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On some recent achievements of earthquake simulation

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Abstract

This paper presents some recent achievements of earthquake simulation, which is divided into the seismic wave propagation simulation and the seismic structure response simulation. These achievements are based on rigorous mathematical treatment of continuum mechanics problems, and numerical algorithms of solving the problems are developed. A multi-scale analysis method is developed for the seismic wave propagation simulation; numerical dispersion is reduced by introducing a new discretization scheme. A smart treatment of crack initiation and propagation is developed for the seismic structure response simulation, so that a numerical experiment is made for failure processes by using numerous samples of one structure.

Keywords: large scale numerical computation, multi-scale analysis, numerical dispersion, particle discretization scheme, crack

1. Introduction

Earthquake simulation is divided into two categories, namely, the simulation of seismic wave propagation and the simulation of seismic structure response. Both the simulations solve an initial-boundary value problem for a vector function of displacement, with the governing equation being linear or non-linear partial differential equations, which are derived from classical continuum mechanics. Spatial and temporal resolution must be high in order to make a reliable prediction about earthquake hazard and disaster, which leads to the need for large scale numerical computation.

The recent seismic wave propagation simulation is capable to reproduce or predict wave components up to around 1 Hz in the frequency range[1, 2, 3, 4]. Current major concerns are thus investigation of rupture processes on a fault from which seismic waves are emitted and modeling of heterogeneous crust structures, in order to make a more reliable model for the simulation. The governing equation is linear, and a finite difference method is usually used as a numerical analysis method. The method of solving the governing equation is matured, except for numerical treatment of the configuration of the crust structures and the ground surface, depending on which concentration of wave amplitudes would take place in particular regions.

The seismic structure response simulation has been used for seismic design of a wide class of buildings and structures[5]. Since the purpose is design, it is sufficient to simulate vibrations of a target structure which are induced by seismic ground motion; a reliable simulation can be made for the structure responses prior to damaging or collapsing by using a commercial finite element method. A remaining task of the seismic structure response simulation is to analyze damage or collapse processes of a structure.

The earthquake simulation is chosen as one of key research themes of the next generation super computer in Japan. The main objective is to improve the spatial and temporal resolution of the seismic wave propagation and structure

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response simulations by taking advantage of a large scale parallel computing environment. The improvement requires much smarter theoretical analysis of continuum mechanics problems, together with development of better algorithms which can take advantage of the large scale computational environment.

2. Solid continuum mechanics

A basic model of solid continuum mechanics is elastic body. Analysis of deformation of this solid starts from three phenomenological assumptions, namely, strain-displacement relation, equilibrium equation, and stress-strain relation. The three equations are mathematically combined, so that an initial-boundary value problem is posed for a displacement function, which is often transformed to the form of Lagrangian. We explain this framework using a Cartesian coordinate (x_1, x_2, x_3). Displacement, strain and stress are denoted by u_i, ϵ_{ij} and σ_{ij}, respectively, which are vector or second-order tensor functions of space $\mathbf{x}$ and time t. The three phenomenological equations are

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \sigma_{ji,j} = \rho \ddot{u}_i, \quad \sigma_{ij} = c_{ijkl}\epsilon_{kl}, \tag{1}$$

and the governing equation, which is often called a wave equation, is

$$(c_{ijkl}u_{k,l})_{,j} = \rho \ddot{u}_i. \tag{2}$$

A Lagrangian for the displacement function is

$$\mathcal{L}[\mathbf{u}, \dot{\mathbf{u}}] = \int_D \frac{1}{2}c_{ijkl}u_{i,j}u_{k,l} - \frac{1}{2}\rho \dot{u}_i \dot{u}_i \, \mathrm{d}\mathbf{x}. \tag{3}$$

Here, c_{ijkl} and ρ are elasticity tensor and density, comma and dot indicate spatial and temporal derivatives, i.e., $(.)_{,i} = \frac{\partial(.)}{\partial x_i}$) and $\dot{(.)} = \frac{\partial(.)}{\partial t}$Cand summation convention is employed. Symbol D denotes a domain of analysis.

