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Interaction of Connected Single-Degree-of-Freedom Systems

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Abstract

In most industrial complexes many structures are connected to each other by secondary elements, since the connections between main structures are carried out by these secondary elements, modeling of links is relinquished in primary and also sometimes main seismic analysis, so their effects on the main structures and elements are not considered. While this negligence will be associated with some errors, modeling a whole set of adjacent-connected structures seems to be impractical in any case. This research investigates the interaction effect between two adjacent and connected single-degree-of-freedom systems through a study of basic parameters and shows the limitations and range of errors in current disconnected analysis practice. As a result of this study engineers can identify when their current analysis with separating structures would be reliable and when accurate analysis should be used. Also, practical graphs are presented making possible to do the analysis separately for each structure and modifying the maximum response using the correction factors given.

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1 INTRODUCTION

In industrial complexes, structures, elements and different equipments with variety stiffness, mass, figure, Degree of Freedom (DOF) and ductility are stand adjacent each other and in many cases these structures are connected to each other by variety connectors, for instance pipes, connecting beams, conductors or other similar links. In primary seismic analysis the effects of these connectors are

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relinquished and each structure is analyzed separately. Although these simplifications are sufficed for primary intent, for accurate analysis and study of seismic behavior, it should be necessary to identify the interaction of Adjacent and Connected Structure (ACS).

Field investigations after earthquakes (Benuska L 1990; Hall J 1995) have revealed that dynamic interaction between the connected equipment items may be responsible for much of the observed damage in electrical substations and similar installations.

Interaction of structure with the vicinity always affects the particular structure behavior. In case of ACS behaviors of particular structure is dependent on adjacent structure and their connecting. For seismic analysis of each structure it should be studied both structures as a connected system.

Practically, connecting of two structures influences the natural frequency of system and the final response. So, maybe the real final response is bigger than it was analyzed separately and so the system would be failed because of loose capacity.

According to following causes, calculating of response spectra for ACS has some problems:

1. Modeling of whole set of connected equipments most often is not practical because of Irregularity, unclearness and complexity in distribution of structures and installations.
2. In many cases, supports of an equipment base on two different structures or on two levels of one. Hence, the input spectra of each support maybe differ from the other.
3. Duo to the difference between R-factors of two adjacent structures, common software can't analyze correctly.

In order to develop a thorough understanding of the effect of dynamic interaction, (Der Kiureghian et al. 1999) carried out extensive parametric studies for two interconnected equipment item with one input spectrum for both equipments using linear, single DOF models for the equipment and linear elastic model for the connector in power plants. Later, a theoretical model was developed to describe the highly non-linear moment-curvature relationship of conductor cables and to investigate the effect of interaction in equipment items connected by flexible bus (Hong et al. 2001; Hong 2003; Hong et al. 2005), another study (Song et al. 2006) investigates the interaction effect between electrical substation equipment items connected by non-linear rigid bus conductors.

This research investigates the interaction effects between two adjacent and connected single DOF systems through a study of basic parameters and shows the limitations and range of errors in current disconnected analysis practice. So, it can be identified when current analysis with separating structures would be reliable and when accurate analysis should be used. Also, practical graphs are presented making possible to do the analysis separately for each structure and modifying the maximum response using the correction factors given.

2 ASSUMPTIONS AND INTRODUCTION OF BASIC PARAMETERS

In a general condition, two equipments with two masses have been jointed together by a connector member. The connector member is linked to interfaces masses in each equipment or structure. Assumption to consider the masses in lumped form is possible for any equipment and to withdraw the connector mass against the lumped masses. Equipment can be located in different levels of structures and connected to each other (Fig. 1).

Another important point that should be noted is the number of DOF in each mass. Basically, in each three dimensional system, each mass have three transitional DOF and three rotational DOF. In two dimensional spaces the whole number of dynamic DOF system is limited to six, and contribution of each mass from this number is two transitional DOF in two perpendicular axes in a plate and one rotational DOF around the vertical axis on the plate (Fig. 2).

