Nominal Model-Based Sliding Mode Control with Backstepping for 3-Axis Flight Table

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Abstract: Based on nominal model, a novel global sliding mode controller (GSMC) with a new control scheme is proposed for a practical uncertain servo system. This control scheme consists of two combined controllers. One is the global sliding mode controller for practical plant, the other is the integral backstepping controller for nominal model. Modeling error between practical plant and nominal model is used to design GSMC. The steady-state control accuracy can be guaranteed by the integral backstepping control law, and the global robustness can be obtained by GSMC. The stability of the proposed controller is proved according to the Lyapunov approach. The simulation results both of sine signal and step signal tracking for 3-axis flight table are investigated to show good position tracking performance and high robustness with respect to large and parameter changes over all the response time.

Key words: nominal model; sliding mode control; backstepping control; robust control; 3-axis flight table

基于名义模型的飞行模拟转台反演滑模控制. 刘金琨, 孙富春. 中国航空学报(英文版), 2006, 19(1): 65-71.

摘　要: 针对飞行模拟转台这一实际的不确定伺服系统, 提出一种新型控制策略, 该控制策略是建立在名义模型基础上的一种新型全鲁棒滑模控制器。控制系统由两种控制器构成, 一种是针对实际对象的全鲁棒滑模控制器, 另一种是针对名义模型的积分反演滑模控制器。采用名义模型与实际对象之间的建模误差设计全鲁棒滑模控制器, 采用积分反演滑模控制器来保证控制精度, 全鲁棒性能由全局滑模控制器来保证。采用 Lyapunov 方法实现了两种控制器的稳定性分析。以飞行模拟转台伺服系统为被控对象, 针对正弦和阶跃响应的仿真结果表明, 采用所提出的控制方法, 可实现全局鲁棒性并保证较高的位置跟踪精度。

关键词: 名义模型; 滑模控制; 反演控制; 鲁棒控制; 飞行模拟转台

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Recently the need for high performance servo system has been increased in many applications, such as 3-axis flight table. In 3-axis flight table servo systems, uncertain factors can reduce the tracking performance involving high-precision, robustness tasks[1]. Moreover, uncertain factors are generally dependent on the hardware architecture of the motors and may also change over time.

Backstepping is a recursive procedure for systematically selecting the control Lyapunov functions that allows the design of controllers for a class of nonlinear processes[2]. It has been demonstrated[3,4] that the controlling linear systems with backstepping controllers, compared to the certainty-equivalence controllers, lead to possible significant improvements of the transient performance, without increasing the control effect. Stability analysis, which represents a major drawback of traditional controllers, can also be easily performed *via* the backstepping method.

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Since the publication of the first paper[5], significant interests in sliding mode control (SMC) have been generated to provide research in the direction of real world control problems solving[6]. After the seventies of the 20th century, the SMC has become more popular and nowadays it enjoys a wide variety of applications in electro-mechanical systems[7]. The main reason of this popularity is the attractive superior properties of SMC, such as good control performance even in the case of uncertain systems. The most favorite property of SMC is its robustness[8]. Generally speaking, a system controlled by a SMC is insensitive to parameter changes or external disturbances.

Despite its invariance properties, the robustness of the conventional sliding mode controller to uncertainties and external disturbances may not be preserved during the reaching phase. To reduce the reaching phase duration, the global sliding mode control was proposed, which ensures sliding behavior throughout an entire response.

There have been some papers in global sliding mode control design for uncertain servo system. For example, a global sliding mode control was proposed for companion nonlinear system[9], the insensitivities of the system to external disturbance and parameter uncertainties are guaranteed from the very beginning of the proposed control action, however, the controller is designed only for certain servo system. Another GSMC was proposed for controlling an uncertain system with torque limits[10,11], but the nominal model is not used.

A new control scheme for uncertain servo system is considered. An integral backstepping controller is designed for nominal model to get high steady-state control accuracy. Modeling error between practical plant and nominal model is used to design global sliding mode control. Global sliding mode controller is designed for practical plant to get global robustness. Moreover, the control input signal of nominal model is used as an input signal of global sliding mode controller.

