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A food chain system with Holling type IV functional response and impulsive perturbations

Hunki Baek [a]

[a] *Department of Mathematics, Kyungpook National University, Daegu, 702-701, South Korea*

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ABSTRACT

In this paper, a three-trophic-level food chain system with Holling type IV functional response and impulsive perturbations is established. We show that this system is uniformly bounded. Using the Floquet theory of impulsive equations and small perturbation skills, we find conditions for the local and global stabilities of the prey and top predator-free periodic solution. Moreover, we obtain sufficient conditions for the system to be permanent via the comparison theorem. We display some numerical examples to substantiate our theoretical results.

1. Introduction

In recent years, it has been of great interest to study dynamical properties of impulsive perturbations on population models. In particular, the impulsive prey–predator population models have been investigated by many researchers [1–8] and there is also a lot of literature on three-species food chain systems with impulsive perturbations [9–15]. For example, Liu et al. [2] investigated the dynamic behaviors of a Holling I predator–prey model with impulsive effect as regards biological and chemical control strategies. Cheng et al. and Liu et al. [1,3] studied the dynamic behaviors of a Holling II functional response predator–prey system as regards an impulsive control strategy—periodic releasing of natural enemies and spraying pesticide at different fixed times. Zhang and Chen [12] observed a three-trophic-level food chain system with Holling II functional responses and periodic constant impulsive perturbations of the top predator. In this context, many authors researched Holling type IV population models with impulsive perturbations [10,4,16,13]. In particular, Li and Tan [4] proposed a predator–prey model with Holling type IV functional response and an impulsive control strategy as follows:

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(t)(a-bx(t)) - \frac{c_1x(t)y(t)}{1+e_1(x(t))^2},\\ y'(t) &= -d_1y(t) + \frac{c_2x(t)y(t)}{1+e_1(x(t))^2}, \end{aligned}\right\} & t \neq (n+\tau-1)T,\, t\neq nT,\\ \left.\begin{aligned} x(t^+) &= (1-p_1)x(t),\\ y(t^+) &= (1-p_2)y(t), \end{aligned}\right\} & t=(n+\tau-1)T,\\ \left.\begin{aligned} x(t^+) &= x(t),\\ y(t^+) &= y(t)+q, \end{aligned}\right\} & t = nT,\\ (x(0^+), y(0^+)) = (x_0, y_0), \end{cases} \tag{1}$$

E-mail address: hkbaek@knu.ac.kr.

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where $x(t)$ and $y(t)$ are functions of time representing population densities of the prey and the predator, respectively, and all parameters are positive constants. The constant a is the intrinsic growth rate of the prey population, b is the coefficient of intra-species competition, c_1 is the per capita rate of predation of the predator, e_1 is the half-saturation constant, d_1 denotes the death rate of the predator, c_2 is the rate of conversion of a consumed prey to a predator, τ and T are the periods of spaying pesticides (harvesting) and the impulsive immigration or stocking of the predator, respectively, $0 \le p_1, p_2 < 1$ represent the fractions of the prey and the predator which die due to the harvesting or pesticides etc., and q is the size of the immigration or stocking of the predator. They found conditions for system (1) to go extinct and established permanence conditions via the comparison theorem involving multiple Lyapunov functions. Also, they illustrated that system (1) has rich dynamical behaviors by using numerical simulations.

In most researches, the authors regarded the prey ($x(t)$) and the predator ($y(t)$) as a pest and a natural enemy of the prey, respectively, and studied the dynamical properties of population systems. It is natural to assume that the predator ($y(t)$) also has a natural enemy ($z(t)$) not a pest for understanding and investigating more complex food chain systems. Thus, in this paper, we develop the following three-species Holling type IV system by introducing spraying pesticide and periodic constant release of natural enemies (mid-level predators) at different fixed times:

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(t)(a - bx(t)) - \frac{c_1 x(t) y(t)}{1 + e_1 (x(t))^2}, \\ y'(t) &= -d_1 y(t) + \frac{c_2 x(t) y(t)}{1 + e_1 (x(t))^2} - \frac{c_3 y(t) z(t)}{1 + e_2 (y(t))^2}, \\ z'(t) &= -d_2 z(t) + \frac{c_4 y(t) z(t)}{1 + e_2 (y(t))^2}, \end{aligned}\right\} & t \ne (n + \tau - 1)T, t \ne nT, \\ \left.\begin{aligned} \Delta x(t) &= -p_1 x(t), \\ \Delta y(t) &= -p_2 y(t), \\ \Delta z(t) &= -p_3 z(t), \end{aligned}\right\} & t = (n + \tau - 1)T, \\ \left.\begin{aligned} x(t^+) &= x(t), \\ y(t^+) &= y(t) + q, \\ z(t^+) &= z(t), \end{aligned}\right\} & t = nT, \\ (x(0^+), y(0^+), z(0^+)) = (x_0, y_0, z_0), \end{cases} \tag{2}$$

where c_3 is the per capita rate of predation of the top predator, e_2 is the half-saturation constant, d_2 denotes the death rate of the predator, c_4 is the rate of conversion of consumed prey to a predator and $0 \le p_3 < 1$ presents the fraction of the top predator dying due to the harvesting or pesticides etc. Such a system is an impulsive differential equation whose theories and applications were greatly developed by the efforts of Bainov and Lakshmikantham et al. [17,18] and, moreover, the theory of impulsive differential equations is being recognized not only to be richer than the corresponding theory of differential equations without impulses, but also to represent a more natural framework for mathematical modeling of real world phenomena.

The main purpose of this paper is to establish conditions for the local and global stabilities of pest, top predator-free periodic solutions and for the permanence of system (2). To achieve our purpose, we make use of Floquet theory for the impulsive equation, comparison techniques and so on.

2. Preliminaries

In this section we shall introduce some notation and definitions together with a few auxiliary results related to the comparison theorem, which will be useful for establishing our main results.

Let $\mathbb{R}_+ = [0, \infty)$, $\mathbb{R}_+^* = (0, \infty)$ and $\mathbb{R}_+^3 = \{\mathbf{x} = (x, y, z) \in \mathbb{R}^3 : x, y, z \ge 0\}$. Denote as $\mathbb{N}$ the set of all of nonnegative integers and as $f = (f_1, f_2, f_3)^T$ the right-hand sides of the first three equations in (2). Let $V : \mathbb{R}_+ \times \mathbb{R}_+^3 \to \mathbb{R}_+$; then V is said to belong to class V_0 if

(1) V is continuous on $((n-1)T, (n+\tau-1)T] \times \mathbb{R}_+^3 \cup ((n+\tau-1)T, nT] \times \mathbb{R}_+^3$ and $\lim_{(t,\mathbf{y})\to(t_0,\mathbf{x})} V(t, \mathbf{y}) = V(t_0, \mathbf{x})$ exist, where $t_0 = (n+\tau-1)T^+$ and nT^+,
(2) V is locally Lipschitzian in $\mathbf{x}$.

