figs. 3 and 4 for the threshold slenderness ratio \( \lambda_y = 1.0 \). The obtained results confirm the above presumable statement. For simply supported elements bent in a double curvature, for the slenderness \( \lambda_y = 2.0 \) and greater ones, up to that corresponding to the successive bifurcation mode, Eq. (4) still holds providing that the buckling reduction factor corresponds to the bifurcation load of the antisymmetric buckling mode. This situation periodically repeats for the higher slenderness ratio ranges and the achievement of the successive bifurcation loads need to be included for the calculation of \( \chi_y \). Practical importance of higher values of slenderness ratio than those in the range \( \lambda_y \leq 2.0 \) is however questionable since so high values of the slenderness ratio are unacceptable in steel frameworks construction.

5. Verification of analytical formulation of beam-column resistance and final remarks

All the cases considered so far dealt with different loading and boundary conditions. Practical assessment of member strength may be done by adopting the analytical formulation given in section 3. Fig. 6 presents the verification of analytical formula (3) of the beam-column strength evaluation using numerical results from the beam model finite element simulations presented in Figs. 3, 4 and 5. The vertical coordinate corresponds to the finite element resistance \( m_{N,y,FEA} \) while the horizontal one corresponds to \( m_{N,y,post} - \) the resistance postulated by use of Eq. (4). The buckling reduction factor \( \chi_y \) is calculated according to Eq. (2) for \( ip = y \) (considered plane of bending) and for the buckling mode corresponding to the member deflected profile under specific loading case. The average of \( m_{N,y,FEA} / m_{N,y,post} \) is equal to 0.99 with the coefficient of variation of 4.3%.

Concluding, the proposed analytical criterion given by Eq. (3) is useful in a quick verification of the beam column resistance. However, a more accurate estimation requires to introduce a correction factor that would be dependent upon the member slenderness.

References