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A Fibonacci collocation method for solving a class of Fredholm–Volterra integral equations in two-dimensional spaces

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ABSTRACT

In this paper, we present a numerical method for solving two-dimensional Fredholm–Volterra integral equations (F-VIE). The method reduces the solution of these integral equations to the solution of a linear system of algebraic equations. The existence and uniqueness of the solution and error analysis of proposed method are discussed. The method is computationally very simple and attractive. Finally, numerical examples illustrate the efficiency and accuracy of the method.

1. Introduction

A sequence of polynomials is a Fibonacci sequence if it satisfies the recursion

$$f_{n+2}(x) = x \cdot f_{n+1}(x) + f_n(x), \quad n \geq 1. \tag{1}$$

Two well-known Fibonacci sequences are the Fibonacci polynomials, $\{F_n(x)\}$, defined using (1) with $F_1(x) = 1$ and $F_2(x) = x$ and the Lucas polynomials, $\{L_n(x)\}$, defined using (1) with $L_1(x) = 2$ and $L_2(x) = x$ (Bergum and Hoggatt, 1974; Bicknell, 1970; Bicknell and Hoggatt, 1973). In addition to being Fibonacci sequences, these polynomials produce Fibonacci and Lucas numbers, respectively, when evaluated at $x = 1$. Fibonacci (from "filius Bonacci"), Italian mathematician of the 13th century, is the best known to the modern world for a number sequence named the Fibonacci numbers after him, which he did not discover but used as an example in his book, Liber Abaci. In Fibonacci's Liber Abaci book, chapter 12, he posed, and solved a problem involving the growth of a population of rabbits based on idealized assumptions. The solution, generation by generation, was a sequence of numbers later known as Fibonacci numbers. The number sequence was known to Indian mathematicians as early as the 6th century, but it was Fibonacci's Liber Abaci that introduced it to the West. In the Fibonacci sequence of numbers, each number is

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the sum of the previous two numbers, starting with 0 and 1. This sequence begins 0, 1, 1, 2, 3, 5, … (Grimm, 1973).

Recently, Mirzaee and Hoseini (2013b; 2014) adapted the matrix method for the Fibonacci polynomials. They have been used the Fibonacci matrix method to find approximate solutions of singularly perturbed differential-difference equations and systems of linear Fredholm integro–differential equations by setting the equations for the Fibonacci polynomials in matrix form as $F(x) = BX(x)$, where $F(x) = [F_1(x), F_2(x), F_3(x), \ldots, F_{N+1}(x)]^T, X(x) = [1, x, x^2, x^3, \ldots, x^N]^T$, and B is the invertible lower triangular matrix with entrances the coefficients appearing in the expansion of the Fibonacci polynomials in increasing powers of x. They approximate the solution of these equations as follows

$$y(x) \approx \sum_{n=1}^{N+1} c_n F_n(x), \quad 0 \le a \le x \le b, \tag{2}$$

where c_n, $n = 1, 2, \cdots, N+1$ are the unknown Fibonacci coefficients, N is any arbitrary positive integer such that $N \ge m$, and $F_n(x)$, $n = 1, 2, \ldots, N+1$ are the Fibonacci polynomials defined by

$$F_{n+1}(x) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-i}{i} x^{n-2i}, \quad n \ge 0, \tag{3}$$

where $\lfloor n/2 \rfloor$ denotes the greatest integer in $n/2$.

Note that $F_{2n}(0) = 0$ and $x = 0$ is the only real root, while $F_{2n+1}(0) = 1$ with no real roots. Also for $x = k \in N$ we obtain the elements of the k-Fibonacci sequences (Falcon and Plaza, 2009).

The Volterra–Fredholm integral equations arise in a variety of applications in many fields including continuum mechanics, potential theory, geophysics, electricity and magnetism, antenna synthesis problem, communication theory, mathematical economics, population genetics, radiation, the particle transport problems of astrophysics and reactor theory, fluid mechanics (Abdou, 2003; Bloom, 1980; Jaswon and Symm, 1977; Jiang and Rokhlin, 2004; Schiavane et al., 2002; Semetanian, 2007).