Definition 1. Let $V \in V_0$. For $(t, \mathbf{x}) \in ((n-1)T, (n+\tau-1)T] \times \mathbb{R}_+^3 \cup ((n+\tau-1)T, nT] \times \mathbb{R}_+^3$, the upper right derivative of V with respect to the impulsive differential system (2) is defined as

$$D^+V(t, \mathbf{x}) = \limsup_{h \to 0+} \frac{1}{h}[V(t+h, \mathbf{x} + hf(t, \mathbf{x})) - V(t, \mathbf{x})].$$

Remark 2. (1) The solution of system (2) is a piecewise continuous function; $\mathbf{x} : \mathbb{R}_+ \to \mathbb{R}_+^3$, $\mathbf{x}(t)$ is continuous on $((n-1)T, (n+\tau-1)T] \cup ((n+\tau-1)T, nT]$ and $\mathbf{x}(t_0^+) = \lim_{t \to t_0^+} \mathbf{x}(t)$ exists, where $t_0 = (n+\tau-1)T^+$ and nT^+. (2) The smoothness properties of f guarantee the global existence and uniqueness of solutions of system (2). (See [18] for the details.)

We will use a comparison result for impulsive differential inequalities. For this, suppose that $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the following hypotheses:

(H) g is continuous on $((n-1)T, (n+\tau-1)T] \times \mathbb{R}_+ \cup ((n+\tau-1)T, nT] \times \mathbb{R}_+$ and the limit $\lim_{(t,y)\to(t_0,x)} g(t,y) = g(t_0, x)$ exists, where $t_0 = (n+\tau-1)T^+$ and nT^+, and is finite for $x \in \mathbb{R}_+$ and $n \in \mathbb{N}$.

Lemma 3 ([18]). *Suppose $V \in V_0$ and*

$$\begin{cases} D^+V(t,\mathbf{x}) \le g(t, V(t,\mathbf{x})), & t \ne (n+\tau-1)T, nT, \\ V(t,\mathbf{x}(t^+)) \le \psi_n^1(V(t,\mathbf{x})), & t = (n+\tau-1)T, \\ V(t,\mathbf{x}(t^+)) \le \psi_n^2(V(t,\mathbf{x})), & t = nT, \end{cases} \tag{3}$$

where $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies (H) *and $\psi_n^1, \psi_n^2 : \mathbb{R}_+ \to \mathbb{R}_+$ are non-decreasing for all $n \in \mathbb{N}$. Let $r(t)$ be the maximal solution for the impulsive Cauchy problem*

$$\begin{cases} u'(t) = g(t, u(t)), & t \ne (n+\tau-1)T, nT, \\ u(t^+) = \psi_n^1(u(t)), & t = (n+\tau-1)T, \\ u(t^+) = \psi_n^2(u(t)), & t = nT, \\ u(0^+) = u_0 \ge 0, \end{cases} \tag{4}$$

defined on $[0, \infty)$. *Then $V(0^+, \mathbf{x}_0) \le u_0$ implies that $V(t, \mathbf{x}(t)) \le r(t)$, $t \ge 0$, where $\mathbf{x}(t)$ is any solution of* (3).

We now indicate a special case of Lemma 3 which provides estimations for the solution of a system of differential inequalities. For this, we let $PC(\mathbb{R}_+, \mathbb{R})(PC^1(\mathbb{R}_+, \mathbb{R}))$ denote the class of real piecewise continuous (real piecewise continuously differentiable) functions defined on $\mathbb{R}_+$.

Lemma 4 ([18]). *Let the function $u(t) \in PC^1(\mathbb{R}^+, \mathbb{R})$ satisfy the inequalities*

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} \le f(t)u(t) + h(t), & t \ne \tau_k, t > 0, \\ u(\tau_k^+) \le \alpha_k u(\tau_k) + \beta_k, & k \ge 0, \\ u(0^+) \le u_0, \end{cases} \tag{5}$$

where $f, h \in PC(\mathbb{R}_+, \mathbb{R})$ and $\alpha_k \ge 0$, β_k and u_0 are constants and $(\tau_k)_{k\ge 0}$ is a strictly increasing sequence of positive real numbers. Then, for $t > 0$,

$$u(t) \le u_0 \left(\prod_{0<\tau_k<t} \alpha_k\right) \exp\left(\int_0^t f(s)\mathrm{d}s\right) + \int_0^t \left(\prod_{0\le\tau_k<t} \alpha_k\right) \exp\left(\int_s^t f(\gamma)\mathrm{d}\gamma\right) h(s)\mathrm{d}s + \sum_{0<\tau_k<t} \left(\prod_{\tau_k<\tau_j<t} \alpha_j\right) \exp\left(\int_{\tau_k}^t f(\gamma)\mathrm{d}\gamma\right) \beta_k.$$

Similar results can be obtained when all conditions of the inequalities in the Lemmas 3 and 4 are reversed. Using Lemma 4, it is possible to prove that the solutions of the Cauchy problem (4) with strictly positive initial value remain strictly positive.

Lemma 5. *The positive octant $(\mathbb{R}_+^*)^3$ is an invariant region for system* (2).

Proof. Let $(x(t), y(t), z(t)) : (0, t_0) \to \mathbb{R}^2$ be a solution of system (2) with a strictly positive initial value (x_0, y_0, z_0). By Lemma 4, we can obtain that, for $0 < t < t_0$,

$$\begin{cases} x(t) \ge x_0(1-p_1)^{[\frac{t}{T}]} \exp\left(\displaystyle\int_0^t f_1(s)\mathrm{d}s\right), \\ y(t) \ge y_0(1-p_2)^{[\frac{t}{T}]} \exp\left(\displaystyle\int_0^t f_2(s)\mathrm{d}s\right), \\ z(t) \ge z_0(1-p_3)^{[\frac{t}{T}]} \exp\left(\displaystyle\int_0^t f_3(s)\mathrm{d}s\right), \end{cases} \tag{6}$$

where $f_1(s) = a - bx(s) - c_1y(s)$, $f_2(s) = -d_1 - c_3z(s)$ and $f_3(s) = -d_2$. Thus, $x(t)$, $y(t)$ and $z(t)$ remain strictly positive on $(0, t_0)$. □

3. Main theorems

3.1. Boundedness

Firstly, we show that all solutions of (2) are uniformly bounded.