The rest of the paper is organized as follows: 3-axis flight table servo system is described in Section 2. A novel control system combined with GSMC and backstepping controllers is designed in Section 3, and the stability analysis is presented. Examples are given to illustrate the results of the proposed controller in Section 4. The paper is concluded in Section 5.

1 System Description

The 3-axis flight table is a typical high performance position and speed servo system used in the hardware-in-the-loop simulation of flight control system. The table is a three-axis servo system with high precision position tracking and speed tracking, which can be used to reproduce the on-ground dynamic moment of many kinds of aircraft. Each axis of the servo system can be considered as an electromechanical servo system.

The experimental frequency response with the analytic ones is shown in Fig. 1. The analytic ones, obtained by HP frequency analyzer, are plotted as dashed lines. In Fig. 1, the deviation between the experimental results and the analytical ones in high-frequency region above 100 Hz is due to the effect of the noise corruption, and that in the low-frequency region below 3 Hz is due to weakened performance of the frequency analyzer at this frequency.

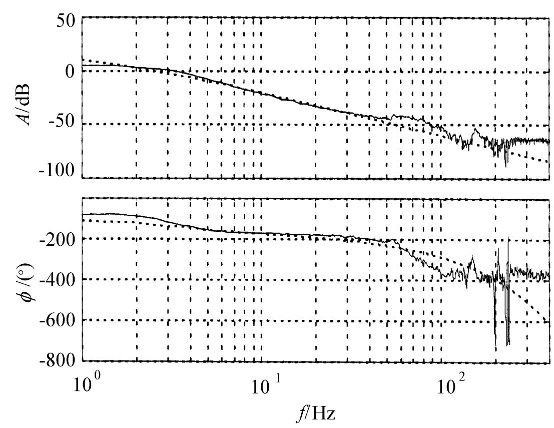

Fig. 1 Comparisons of analytic and experimental characteristics of 3-axis flight table

From Fig. 1, the practical plant can be assumed as a classical uncertain servo system,

$$J\ddot{\theta} + B\dot{\theta} = u + d \qquad (1)$$

Where J is the moment of inertia, B is the damping coefficient, u is the control input, and θ represents position, $J > 0$, $B > 0$. In the practical 3-axis flight table, the moment of inertia J is time-varying parameter, therefore, the system function Eq. (1) is uncertain, and d represents the external disturbance, uncertainty as well as unmodeled dynamics.

According to Fig. 1, using transfer function approaching method, the nominal model of the plant can be obtained as

$$J_n\ddot{\theta} + B_n\dot{\theta}_n = \mu \tag{2}$$

where J_n is the nominal moment of inertia, B_n is the nominal damping coefficient, μ is the control input, and θ_n represents position. $J_n > 0$, $B_n > 0$.

2 Controller Design

The control system structure is shown in Fig. 2. The system has two controllers, one is the global sliding mode controller for practical plant, and the other is the integral backstepping controller design for nominal model.

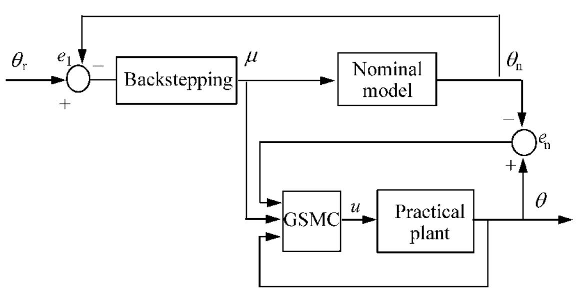

Fig. 2 Control system structure

From Fig. 2, it can be seen that nominal model is controlled by backstepping, and the modeling error between practical plant and nominal model can be decreased by the global sliding mode controller, thus high tracking precision and high robustness can be obtained.

2.1 Backstepping controller design for nominal model

High precision performance of position tracking for servo system can be obtained by using the integral backstepping control method[4]. In this design, the integral action is ultilized to design the backstepping controller.