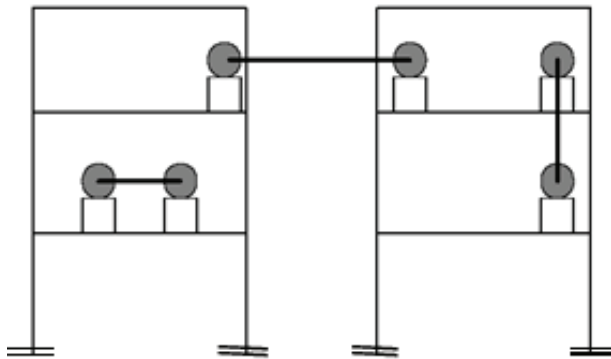


Figure 1: Different placement of two ACS

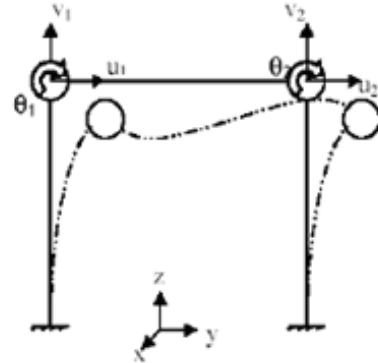


Figure 2: Degrees of freedom of system

Since in most industrial structures, the base of structure has a considerable axial stiffness, then because of very small displacement in the system we can withdraw the transitional freedom in vertical axis and rotational freedom of each mass. Thus, the dynamic DOF will be reduced to two degrees in horizontal axis, as in Fig. 3.

In Fig. 3 the masses (M_1, M_2) are assumed as concentrated in two blocks. The rollers had restricted the blocks in a way that they can have only a simple horizontal displacement. Then the two displacements coordinate (u_1, u_2) can explain the masses condition completely. The linear elastic resistance has been provided against the horizontal displacement by three mass-less springs (K_1, K_2, K_L) and the energy damping has been provided by C_1, C_2 and C_L . In this system the external time dependent loads (F_1, F_2) force the masses for dynamic response.

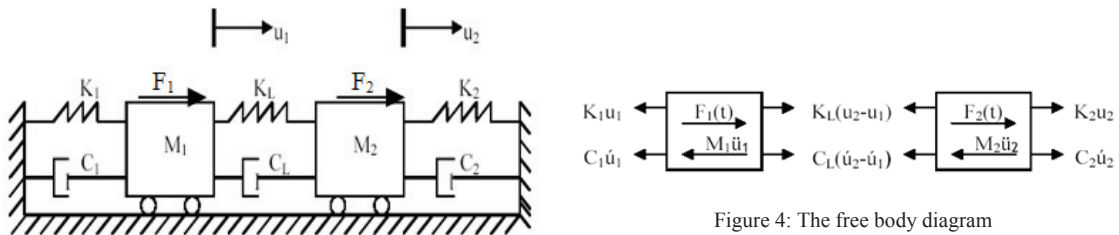


Figure 3: Equivalent spring-mass-damper system

Figure 4: The free body diagram

3 EQUATIONS OF MOTION AND NATURAL FREQUENCIES

The formulation of the case could be done by giving the direct dynamic equilibrium of all acting forces on the masses. The forces acting on the masses at some instant of time are shown in Fig.4. These include the external forces $F(t)$, the elastic (or inelastic) resisting forces ku , the damping resisting forces $C\dot{u}$, and the inertia forces $m\ddot{u}$. The equation of motion can be written by dynamic equilibrium compactly in matrix form:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \tag{1}$$

by introducing the following notation:

$$[M] = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}, [C] = \begin{bmatrix} C_1 + C_L & -C_L \\ -C_L & C_2 + C_L \end{bmatrix}, [K] = \begin{bmatrix} K_1 + K_L & -K_L \\ -K_L & K_2 + K_L \end{bmatrix} \quad (2a,b,c)$$

$$\{u\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad \{\dot{u}\} = \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} \quad \{\ddot{u}\} = \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} \quad \{F(t)\} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} \quad (2d,e,f,g)$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the lateral stiffness matrix, $\{F(t)\}$ vectors, indicates the value of forces to the masses, $\{u\}$ vectors, and its first and second derivatives are showing the value of displacement, velocity and acceleration of mass, respectively.