In recent years, many different basic functions have used to estimate the solution of linear and nonlinear Volterra–-Fredholm integral equations, such as orthonormal bases and wavelets (Brunner, 1990; Ghasemi et al. (2007); Maleknejad et al. (2010); Ordokhani, 2006; Ordokhani and Razzaghi, 2008; Yalcinbas, 2002; Yousefi and Razzaghi, 2005). Mirzaee and Hoseini (2013a) applied hybrid of block-pulse functions and Taylor series to approximate the solution of a nonlinear Fredholm–Volterra integral equation of the form

$$y(t) = x(t) + \lambda_1 \int_0^t k_1(t,s) F_1(y(s)) ds + \lambda_2 \int_0^1 k_2(t,s) F_2(y(s)) ds,$$
$$t \in I = [0, 1],$$

where λ_1 and λ_2 are constant, $x(t) \in L^2(I), k_1(t,s)$ and $k_2(t,s) \in L^2(I \times I)$ and $F_1(y(s))$ and $F_2(y(s))$ are given continuous functions which are nonlinear with respect to $y(t)$, and $y(t)$ is an unknown function.

In this work, we consider the two dimensional Fredhom–Volterra integral equation that given by the form

$$g(s,t) + \int_0^1 q(s,y) g(y,t) dy + \int_0^t k(t,x) g(s,x) dx = f(s,t), \quad s,t \in I, \tag{4}$$

where q, k, f are known and g is unknown. Moreover, functions q, k, f and g belong to $L^2(\Gamma)$ that $\Gamma = I \times I$. Hendi and Bakodah (2013) have been used the Adomian decomposition method to find approximate solution of nonlinear Fredholm–Volterra integral equations. Babolian et al. (2011) applied block-pulse functions to solve Eq. (4). In this manuscript, we propose a method based on series of Fibonacci polynomials to solve the Fredholm–Volterra integral equation (4).

2. Method of solution

The aim of our method is to get solution as Fibonacci series defined by

$$g(s,t) \simeq g_{N+1,M+1}(s,t) = \sum_{n=1}^{N+1} \sum_{m=1}^{M+1} c_{nm} F_n(s) F_m(t) = F(s,t) C, \tag{5}$$

where c_{nm}, $n = 1, 2, \ldots, N+1$, $m = 1, 2, \ldots, M+1$ are the unknown Fibonacci coefficients,

$$C = \left[c_{11}, \ldots, c_{1(M+1)}, c_{21}, \ldots, c_{2(M+1)}, \ldots, c_{(N+1)1}, \ldots, c_{(N+1)(M+1)}\right]^T,$$

N is any arbitrary positive integer, $F_n(x), n = 1, 2, \ldots, N+1$ are the Fibonacci polynomials defined in Eq. (3) and $F(s,t)$ is $1 \times (N+1)(M+1)$ matrix introduced as follows

$$F(s,t) = \left[h_{11}(s,t), \ldots, h_{1(M+1)}(s,t), h_{21}(s,t), \ldots, h_{2(M+1)}(s,t), \ldots, h_{(N+1)1}(s,t), \ldots, h_{(N+1)(M+1)}(s,t)\right],$$

where

$$h_{nm}(s,t) = F_n(s) F_m(t), \quad n = 1, 2, \ldots, N+1, m = 1, 2, \ldots, M+1.$$

The method of collocation solves the F-VIE (4) using the approximation (5) through the equations

$$r_{N+1,M+1}(s_i,t_j) = g_{N+1,M+1}(s_i,t_j) + \int_0^1 q(s_i,y) g_{N+1,M+1}(y,t_j) dy + \int_0^{t_j} k(t_j,x) g_{N+1,M+1}(s_i,x) dx - f(s_i,t_j). \tag{6}$$

that for a suitable set of collocation points, we choose Newton-Cotes nodes as $(s_i, t_j) = (2i - 1/2(N+1), 2j - 1/2(M+1))$ for all $i = 1, 2, \ldots, N+1$, $j = 1, 2, \ldots, M+1$.