Firstly, we define the position tracking error $e_1 = \theta_r - \theta_n$ and compute its dynamics as

$$\frac{de_1}{dt} = \dot{\theta}_r - \dot{\theta}_n \tag{3}$$

This definition is directly related to the control objective: minimizing the position tracking error e_1. Through step-by-step backstepping design, the tracking error will be made to converge to zero asymptotically. If the angular velocity ω is the control input, it is possible to choose ω to render the exponential convergence for the system. One example of such a choice is to pick

$$\omega = c_1 e_1 + \dot{\theta}_r \tag{4}$$

where c_1 is a positive constant. This generate the desired exponential behavior for the tracking error e_1 as

$$\frac{de_1}{dt} = -c_1 e_1 \tag{5}$$

Since the angular velocity ω is only a system variable which has its own dynamic behavior, the angular velocity can not be specified as in Eq. (4). But it is still possible to specify a desired behavior for $\dot{\theta}_n$ and therefore to take the angular velocity $\dot{\theta}_n$ as the "virtual" control. This desired behavior for the "virtual" control ω is chosen as following

$$\omega_r = c_1 e_1 + \dot{\theta}_r + \lambda_1 z_1 \tag{6}$$

where λ_1 is a positive constant at the disposal of the designer, and the integral item z_1 is defined as the integral of the position tracking error as

$$z_1 = \int_0^t e_1(t)\,dt \tag{7}$$

In Eq. (6), the integral action is introduced into the desired behavior for the angular velocity ω. The idea of using integral action in the backstepping design was first proposed in Ref. [2]. By integrating this integral action into the stabilizing function, the convergence of the tracking error to zero at the steady state can be enforced. Since the angular velocity ω is unable to control, there is a dynamic error between the angular velocity ω_n and its desired behavior ω_r, which can be computed as,

$$e_2 = \omega_r - \omega_n \tag{8}$$

$$\frac{de_2}{dt} = c_1\frac{de_1}{dt} + \ddot{\theta}_r + \lambda_1 e_1 - \frac{\mu}{J_n} + \frac{B_n}{J_n}\dot{\theta}_n \tag{9}$$

With the definition of e_2, the position tracking error dynamics in Eq. (3) can be rewritten as

$$\frac{\mathrm{d}e_1}{\mathrm{d}t} = -c_1 e_1 - \lambda_1 z_1 + e_2 \tag{10}$$

Substituting Eq. (10) into Eq. (9), it is obtained that

$$\frac{\mathrm{d}e_2}{\mathrm{d}t} = c_1(-c_1 e_1 - \lambda_1 z_1 + e_2) + \ddot{\theta}_r + \lambda_1 e_1 - \frac{\mu}{J_n} + \frac{B_n}{J_n}\dot{\theta}_n \tag{11}$$

Choosing a Lyapunov function candidate as

$$V_1 = \frac{\lambda_1}{2} z_1^2 + \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \tag{12}$$

And compute its derivative $\dot{V}_1$

$$\dot{V}_1 = \lambda_1 z_1 \dot{z}_1 + e_1 \frac{\mathrm{d}e_1}{\mathrm{d}t} + e_2 \frac{\mathrm{d}e_2}{\mathrm{d}t} =$$

$$\lambda_1 z_1 e_1 + e_1(-c_1 e_1 - \lambda_1 z_1 + e_2) + e_2 \cdot \left[c_1\left(-c_1 e_1 - \lambda_1 z_1 + e_2\right) + \ddot{\theta}_r + \lambda_1 e_1 - \frac{\mu}{J_n} + \frac{B_n}{J_n}\dot{\theta}_n\right] =$$

$$e_1(-c_1 e_1 + e_2) + e_2\left[c_1(-c_1 e_1 - \lambda_1 z_1 + e_2) + \ddot{\theta}_r + \lambda_1 e_1 - \frac{\mu}{J_n} + \frac{B_n}{J_n}\dot{\theta}_n\right] \tag{13}$$

Now it is ready to choose the controller to cancel some undesirable dynamics. Choose the backstepping control law as

$$\mu = J_n\{(1 - c_1^2 + \lambda_1) e_1 + (c_1 + c_2) e_2 - c_1 \lambda_1 z_1 + \ddot{\theta}_r\} + B_n \dot{\theta}_n \tag{14}$$

where c_2 is a positive constant at the disposal of the designer and it actually determines the convergence speed of the velocity tracking loop.

Substituting Eq. (14) into $\dot{V}_1$, gives

$$\dot{V}_1 = -c_1 e_1^2 - c_2 e_2^2 \leqslant 0 \tag{15}$$

According to the Lyapunov stability analysis, the developed control law Eq. (14) can guarantee $e_1 = 0$ and $e_2 = 0$.