Theorem 6. *There is an $R > 0$ such that $x(t) \leq R$, $y(t) \leq R$ and $z(t) \leq R$ for all t large enough, where $(x(t), y(t), z(t))$ is a solution of system (2).*

Proof. Let $(x(t), y(t), z(t))$ be a solution of (2) with an initial value (x_0, y_0, z_0) and let $u(t) = \frac{c_2}{c_1}x(t) + y(t) + \frac{c_3}{c_4}z(t)$ for $t \geq 0$. Then, if $t \neq nT$, $t \neq (n + \tau - 1)T$ and $t > 0$, we obtain that

$$u'(t) = -\frac{c_2 b}{c_1}x^2(t) + \frac{c_2 a}{c_1}x(t) - d_1 y(t) - \frac{d_2 c_3}{c_4}z(t). \tag{7}$$

From choosing $0 < \beta_0 < \min\{d_1, d_2\}$, we get

$$u'(t) + \beta_0 u(t) \leq -\frac{c_2 b}{c_1}x^2(t) + \frac{c_2}{c_1}(a + \beta_0)x(t), \quad t \neq nT, t \neq (n + \tau - 1)T, t > 0. \tag{8}$$

As the right-hand side of (8) is bounded from above by $R_0 = \frac{c_2(a+\beta_0)^2}{4bc_1}$, it follows that

$$u'(t) + \beta_0 u(t) \leq R_0, \quad t \neq nT, t \neq (n + \tau - 1)T, t > 0.$$

If $t = nT$, then $u(t^+) = u(t) + q$ and if $t = (n + \tau - 1)T$, then $u(t^+) \leq (1 - p)u(t)$, where $p = \min\{p_1, p_2, p_3\}$. From Lemma 4, we get that

$$\begin{aligned}
u(t) &\leq u(0^+)\left(\prod_{0<kT<t}(1-p)\right)\exp\left(\int_0^t -\beta_0 \mathrm{d}s\right) + \int_0^t \left(\prod_{0\leq kT<t}(1-p)\right)\exp\left(\int_s^t -\beta_0 d\gamma\right)R_0 \mathrm{d}s \\
&\quad + \sum_{0<kT<t}\left(\prod_{kT<jT<t}(1-p)\right)\exp\left(\int_{kT}^t -\beta_0 \mathrm{d}\gamma\right)q \\
&\leq u(0^+)\exp(-\beta_0 t) + \frac{R_0}{\beta_0}(1 - \exp(-\beta_0 t)) + \frac{q\exp(\beta_0 T)}{\exp(\beta_0 T) - 1}.
\end{aligned} \tag{9}$$

Since the limit of the right-hand side of (9) as $t \to \infty$ is

$$R \equiv \frac{R_0}{\beta_0} + \frac{q\exp(\beta_0 T)}{\exp(\beta_0 T) - 1} < \infty,$$

it follows that $u(t)$ is bounded for sufficiently large t. Therefore, $x(t)$, $y(t)$ and $z(t)$ are bounded by a constant R for sufficiently large t. □

3.2. Stability of the prey and top predator-free periodic solutions

First, we will give the basic properties of the following impulsive differential equation considered the absence of the prey and the top predator.

Lemma 7 ([9]). *If $aT + \ln(1 - p_1) \leq 0$, then $x(t) \to 0$ as $t \to \infty$ for any solution $x(t)$ of the following impulsive differential equation:*

$$\begin{cases} x'(t) = x(t)(a - bx(t)), & t \neq (n + \tau - 1)T, t \neq nT, \\ x(t^+) = (1 - p_1)x(t), & t = (n + \tau - 1)T, \\ x(t^+) = x(t), & t = nT. \end{cases} \tag{10}$$

Next, we give the basic properties of an impulsive differential equation as follows:

$$\begin{cases} y'(t) = -d_1 y(t), & t \neq (n + \tau - 1)T, t \neq nT, \\ y(t^+) = (1 - p_2)y(t), & t = (n + \tau - 1)T, \\ y(t^+) = y(t) + q, & t = nT. \end{cases} \tag{11}$$

System (11) is a periodically forced linear system. It is easy to obtain that

$$y^*(t) = \begin{cases} \dfrac{q\exp(-d_1(t - (n-1)T))}{1 - (1 - p_2)\exp(-d_1 T)}, & (n-1)T < t \leq (n + \tau - 1)T, \\ \dfrac{q(1 - p_2)\exp(-d_1(t - (n-1)T))}{1 - (1 - p_2)\exp(-d_1 T)}, & (n + \tau - 1)T < t \leq nT, \end{cases} \tag{12}$$

$y^*(0^+) = y^*(nT^+) = \frac{q}{1-(1-p_2)\exp(-d_1T)}$ and $y^*((n+\tau-1)T^+) = \frac{q(1-p_2)\exp(-d_1\tau T)}{1-(1-p_2)\exp(-d_1T)}$ is a positive periodic solution of (11). Moreover, we can obtain that

$$y(t) = \begin{cases} (1-p_2)^{n-1}\left(y(0^+) - \dfrac{q(1-p_2)e^{-T}}{1-(1-p_2)\exp(-d_1T)}\right)\exp(-d_1t) + y^*(t), \\ \quad (n-1)T < t \le (n+\tau-1)T, \\ (1-p_2)^{n}\left(y(0^+) - \dfrac{q(1-p_2)e^{-T}}{1-(1-p_2)\exp(-d_1T)}\right)\exp(-d_1t) + y^*(t), \\ \quad (n+\tau-1)T < t \le nT, \end{cases} \tag{13}$$

is a solution of (11). From (12) and (13), we get easily the following result.

Lemma 8. *For every solution $y(t)$ and every positive periodic solution $y^*(t)$ of system (2), it follows that $y(t)$ tends to $y^*(t)$ as $t \to \infty$. Thus, the complete expression for the prey and top predator-free periodic solution of system (2) is obtained:* $(0, y^*(t), 0)$.

Now, it is time to state the stability of the periodic solution of system (2).

Theorem 9. (1) *The periodic solution* $(0, y^*(t), 0)$ *is locally asymptotically stable if*

$$aT + \ln(1-p_1) < c_1\Gamma \tag{14}$$

and

$$\frac{c_4}{d_1\sqrt{e_2}}(\Lambda_1 + \Lambda_2 - \Lambda_3 - \Lambda_4) + \ln(1-p_3) < d_2T, \tag{15}$$

where $\Gamma = \frac{q(1-(1-p_2)\exp(-d_1T)-p_2\exp(-d_1\tau T))}{d_1(1-(1-p_2)\exp(-d_1T))}$,

$$\Lambda_1 = \tan^{-1}\left(\frac{\exp(d_1T)}{\mu\sqrt{e_2}(1-p_2)}\right), \qquad \Lambda_2 = \tan^{-1}\left(\frac{\exp(d_1\tau T)}{\mu\sqrt{e_2}}\right),$$

$$\Lambda_3 = \tan^{-1}\left(\frac{\exp(d_1\tau T)}{\mu\sqrt{e_2}(1-p_2)}\right), \qquad \Lambda_4 = \tan^{-1}\left(\frac{1}{\mu\sqrt{e_2}}\right) \quad \textit{and}$$

$$\mu = \frac{q}{d_1(1-(1-p_2)\exp(-d_1T))}.$$

(2) *Suppose that $aT + \ln(1-p_1) \le 0$. Then the periodic solution $(0, y^*(t), 0)$ is globally stable if (14), (15) and $c_4\Gamma + \ln(1-p_3) < d_2T$ hold.*