Now, we find Fibonacci coefficients $g_{n,m}$ introduced in matrix form (5). Using (5) for (s_i, t_j) and (4) we have

$$F(s_i,t_j)C+\int_0^1 q(s_i,y)F(y,t_j)C\,dy+\int_0^{t_j} k(t_j,x)F(s_i,x)C\,dx=f(s_i,t_j). \tag{7}$$

for $i=1,2,\ldots,N+1,\ j=1,2,\ldots,M+1$ Then Eq. (7) can be written in the matrix form as follows

$$(\mathbf{F}+\mathbf{Q}+\mathbf{K})C=\mathbf{A}, \tag{8}$$

where

Eq. (7) gives $(N+1)(M+1)$ algebraic equations with $(N+1)(M+1)$ unknowns as coefficients of vector C. After solving this linear system, we can approximate the solution of equation (4) with substituting C in (5).

3. Convergence and error analysis

Assume that $(C[\Gamma], \|\,.\,\|)$ is the Banach space of all continuous functions on Γ with norm

$$\mathbf{F}=\begin{bmatrix} F(s_1,t_1)\\ \vdots\\ F(s_1,t_{M+1})\\ F(s_2,t_1)\\ \vdots\\ F(s_2,t_{M+1})\\ \vdots\\ F(s_{N+1},t_1)\\ \vdots\\ F(s_{N+1},t_{M+1})\end{bmatrix}_{(N+1)(M+1)\times(N+1)(M+1)},\quad \mathbf{Q}=\begin{bmatrix} \int_0^1 q(s_1,y)F(y,t_1)dy\\ \vdots\\ \int_0^1 q(s_1,y)F(y,t_{M+1})dy\\ \int_0^1 q(s_2,y)F(y,t_1)dy\\ \vdots\\ \int_0^1 q(s_2,y)F(y,t_{M+1})dy\\ \vdots\\ \int_0^1 q(s_{N+1},y)F(y,t_1)dy\\ \vdots\\ \int_0^1 q(s_{N+1},y)F(y,t_{M+1})dy\end{bmatrix}_{(N+1)(M+1)\times(N+1)(M+1)},$$

$$\mathbf{A}=\begin{bmatrix} f(s_1,t_1)\\ \vdots\\ f(s_1,t_{M+1})\\ f(s_2,t_1)\\ \vdots\\ f(s_2,t_{M+1})\\ \vdots\\ f(s_{N+1},t_1)\\ \vdots\\ f(s_{N+1},t_{M+1})\end{bmatrix}_{(N+1)(M+1)\times 1},\quad \mathbf{K}=\begin{bmatrix} \int_0^{t_1} k(s_1,y)F(y,t_1)dy\\ \vdots\\ \int_0^{t_{M+1}} k(s_1,y)F(y,t_{M+1})dy\\ \int_0^{t_1} k(s_2,y)F(y,t_1)dy\\ \vdots\\ \int_0^{t_{M+1}} k(s_2,y)F(y,t_{M+1})dy\\ \vdots\\ \int_0^{t_1} k(s_{N+1},y)F(y,t_1)dy\\ \vdots\\ \int_0^{t_{M+1}} k(s_{N+1},y)F(y,t_{M+1})dy\end{bmatrix}_{(N+1)(M+1)\times(N+1)(M+1)}.$$

$$\|g(s,t)\| = \max_{(s,t)\in\Gamma}|g(s,t)|. \tag{9}$$

Furthermore, we denote the error by

$$e_{N+1,M+1}(s,t) = \left|g_{N+1,M+1}(s,t) - g(s,t)\right|, \tag{10}$$

where $g(s,t)$, $g_{N+1,M+1}(s,t)$ show the exact and approximate solutions of the two-dimensional Fredholm–Volterra integral equation, respectively.