In Eq. (1), the strain-displacement relation is linear. This is called an assumption of infinitesimally small deformation. As deformation becomes larger, say, $|\epsilon_{ij}|$ exceeds 10^{-3}, this assumption is not suitable and non-linear strain-displacement relation should be used. Similarly, the stress-strain relation in Eq. (1) is linear, which is called an assumption of linear elasticity. Depending on the kind of a material, as deformation becomes larger, the stress-strain relation tends to be non-linear or stress tends to depend on the history of strain (or the history of stress). Such extensions of the strain-displace6ment and stress-strain relations, which usually lead to non-linearity, have been studied in order to generalize continuum mechanics.

Specialization has been made for continuum mechanics, on the contrary to the above generalization. It provides specific constrains for displacement, which are related to the configuration of a target solid. For instance, a displacement function is approximated as a function of one variables for structure member of bar or beam, and a function of two variables for structure member of plate or shell. Even though some additional approximations are made, such specialization converts to Eq. (2) in a simpler form, so that an analytic solution can be obtained. Continuum mechanics of this specialization is understood as structure mechanics, which is the foundation of earthquake engineering. It should be noted that the simplified governing equation is readily derived by substituting constrained displacement functions into $\mathcal{L}$ of Eq. (3).

As briefly mentioned, it is a finite difference method that is usually used for the seismic wave propagation simulation of solving Eq. (2). The domain D is a heterogeneous semi-infinite space which models the crust structure. As the nature of the wave equation, higher temporal resolution is needed in order to increase the spatial resolution of solving Eq. (2). Hence, it is a smart choice to use finer grid in the finite difference method, with sacrifice of accuracy in modeling the crust structure, for the sake of large scale and efficient numerical computation.

The seismic structure response simulation usually uses a finite element method to solve a Lagrange equation of Eq. (3) even though the form of Lagrangian changes depending on a structure. This is because non-linear analysis due to the generalization of continuum mechanics is required. The specialization of continuum mechanics provides smart elements which are relevant to a structure member of particular configuration. The use of the smart elements drastically reduces the degree-of-freedom that is needed in numerical computation, even though it often needs experiments to tune up non-linear properties of the element.

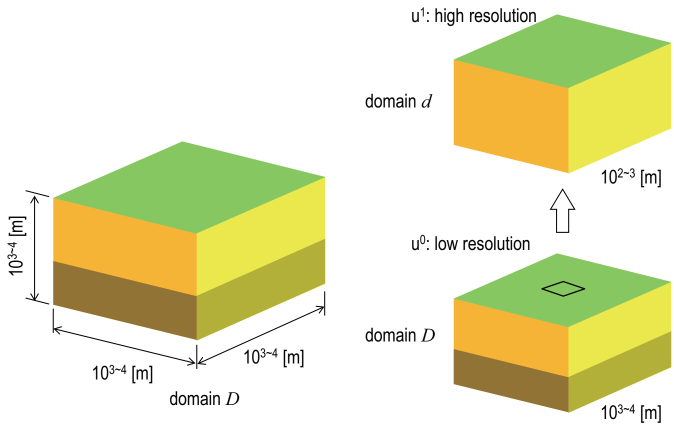


Figure 1: Schematic view of $\mathbf{u}^0$ and $\mathbf{u}^1$ of multi-scale analysis for seismic wave propagation simulation.

3. Seismic wave propagation simulation

Seismic waves are generated by the failure of a fault which is located at the depth of $10^{3\sim4}$ m, and propagate in the crust. The crust consists of rock, and has a wave velocity of around 10^3 m/s when modeled as an elastic body. Due to the on-going sedimentation process, the crust has strong heterogeneity near the ground surface, and soft soil structures appear above around 10^1 m. The wave velocity of ground is smaller by the order of $10^{1\sim2}$ m/s, which changes the direction of the seismic wave to the vertical and amplifies the amplitude of the seismic waves; the amplification reaches 10 times for wave components of $10^{-1\sim0}$ s which are the range of the period of structure vibration. Smaller wave speed means shorter wave length; the wave length that corresponds to the period of $10^{-1\sim0}$ s is $10^{1\sim2}$ m for the surface ground layer but $10^{2\sim3}$ m for the deeper part of the crust. In numerical computation, therefore, higher spatial resolution in modeling the heterogeneous ground structure is needed for the surface ground layer.

