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## Dynamic Modeling of Lift Hoisting Mechanism Block Pulley

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### Abstract

The mathematical model of loaded block pulley trolley movement was developed based on Lagrange equation of the second genus. The differential equations of loaded trolley motion were given along with the relations for viscoelastic block pulley characteristics based on empirical observations. It was shown that the modeling block pulley dynamics could be substantially simplified by means of universal program complexes when considering the mass variation and the rope rigidity of block pulley negligible by changing its length.

It was demonstrated that the loaded block pulley model could be geared as a subsystem for the computer dynamic model of lifting crane, when executing the axial force of upper block pulley, the movement of exit rope section and the rope tension force at this section by means of subsystem output parameters.

The example of trolley motion start model was given for the simultaneous load lifting with at a constant speed with the use of three-mass model.

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*Keywords:* mathematical model; equations of motion; hoisting mechanism; block pulley; rope.

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### 1. Introduction

The lifting cranes are the complex dynamic systems that include steel elements, travel and hoisting mechanisms, shafts, control system. The calculation procedures for lifting crane are continually improved, tending to accurate and complete assessment of dynamic effects, occurring during the crane operation. The block pulley is the compulsory element of lifting cranes. In the loading mechanisms, the block pulleys are used to decrease the tension force of rope and driving torque. In the course of the dynamic modeling for loading cranes, the dynamic model of block pulley is

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used as one of the crane model subsystems. The complexity of block pulley modeling is associated with large deformations and rope motion, mass variation [1, 2].

The need of improvements in calculating methods and techniques for loading cranes is conditioned by high breakdown consequences through the design and use errors. While loading crane calculations GOST 29609-90 require taking into account the loads induced by the acceleration of load and crane construction elements [3]. In the course of crane operating, the dynamic loads significantly outweigh the static loads. The detailed calculation procedures for dynamic loads are given in [4, 5].

In [6] it is seen that vertical and horizontal motions of crane bridge always occur due to load lifting with pick-up. The calculation procedure is based on the principles of structural mechanics with account for inertial forces of concentrated reduced mass joined with elastic bracing.

The mathematical motion model for tower crane adjusted for load movement is developed in [7]. The authors outlined the necessity to account load pendular oscillation and to compare load oscillation frequency with the natural frequency spectrum of tower crane. The possibility to limit the oscillation of load moved by cranes is founded in [8].

The performance review of strength and crane stability calculation methods [8] demonstrates the necessity to apply the refined dynamic models of external excitation, finite-element methods.

The refined dynamic model of crane shall comprise the description of loading gripper and the load itself. The constructions of loading grippers for constructive loads are given in [9].

To model the dynamics of complex technical systems since 60's there has been implemented the development of software environment in which the prototype object is represented by the system of perfectly rigid or deformable bodies bounded with kinematic couples of different types and load-bearing member. Comprehensive facilities for the dynamic modeling of complex technical systems represent the universal program complexes [9, 10]. The formation of motion equation in symbol formula has the significant advantages in the process reaction rate of modeling, and the numerical iteration algorithms enable to organize simpler modeling [11].

The development of complex dynamic models with the modules Matlab/Simulink switched to the models of mechanical systems in software "Universal Mechanism" that account the mechanical and electrical shaft components, is represented in [12-14].

## 2. Lagrange equations of the trolley and block pulleys motion

During the motion of loading crane, the upper block pulleys are moved with trolley (fig. 1) or crane jib.

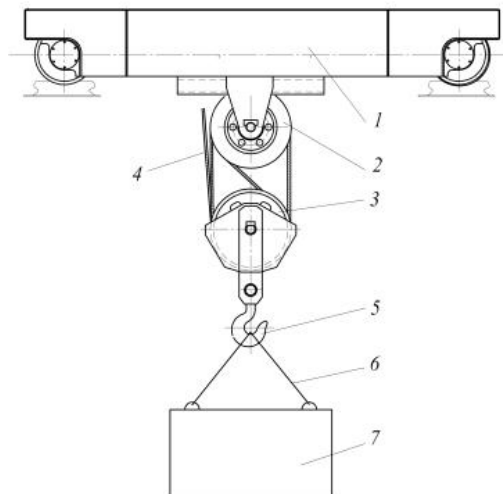


Fig. 1. Crane trolley with block pulley: 1 – trolley, 2 – upper blocks, 3 – lower blocks, 4 – rope, 5 – hook block, 6 – slings, 7 – load

The distance between upper and lower blocks decreases or increases in load lifting or lowering correspondingly, the load with hook block oscillates about vertical axis.

Let us consider the block pulley mechanism of load lifting as the variable-length pendulum fixed on the movable bearing, namely, at the trolley (fig. 2).