Proof. (1) The local stability of the periodic solution $(0, y^*(t), 0)$ of system (2) may be determined by considering the behavior of small amplitude perturbations of the solution. Let $(x(t), y(t), z(t))$ be any solution of system (2). Define $u(t) = x(t)$, $v(t) = y(t) - y^*(t)$, $w(t) = z(t)$. Then they may be written as

$$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \Phi(t)\begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix}$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} a - c_1y^*(t) & 0 & 0 \\ c_2y^*(t) & -d_1 & -\dfrac{c_3y^*(t)}{1+e_2(y^*(t))^2} \\ 0 & 0 & -d_2 + \dfrac{c_4y^*(t)}{1+e_2(y^*(t))^2} \end{pmatrix}\Phi(t)$$

and $\Phi(0) = I$, where I is the identity matrix. So the fundamental solution matrix is

$$\Phi(t) = \begin{pmatrix} \exp\left(\int_0^t a - c_1y^*(s)ds\right) & 0 & 0 \\ \exp\left(c_2\int_0^t y^*(s)ds\right) & \exp(-d_1t) & \exp\left(\int_0^t -\dfrac{c_3y^*(s)}{1+e_2(y^*(s))^2}ds\right) \\ 0 & 0 & \exp\left(\int_0^t -d_2 + \dfrac{c_4y^*(s)}{1+e_2(y^*(s))^2}ds\right) \end{pmatrix}.$$

The resetting impulsive conditions of system (2) become

$$\begin{pmatrix} u((n+\tau-1)T^+) \\ v((n+\tau-1)T^+) \\ u((n+\tau-1)T^+) \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} u((n+\tau-1)T) \\ v((n+\tau-1)T) \\ w((n+\tau-1)T) \end{pmatrix}$$

and

$$\begin{pmatrix} u(nT^+) \\ v(nT^+) \\ w(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}.$$

All of the eigenvalues of

$$S = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T)$$

are $\mu_1 = (1-p_1)\exp\left(\int_0^T a - c_1 y^*(t)\mathrm{d}t\right)$, $\mu_2 = (1-p_2)\exp(-d_1T) < 0$ and $\mu_3 = (1-p_3)\exp\left(\int_0^T -d_2 + \frac{c_4 y^*(t)}{1+e_2(y^*(t))^2}\mathrm{d}t\right)$. Since

$$\int_0^T y^*(t)\mathrm{d}t = \frac{q(1-(1-p_2)\exp(-d_1T) - p_2\exp(-d_1\tau T))}{d_1(1-(1-p_2)\exp(-d_1T))} \tag{16}$$

and

$$\int_0^T \frac{y^*(t)}{1+e_2(y^*(t))^2}\mathrm{d}t = \frac{1}{d_1\sqrt{e_2}}(\Lambda_1 + \Lambda_2 - \Lambda_3 - \Lambda_4), \tag{17}$$

where

$$\Lambda_1 = \tan^{-1}\left(\frac{\exp(d_1T)}{\mu\sqrt{e_2}(1-p_2)}\right), \qquad \Lambda_2 = \tan^{-1}\left(\frac{\exp(d_1\tau T)}{\mu\sqrt{e_2}}\right),$$
$$\Lambda_3 = \tan^{-1}\left(\frac{\exp(d_1\tau T)}{\mu\sqrt{e_2}(1-p_2)}\right), \qquad \Lambda_4 = \tan^{-1}\left(\frac{1}{\mu\sqrt{e_2}}\right) \quad \text{and}$$
$$\mu = \frac{q}{d_1(1-(1-p_2)\exp(-d_1T))},$$

the conditions $|\mu_1| < 1$ and $|\mu_3| < 1$ are equivalent to Eqs. (14) and (15), respectively. By Floquet theory [18], we obtain that $(0, y^*(t), 0)$ is locally asymptotically stable.

(2) Now we will prove the global stability. Since $c_4\Gamma + \ln(1-p_3) < d_2T$, we take a sufficiently small number $\epsilon_1 > 0$ satisfying

$$\phi \equiv (1-p_3)\exp\left(-d_2T + c_4\Gamma_{\epsilon_1} + c_4\epsilon_1 T\right) < 1,$$

where $\Gamma_{\epsilon_1} = \frac{q(1-(1-p_2)\exp(-(d_1-c_2\epsilon_1)T) - p_2\exp(-(d_1-c_2\epsilon_1)\tau T))}{(d_1-c_2\epsilon_1)(1-(1-p_2)\exp(-(d_1-c_2\epsilon_1)T))}$. From the first equation in (2), we obtain that $x'(t) = x(t)(a - bx(t) - \frac{c_1x(t)y(t)}{1+e_1(x(t))^2}) \le x(t)(a-bx(t))$ for $t \neq (n+\tau-1)T$, $t \neq nT$. By Lemma 3, $x(t) \le \tilde{x}(t)$ for $t \ge 0$, where $\tilde{x}(t)$ is the solution of (10) with the initial value $x(0^+)$. By Lemma 7, we get $\tilde{x}(t) \to 0$ as $t \to \infty$ which implies that there is $T_1 > 0$ such that $x(t) \le \epsilon_1$ for $t \ge T_1$. For the sake of simplicity, we suppose that $x(t) \le \epsilon_1$ for all $t \ge 0$. We can infer from the second equation in (2) that $y'(t) = y(t)\left(-d_1 + \frac{c_2x(t)y(t)}{1+e_1(x(t))^2} - \frac{c_3y(t)z(t)}{1+e_2(y(t))^2}\right) \le y(t)(-d_1 + c_2x(t)) \le y(t)(-d_1 + c_2\epsilon_1)$ for $t \neq nT$, $t \neq (n+\tau-1)T$. Let $\tilde{y}_1(t)$ be the solution of the following equation:

$$\begin{cases} \tilde{y}_1'(t) = -(d_1 - c_2\epsilon_1)\tilde{y}_1(t), & t \neq (n+\tau-1)T, t \neq nT, \\ \tilde{y}_1(t^+) = (1-p_2)\tilde{y}_1(t), & t = (n+\tau-1)T, \\ \tilde{y}_1(t^+) = \tilde{y}_1(t) + q, & t = nT, \\ \tilde{y}_1(0^+) = y_0. \end{cases} \tag{18}$$