Setting $F(t)=0$ gives the differential equation governing free vibration of the system, which for systems without damping ($c=0$) specializes to:

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \quad (3)$$

The solution to the homogeneous differential equation is obtained by standard methods:

$$\{u\} = \{a\} \sin(\omega t - \alpha) \quad (4)$$

where; a_i is the amplitude of motion, n is the numbers of DOF, α is the phase angle factor and ω is the natural frequency of system.

By substituting Eq. (4) into Eq. (3) result in a form of the eigenvalue equation is obtained:

$$[[K] - \omega^2[M]] \cdot \{a\} = 0 \quad (5)$$

Eq. (5) is a homogeneous arithmetic system equation, with n unknown quantity a_i and the unknown quantity parameter of ω^2 . The classical solution to the above equation derives from the fact that in order for a set of homogeneous equilibrium equations to have a nontrivial solution, the determinant of the coefficient matrix must be zero:

$$\det([K] - \omega^2[M]) = 0 \rightarrow \det \begin{bmatrix} K_1 + K_L - \omega^2 M_1 & -K_L \\ -K_L & K_2 + K_L - \omega^2 M_2 \end{bmatrix} = 0 \quad (6)$$

Determinant expansion will result to solve the system characteristic equation from level 2 according to ω^2 (Eq. 7).

$$\omega^4 - \left(\frac{K_L}{M_2} + \frac{K_1}{M_1} + \frac{K_2}{M_2} + \frac{K_L}{M_1} \right) \omega^2 + \left(\frac{K_1}{M_1} \cdot \frac{K_2}{M_2} + \frac{K_1}{M_1} \cdot \frac{K_L}{M_2} + \frac{K_2}{M_2} \cdot \frac{K_L}{M_1} \right) = 0, \quad M_1, M_2 \neq 0 \quad (7)$$

With result of Eq. (7) two measures for ω^2 will be found in Eq. (8).

$$\omega_i^2 = \frac{1}{2} \times \left(\left[\frac{K_1}{M_1} + \frac{K_2}{M_2} + \frac{K_L}{M_1} + \frac{K_L}{M_2} \right] + (-1)^i \times \sqrt{\left[\frac{K_1}{M_1} - \frac{K_2}{M_2} + \frac{K_L}{M_1} - \frac{K_L}{M_2} \right]^2 + \frac{4K_L^2}{M_1 M_2}} \right) \quad (8)$$

In this equation; i , is the number of modes and it is equal to 1 for first mode and 2 for second mode and thus the relation $\omega_1 < \omega_2$ is established permanently. Equation (8) is arranged in a way that K_1 and M_1 are the parameters of more flexible structure and K_2 and M_2 are the parameters of more rigid structure. It means that the more rigid structure is numbered with 2 and the other one is numbered with 1.

4 DIMENSIONAL RELATIONS OF FREQUENCY

Through dividing the Eq. (8) for $i=1$ by $\omega_1^2 = K_1 / M_1$ a dimensionless equation (Eq.9.A) is obtained and in a same condition by dividing Eq. (8) for $i=2$ by $\omega_2^2 = K_2 / M_2$ another dimensionless equation is obtained (Eq. 9.B):

$$\Omega_1 = \frac{1}{2} \times \left([1 + mk + s_1 + ms_1] - \sqrt{(1 - mk + s_1 - ms_1)^2 + 4ms_1^2} \right) \tag{9.A}$$

$$\Omega_2 = \frac{1}{2} \times \left(\left[\frac{1}{mk} + 1 + \frac{s_2}{m} + s_2 \right] + \sqrt{\left(\frac{1}{mk} - 1 + \frac{s_2}{m} - s_2 \right)^2 + \frac{4s_2^2}{m}} \right) \tag{9.B}$$