The authors apply a multi-scale analysis in order to efficiently make the seismic wave propagation simulation in a designated temporal resolution[6, 7]. The multi-scale analysis is based on singular perturbation expansion. That is, denoting by $\varepsilon(\ll 1)$ the ratio between the spatial resolution for the rock crust and the soft ground layer, we expand $\mathbf{u}$ in the following form:

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}^0(\mathbf{x}, t) + \varepsilon\mathbf{u}^1(\mathbf{x}, \frac{1}{\varepsilon}\mathbf{x}, t) \tag{4}$$

The first term, $\mathbf{u}^0$, is a solution of low resolution, and the second term, $\mathbf{u}^1$, is its correction of high resolution; see Fig. 1. Unlike regular perturbation expansion, it is assumed that $\mathbf{u}^1$ is a function of $\frac{1}{\varepsilon}\mathbf{x}$ as well as $\mathbf{x}$, and hence Eq. (4) is called singular; $\mathbf{x}$ and $\frac{1}{\varepsilon}\mathbf{x}$ correspond to the spatial scale of the low and high spatial resolution, respectively. A set of coupled governing equations are derived for both $\mathbf{u}^0$ and $\mathbf{u}^1$ by substituting the expansion into Eq. (2). $\mathbf{u}^0$ is computed for the entire domain D that includes the fault, and then $\mathbf{u}^1$ is computed for a small domain which is located near the ground surface and includes the target structure in it. For simplicity, this small domain is denoted by d.

In the seismic wave propagation simulation, a spatial resolution depends on the temporal resolution. This implies that a smaller domain is employed for d as the temporal resolution increases. However, the simulation requires wave components in longer periods, and such a small domain is not suitable to accurately compute these components in d. The multi-scale analysis based on the singular perturbation expansion must use a larger domain even though the temporal resolution is increased, in order to obtain an accurate solution in a wide range of periods. Thus, the authors model d using the spatial resolution that corresponds to the highest temporal resolution, say, 10^1 m, but the size of d is in the order of $10^{2\sim3}$ m. With this size of d,

> $\mathbf{u}^0$ is used as approximate boundary conditions acting on the boundary of d, and $\mathbf{u}^0 + \mathbf{u}^1$ which is regarded as a function of $\frac{1}{\varepsilon}\mathbf{x}$ is solved in d.

This is the authors' problem setting of the numerical computation that is made for the seismic wave propagation simulation.

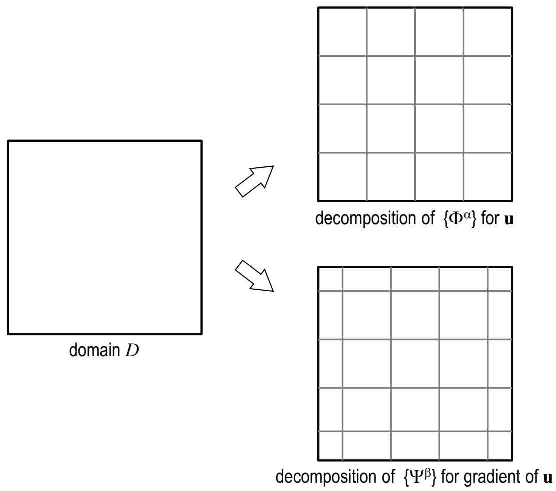


Figure 2: Schematic view of PDS; two domain decompositions, $\{\Phi^\alpha\}$ and $\{\Psi^\beta\}$, are applied to domain D.

Besides the amplification due to the difference in the material properties of rock and ground, seismic waves are amplified due to the configuration of heterogeneous crust and ground structures. For instance, surface soils make layered structures, and the width of each layer is not uniform in a segmented plane. Sometime there occurs large concentration at the part where the layer configuration wildly changes[8, 9]. As an extreme case, the concentration of strain becomes infinite at the edge of the layer, if exists, when linear elasticity is assumed. Accurate evaluation of such wave amplification requires a model of higher spatial resolution, which may exceed the spatial resolution that corresponds to the required temporal resolution.