Theorem 1. *Assume that $f(s,t)$ in Eq (4) is bounded, for all $(s,t)\in\Gamma$, the kernel of Volterra term is bounded such that $|k(t,x)| \le M_1, (x,t)\in\Gamma$ and the kernel of the Fredholm term is bounded such that $|q(s,y))| \le M_2, (s,y)\in\Gamma$. Then the problem (4) has an unique solution whenever*

$$0<\alpha<1, \alpha = M_1 + M_2.$$

Proof. *Let*

$$\begin{aligned}
\left\|e_{N+1,M+1}(s,t)\right\| &= \left\|g_{N+1,M+1}(s,t) - g(s,t)\right\| = \max_{(s,t)\in\Gamma}\left|g_{N+1,M+1}(s,t) - g(s,t)\right| \\
&= \max_{(s,t)\in\Gamma}\left|\int_0^1 q(s,y)\left(g_{N+1,M+1}(y,t) - g(y,t)\right)dy + \int_0^t k(t,x)\left(g_{N+1,M+1}(s,x) - g(s,x)\right)dx\right| \\
&\le \left\|g_{N+1,M+1}(s,t) - g(s,t)\right\| \max_{(s,t)\in\Gamma}\left|\int_0^1 q(s,y)dy + \int_0^t k(t,x)dx\right| \\
&\le \left\|e_{N+1,M+1}(s,t)\right\|(M_1 + M_2),
\end{aligned}$$

therefore

$$(1-\alpha)\left\|e_{N+1,M+1}(s,t)\right\| \le 0$$

So if $0<\alpha<1$ we have $||e_{N+1,M+1}(s,t)||\to 0$ as $N, M\to\infty$ and this completes the proof of theorem.□

The Fibonacci polynomials can be expressed in terms of some orthogonal polynomials, such as Chebychev polynomial $u_n(x)$ of second kind (Rainville, 1960). It can be shown that

$$F_{n+1}(x) = i^n u_n\left(\frac{ix}{n}\right), \tag{11}$$

that $i^2 = -1, n \ge 0$.

Therefor Expansion of $g(s,t)$ in the approximated form of Fibonacci polynomials in Eq. (5) can be eventually written as

$$g_{N+1,M+1}(s,t) = \sum_{n=0}^{N}\sum_{m=0}^{M} b_{nm}u_n(s)u_m(t), \tag{12}$$

where b_{nm} can be expressed in terms of c_{nm}, $n = 1,2,\cdots,N+1$, $m = 1,2,\cdots,M+1$.

Preposition 1

$$T_n(x) = \frac{1}{2}\left(u_n(x) - u_{n-2}(x)\right), \tag{13}$$

where $T_n(x)$ is the Chebyshev polynomial of the first kind. Thus a second-kind expansion can be derived directly from a first-kind expansion (but not vice versa) (Mason and Handscomb, 2003).

Theorem 2. *If $f(x,y)$ be a continuous and bounded variation in*

$$S = \{-1 \le x, y \le 1\},$$

and if one of its partial derivatives be bounded in S, then f has a double Chebyshev expansion, uniformly convergent on S, of the form

$$f(x,y) = \sum_{i=0}^{\infty}{}'\sum_{j=0}^{\infty}{}' a_{ij}T_i(x)T_j(x), \tag{14}$$

where the primes in (14) indicate that (i) the first term is $1/4a_{00}$, (ii) a_{i0} and a_{0j} are to be taken as $\frac{1}{2}a_{i0}$ and $\frac{1}{2}a_{0j}$ for i,j > 0, respectively (Mason and Handscomb, 2003).

For an error estimation of the approximation solution of Eq. (4), we consider

Table 1 – The maximum error of Eq. (16).