Then we know that $y(t) \le \tilde{y}_1(t)$ by Lemma 3. Thus, from the third equation in (2) and Lemma 8, we obtain that

$$\begin{aligned} z'(t) &\le z(t)(-d_2 + c_4\tilde{y}_1(t)) \\ &\le z(t)(-d_2 + c_4\tilde{y}_1^*(t) + c_4\epsilon_1) \end{aligned} \tag{19}$$

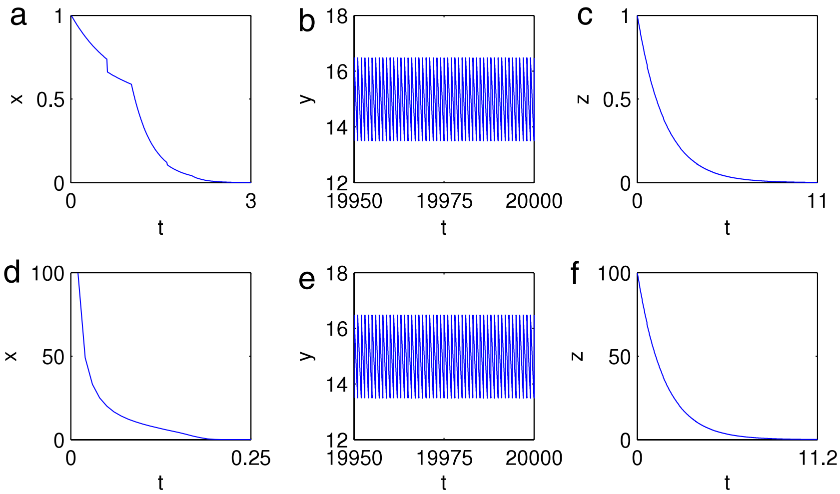

Fig. 1. $a = 1.1, b = 1, c_1 = 0.9, c_2 = 1, c_3 = 0.9, c_4 = 0.01, d_1 = 0.2, d_2 = 0.6, e_1 = 0.2, e_2 = 0.2, p_1 = 0.1, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 1$ and $q = 3$. ((a)–(c)) Time series of system (2) when $(x_0, y_0, z_0) = (1, 1, 1)$ and ((d)–(f)) time series of system (2) when $(x_0, y_0, z_0) = (100, 100, 100)$.

where

$$\tilde{y}_1^*(t) = \begin{cases} \dfrac{q\exp(-(d_1 - c_2\epsilon_1)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1 - c_2\epsilon_1)T)}, & (n-1)T < t \le (n+\tau-1)T, \\ \dfrac{q(1-p_2)\exp(-(d_1 - c_2\epsilon_1)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1 - c_2\epsilon_1)T)}, & (n+\tau-1)T < t \le nT \end{cases}$$

is the periodic solution of (18). Integrating (19) on $((n+\tau-1)T, (n+\tau)T]$, we obtain

$$\begin{aligned} z((n+\tau)T) &\le z((n+\tau-1)T^+)\exp\left(\int_{(n+\tau-1)T}^{(n+\tau)T} -d_2 + c_4\tilde{y}_1^*(t) + c_4\epsilon_1 \mathrm{d}t\right) \\ &= z((n+\tau-1)T)\phi. \end{aligned}$$

Therefore we obtain $z((n+\tau)T) \le z(\tau T)\phi^n \to 0$ as $n \to \infty$. Also, for $t \in ((n+\tau-1)T, (n+\tau)T]$,

$$\begin{aligned} z(t) &\le z((n+\tau-1)T^+)\exp\left(\int_{(n+\tau-1)T}^{t} -d_2 + c_4\tilde{y}_1^*(t) + c_4\epsilon_1 \mathrm{d}t\right) \\ &\le z((n+\tau-1)T)\exp\left(\frac{qc_4}{1-(1-p_2)\exp(-(d_1-c_2\epsilon_1)T)} + c_4\epsilon_1 T\right) \end{aligned}$$

which implies that $z(t) \to 0$ as $t \to \infty$. Thus, we may assume that $z(t) \le \epsilon_2$ for $t > 0$. Again, from the second equation in (2), we obtain that $y'(t) \ge y(t)(-d_1 - c_3\epsilon_2)$. Let $\tilde{y}_2(t)$ and $\tilde{y}_2^*(t)$ be the solution and the periodic solution of (11), respectively, with d_1 changed into $d_1 + c_3\epsilon_2$ and the same initial value y_0. Then we can infer from Lemmas 3 and 7 that $\tilde{y}_2(t) \le y(t) \le \tilde{y}_1(t)$ and $\tilde{y}_i(t)$ $(i = 1, 2)$ become close to $\tilde{y}_i^*(t)$ $(i = 1, 2)$ as $t \to \infty$, respectively. Note that the $\tilde{y}_i^*(t)$ $(i = 1, 2)$ are close to $y^*(t)$ as ϵ_1 and $\epsilon_2 \to 0$. Therefore we obtain $y(t) \to y^*(t)$ as $t \to \infty$. □

Example 1. It follows from Theorem 9(1) that the periodic solution $(0, y^*(t), 0)$ is locally stable if we take $a = 1.1, b = 1, c_1 = 0.9, c_2 = 1, c_3 = 0.9, c_4 = 0.01, d_1 = 0.2, d_2 = 0.6, e_1 = 0.2, e_2 = 0.2, p_1 = 0.1, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 1$ and $q = 3$. Fig. 1 shows this phenomenon and exhibits that this periodic solution may be globally stable even though $aT + \ln(1 - p - 1) > 0$. Thus we conjecture that the periodic solutions of system (2) can be globally stable if the conditions (14) and (15) hold.

Example 2. If we let $a = 1.1, b = 1, c_1 = 0.9, c_2 = 1, c_3 = 0.9, c_4 = 0.01, d_1 = 0.2, d_2 = 0.6, e_1 = 0.2, e_2 = 0.2, p_1 = 0.7, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 1$ and $q = 3$, then these parameters satisfy the condition of Theorem 9(2). Thus the periodic solution $(0, y^*(t), 0)$ is globally asymptotically stable. (See Fig. 2.)

From the proof of Theorem 9 and Theorem 3.2 in [4], we obtain a condition for the global stability of the periodic solution $(0, y^*(t))$ of system (1).

