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Physics Letters B 527 (2002) 193–198
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Radiative $\phi \to f_0(980)\gamma$ decay in light cone QCD sum rules

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Received 13 November 2001; received in revised form 13 December 2001; accepted 7 January 2002
Editor: G.F. Giudice

Abstract

The light cone QCD sum rules method is used to calculate the transition form factor for the radiative $\phi \to f_0\gamma$ decay, assuming that the quark content of the f_0 meson is pure $\bar{s}s$ state. The branching ratio is estimated to be $\mathcal{B}(\phi \to f_0\gamma) = 3.5 \times (1 \pm 0.3) \times 10^{-4}$. A comparison of our prediction on branching ratio with the theoretical results and experimental data existing in literature is presented.

PACS: 11.55.Hx; 13.40.Hq; 14.40.Ev

1. Introduction

According to the quark model, mesons are interpreted as pure $\bar{q}q$ states. Scalar mesons constitute a remarkable exception to this systematization and their nature is not well established yet [1–4].

In the naive $\bar{q}q$ picture, one can treat the isoscalar $f_0(980)$ either as the meson that exists mostly as nonstrange and almost degenerate with the isovector $a_0(980)$ or as mainly $\bar{s}s$, in analogy to the pure $\bar{s}s$ vector meson $\phi(1020)$.

In order to understand the content of the f_0 meson several alternatives have been suggested, such as, the analysis of the $f_0 \to 2\gamma$ decay [5,6]; study of the ratio $\Gamma(a_0 \to f_0\gamma)/\Gamma(\phi \to f_0\gamma)$ [7]. However, among these, the $(\phi \to f_0\gamma)$ decay occupies a special place, since the branching ratio expected of this decay, is essentially dependent on the content of f_0. For example, $\mathcal{B}(\phi \to f_0\gamma)$ is as high as $\sim 10^{-4}$ if it were composed of $\bar{q}q\bar{q}q$, and $\sim 10^{-5}$ if f_0 were a pure $\bar{s}s$ state.

It has been known for a long time that $f_0(980)$ couples significantly through its $\bar{s}s$ content, from its detection as a peak in the $J/\psi \to \phi f_0$ [8] and $D_s \to \pi f_0$ [9] decays, as discussed in [10] and [11] (see also [12]). For this reason, in this work we assume that quark content of both ϕ and f_0 mesons are pure $\bar{s}s$. In the present Letter we analyze the radiative $\phi \to f_0\gamma$ decay in framework of the light cone QCD sum rules (about light cone QCD sum rules and its applications, see, for example, [13]). Note also that the $\phi \to f_0\gamma$ decay is analyzed in framework of the 3-point sum rules in [14]. In order to calculate the transition form factor describing the $\phi \to f_0\gamma$ decay in light cone QCD sum rules, we consider the following correlator

$$\Pi_\mu = i \int d^4x\, e^{ipx} \langle 0 | T\{J^s(x) J^\phi_\mu(0)\} | 0 \rangle_\gamma, \qquad (1)$$

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PII: S0370-2693(02)01169-3

where $J^s = \bar{s}s$ and $J_\mu^\phi = \bar{s}\gamma_\mu s$ are interpolating currents for f_0 and ϕ mesons, respectively, and γ is the background electromagnetic field (for more about external field technique in QCD see [15,16]).

The physical part of the correlator can be obtained by inserting a complete set of one meson states into the correlator,

$$\Pi_\mu = \sum \frac{\langle 0|J^s(x)|f_0(p)\rangle\langle f_0(p)|\phi(p_1)\rangle_\gamma\langle\phi(p_1)|J_\mu^\phi(0)|0\rangle}{(p^2-m_{f_0}^2)(p_1^2-m_\phi^2)}, \quad (2)$$

where ϕ and f_0 are the quantum numbers and $p_1 = p + q$ with q being the photon momentum.