In the multi-scale analysis mentioned above, Eq. (2) with slight modification is solved for the two models of D and d by applying large scale numerical computation. Even though the equation is linear, the profile of seismic waves that are numerically computed is often broken. This is because the accuracy of computing wave components of shorter periods is worse than that of computing wave components of longer periods. This numerical error is usually referred as numerical dispersion. There have been made a number of attempts to reduce the numerical dissipation; for instance, smart discretization of functions have been proposed. In particular, computational seismology that uses a finite different method takes advantages of staggered grid in which displacement and stress are discretized in different grids.

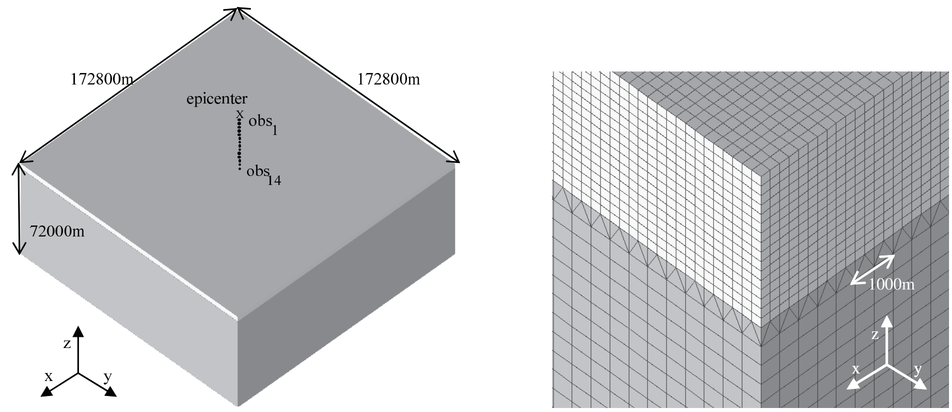
The authors apply the same idea as the staggered grid to a finite element method[10]. This attempt applies two sets of basis functions to discretize a function and its derivatives; see Fig. 2. In solving Eq. (2) for D, a displacement function $\mathbf{u}$ is dicretized by decomposing D into a set of small subdomains $\{\Phi^\alpha\}$ and using a characteristic function of Φ^α, denoted by ϕ^α, as follows:

$$\mathbf{u}(\mathbf{x}, t) = \sum_\alpha \mathbf{u}^\alpha(t)\,\phi^\alpha(\mathbf{x}). \tag{5}$$

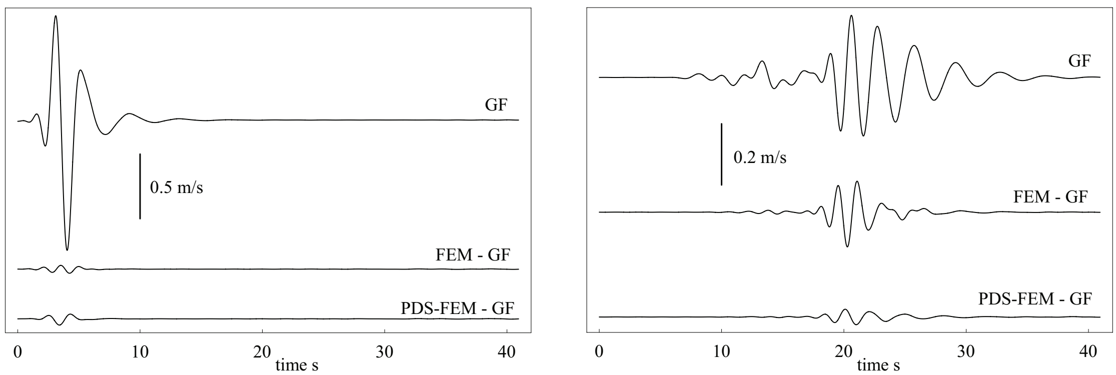
The gradient of the displacement function, $\nabla\mathbf{u}$, uses another domain decomposition, $\{\Psi^\beta\}$. Denoting by $\psi^{\beta p}$ a polynomial in Ψ^β which serves as a basic function, the discretization of $\nabla\mathbf{u}$ is