Present method N = 1	Method of [2] m = 32	Method of [2] m = 64	Method of [11] m = 5, n = 8
0	0.02	0.01	1.27255e-1

Table 2 – Absolute errors of Eq. (17) for N = 3.

s	x	Absolute error
0.1250	0.1250	0.0303e-14
0.1250	0.3750	0.0261e-14
0.1250	0.6250	0.1269e-14
0.1250	0.8750	0.0578e-14
0.3750	0.1250	0.0303e-14
0.3750	0.3750	0.0261e-14
0.3750	0.6250	0.1269e-14
0.3750	0.8750	0.0578e-14
0.6250	0.1250	0.0303e-14
0.6250	0.3750	0.0261e-14
0.6250	0.6250	0.1269e-14
0.6250	0.8750	0.0578e-14
0.8750	0.1250	0.0261e-14
0.8750	0.3750	0.0261e-14
0.8750	0.6250	0.1269e-14
0.8750	0.8750	0.0578e-14

Table 3 – The maximum error of Eq. (17).

Present method N = 2	Present method N = 3	Method of [2] m = 32	Method of [2] m = 64
0.0248	1.2690e-15	0.1	0.04

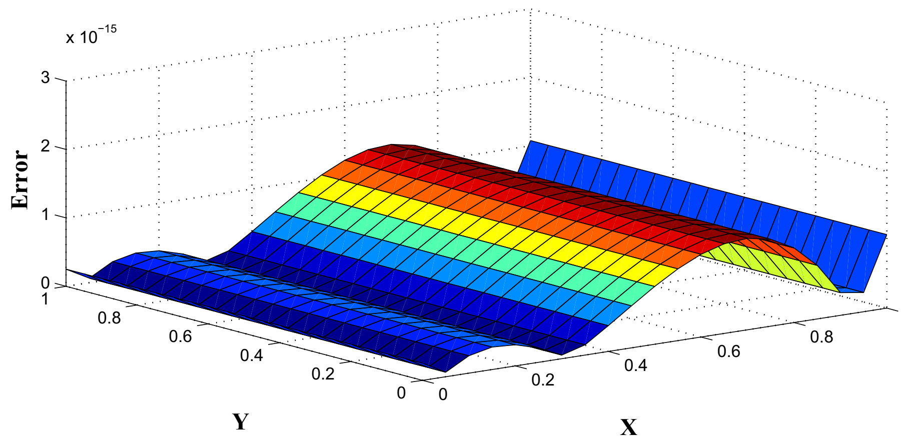

Fig. 1 – Absolute error functions obtained by the present method for N = 3 of Eq. (17).

$$r_{N+1,M+1}(s,t)+f(s,t)=g_{N+1,M+1}(s,t)+\int_0^1 q(s,y)g_{N+1,M+1}(y,t)dy$$
$$+\int_0^t k(t,x)g_{N+1,M+1}(s,x)dx. \quad (15)$$

$r_{N+1,M+1}(s,t)$ *is the perturbation function that depends only on* $<g_{N+1,M+1}(s,t)$. *By subtracting Eq.* (15) *form* (4) *we have*

$$\|r_{N+1,M+1}(s,t)\| \le \|e_{N+1,M+1}(s,t)\| + M_1\|e_{N+1,M+1}(s,t)\| + M_2\|e_{N+1,M+1}(s,t)\| = (1+M_1+M_2)\|e_{N+1,M+1}(s,t)\|.$$

then $r_{N+1,M+1}(s,t)$ *is bounded for* $0 \le s,t \le 1$. *So from above equation and Eqs.* (11)–(13) *and Theorem* 2 *we conclude that our proposed method is convergent for each* $f(x,y)$ *that is satisfying in Theorem* 2.

4. Numerical examples

In this section, three numerical examples are included to demonstrate the validity and applicability of the proposed technique. All results are computed by using a program written in the Matlab. In this regard, we have presented with tables and figures. In order to demonstrate the error of the method, we introduce the notation of the absolute error function $e_{N+1,M+1}(s,t)$ as Eq. (10) at the selected points of the given interval. In addition, we define the maximum error for $g_{N+1,M+1}(s,t)$ as in Eq. (9). In following examples we supposed that $M = N$ for convenience .