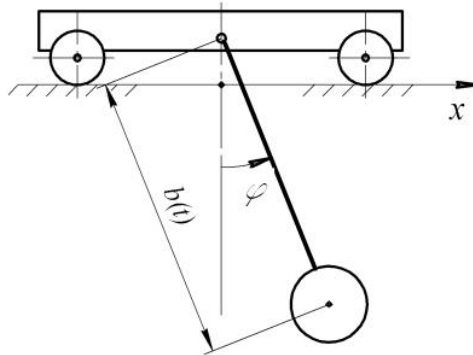


Fig. 2. The scheme of pendulum on the movable bearing

The length of pendulum  $b(t)$  equals to the distance from the upper block axis to the load mass center with hook block, lower blocks and the block pulley rope. The length of pendulum  $b(t)$  varies with the course time  $t$  in the load lifting and lowering. The mass variation of block pulley will be neglected due to the rope winding and withdrawing.

Let us write the differential equations of load motion with block pulley using Lagrange equations of the second genus. The kinetic system energy comprising the trolley, block pulley, loaded hook block, is defined by the following expression

$$T = (m_1 + m_2)\dot{x}^2 / 2 + J_2(t)\dot{\phi}^2 / 2 + m_2 b(t)\dot{\phi}\dot{x} \cos \phi \quad (1)$$

where  $x$  – is the axial coordinate of pendulum,  $\phi$  – is the rotation angle of block pulley about an axis of upper blocks,  $\dot{x} = dx / dt$ ,  $\dot{\phi} = d\phi / dt$ ,  $m_1$  – is the trolley weight with upper blocks,  $m_2$  – is the load weight with hook block and lower blocks,  $J_2(t)$  – is the moment of inertia of the block pulley with hook block and load about the upper block axis,  $b(t)$  – is the distance from the center of load mass with hook block and lower blocks to the upper block axis.

The equation (1) considers the translational movement of trolley and plane motion of block pulley with hook block and load. The relationships between  $J_2(t)$  and  $b(t)$  is determined by the shaft control of load weighting mechanisms.

Without considering the rope stretch, the motion equations of movable bearing and pendulum take on the following form:

$$(m_1 + m_2)\ddot{x} + m_2 b\ddot{\phi} \cos \phi + m_2 \dot{b}\dot{\phi} \cos \phi - m_2 b\dot{\phi}^2 \sin \phi = Q_x \quad (2)$$

$$J_2\dot{\phi} + J_2\ddot{\phi} + m_2 \dot{b}\dot{x} \cos \phi + m_2 b\ddot{x} \cos \phi = Q_\phi - m_2 g b \sin \phi \quad (3)$$

where  $Q_x$  – is the summarized force that takes into account the pulling force of load shaft and the trolley motion resistance force,  $Q_\phi$  – is the summarized force that takes into account the resistance of block pulley rotation about the upper block axis,  $g$  – is the free-fall acceleration.

Even in the two-mass mechanical set of equations (2), (3), the trolley and load movements take the essentially nonlinear form. With the increase of the amount of bodies, the complexity of the set of equations grows nonlinearly.

The analytical study of these equations is quite problematic, but they can be simplified, if to set  $\sin\varphi \approx \varphi$  and  $\cos\varphi \approx 1$  for the small values of  $\varphi$  angles.

The numerical solution of the equations (2), (3) is possible in the MathCAD, MathLAB environment, but the motion modeling of block pulley mechanism is more convenient in the universal software “Universal Mechanism”, MSC.ADAMS and others.

### 3. Computer model of block pulley mechanism

The dynamic modeling of lifting crane mechanisms in universal program requires the representation of block pulley by means of the elements accepted in the program complex (bodies, hinges etc.) [12]. The rope modeling by means of the circuit of elastic and viscous elements and bodies is associated with the interaction difficulties for the pulley blocks, rope mass and rigidity variation in its length changes.

The block pulley modeling can be significantly simplified if not to consider the rope mass and rigidity variation while changing its length [18]. In these circumstances, it is sufficient to consider the length violation of block pulley, the movement of upper blocks with the trolley or crane jib, the load oscillation with hook block about the vertical axis.

The scheme of the dynamic model of block pulley mechanism for load lifting with trolley is represented in fig. 3. The model consists of three bodies (trolley with upper block 1, hook block with lower block 2 and load 3), rotating 4 and two translational hinges 5 and 6, spring element 7 and two damping elements 8, 9. Translational hinge 5 models the length change of block pulley. The translational hinge 6, spring element 7 and damping element 8 models the viscoelastic characteristics of the block pulley rope and slings. The damping element 9 includes the resistance to the block pulley vertical oscillation with load.