The matrix element $\langle\phi(p_1)|J_\mu^\phi(0)|0\rangle$ in Eq. (1) is defined as

$$\langle\phi(p_1)|J_\mu^\phi(0)|0\rangle = m_\phi f_\phi \varepsilon_\mu^\phi, \quad (3)$$

where ε_μ^ϕ is the ϕ meson polarization vector. The coupling of the $f_0(980)$ to the scalar current $J^s = \bar{s}s$ is defined in terms of a constant λ_f

$$\langle 0|J^s|f_0(p)\rangle = m_{f_0}\lambda_f. \quad (4)$$

The relevant matrix element describing the transition $\phi \to f_0$ induced by an external electromagnetic current can be parametrized in the following form:

$$\big\langle f_0(p)\big|\phi\big(p_1,\varepsilon^\phi\big)\big\rangle_\gamma = e\varepsilon^\mu\big[F_1\big(q^2\big)(p_1q)\varepsilon_\mu^\phi + F_2\big(q^2\big)\big(\varepsilon^\phi q\big)p_{1\mu}\big], \quad (5)$$

where ε is the photon polarization and we have used $(\varepsilon q) = 0$. From gauge invariance we have

$$F_1\big(q^2\big) = -F_2\big(q^2\big), \quad (6)$$

and since the photon is real in the decay under consideration, we need the values of the form factors only at the point $q^2 = 0$. Using Eq. (6) the matrix element $\langle f_0|\phi\rangle_\gamma$ takes the following gauge-invariant form,

$$\langle f_0|\phi\rangle_\gamma = e\varepsilon^\mu F_1(0)\big[(p_1q)\varepsilon_\mu^\phi - \big(\varepsilon^\phi q\big)p_{1\mu}\big]. \quad (7)$$

Using Eqs. (1)–(4) and (7), for the phenomenological part of the correlator we have

$$\Gamma_\mu^{\text{phen}} = eF_1(0)\varepsilon^\nu\big[-(p_1q)g_{\mu\nu} + p_{1\nu}q_\mu\big] \times \frac{\lambda_f f_\phi m_{f_0} m_\phi}{(p^2-m_{f_0}^2)(p_1^2-m_\phi^2)}. \quad (8)$$

In order to construct the sum rule, calculation of the correlator from QCD side (theoretical part) is needed. From Eq. (1) we get

$$\Pi_\mu = \int d^4x\, e^{ipx}\big\langle 0\big|\text{Tr}\big\{-\gamma_\mu S_s(-x)S_s(x)\big\}\big|0\big\rangle_\gamma, \quad (9)$$

where S_s is the full propagator of the strange quark (see below). Theoretical part of the correlator contains two pieces, perturbative and nonperturbative. Perturbative part corresponds to the case when photon is radiated from the freely propagating quarks. Its expression can be obtained by making the following replacement in each one of the quark propagators in Eq. (9)

$$(S_s)_{\alpha\beta}^{ab} \to 2ee_q\big(dy\, F_{\mu\nu}y^\nu S_s^{\text{free}}(x-y)\gamma^\mu S_s^{\text{free}}(y)\big)_{\alpha\beta}^{ab}, \quad (10)$$

where the Fock–Schwinger gauge $x^\mu A_\mu(x) = 0$ is used and S_s^{free} is the free s-quark propagator $S_s^{\text{free}}(x) = i\not{x}/(2\pi^2x^4)$ and the remaining one is the full quark propagator.

The nonperturbative piece of the theoretical part can be obtained from Eq. (9) by replacing each one of the propagators with

$$(S_s)_{\alpha\beta}^{ab} = -\frac{1}{4}\bar{q}^a A_i q^b (A_i)_{\alpha\beta}, \quad (11)$$

where A_i is the full set of Dirac matrices and sum over A_i is implied and the other quark propagator is the full propagator, involving perturbative and nonperturbative contributions. In order to calculate perturbative and nonperturbative parts to the correlator function (1), expression of the s-quark propagator in external field is needed.

The complete light cone expansion of the light quark operator in external field is presented in [16]. The propagator receives contributions from the nonlocal operators $\bar{q}Gq$, $\bar{q}GGq$, $\bar{q}q\bar{q}q$, where G is the gluon field strength tensor. In the present work we consider operators with only one gluon field and neglect terms with two gluons $\bar{q}GGq$, and four quarks $\bar{q}q\bar{q}q$ and formal neglect of these these terms can be justified on the basis of an expansion in conformal spin [17]. In this approximation full propagator of the s-quark is given as

$$S_s(x) = \frac{i\not{x}}{2\pi^2x^4} - \frac{\langle\bar{s}s\rangle}{12}\left(1 + \frac{x^2}{16}m_0^2\right) + \frac{im_s\langle\bar{s}s\rangle}{48}\not{x} - \frac{im_0^2m_s}{2^7 3^2}x^2\not{x}$$

$$- i g_s \int_0^1 dv \left[\frac{\not x}{16\pi^2 x^2} G_{\mu\nu}(vx)\sigma^{\mu\nu} - \frac{i}{4\pi^2 x^2} v x^\mu G_{\mu\nu}\gamma^\nu \right]. \quad (12)$$