$$\nabla\mathbf{u}(\mathbf{x}) = \sum_\beta \sum_p (\nabla\mathbf{u})^{\beta p}(t)\,\psi^{\beta p}(\mathbf{x}), \tag{6}$$

where $(\nabla\mathbf{u})^{\beta p}$ stands for the coefficient of the discretization. For a given $\mathbf{u}$ or $\nabla\mathbf{u}$, the discretization coefficient, $\{\mathbf{u}^\alpha\}$ or $\{(\nabla\mathbf{u})^{\beta p}\}$ is determined by minimizing $\int_D |\mathbf{u} - \sum_\alpha \mathbf{u}^\alpha\phi^\alpha|^2 d\mathbf{x}$ or $\int_D |\nabla\mathbf{u} - \sum_\beta\sum_p (\nabla\mathbf{u})^{\beta p}\psi^{\beta p}|^2 d\mathbf{x}$, respectively, where $|(.)|^2$ stands for the square of the norm of a vector or a second-order tensor, (.). As shown in Eq. (5), $\mathbf{u}$ is discretized as a sum of characteristic functions. This discretization is interpreted as modeling of a body as an assembly of small particles each of which moves with rigid-body motion, i.e., Φ^α moves with $\mathbf{u}^\alpha$. In view of this interpretation, the discretization of Eqs. (5) and (6)is called Particle Discretization Scheme (PDS).



a) model of numerical experiment



b) comparison of exact solution (GF) with ordinary FEM (FEM) and PDS-FEM at observation points 1 and 14

Figure 3: Example of reduction of numerical dispersion by using PDS-FEM.

The authors apply a finite element method which is implemented with PDS (PDS-FEM) to the seismic wave propagation simulation. It is shown that the numerical dispersion is reduced by using PDS as a discretization scheme; PDS-FEM, just like the staggered grid, $\{\Phi^\alpha\}$ and $\{\Psi^\beta\}$ are a set of grids which are shifted by the half length of the grid edge length, and $\{\psi^{\beta\, p}\}$ is third order polynomials. The simulation is made for a two-layered D to which Green's function is analytically computed. It is shown that the numerical dispersion is reduced to around 1/5 of the ordinary FEM, for the part of D where larger numerical dispersion takes place[7]; see Fig. 3. Beside the use of the two sets of basis functions for the discretization, PDS-FEM regards a continuum as an assembly of rigid-body particles, so that the mass matrix automatically becomes diagonal[11, 12]; unlike an ordinary finite element method which makes approximations to convert a mass matrix to a diagonal one, PDS-FEM rigorously derives a diagonal mass matrix.

4. Seismic structure response simulation

The mechanism of seismic structure response is more complicated than usually considered. Seismic ground motions are waves which propagate in the grounds. When the motions are input to a structure, it is shaken as if it is a rigid body. This is because the structure is fixed to the ground; while some portions of seismic waves propagate in the structure, the rigid body motion is dominant in shaking the structure. The motion of the rigid body instantaneously induces acceleration at every point in the structure, and the gradient of stress is generated to satisfy equilibrium. The stress satisfies the traction free conditions on the structure surface, so that its value is determined. As shown in the preceding section, a set of fourth dimensional partial differential equations, Eq. (2), are posed for displacement which accompanies the stress; this deformation is induced by the rigid boy motion of the structure that is transformed from the seismic ground motion.

The above mechanism is described in the viewpoint of continuum mechanics. While the information of the boundary conditions is delivered to the point inside the structure at the elastic wave speed, the information of the seismic ground motion which is transformed as rigid body motion is delivered instantaneously. This is strange in the viewpoint of modern physics, showing the limitation of continuum mechanics; this limitation that the infinite speed of delivering ground motion information is never fatal for earthquake engineering.