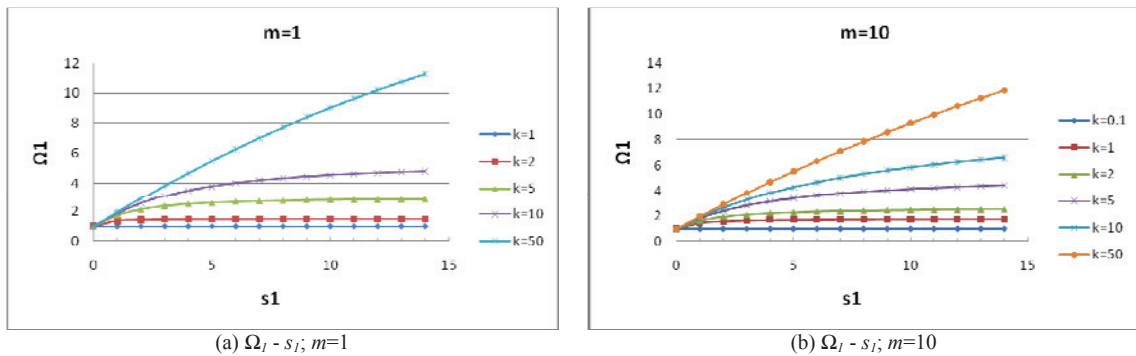


Figure 5: Variation of first mode frequency ratio with rigidity ratio of structures with m=1 and 10

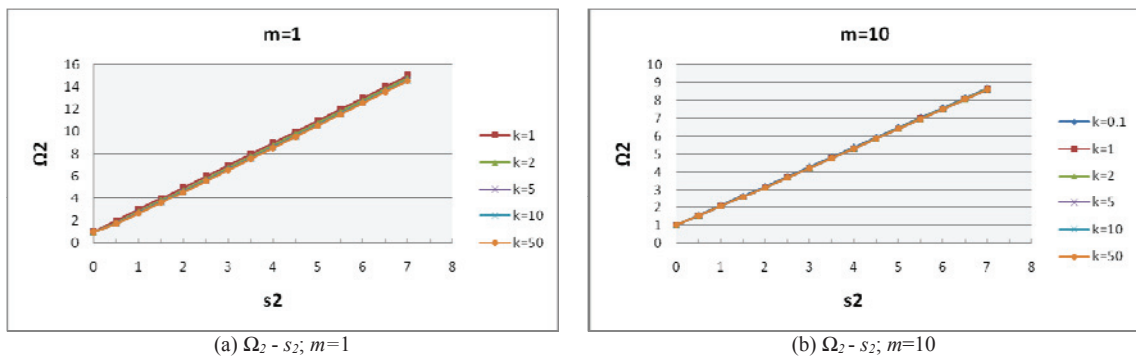


Figure 6: Variation of second mode frequency ratio with rigidity ratio of structures with m=1 and 10

The parameters of Ω_1 , Ω_2 , s_1 , s_2 , m , and k are positive dimensionless values; where:

$$\Omega_1 = \left(\frac{\varpi_1}{\omega_1} \right)^2, \Omega_2 = \left(\frac{\varpi_2}{\omega_2} \right)^2, s_1 = \frac{K_L}{K_1}, s_2 = \frac{K_L}{K_2}, m = \frac{M_1}{M_2}, k = \frac{K_2}{K_1}, mk = \left(\frac{\omega_2}{\omega_1} \right)^2 \tag{10}$$

In figures (5) and (6) the variation of Ω_1 and Ω_2 are given with variety value of $m \cdot k$, s_1 and s_2 .

As noticed, for different values of m , by increasing the values of s_1 , s_2 and k , the values of Ω_1 and Ω_2 also increase. In other words, in range of function, the Ω_1 and Ω_2 are ascending functions and their quantities always equal or larger than 1. Charts are also compatible with the physics problem, example of extreme case $K_L = 0$ shows that the amounts of structural frequencies ratios are equal to 1 ($\Omega_1=1, \Omega_2=1$).

Also, it is recognized the variation of rigidity ratio and mass ratio of both structures to first system mode has more influence and in ratio of second system mode, only the value of mass ratio has more influence.