It follows from Eqs. (11) and (9) that in calculating the QCD part of the correlator, as is generally the case, we are left with the matrix elements of the gauge-invariant nonlocal operators, sandwiched in between the photon and the vacuum states $\langle\gamma(q)|\bar{s}A_i s|0\rangle$. These matrix elements define the light cone photon wave functions. The photon wave functions up to twist-4 are [17,18]

$$\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = i e e_q \langle\bar{q}q\rangle \times \int_0^1 du\, e^{iqx}\big\{(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \times \big[\chi\phi(u) + x^2\big(g_1(u) - g_2(u)\big)\big] + \big[(qx)(\varepsilon_\mu x_\nu - \varepsilon_\nu x_\mu) + (\varepsilon x)(x_\mu q_\nu - x_\nu q_\mu)\big] g_2(u)\big\}, \quad (13)$$

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = \frac{ef}{4} e_q \epsilon_{\alpha\beta\rho\sigma}\varepsilon^\beta q^\rho x^\sigma \int_0^1 du\, e^{iuqx}\psi(u). \quad (14)$$

The path-ordered gauge factor $\mathcal{P}\exp(ig_s\int_0^1 du\, x^\mu \times A_\mu(ux))$ is emitted since the Schwinger–Fock gauge $x^\mu A_\mu(x) = 0$ is used. The functions $\phi(u)$, $\psi(u)$ are the leading twist-2 photon wave functions, while $g_1(u)$ and $g_2(u)$ are the twist-4 photon wave functions. Note that twist-3 photon wave functions are neglected in the calculations, since their contributions are small and change the result by 5%. In Eq. (13) χ is the magnetic susceptibility of the quark condensate and e_q is the quark charge. The theoretical part is obtained by substituting photon wave functions and expression for the s-quark propagators into Eq. (9). The sum rules is obtained by equating the phenomenological and theoretical parts of the correlator. In order to suppress higher states and continuum contribution (for more details see [19,20]) double Borel transformations of the variables $p_1^2 = p^2$ and $p_2^2 = (p+q)^2$ are performed on both sides of the correlator, after which the following sum rule is obtained

$$F_1(0) = e^{m_{f_0}^2/M_2^2} e^{m_\phi^2/M_1^2} \frac{e_s}{\lambda_{f_0} f_\phi m_{f_0} m_\phi} \times \Bigg\{ \bigg[2\chi\langle\bar{s}s\rangle\phi(u_0) - \frac{3m_s}{2\pi^2}(1+\gamma_E) \bigg] M^2 E_0\big(s_0^2/M^2\big) + \frac{1}{24}\langle\bar{s}s\rangle\big[-192 g_1(u_0) + m_s\phi(u_0)\langle\bar{s}s\rangle\big] + \frac{3m_s}{2\pi^2}\bigg[M^2\bigg(\gamma_E + \ln\frac{M^2}{\Lambda^2}\bigg) E_0\big(s_0/M^2\big) + M^2 f\big(s_0/M^2\big)\bigg]\Bigg\}, \quad (15)$$

where s_0 is the continuum threshold

$$E_0\big(s_0/M^2\big) = 1 - e^{-s_0/M^2},$$

$$f\big(s_0/M^2\big) = \int_0^{s_0/M^2} dy\, \ln y e^{-y},$$

which have been used to subtract continuum, and

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2}, \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2},$$

where M_1^2 and M_2^2 are the Borel parameters in ϕ and f_0 channels, respectively, Λ is the QCD scale parameter and γ_E is the Euler constant. Since the masses of ϕ and f_0 are very close to each other we will set $M_1^2 = M_2^2 \equiv 2M^2$, obviously from which it follows that $u_0 = 1/2$.