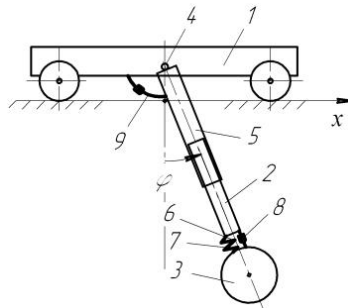


Fig. 3. The model structure of block pulley

The characteristics of spring element 7 and damping element 8 are defined as the averaged values of the studied block pulley running. The workload range of rope  $S_r \leq 0,1S_{break}$  the dependence of rope tension force  $S_r$  on the tension strain  $\varepsilon_r$  we will take as linear

$$S_r = E_r \varepsilon_r + \eta_r \frac{d\varepsilon_r}{dt} (N),$$

where  $S_{break}$  – is the rope breaking force,  $\varepsilon_r$  – is the relative tensile strain of rope,  $\eta_r$  – is the viscosity rate.

The rope tensile stiffness  $E_r$  is assessed based on the experimental dependencies of the rope tension force under static deformation.

$$E_r = \gamma_r E_0 \frac{\pi d_r^2}{4} (N),$$

where  $\gamma_r$  – is the coefficient that considers the rope structure,  $E_0$  – is the modulus of elasticity for the rope wire,  $d_r$  – is the rope section. For the double-lay rope  $\gamma_r = 0,333 \dots 0,65$  [20].

The rope tensile stiffness of block pulley and slings is represented by one elastic element with the rigidity  $C_1$

$$C_1 = C_K C_C / (C_K + C_C) .$$

Tensile stiffness of the block pulley rope

$$C_K = \xi K_n E_k / l_n \text{ (N/m)} ,$$

where  $K_n$  – is the block pulley rate,  $\xi = 1$  for single block pulley,  $\xi = 2$  for double block pulley,  $l_n$  – the average length of block pulley for the reviewed task.

Tensile stiffness of the rope slings

$$C_C = N_C E_C / l_C \text{ (N/m)} ,$$

where  $N_C$  – is the number of slings,  $E_C$  – is tensile stiffness of the rope sling,  $l_C$  – is the sling length.

The value of viscosity grade  $\eta_k$  for the block pulley ropes and slings is determined by the experimental data on decaying axial oscillations of the rope sample with the length  $l_0$  with load at the end with the mass  $m_0$  [2].

$$\eta_k = \frac{4m_0 l_0}{T_c} \ln\left(\frac{a_i}{a_{i+1}}\right) \text{ (N/s)},$$

where  $a_i, a_{i+1}$  – are the amplitudes of oscillations for the  $i$ -st,  $i+1$  oscillations,  $T_c$  – is the time of load oscillations.

The resistance coefficient  $\mu_1$  of damping element 8 is determined according to the formula

$$\mu_1 = \xi K_n \eta_k / l_n \text{ (N·s/m)} .$$

The resistance coefficient  $\mu_2$  of damping element 9 is determined according to the formula

$$\mu_2 = \xi K_n \mu_k \text{ (N·s/rad)} ,$$

where  $\mu_k$  – is the experimental value of the resistance coefficient for one rope (during the vertical oscillations).

The mass of block pulley ropes  $m_k$  is added to the mass of hook block.

After the integration of subsystem “Loaded block pulley” with the dynamic crane model, the outcome subsystem parameters include the hinge force 4, the movement of rope section, that comes out from the block pulley  $\delta_k(t)$  (fig. 4), and the rope tension force  $S_k(t)$  in this cross-section:  $\delta_k(t) = \delta_2(t) K_n$ .

During the load lifting  $S_k(t) = (F_6 + F_7) / \xi K_n \eta_n$  ;

during the load lowering  $S_k(t) = (F_6 + F_7) \eta_n / \xi K_n$  ,

where  $\delta_2$  – is the motion of body 3 relative to body 2,  $F_6$  and  $F_7$  – are the forces of spring element 7 and damping element 8.

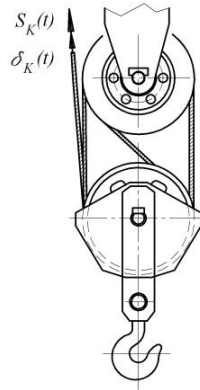


Fig. 4. The motion direction  $\delta_k(t)$  and tensile force  $S_k(t)$  in the rope section, that comes out from the block pulley.

In order to perform the load lifting and lowering, the rope is spooled on the rope drum or reeled out of it. The drive of lifting mechanism has own control, for that reason the parameters  $\delta_k(t)$  and  $S_k(t)$  unite the subsystems “Loaded block pulley” and “Drive of hoist”.

If to consider the subsystem “Loaded block pulley” separately, the variation of block pulley can be kinematically defined as the motion

$$\delta_2(t) = \delta_k(t) / K_n \quad (m).$$

The load movement in this case is conditioned by the primary settings.