5 DIMENSIONAL RELATIONS OF MODE SHAPES

With solving the Eq. (5) for values of Eq. (8) with $i=1$ and $i=2$, after simplifying of relation and its expansion to a dimensionless shape the Eq. (11.A) and (11.B) will be found, respectively:

$$\frac{a_{11}}{a_{21}} = \frac{1}{s_2} + 1 - \frac{1}{2} \left[\left(\frac{1}{mks_2} + \frac{1}{s_2} + \frac{1}{m} + 1 \right) - \sqrt{\left(\frac{1}{mks_2} - \frac{1}{s_2} + \frac{1}{m} - 1 \right)^2 + \frac{4}{m}} \right] \tag{11.A}$$

$$\frac{a_{22}}{a_{12}} = \frac{1}{s_1} + 1 - \frac{1}{2} \left[\left(\frac{1}{s_1} + \frac{mk}{s_1} + 1 + m \right) + \sqrt{\left(\frac{1}{s_1} - \frac{mk}{s_1} + 1 - m \right)^2 + 4m} \right] \tag{11.B}$$

The parameters s_1, s_2, m and k are positive and dimensionless and their values are given in Eq. (10) also the first and second index of a represent the number of structures and modes respectively.

Similar to previous section for different parameters, the charts of mode shapes ratio are traced against the rigidity ratio in figures (7) and (8) for first and second mode of system, respectively.

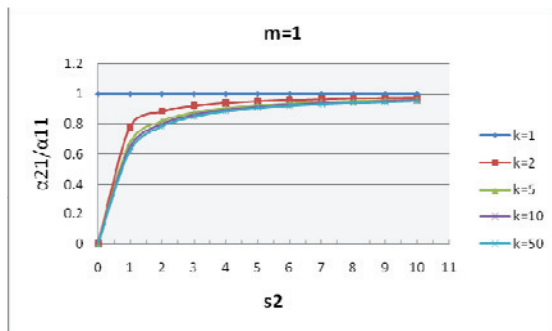


Fig.7.a: $\alpha_{21}/\alpha_{11} - s_2; m=1$

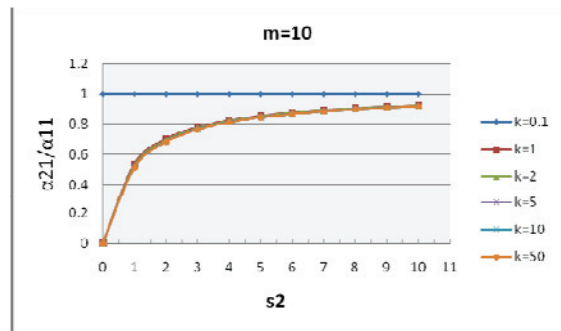


Fig.7.b: $\alpha_{21}/\alpha_{11} - s_2; m=10$

Figure 7: Variation of first mode shapes ratio with rigidity ratio of structures with $m=1$ and 10

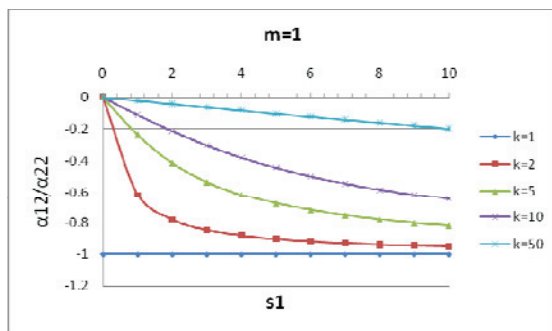


Fig.8.a: $\alpha_{12}/\alpha_{22} - s_1; m=1$

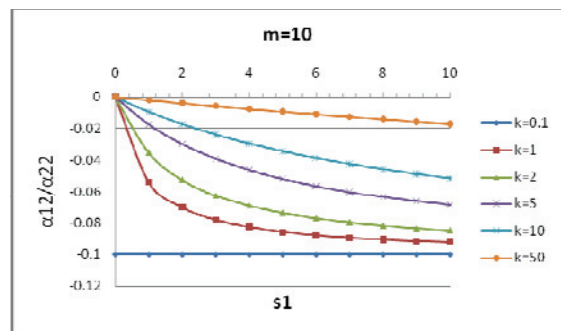


Fig.8.b: $\alpha_{12}/\alpha_{22} - s_1; m=10$

Figure 8: Variation of second mode shapes ratio with rigidity ratio of structures with $m=1$ and 10

As it is seen, in first mode with increase of rigidity ratio and increase of two structures mass ratio, the mode shape of both structures are approximating to each other. Nevertheless, in second mode, the mode shapes ratios of systems are completely different and have no compatibility to each other.