It is clear from Eq. (14) that the values of λ_{f_0} and f_ϕ are needed in order to determine $F(0)$. The coupling of the $f_0(980)$ to the scalar $\bar{s}s$ current is determined by the constant λ_{f_0} and in the two-point QCD sum rules its value is found to be $\lambda_{f_0} = (0.18 \pm 0.0015)$ GeV [14]. In further numerical analysis we will use $f_\phi = 0.234$ GeV which is obtained from the experimental analysis of the $\phi \to e^+e^-$ decay [21].

Having the values of λ_{f_0} and f_ϕ, our next and final attempt is the calculation of transition form factor $F_1(0)$. As we can easily see from Eq. (15) the main input parameters of the light cone QCD sum rules is the photon wave function. It is known that the leading photon wave function receive only small corrections from the higher conformal spin [17,19,22], so that they do not deviate much from the asymptotic form. The photon wave functions we use in our numerical analysis are given as

$$\phi(u) = 6u(1-u), \qquad \psi(u) = 1,$$
$$g_1(u) = -\frac{1}{8}(1-u)(3-u).$$

Furthermore, the values of the input parameters that are used in the numerical calculations are: $f = 0.028$ GeV^2, $\chi = -4.4$ GeV^{-2} [23] (in [24] this quantity is predicted to have the value $\chi = -3.3$ GeV^{-2}), $\langle \bar{s}s(1\ \text{GeV})\rangle = -0.8 \times (0.243)^3$ GeV^3 and the QCD scale parameter is taken as $\Lambda = 0.2$ GeV. The strange quark mass is chosen in the range $m_s = 0.125$–0.16 GeV, obtained in the QCD sum rules approach [25]. The masses of the ϕ and f_0 mesons are $m_\phi = 1.02$ GeV, $m_{f_0} = 0.98$ GeV. The transition form factor is a physical quantity and therefore it must be independent of the auxiliary continuum threshold s_0 and and the Borel mass M^2 parameters. So our main concern is to find a region where the transition form factor $F_1(0)$ is practically independent of the parameters s_0 and M^2. For this aim in Fig. 1 we present the dependence of the transition form factor $F_1(0)$ on the Borel parameter M^2 at three different values of the continuum threshold: $s_0 = 2.0$ GeV^2, 2.2 GeV^2 and 2.4 GeV^2. It follows from this figure that for the choice of the continuum thresholds in the above-mentioned range, the variation of the result on the transition form factor $F_1(0)$ is about 10%. In other words, we can conclude that $F_1(0)$ is practically independent of the continuum threshold. Furthermore, we observe that when $1.4 \leqslant M^2 \leqslant 2.0$ GeV^2, $F_1(0)$ is quite stable with respect to the variations of the Borel parameter M^2. As a result, one can directly read from this figure

$$F_1(0) = (3.25 \pm 0.20)\ \text{GeV}^{-1},$$

where the resulting error is due to the variations in s_0 and M^2. The other sources of errors contributing to the numerical analysis of the transition form factor come from the strange quark mass and the uncertainties in values of various condensates. Hence, our final prediction on the transition form factor is

$$F_1(0) = (3.25 \pm 0.50)\ \text{GeV}^{-1}. \tag{16}$$

Using the matrix element (7) for the decay width of the considered process, we obtain

$$\Gamma(\phi \to f_0\gamma) = \alpha \left|F_1(0)\right|^2 \frac{(m_\phi^2 - m_{f_0}^2)^3}{24 m_\phi^3}. \tag{17}$$

Using the experimental value $\Gamma_{\text{tot}}(\phi) = 4.458$ MeV [21], and Eqs. (16) and (17), we get for the branching ratio

$$\mathcal{B}(\phi \to f_0\gamma) = 3.5 \times (1.0 \pm 0.3) \times 10^{-4}. \tag{18}$$

Our result on the branching ratio is obtained under the assumption that f_0 meson is represented as a