Theorem 10. *The periodic solution $(0, y^*(t))$ of system (1) is globally asymptotically stable if $\Gamma \le 0$ and $aT + \ln(1 - p_1) < c_1\Gamma$, or $\Gamma > 0$ and $aT + \ln(1 - p_1) \le 0$.*

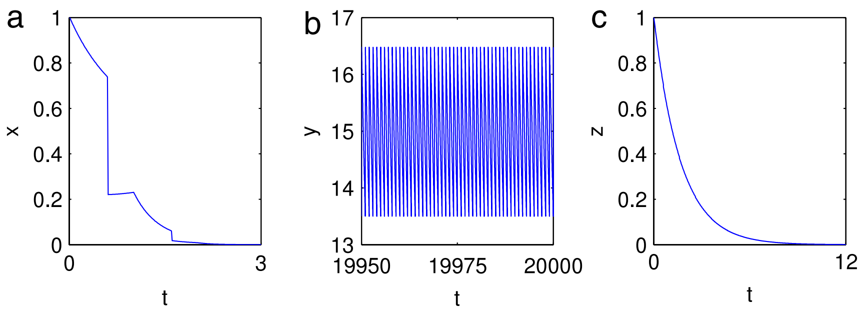

Fig. 2. $a = 1.1, b = 1, c_1 = 0.9, c_2 = 1, c_3 = 0.9, c_4 = 0.01, d_1 = 0.2, d_2 = 0.6, e_1 = 0.2, e_2 = 0.2, p_1 = 0.7, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 1$ and $q = 3$. ((a)–(c)) Time series of system (2) when $(x_0, y_0, z_0) = (1, 1, 1)$.

3.3. Permanence

We make mention of the definition of permanence before stating the permanence of system (2).

Definition 11. System (2) is said to be permanent if there exist two positive constants m and M such that every positive solution $(x(t), y(t), z(t))$ of system (2) with $x_0, y_0, z_0 > 0$ satisfies $m \le x(t) \le M$, $m \le y(t) \le M$ and $m \le z(t) \le M$ for sufficiently large t.

To prove the permanence of system (2), we consider the following two subsystems. If the top predator is absent i.e., $z(t) = 0$, then system (2) can be expressed as

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(t)(a - bx(t)) - \frac{c_1 x(t) y(t)}{1 + e_1 (x(t))^2}, \\ y'(t) &= -d_1 y(t) + \frac{c_2 x(t) y(t)}{1 + e_1 (x(t))^2}, \end{aligned}\right\} & t \ne (n + \tau - 1)T,\ t \ne nT, \\ \left.\begin{aligned} x(t^+) &= (1 - p_1) x(t), \\ y(t^+) &= (1 - p_2) y(t), \end{aligned}\right\} & t \ne (n + \tau - 1)T, \\ \left.\begin{aligned} x(t^+) &= x(t), \\ y(t^+) &= y(t) + p, \end{aligned}\right\} & t = nT, \\ (x(0^+), y(0^+)) = (x_0, y_0). \end{cases} \tag{20}$$

In fact, subsystem (20) is the same as system (1). If the prey is extinct, then system (2) can be expressed as

$$\begin{cases} \left.\begin{aligned} y'(t) &= -d_1 y(t) - \frac{c_3 y(t) z(t)}{1 + e_2 (y(t))^2}, \\ z'(t) &= -d_2 z(t) + \frac{c_4 y(t) z(t)}{1 + e_2 (y(t))^2}, \end{aligned}\right\} & t \ne (n + \tau - 1)T,\ t \ne nT, \\ \left.\begin{aligned} y(t^+) &= (1 - p_2) y(t), \\ z(t^+) &= (1 - p_3) z(t), \end{aligned}\right\} & t \ne (n + \tau - 1)T, \\ \left.\begin{aligned} y(t^+) &= y(t) + p, \\ z(t^+) &= z(t), \end{aligned}\right\} & t = nT, \\ (y(0^+), z(0^+)) = (y_0, z_0). \end{cases} \tag{21}$$

In particular, Li and Tan [4] gave a condition for the permanence of subsystem (20).

Theorem 12 ([4]). *Subsystem (20) is permanent if*

$$aT + \ln(1 - p_1) > c_1 \Gamma.$$

Theorem 13. *Subsystem (21) is permanent if*

$$\frac{c_4}{d_1 \sqrt{e_2}} (\Lambda_1 + \Lambda_2 - \Lambda_3 - \Lambda_4) + \ln(1 - p_3) > d_2 T. \tag{22}$$

Proof. Let $(y(t), z(t))$ be a solution of subsystem (21) with $y_0 > 0$, $z_0 > 0$. From Theorem 6, we may assume that $y(t) \le M$ and $z(t) \le \frac{M}{c_4}$ for some $M > 0$. Then $y'(t) \ge -(d_1 + M) y(t)$. From Lemmas 3 and 8, we have $y(t) \ge u^*(t) - \epsilon$ for $\epsilon > 0$, where

$$u^*(t)=\begin{cases}\dfrac{q\exp(-(d_1+M)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1+M)T)}, & (n-1)T<t\le(n+\tau-1)T,\\ \dfrac{q(1-p_2)\exp(-(d_1+M)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1+M)T)}, & (n+\tau-1)T<t\le nT.\end{cases}$$

Thus, we obtain that $y(t)\ge\frac{q(1-p_2)(\exp(-(d_1+M)T))}{1-(1-p_2)\exp(-(d_1+M)T)}-\epsilon\equiv m_0$ for sufficiently large t. Therefore, we only need to find an $m_2>0$ such that $z(t)\ge m_2$ for large enough t. We will do this in the following two steps.

(Step 1) From (22), we can choose $m_1>0$, $\epsilon_1>0$ small enough that

$$\phi\equiv(1-p_3)\exp\left(-d_2T-c_4\epsilon_1T+\frac{c_4}{(d_1+c_3m_1)\sqrt{e_2}}(\Lambda_1^{\epsilon_1}+\Lambda_2^{\epsilon_1}-\Lambda_3^{\epsilon_1}-\Lambda_4^{\epsilon_1})\right)>1,$$

where

$$\Lambda_1^{\epsilon_1}=\tan^{-1}\left(\frac{(1+e_2\epsilon_1^2)\exp((d_1+c_3m_1)T)+e_2\epsilon_1\mu(1-p_2)}{\mu\sqrt{e_2}(1-p_2)}\right),$$

$$\Lambda_2^{\epsilon_1}=\tan^{-1}\left(\frac{(1+e_2\epsilon_1^2)\exp((d_1+c_3m_1)\tau T)+e_2\epsilon_1\mu(1-p_2)}{\mu\sqrt{e_2}}\right),$$

$$\Lambda_3^{\epsilon_1}=\tan^{-1}\left(\frac{(1+e_2\epsilon_1^2)\exp((d_1+c_3m_1)\tau T)+e_2\epsilon_1}{\mu\sqrt{e_2}(1-p_2)}\right)\quad\text{and}$$

$$\Lambda_4^{\epsilon_1}=\tan^{-1}\left(\frac{\psi 1+e_2\epsilon_1^2+e_2\epsilon_1}{\mu\sqrt{e_2}}\right).$$