6 RESPONSE SPECTRA

According to principles of dynamic of structures and with having the input spectra of each support, the response to maximum displacement value of each mass will be evaluated with using the square root of sum of squares (SRSS) method in structure number i as below:

$$u_{i \max} = \sqrt{\sum_{j=1,2} (a_{ij} \gamma_j \bar{S}_{dj})^2} \quad (12)$$

In Eq. (12), i denotes as the number of structure, j indicates the mode number of system, \bar{S}_{dj} is the support displacement spectrum, $\bar{\omega}_j$ shows the frequency of system in j mode, a_{ij} is the mode shapes values according to Eq. (11) and γ_j is the participation factor in j mode according to Eq. (13).

$$\gamma_j = \frac{M_1 a_{1j} + M_2 a_{2j}}{M_1 a_{1j}^2 + M_2 a_{2j}^2} \quad (13)$$

If the condition is in a way that the values of Ω_1 and Ω_2 (Eq. 9) trended to number 1, it means connector member is so flexible that according to figures (5) and (6) for all values of a_{11} and a_{22} , the value of mode shapes of a_{12} and a_{21} are in order values of approximately zero. Then, by Substituting into Eq. (13), participation factors are:

$$\gamma_1 = 1/a_{11}, \gamma_2 = 1/a_{22} \quad (14)$$

By substituting Eq. (14) into Eq. (12); results are obtained:

$$u_{1 \max} = |\bar{S}_{d1}|, u_{2 \max} = |\bar{S}_{d2}| \quad (15)$$

Regarding the definition of spectrum, value of a distinct spectrum from equal frequencies will be equal. It means when $\bar{w}_j = w_i$ then $\bar{S}_{dj} = S_{di}$, so by noticing Eq.(15) the system response is equal in both connected and individual condition. In other words, the connected structures may be analyzed alone if:

$$\Omega_1 \rightarrow 1 \& \Omega_2 \rightarrow 1 \Leftrightarrow u_{1 \max} = v_1, u_{2 \max} = v_2 \quad (16)$$

where; v_1 and v_2 are the response of first and second structures under an individual analysis.

Now, with increase of frequency of each structure to $\Omega_i^{1/2} w_i$, the frequencies can be modified in a way that its main response to the basic input spectrum becomes equal to the system response in the related mode. Then we can calculate each structure modified frequency with help of equations (9) and (10):

$$\bar{w}_j^2 = \Omega_i w_i^2 \rightarrow \bar{w}_j = \Omega_i^{1/2} w_i \Rightarrow \bar{w}_1 = \Omega_1^{1/2} w_1 \text{ and } \bar{w}_2 = \Omega_2^{1/2} w_2 \quad (17)$$

After the modified frequencies are gained from Eq. (17) responses are calculated by Eq. (12).

7 THE INDIVIDUALLY ANALYZING LIMITATIONS

Regarding the described cases for detachment of the connected structures, the range of changes which caused the necessary condition of Eq. (16) should be determined. The influencing parameters in this relation were distinguished before (Figure 5 and 6).

Although the situation $\Omega=1$ seldom be happen, in practical condition, some error should be accepted and assume Ω close to 1. If the inverse Eq. (9.A) is written related to s_1 as Eq. (18):

$$s_1 = \frac{\Omega_1^2 - (1 + mk)\Omega_1 + mk}{(1 + m)\Omega_1 - (1 + k)m} \quad (18)$$

Then in ideal case, by substituting $\Omega_1 = 1 + \varepsilon$ in to Eq. 18:

$$s_1 = \frac{(1 + \varepsilon)^2 - (1 + mk)(1 + \varepsilon) + mk}{(1 + m)(1 + \varepsilon) - (1 + k)m} \quad (19)$$