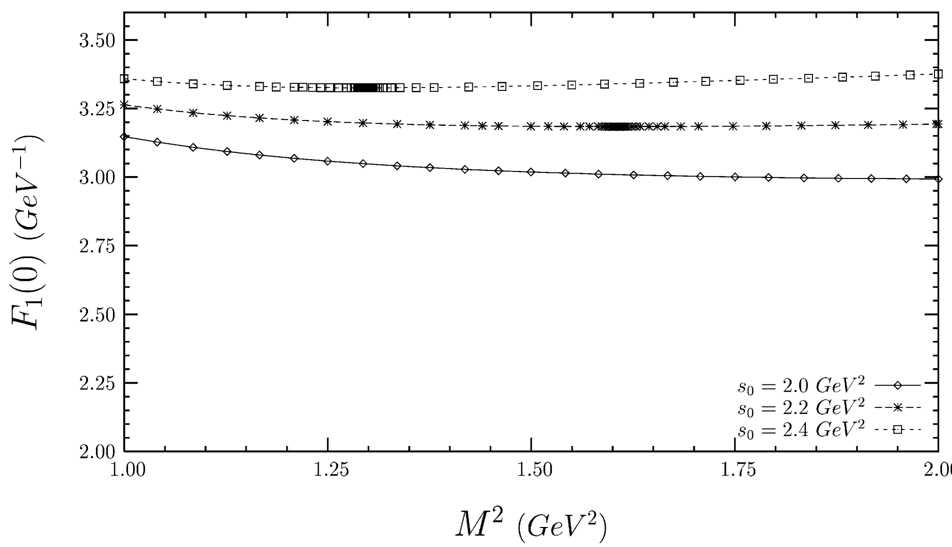

Fig. 1. The dependence of the transition form factor $F_1(0)$ for the radiative $\phi \to f_0\gamma$ decay on M^2 at three different values of the continuum threshold $s_0 = 2.0$ GeV^2, 2.2 GeV^2 and 2.4 GeV^2.

pure $\bar{s}s$ component. How does the result change if we assume that ϕ and f_0 mesons can be represented as a mixing of $\bar{s}s$ and $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$ state, i.e.,

$$\phi = \cos\alpha\, \bar{s}s + \sin\alpha\, \bar{n}n,$$

$$f_0 = \sin\beta\, \bar{s}s + \cos\beta\, \bar{n}n?$$

Analysis of the process $\phi \to \pi^0\gamma$ and combined analysis of the $\phi \to f_0\gamma$ and $f_0 \to 2\gamma$ decays show that $|\alpha| \leqslant 4^\circ$ and two solutions are found for β, i.e., $\beta = -48^\circ \pm 6^\circ$ or $\beta = 85^\circ \pm 5^\circ$, respectively (see, for example, [5]). In other words, quark content of ϕ meson is pure $\bar{s}s$ state, while in f_0 meson there might be sizable $\bar{n}n$ component. Obviously, when $F_1(0)$ is calculated from QCD side, only $\sin\beta\bar{s}s$ component operates (see Eq. (1)) and, therefore, the decay width $\Gamma(\phi \to f_0\gamma)$, and hence the corresponding branching ratio, contains an extra factor $\sin^2\beta$. If $\beta = 85^\circ \pm 5^\circ$, then prediction for the branching ratio given in Eq. (18) is practically unchanged, but when $\beta = -48^\circ \pm 6^\circ$ $\mathcal{B}(\phi \to f_0\gamma)$ decreases by about a factor of 2.

Finally, let us compare our prediction on branching ratio with the existing theoretical results and experimental data in the literature. Obviously, our result is slightly larger compare to the 3-point QCD sum rule result which predicts $\mathcal{B}(\phi \to f_0\gamma) \simeq (2.7 \pm 1.1) \times 10^{-4}$ [14], and approximately three times larger compared to the prediction of the spectral QCD sum rules and chiral unitary approaches, whose predictions are $\mathcal{B}(\phi \to f_0\gamma) = 1.3 \times 10^{-4}$ [26] and $\mathcal{B}(\phi \to f_0\gamma) = 1.6 \times 10^{-4}$ [27], respectively. It is interesting to note that this value of the branching ratio is closer to our prediction when the mixing angle is chosen to be $\beta = -48^\circ \pm 6^\circ$. Our result, which is given in Eq. (18), is larger compared to the predictions of [7,28], whose results are $\mathcal{B}(\phi \to f_0\gamma) = 1.9 \times 10^{-4}$ [28], and $\mathcal{B}(\phi \to f_0\gamma) = 1.35 \times 10^{-4}$ [7], respectively.

As the final words we would like to point out that our prediction given in Eq. (18), is in a very good agreement with the existing experimental result $\mathcal{B}(\phi \to f_0\gamma) \simeq (3.4 \pm 1.1) \times 10^{-4}$ [21].

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