2.2 Global sliding mode controller design for practical plant

Partly reference to Ref. [10], a global sliding mode controller is designed as follows.

Assumption 1: the known ranges of parameter variation in plant Eq. (1) are

$$J_m \leqslant J \leqslant J_M \tag{16}$$

$$B_m \leqslant B \leqslant B_M \tag{17}$$

$$|d| \leqslant d_M \tag{18}$$

Define the tracking error between nominal model and practical plant as

$$e_n = \theta - \theta_n \tag{19}$$

Define the global sliding line $s = 0$ to have the form

$$s = \dot{e}_n + \lambda e_n - f(t) \tag{20}$$

where $\lambda > 0$.

The term $f(t)$ is the forcing function in the sliding dynamics and is designed to satisfy the following conditions

(a) $f(0) = \dot{e}_{n0} + \lambda e_{n0}$;

(b) $f(t) \to 0$ as $t \to \infty$;

(c) $f(t)$ has a bounded first derivative with respect to time.

Here e_{n0} and $\dot{e}_{n0}$ are the position error and velocity error at $t = 0$, respectively. Condition (a) implies that the system state is initially located in the sliding regime. Condition (b) implies the asymptotic stability of the closed-loop system. Condition (c) is required for the existence of a sliding mode.

If conditions (a)-(c) are satisfied and the control law is designed such that the sliding condition holds near the sliding regime, the asymptotic stability is ensured and the sliding mode exists continually so that the robustness is ensured throughout the entire response.

According to the above analysis, $f(t)$ can be defined as

$$f(t) = s(0)\exp(-\eta t) \tag{21}$$

where $\eta > 0$, and $s(0)$ is the initial value of s.

Let λ be

$$\lambda = \frac{B_n}{J_n} \tag{22}$$

Define the medium value

$$J_a = \frac{1}{2}(J_m + J_M) \tag{23}$$

$$B_a = \frac{1}{2}(B_m + B_M) \tag{24}$$

To maintain $s = 0$ continually, the following global sliding mode control law is indroduced,

$$u = -Ks - h(\theta, \dot{\theta}, t) \cdot \mathrm{sgn}(s) + J_a\left(\frac{1}{J_n}\mu - \lambda\dot{\theta}\right) + B_a\dot{\theta} \tag{25}$$

where $K > 0$. The item $h(\theta, \dot\theta, t)$ is defined as

$$h(\theta, \dot\theta, t) = d_M + \frac{1}{2}(J_M - J_m)\left|\frac{1}{J_n}u - \lambda\dot\theta\right| + \frac{1}{2}(B_M - B_m)|\dot\theta| + J_M\eta|s(0)|\exp(-\eta t) \tag{26}$$

Theorem 1. For system (1), if the control law is designed as Eq.(25), the sliding mode will reach to zero and the tracking error will asymptotically converge to zero.

Proof.

Define Lyapunov function as

$$V_2 = \frac{1}{2}Js^2 \tag{27}$$

From Eq.(20),

$$J\dot s = J[(\ddot\theta - \ddot\theta_n) + \lambda(\dot\theta - \dot\theta_n) + \eta s(0)\exp(-\eta t)] = (J\ddot\theta + B\dot\theta) - \frac{J}{J_n}(J_n\ddot\theta_n + B_n\dot\theta_n) - B\dot\theta + \lambda J\dot\theta + J\eta s(0)\exp(-\eta t) = u + d - \frac{J}{J_n}u - B\dot\theta + \lambda J\dot\theta + J\eta s(0)\exp(-\eta t) \tag{28}$$

Substituting control law Eq.(25) into Eq.(28) yields

$$J\dot s = -Ks - h(\theta, \dot\theta, t)\mathrm{sgn}(s) + J_a\left(\frac{1}{J_n}u - \lambda\dot\theta\right) + B_a\dot\theta + d - B\dot\theta - \frac{J}{J_n}u + \lambda J\dot\theta + J\eta s(0)\exp(-\eta t) = -Ks - h(\theta, \dot\theta, t)\mathrm{sgn}(s) + d + (J_a - J)\cdot\left(\frac{1}{J_n}u - \lambda\dot\theta\right) + (B_a - B)\dot\theta + J\eta s(0)\exp(-\eta t) \tag{29}$$