Example 1. *Consider the following two-dimensional Fredholm–Volterra integral equation* (Babolian et al., 2011; Hendi and Bakodah, 2013)

$$g(s,t)+\int_0^1 (5s+2y^2-5)g(y,t)dy+\int_0^t (-4t^2+xt)g(s,x)dx$$
$$=-10+st-2t+\frac{5}{2}s(6+t)-\frac{1}{6}t^3(63+10st), \quad (16)$$

with the exact solution $g(s,t)=st+3$.

Let $N=1$, *so* $F(s,t)=[1,s,t,st]$ *and the collocation points are* $(1/4,1/4),(1/4,3/4),(3/4,1/4)$ *and* $(3/4,3/4)$. *By applying presented method for the Eq.* (16) *we have*

$$\mathbf{F}=\begin{bmatrix}1.0000 & 0.2500 & 0.2500 & 0.0625\\ 1.0000 & 0.2500 & 0.2500 & 0.1875\\ 1.0000 & 0.2500 & 0.7500 & 0.1875\\ 1.0000 & 0.7500 & 0.7500 & 0.5625\end{bmatrix},\quad \mathbf{Q}=\begin{bmatrix}\frac{-37}{12} & \frac{-37}{48} & \frac{-11}{8} & \frac{-11}{32}\\ \frac{-37}{12} & \frac{-37}{16} & \frac{-11}{8} & \frac{-33}{32}\\ \frac{-7}{12} & \frac{-7}{48} & \frac{-1}{8} & \frac{-1}{32}\\ \frac{-7}{12} & \frac{-7}{16} & \frac{-1}{8} & \frac{-3}{32}\end{bmatrix},$$

$$\mathbf{K}=\begin{bmatrix}\frac{-7}{128} & \frac{-5}{768} & \frac{-7}{512} & \frac{-5}{3072}\\ \frac{-189}{128} & \frac{-135}{256} & \frac{-189}{512} & \frac{-135}{1024}\\ \frac{-7}{128} & \frac{-5}{768} & \frac{-21}{512} & \frac{-5}{1024}\\ \frac{-189}{128} & \frac{-135}{256} & \frac{-562}{512} & \frac{-405}{1024}\end{bmatrix},\quad \mathbf{A}=\begin{bmatrix}-6.6969\\ -11.6553\\ 1.2373\\ -3.1064\end{bmatrix}.$$

By solving the system (7), *we have*

$C=[3,0,0,1]$.

By substituting the elements of this vector into Eq. (5), *we have* $g(s,t)=st+3$ *which is the exact solution of the equation* (16).

Table 1, *gives the comparison of the result of the maximum error obtained by the present method, the method of* Babolian et al. (2011) *and the method of* Hendi and Bakodah, 2013.

Example 2. *Consider the following two-dimensional Volterra integral equation* (Babolian et al. (2011))

$$\int_0^t (x^2+xt-1)g(s,x)dx=\frac{1}{60}t\left(-120+100t^2-15t^3+22t^5-15st(-6+7t^2)\right), \quad (17)$$

with the exact solution $g(s,x)=x^3-3xs+2$.

Let $N=3$. *By following the method given in* Section 2, *we obtain the approximate solution of the problem by the Fibonacci polynomials as follows,*

Table 4 – Absolute errors of Eq. (18) for N = 3.

s	t	*Absolute error*
0.1250	0.1250	0.4650e-04
0.1250	0.3750	0.1719e-04
0.1250	0.6250	0.2280e-04
0.1250	0.8750	0.3620e-04
0.3750	0.1250	0.5597e-04
0.3750	0.3750	0.1825e-04
0.3750	0.6250	0.3065e-04
0.3750	0.8750	0.5289e-04
0.6250	0.1250	0.6862e-04
0.6250	0.3750	0.2010e-04
0.6250	0.6250	0.4054e-04
0.6250	0.8750	0.7349e-04
0.8750	0.1250	0.8524e-04
0.8750	0.3750	0.2287e-04
0.8750	0.6250	0.5311e-04
0.8750	0.8750	0.9930e-04

Table 5 – The maximum error of Eq. (18).