As is seen, it is obvious that a model which precisely describes the structure configuration is needed, and a large scale computation is inevitable to numerically solve the differential equations of such a model. When the computer environment was poor, various approximations were made to obtain reasonably accurate approximate solutions; as mentioned, a non-linear model was developed for a structure member, and it was tuned up by carrying out various experiments. The approximate solutions are actually useful for the purpose of seismic design, because they are simple. However, they are not sufficient to analyze the processes in which a structure is damaged or collapsed by the seismic ground motion, or to estimate malfunctioning of structure members and facilities. Furthermore, a newly tuned-up model must be developed when the design of a structure is changed. The current computer environment gives us an opportunity to accurately analyze the seismic structure response[13]. The analysis is made by applying a finite element method to a detailed model which is constructed in terms of solid elements; the method uses massive numerical computation to solve the Lagrangian equation of Eq. (3), instead of Eq. (2).

It should be emphasized that the seismic structure response is regarded as vibration phenomena, rather than wave propagation phenomena. For small deformation, all the equations shown in Eq. (1) are linear, and hence the structure response is given as a sum of several distinct modes of vibration. When the deformation induced by the seismic ground motion becomes large, the equations become non-linear and some residual deformation is generated after the seismic ground motion is finished. This residual deformation corresponds to local damage of the structure. When the induced deformation becomes larger, it results in local or overall failure due to the initiation and propagation of cracks.

A crack is separation of solid. Mathematically, it is expressed as discontinuity in a displacement function. In numerically solving differential equations, a target function is required to be continuous and differentiable, and it is necessary to introduce a new surface or a special basis function every time a crack is initiated or propagates. The degree-of-freedom of the numerical computation increases. Moreover, it is a difficult task to determine the configuration of the propagating crack. A numerical analysis which is based on fracture mechanics determines an increment of crack surface using a suitable fracture criterion for a given increment of loading, but a two-dimensional curved surface is not easily determined for a crack which propagates in a three-dimensional body; the determination is really difficult, if branching and kinking are further considered.

When PDS is applied to discretize a displacement function, as shown in Eq. (5), the function has discontinuity across boundary $\partial\Phi^\alpha$ of a decomposed subdomain Φ^α. Assuming that a set of $\partial\Phi^\alpha$ are candidates of facets of an initiating or growing crack, we replace a problem of determining the crack surface configuration with a problem of choosing a suitable facet among the candidates, which can be solved with much less numerical computation. This assumption corresponds to the spatial heterogeneity of local material strength or fracture properties, i.e., $\partial\Phi^\alpha$ is regarded as a facet on which the local material is weaker than the surroundings. It is natural to employ non-structured domain decomposition such as Voronoi tessellation. Indeed, if $\{\Phi^\alpha\}$ is chosen as a Voronoi tessellation of the domain D and $\{\Psi^\beta\}$ is the dual Delaunay tessellation with $\psi^{\beta p}$ being a 0-th order polynomial or the characteristic function of Ψ^β, then, a stiffness matrix of PDS-FEM coincides with that of ordinary FEM with linear elements[10, 14]. The stiffness matrix of PDS-FEM is given as

$$K_{ij}^{\beta\alpha\alpha'} = \left(\int \phi^{\alpha}_{,k}\psi^{\beta}\mathrm{d}\mathbf{x}\right) c_{ikjl} \left(\int \phi^{\alpha'}_{,l}\psi^{\beta}\mathrm{d}\mathbf{x}\right), \tag{7}$$

and this coincides with the stiffness matrix of linear elements when Ψ^β is used as a tetrahedron element; ψ^β is the characteristic function of Ψ^β. The coincidence of the stiffness matrix means that the speed of the convergence of the PDS-FEM solution is the same as that of the ordinary FEM with linear elements. When a crack propagates on $\partial\Psi^\beta$, the corresponding stiffness matrix changes the value of its components since the contribution of $\nabla\phi^{\alpha'}\psi^\beta$ to the integration