The derivative of V_2 is

$$\dot V_2 = Js\dot s = -Ks^2 - h(\theta, \dot\theta, t)|s| + s\left[d + (J_a - J)\left(\frac{1}{J_a}u - \lambda\dot\theta\right) + (B_a - B)\dot\theta\right] + sJ\eta s(0)\cdot\exp(-\eta t) \leqslant -Ks^2 - h(\theta, \dot\theta, t)|s| + |s|\cdot\left[|d| + |J_a - J|\left|\frac{1}{J_n}u - \lambda\dot\theta\right| + |B_a - B|\cdot|\dot\theta| + J\eta|s(0)|\exp(-\eta t)\right] \tag{30}$$

According to Eq.(23) and Eq.(24),

$$|J_a - J| \leqslant \frac{1}{2}(J_M - J_m) \tag{31}$$

$$|B_a - B| \leqslant \frac{1}{2}(B_M - B_m) \tag{32}$$

Since

$$h(\theta, \dot\theta, t) \geqslant |d| + |J_a - J|\left|\frac{1}{J_n}u - \lambda\dot\theta\right| + |B_a - B||\dot\theta| + J\eta|s(0)|\exp(-\eta t) \tag{33}$$

Hence the following relation is obtained,

$$\dot V_2 \leqslant -Ks^2 \tag{34}$$

Utilizing Eq.(27) and Eq.(34) gives

$$s(t) \leqslant |s(0)|\exp\left(-\frac{K}{J}t\right) \tag{35}$$

Eq.(35) indicates that $s(t)$ will converge to zero exponentially, thus, the tracking error will converge to zero.

Considering Eqs.(15) and (35), it can be seen that the practical position output will track the position command by adopting the proposed controller Eq.(14) and Eq.(25). In addition, if the practical plant modeling is precise enough, the gain coefficient of switch term sign(s) can become small, therefore, the chattering phenomenon can be decreased greatly.

3 Simulation

Consider a practical 3-axis flight table servo system as follows

$$J\ddot\theta + B\dot\theta = u + d \tag{36}$$

where J(N·s/(°)) represents the equivalent inertia, B(N·s^2/(°)) represents the equivalent damp coefficient, u(N) represents the control input signal, d(N) represents the equivalent disturbance, and θ(°) represents the angle.

Assuming $d(t)$ is symmetric random weight value whose maximum value is 1.0.

In controller Eq.(14), assuming $J_n = \frac{1}{133}$, $B_n = \frac{25}{133}$, the parameters are chosen as $c_1 = 100$, $c_2 = 100$, $\lambda_1 = 5$.

In controller Eq.(25), assuming $J_m = \frac{1}{163}$, $J_M = \frac{1}{103}$, $B_m = \frac{15}{163}$, $B_M = \frac{35}{103}$.

Therefore, $d_M = 1.0$, $\lambda = \frac{B_n}{J_n} = 25$, choose $K = 20$, $\eta = 10$.

Assuming the initial state of the practical plant is [0.5, 0]. Choose the reference signal as $\theta_r(t) =$

$A \sin(2\pi Ft)$, $A = 0.5$, and $F = 3.0$ Hz. The simulation results are shown in Fig. 3-Fig. 5. Fig. 3 shows position tracking with proposed control method, Fig. 4 shows its position tracking error, and Fig. 5 shows its control input signal.