In this step, we will show that $z(t_1)\ge m_1$ for some $t_1>0$. Suppose not; i.e., $z(t)<m_1$ for $t>0$. Consider the following system:

$$\begin{cases}\left.\begin{array}{l}v'(t)=v(t)(-d_1-c_3m_1),\\ w'(t)=w(t)\left(-d_2+\dfrac{c_4v(t)}{1+e_2(v(t))^2}\right),\end{array}\right\} & t\ne(n+\tau-1)T,\ t\ne nT,\\ \left.\begin{array}{l}v(t^+)=(1-p_2)v(t),\\ w(t^+)=(1-p_3)w(t),\end{array}\right\} & t=(n+\tau-1)T,\\ \left.\begin{array}{l}v(t^+)=v(t)+p,\\ w(t+)=w(t),\end{array}\right\} & t=nT,\\ (v(0^+),w(0^+))=(y_0,z_0).\end{cases}\tag{23}$$

Then, by Lemma 3, we obtain $y(t)\ge v(t)$ and $z(t)\ge w(t)$. By Lemma 8, we have $v(t)\ge v^*(t)-\epsilon_1$, where, for $t\in((n-1)T,nT]$,

$$v^*(t)=\begin{cases}\dfrac{q\exp(-(d_1+c_3m_1)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1+c_3m_1)T)}, & (n-1)T<t\le(n+\tau-1)T,\\ \dfrac{q(1-p_2)\exp(-(d_1+c_3m_1)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1+c_3m_1)T)}, & (n+\tau-1)T<t\le nT.\end{cases}$$

Thus

$$\begin{aligned}w'(t)&\ge\left(-d_2+\frac{c_4(v^*(t)-\epsilon_1)}{1+e_2(v^*(t)-\epsilon_1)^2}\right)w(t)\\&\ge\left(-d_2-c_4\epsilon_1+\frac{c_4v^*(t)}{1+e_2(v^*(t)-\epsilon_1)^2}\right)w(t).\end{aligned}\tag{24}$$

Integrating (24) on $((n+\tau-1)T,(n+\tau)T]$, we get

$$\ln\left(\frac{w((n+\tau)T)}{w((n+\tau-1)T^+)}\right)\ge\int_{(n+\tau-1)T}^{(n+\tau)T}-d_2-c_4\epsilon_1+\frac{c_4v^*(t)}{1+e_2(v^*(t)-\epsilon_1)^2}\mathrm{d}t.$$

Note that

$$\int_{(n+\tau-1)T}^{(n+\tau)T}\frac{v^*(t)}{1+e_2(v^*(t)-\epsilon_1)^2}\mathrm{d}t=\frac{1}{(d_1+c_3m_1)\sqrt{e_2}}(\Lambda_1^{\epsilon_1}+\Lambda_2^{\epsilon_1}-\Lambda_3^{\epsilon_1}-\Lambda_4^{\epsilon_1}).$$

Then we get $w((n+\tau)T) \geq w((n+\tau-1)T)\phi$. Therefore $z((n+\tau+k)T) \geq w((n+\tau+k)T) \geq w((n+\tau-1)T)\phi^{k+1} \to \infty$ as $k \to \infty$ which is a contradiction to the boundedness of $z(t)$.

(Step 2) Without loss of generality, we may let $z(t_1) = m_1$. If $z(t) \geq m_1$ for all $t > t_1$, then subsystem (21) is permanent. If not, we may let $t_2 = \inf_{t>t_1}\{z(t) < m_1\}$. Then $z(t) \geq m_1$ for $t_1 \leq t \leq t_2$ and, by continuity of $z(t)$, we have $z(t_2) = m_1$ and $t_1 < t_2$. There exists a t' $(>t_2)$ such that $z(t') \geq m_1$ by step 1. Set $t_3 = \inf_{t>t_2}\{z(t) \geq m_1\}$. Then $z(t) < m_1$ for $t_2 < t < t_3$ and $z(t_3) = m_1$. We can continue this process by using step 1. If the process is stopped in finite times, we complete the proof. Otherwise, there exists an interval sequence $[t_{2k}, t_{2k+1}]$, $k \in \mathbb{N}$, which has the following property: $z(t) < m_1, t \in (t_{2k}, t_{2k+1}), t_{2k-1} < t_{2k} \leq t_{2k+1}$ and $z(t_n) = m_1$, where $k, n \in \mathbb{N}$. Let $T_0 = \sup\{t_{2k+1} - t_{2k} | k \in \mathbb{N}\}$. If $T_0 = \infty$, then we can take a subsequence $\{t_{2k_i}\}$ satisfying $t_{2k_i+1} - t_{2k_i} \to \infty$ as $k_i \to \infty$. As in the proof of the first step, this will lead to a contradiction to the boundedness of $z(t)$. Then we obtain $T_0 < \infty$. Note that

$$\begin{aligned} z(t) &\geq z(t_{2k}) \exp\left(\int_{t_{2k}}^{t} -d_2 + \frac{c_4(v^*(s)-\epsilon_1)}{1+e_2(v^*(s)-\epsilon_1)^2}\mathrm{d}s\right) \\ &\geq m_1 \exp(-d_2T_0) \equiv m_2, \quad t \in (t_{2k}, t_{2k+1}], k \in \mathbb{N}. \end{aligned}$$

So, we obtain that $\liminf_{t\to\infty} z(t) \geq m_2$. Therefore we complete the proof. □

Theorem 14. *System (2) is permanent if the conditions*

$$aT + \ln(1-p_1) > c_1\Gamma$$

and

$$\frac{c_4}{d_1\sqrt{e_2}}(\Lambda_1 + \Lambda_2 - \Lambda_3 - \Lambda_4) + \ln(1-p_3) > d_2T$$

hold.

Proof. Consider the following two subsystems of system (2):

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= x_1(t)(a - bx_1(t)) - \frac{c_1x_1(t)y_1(t)}{1+e_1(x_1(t))^2}, \\ y_1'(t) &= -d_1y_1(t) + \frac{c_2x_1(t)y_1(t)}{1+e_1(x_1(t))^2}, \end{aligned}\right\} & t \neq (n+\tau-1)T, t \neq nT, \\ \left.\begin{aligned} x(_1t^+) &= (1-p_1)x_1(t), \\ y_1(t^+) &= (1-p_2)y_1(t), \end{aligned}\right\} & t \neq (n+\tau-1)T, \\ \left.\begin{aligned} x_1(t^+) &= x_1(t), \\ y_1(t^+) &= y_1(t) + p, \end{aligned}\right\} & t = nT, \\ (x_1(0^+), y_1(0^+)) = (x_0, y_0). \end{cases} \tag{25}$$

and

$$\begin{cases} \left.\begin{aligned} y_2'(t) &= -d_1y_2(t) - \frac{c_3y_2(t)z_2(t)}{1+e_2(y_2(t))^2}, \\ z_2'(t) &= -d_2z_2(t) + \frac{c_4y_2(t)z_2(t)}{1+e_2(y_2(t))^2}, \end{aligned}\right\} & t \neq (n+\tau-1)T, \ t \neq nT, \\ \left.\begin{aligned} y_2(t^+) &= (1-p_2)y_2(t), \\ z_2(t^+) &= (1-p_3)z_2(t), \end{aligned}\right\} & t \neq (n+\tau-1)T, \\ \left.\begin{aligned} y_2(t^+) &= y_2(t) + p, \\ z_2(t^+) &= z_2(t), \end{aligned}\right\} & t = nT, \\ (y_2(0^+), z_2(0^+)) = (y_0, z_0). \end{cases} \tag{26}$$