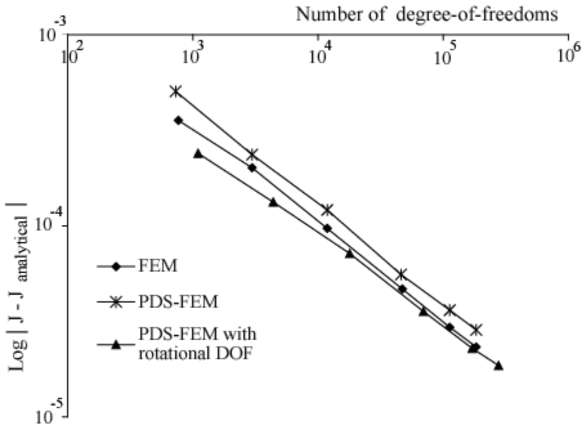


Figure 4: Convergence of Mode I stress intensity factor at a tip of semi-infinite crack.

is dropped. Instead of introducing a new degree-of-freedom, PDS-FEM treats the crack propagation as the reduction of the stiffness matrix. The accuracy of this treatment is not bad; it is shown that the accuracy of computing the strain energy stored at a crack by PDS-FEM is similar to the FEM with linear element, and that the accuracy is increased by adding rotational motion to the motion of Φ^{α}; see Fig. 4.

There are many Voronoi tessellations for one common *D*. One model of a Voronoi tessellation is regarded as one sample since it has a distinct set of possible crack surfaces and shows distinct cracking processes when analyzed by PDS-FEM. A solution is unique if ideal homogeneity is assumed for the parameters of strength or fracture. In reality, however, there exists material heterogeneity. Thus, if the solution of ideal homogeneity is stable or instable, the presence of such local heterogeneity does not or does change the cracking process drastically, respectively. It is also intuitively acceptable to expect that the solution of ideal homogeneity is stable or instable before or after cracking takes place, respectively, since local cracking depends on local material properties. The stability of the solution of the ideally homogeneous body is examined by a numerical experiment which uses many samples of different tessellations. As an illustrative example, a numerical experiment is conducted for two anti-symmetric cracks which are located in a plate subjected to dynamic loading; see Fig. 5. It is shown[15] that

- the anti-symmetry is lost during the cracking processes, and there are many patterns of the processes.
- the crack configuration is likely to remain anti-symmetric for lower loading rate, but it loses anti-symmetry for higher loading rate
- the configuration is not random unlike the process, and, in most samples, one cracks grows long and straight while the other curves.
- the branching is often observed for the crack configuration at highest loading rate.

See Fig. 6 for examples of crack propagation processes and for the distribution of crack configuration; the distribution is expressed as a probability density function at a point that a crack passes the point, and the function is computed by using 200 samples of different Voronoi tessellations.

5. Concluding remarks

Numerical computation of solving non-linear problems of solid continuum mechanics is well established. The authors regard the numerical computation that is needed for the seismic wave propagation and structure response simulations as the last tough problem. This is because an efficient multi-scale analysis is needed to compute a large domain in high spatial resolution, and because smart treatment is required to compute crack initiation and propagation. Commercial finite element methods are well matured, and sufficient for seismic design. The numerical computation

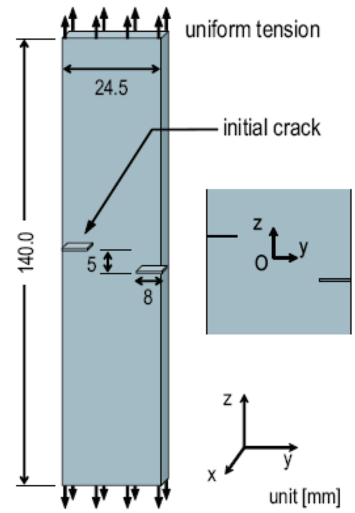


Figure 5: Model for two anti-symmetric cracks located in plate.

discussed in the present paper is needed only to advance seismic design, so that structure responses induced by very strong ground motions caused by a nearby fault is predicted or the seismic safety of various functionalities of a structure are examined.