Present method		Method of [2]	
$N=2$	$N=3$	$m=32$	$m=64$
0.0017	0.9930e-04	0.04	0.02

$$g(s,x)=\frac{3069398723}{397088451422521397346304}x-3sx-\frac{4357066177}{148908169283445524004864}x^2+\frac{124090141069540772865 25}{124090141069537936670 72}x^3+\frac{190602456682810246856173559}{953012283414051353631129 60}.$$

Absolute errors of the proposed procedure at the grid points, for $N=3$, *are tabulated in* Table 2. Table 3 *gives the comparison of the result of the maximum error obtained by the present method and the method of* Babolian et al. (2011). *From* Table 3, *we see the errors decrease rapidly as N increases.* Fig. 1 *displays the absolute error functions obtained by the present method for* $N=3$.

Example 3. *Consider the following two-dimensional Fredholm–Volterra integral equation* (Babolian et al. (2011))

$$g(s,t)+\int_0^1\frac{y}{s+2}g(y,t)dy+\int_0^t e^{t+x}g(s,x)dx=e^{s-t}+\frac{e^{-t}}{2+s}+te^{s+t}, \tag{18}$$

with the exact solution $g(s,t)=e^{s-t}$.

Let $N=3$. *By following the method given in Section* 2, *we obtain the approximate solution by the Fibonacci polynomials of the problem. Absolute errors of the proposed procedure at the grid points, for* $N=3$, *are tabulated in* Table 4. Table 5 *gives the comparison of the result of the maximum error obtained by the present method and the method of* Babolian et al. (2011). *From* Table 5, *we see the errors decrease rapidly as N increases.* Fig. 2 *displays the absolute error functions obtained by the present method for* $N=3$.

5. Conclusion

In this article, we have studied a numerical scheme to solve Fredholm–Volterra integral equations in two-dimensional spaces, based on the expansion of the solution as a series of Fibonacci polynomials beside the collocation method to transform an F-VIE to a linear system of algebraic equations that can be solved easily. We can develop this method for solving two-dimensional nonlinear Fredholm–Volterra integral equations by some modifications. To obtain the best approximating solution of the system, we take more forms from the Fibonacci expansion of functions, that is, the truncation limit N must be chosen large enough. Illustrative examples are given to demonstrate the validity and applicability of the proposed method.

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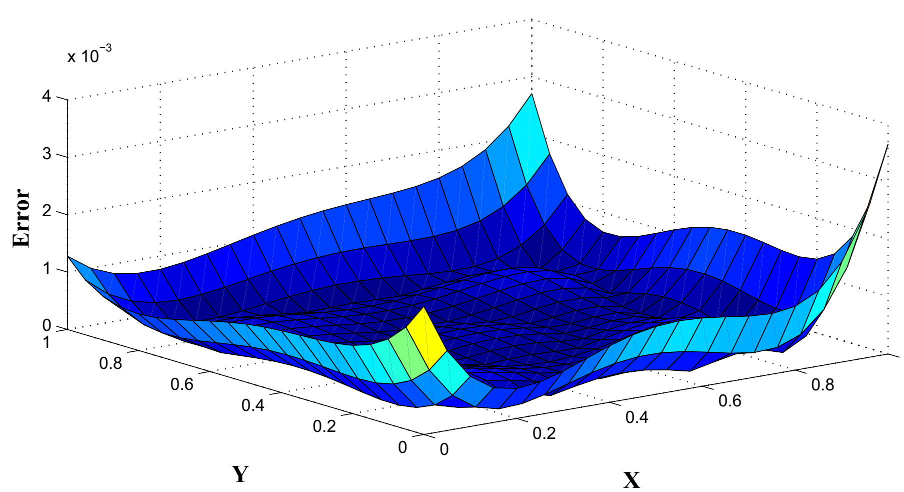

Fig. 2 – Absolute error functions obtained by the present method for N = 3 of Eq. (18).

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