It follows from Lemma 3 that $x_1(t) \leq x(t), y_1(t) \geq y(t), y_2(t) \leq y(t)$ and $z_2(t) \leq z(t)$. If $aT + \ln(1-p_1) > c_1\Gamma$, by Theorem 12, subsystem (25) is permanent. Thus we can take $T_1 > 0$ and $m_1 > 0$ such that $x(t) \geq m_1$ for $t \geq T_1$. Further, if $\frac{c_4}{d_1\sqrt{e_2}}(\Lambda_1 + \Lambda_2 - \Lambda_3 - \Lambda_4) + \ln(1-p_3) > d_2T$, by Theorem 13, subsystem (26) is also permanent. Therefore, there exist $T_2 > 0$ and $m_2, m_3 > 0$ such that $y(t) \geq m_2$ and $z(t) \geq m_3$ for $t \geq T_2$. The proof is complete. □

Example 3. Let $a = 4, b = 1, c_1 = 0.1, c_2 = 1, c_3 = 0.9, c_4 = 1, d_1 = 0.1, d_2 = 0.1, e_1 = 0.2, e_2 = 0.1, p_1 = 0.8, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 4$ and $q = 3$. Then, it follows from Theorem 14 that system (2) is permanent. (See Fig. 3.)

4. Conclusion

In this paper, we have investigated effects of impulsive perturbations on a Holling type IV food chain system. Conditions for system (2) to be extinct are established by using the Floquet theory of impulsive differential equation and small amplitude

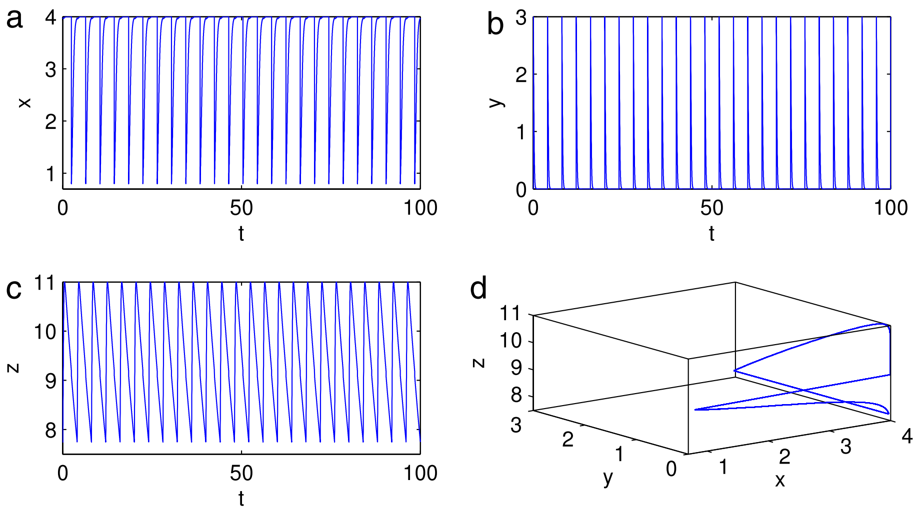

Fig. 3. $a = 4, b = 1, c_1 = 0.1, c_2 = 1, c_3 = 0.9, c_4 = 1, d_1 = 0.1, d_2 = 0.1, e_1 = 0.2, e_2 = 0.1, p_1 = 0.8, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 4$ and $q = 3$. ((a)–(c)) Time series. (d) The trajectory of system (2) with an initial value $(1, 1, 1)$.

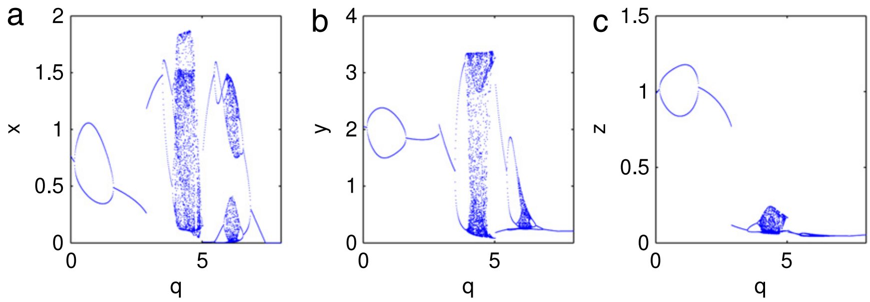

Fig. 4. $a = 2.1, b = 1, c_1 = 0.9, c_2 = 1, c_3 = 0.9, c_4 = 0.4, d_1 = 0.2, d_2 = 0.3, e_1 = 0.2, e_2 = 0.2, p_1 = 0.7, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 15$ and $0 < q < 8$. Bifurcation diagrams of system (2) when $(x_0, y_0, z_0) = (1, 1, 1)$. ((a)–(c)) x, y and z are plotted for q.

perturbation skills, and the boundary of system (2) is given. In addition, it is shown that system (2) can be permanent under some conditions via the comparison theorem. We illustrate some examples. In [4], the authors studied the dynamic complexities of system (1) by using numerical simulations. In fact, system (1) has various dynamical behaviors such as quasi-periodic and periodic windows, strange attractors, and period-doubling and period-halving phenomena etc. (See [4] for more details.) We can also catch sight of such phenomena for system (2). Fig. 4 displays the bifurcation diagrams of system (2) with the parameters as follows:

$a = 2.1, b = 1, c_1 = 0.9, c_2 = 1, c_3 = 0.9, c_4 = 0.4, d_1 = 0.2, d_2 = 0.3, e_1 = 0.2, e_2 = 0.2, p_1 = 0.7, p_2 = 0.001, p_3 = 0.01, \tau = 0.6, T = 15$ and $0 < q < 8$.

This figure indicates that system (2) experiences quasi-periodic oscillation, period-doubling cascades, periodic windows, period-halving cascades, chaos, chaos crisis, and so on. Thus system (2) also has complex dynamical behaviors.

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