Comparison with observations and experiments is of essential importance to advance numerical computation. In seismology and earthquake engineering, comparison of numerical computation is made by using a dense network of seismographs or a full scale experiment, and there is a need to advance observation and experiment techniques. The numerical computation also needs a new benchmark testing to examine the accuracy of a solution. A next generation supercomputer will serve as a computer environment in which such a benchmark testing is made for the seismic wave propagation and structure response simulations.

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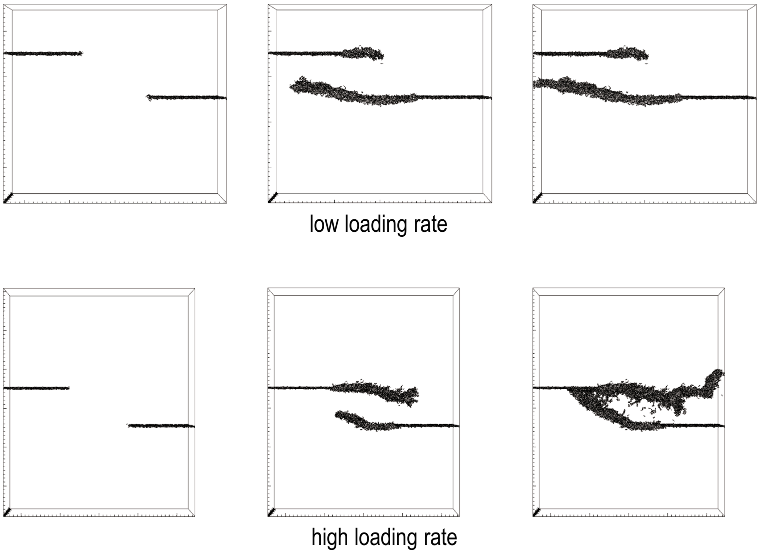
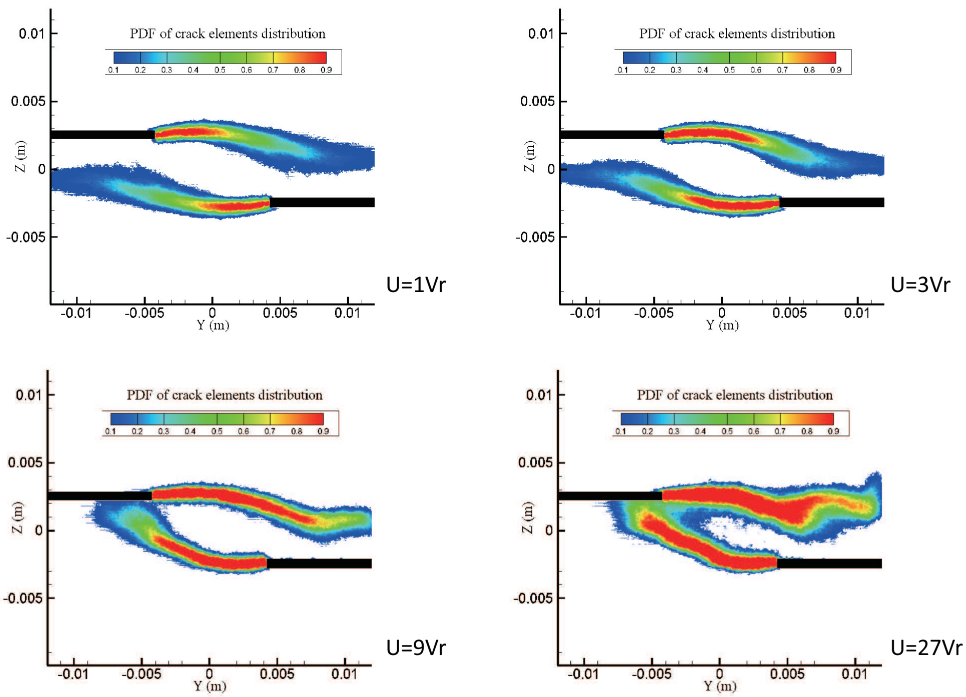


a) examples of crack propagation processes



b) distribution of crack configuration; U indicates loading rate with V_r being reference speed.

Figure 6: Propagation process and configuration of anti-symmetric cracks.

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