By way of example, let us study the initiation of trolley movement combined with load lifting at a constant speed (fig. 5).

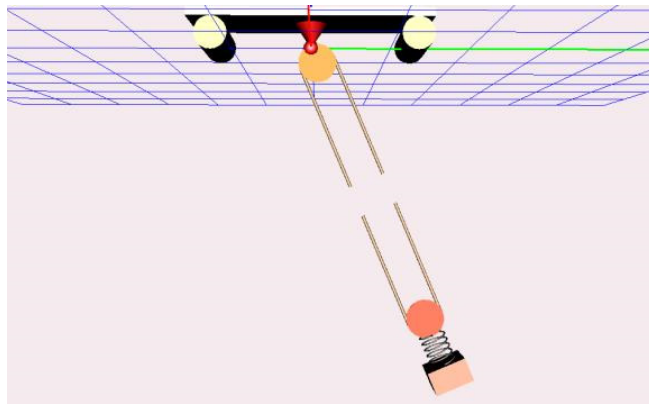


Fig. 5. The block pulley model in the software “Universal machine”

The initial conditions: if  $t = 0$ , the trolley is stationary  $x_1 = 0$ ,  $\dot{x}_1 = 0$ ;

the block pulley is placed in the vertical position  $x_2 = 0$ ,  $\dot{x}_2 = 0$ ;

the hook block is moved upward  $z_2 = b_0 - b_1 t$ ;

the load is fixed in the position of standing equilibrium  $z_4 = 0$ ,  $\dot{z}_4 = 0$ .

In fig. 6 there are the speed variation charts for trolley and load relating to the trolley in the direction of  $y$ -axis.

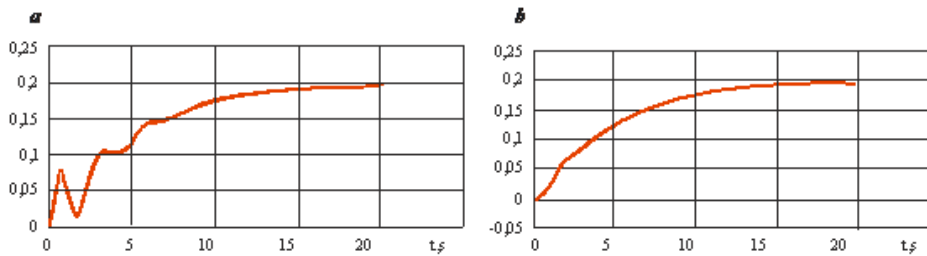


Fig. 6. The charts of speed variation (m/s) of trolley (a) and the speed of load (m/s) relating to the trolley (b) in the direction of y-axis

The initial data: the trolley weight  $m_1 = 2000$  kg; the weight of hook block with lower block and ropes  $m_2 = 600$  kg; the load weight  $m_3 = 10^4$  kg; the block pulley rate  $K_n = 4$ , for the double block pulley  $\xi = 2$ ; the initial rope length – 10 m; the average block pulley length for this task  $l_n = 5$  m; the rope of ЖК–Р type 6x19(1+6+6/6)+1 fiber core, GOST (All-Union State Standard) 2688–80 with the diameter  $d_r = 21$  mm, the breaking force  $S_{break} = 236$  kN, the rope bulk weight  $\rho = 1,635$  kg/m. The tensile stiffness of block pulley ropes  $C_1 = 46,5$  MN/m; the damping rate of block pulley ropes  $\mu_2 = 65$  Ns/m; the experimental value of resistance coefficient for one rope under vertical oscillations  $\mu_k = 9$  Ns/m. The resistance coefficient  $\mu_3$  of the block pulley damping element  $\mu_3 = 36$  (N·s/rad).

The initial distance between load and upper block  $b_0 = 10,0$  m; the load lofting speed  $b_1 = 0,04$  m/s. The pulling force of trolley shaft is determined by the dependence  $F_1 = 500 - 2500\dot{x}_1 t$  (H).

As it is seen on the fig. 6 the trolley speed and load oscillations related to the trolley are stabilized about the value 0,2 m/s in 18...20 s after the initiation of movement.

### 3. Conclusion

The mathematical movement model of loaded block pulley trolley was developed based on Lagrange equation of the second genus. The differential equations of loaded trolley motion were acquired, and the relations for viscoelastic block pulley characteristics based on empirical observations.

The computer dynamic model of loaded block pulley in the software “Universal Mechanism” environment that can be switched as the subsystem to the computer model of lifting crane. It was shown that the modeling block pulley dynamics could be substantially simplified, if to neglect the mass variation and the rope rigidity of block pulley by changing its length.

The studied example demonstrates the opportunity of the use of two-mass model of loaded block pulley for the dynamic computer modeling of lifting cranes.

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