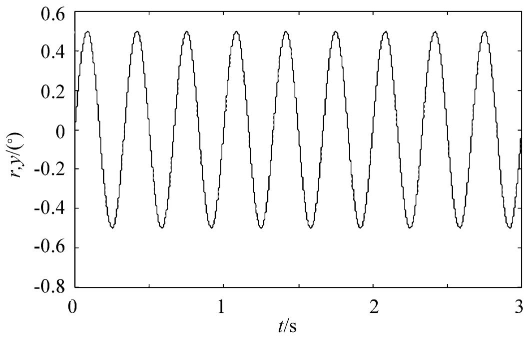

Fig. 3 Position tracking with proposed control method

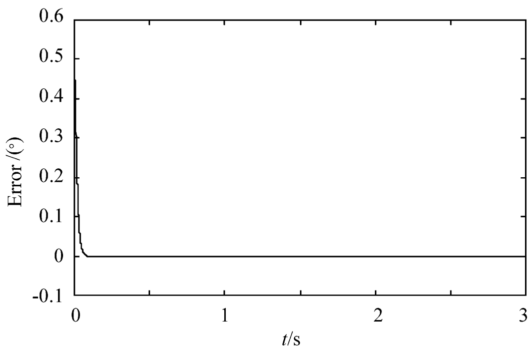

Fig. 4 Position tracking error

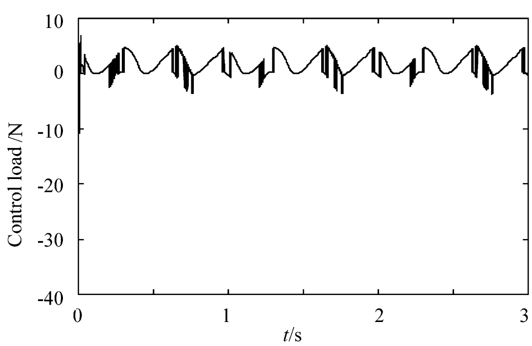

Fig. 5 Control input signal

In order to display the effect of static tracking, choose the reference signal as $\theta_r(t) = 10$. The simulation results are shown in Fig. 6-Fig. 8. Fig. 6 shows the position tracking with proposed control method, Fig. 7 shows the position tracking error, and Fig. 8 shows its control input signal.

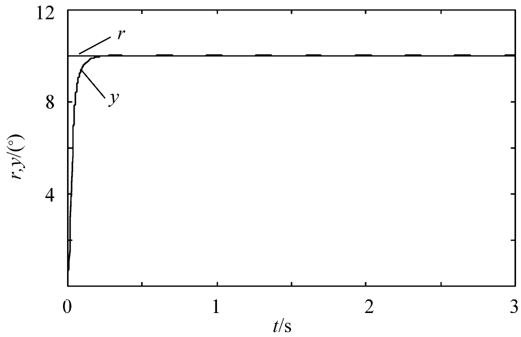

Fig. 6 Position tracking with proposed control method

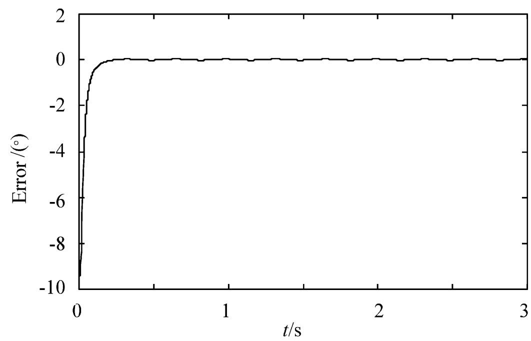

Fig. 7 Position tracking error

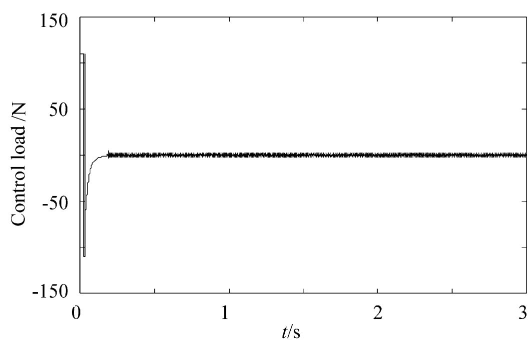

Fig. 8 Control input signal

The simulation results of sine signal tracking and step signal tracking indicate that the proposed controller have good performance for both dynamic position tracking and position static tracking. High precision tracking performance can be obtained, and high robustness with respect to large and parameter changes can be received over all the time.

4 Conclusions

A new control scheme has been presented for 3-axis flight table servo system. In this scheme, nominal model is controlled by backstepping. In order to decrease the modeling error between practical plant and nominal model, global sliding mode controller is designed. The steady-state control accuracy can be guaranteed by integral backstepping control law, and global robustness is obtained by GSMC. Simulation examples for sine signal and step signal tracking have indicated that good position tracking performance and high robustness with respect to large and parameter changes can be received over all the response time.

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