When ε is very small, then ε^2 will be ignored. Therefore, Eq. (19) will be:

$$\varepsilon_1 = \frac{s_1}{1 - \frac{1+m}{(1-mk)} s_1}, mk > 1, \varepsilon \geq 0 \quad (20.A)$$

$$s_1 = \frac{\varepsilon_1}{1 + \frac{1+m}{(1-mk)} \varepsilon_1}, mk > 1, \varepsilon \geq 0 \quad (20.B)$$

For the second structure similar equations will be obtained from Eq. (9.B):

$$\varepsilon_2 = \frac{s_2}{1 - \frac{(1+1/m)}{(1-1/mk)} s_2}, mk > 1, \varepsilon \geq 0 \quad (21.A)$$

$$s_2 = \frac{\varepsilon_2}{1 + \frac{(1+1/m)}{(1-1/mk)} \varepsilon_2}, mk > 1, \varepsilon_2 \geq 0 \quad (21.B)$$

In Eq. (20.A) and Eq. (21.B) the value of error in separate analysis are calculated. If this error could be acceptable, separate analysis can be used instead of exact analysis. Also, in using Eq. (20.B) and Eq. (21.B) with determining the allowable limits of " ε ", the maximum values of s_1 and s_2 for individual analysis will be obtained.

In this study, tables of s_1 and s_2 are presented for 10% error. Table (1) and (2) respectively show the maximum values of s_1 and s_2 that for smaller values of it the system can be modeled separately.

Considering the condition that the natural frequencies of structures No.1 is smaller than No.2, the limit groups that contravene this condition, are shown on the tables marked with NA (Not Applicable). Also under the condition that the value of $mk=1$ or tends to it, error values in Eq. (20.A) and (21.A) are equal to zero or approximate to it and therefore there is not any limitations on s_{1max} and s_{2max} . These cases are shown on the tables marked with NL (Not Limited).

Table (1): values of s_1 maximum , $\varepsilon_1=10\%$

k=K ₂ /K ₁							m
10	5	2	1	0.5	0.2	0.1	
NL	NA	NA	NA	NA	NA	NA	0.1
0.11	NL	NA	NA	NA	NA	NA	0.2
0.10	0.11	NL	NA	NA	NA	NA	0.5
0.10	0.11	0.13	NL	NA	NA	NA	1
0.10	0.10	0.11	0.14	NL	NA	NA	2
0.10	0.10	0.11	0.12	0.17	NL	NA	5
0.10	0.10	0.11	0.11	0.14	0.19	NL	10

Table (2): values of s_2 maximum , $\varepsilon_2=10\%$

k=K ₂ /K ₁							m
10	5	2	1	0.5	0.2	0.1	
NL	NA	NA	NA	NA	NA	NA	0.1
0.05	NL	NA	NA	NA	NA	NA	0.2
0.07	0.07	NL	NA	NA	NA	NA	0.5
0.08	0.08	0.07	NL	NA	NA	NA	1
0.09	0.09	0.08	0.08	NL	NA	NA	2
0.09	0.09	0.09	0.09	0.08	NL	NA	5
0.09	0.09	0.09	0.09	0.09	0.08	NL	10

8 CONCLUSIONS

As per was observed, the structure of industrial complexes and various components are connected by connector members. For different reasons modeling a whole set of ACS seems to be impractical in any case. Only if the connector member was very rigid, the value of response is the same for the two structures. In such cases the system will be behaved as a single collection, with lateral stiffness equal to sum of both lateral stiffness of the main structures and the mass equivalent to the total masses.

In other cases, the final response calculation is associated with using a separate analysis of each structure method with error and proximity. According to tables that have been given usage of separate analysis will have a negligible error when the two connected structures have equal frequency and if the lateral rigidity ratio of connector member against the lateral rigidity of two main structures is negligible; in such cases, any structure can be analyzed independently from the connector member with accepting the engineering error for simplification.

As it is clear in tables due to smallness of values, the number of systems that could be analyzed separately is limited. However, if the ratios of the connector member lateral stiffness to the main structures lateral stiffness is less than 0.1 then it is possible to analyze the system separately by accepting 10% error. In other cases, when we are not able to simplify system, the whole system should be analyzed or the final response should be modified with appropriate methods.

Acknowledgements

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