Finite-deformation analysis of the crack-tip fields under cyclic loading

J. Toribio *, V. Kharin

Department of Materials Engineering, University of Salamanca, E. P. S., Campus Viriato, 49022 Zamora, Spain

A R T I C L E   I N F O

Article history:
Received 10 March 2008
Received in revised form 20 December 2008
Available online 18 January 2009

Keywords:
Crack tip
Fatigue
Elastoplasticity
Large deformations
Finite elements

A B S T R A C T

The plane-strain crack subjected to mode I cyclic loading under small scale yielding was analysed. The influence of the load range, load ratio and overload on the near-tip deformation-, stress- and strain-fields was studied. Although the near-tip zones of appreciable cyclic plastic flow for all loading regimes were matched closely one another, when scaled with $\Delta K/\sigma_Y$, the activities of plastic flow within them manifested dependence on $K_{\text{max}}$ and $K_{\text{min}}$, as well as on overload. Cyclic trajectories of the crack-tip opening displacement (CTOD) converged to stable self-similar loops of the sizes proportional to $\Delta K^2$ and positions in CTOD-K plane dependent on the maximum $K$ along the whole loading route, including an overload. Computed near-tip deformation evidenced plastic crack advance, this way visualising of the Laird–Smith concept of fatigue cracking. This crack growth by blunting-resharpening accelerated with rising $\Delta K$ and was halted by an overload. Crack closure upon unloading had no place. The affinities were revealed between computed near-tip stress–strain variables and the experimental trends of the fatigue crack growth rate, such as its dependence on $K_{\text{max}}$ and $K_{\text{min}}$ (or $\Delta K$ and $K_{\text{max}}$), and retardation by overload. Thus, the effects of loading parameters on fatigue cracking, hitherto associated with crack closure, are attributable to the stress–strain fields in front of it as the direct drives of the key fatigue constituents – damage accumulation and bond breaking.

© 2009 Elsevier Ltd. All rights reserved.

1. Introduction

The process of fatigue fracture is seen as initiation and growth of cracks under alternating loading, so that evaluation and prediction of fatigue crack growth (FCG) forms essential part of failure prevention and control. In this context, the rate of crack growth per cycle $da/dN$ as a function of appropriate mechanical variables, which control crack propagation in a material irrespectively of particular geometry-and-loading circumstances, acquires the significance of a fatigue cracking “law”. In small scale yielding (SSY), the stress intensity factor (SIF) $K$ is considered to work well as controlling mechanical variable (Hertzberg, 1989; Kanninen and Popelar, 1985; Suresh, 1991). For the rate/time-independent material behaviour, the near-tip states at constant-amplitude cyclic loadings are determined by the maximum and minimum values of alternating SIF, $K_{\text{max}}$ and $K_{\text{min}}$, respectively, and the cycle number $N$. Other equivalent couples of variables may be employed, too, such as the SIF range $\Delta K = K_{\text{max}} - K_{\text{min}}$ and the load ratio $R = K_{\text{min}}/K_{\text{max}}$ and so on. The basic FCG law may be then represented as follows:

$$\frac{da}{dN} = \Phi(K_{\text{max}}, K_{\text{min}}) \quad \text{or} \quad \frac{da}{dN} = \Psi(\Delta K, R),$$

where the right-hand parts are considered to be material-only dependent functions. Evidently, load-path dependent effects under variable-amplitude loading cannot be captured within this framework, and then functions over loading histories are required in the right-hand parts of Eq. (1). Although numerous authors, starting with Paris and Erdogan, and followed by Walker, Donahue et al., Erdogan and Ratwani, Forman, Klesni and Lukas, McEvily and Groeger, Priddle, Weertman, etc., proposed and supported experimentally a variety of explicit forms of Eq. (1), which are reviewed elsewhere (Cherepanov, 1974; Gdoutos, 1993; Hertzberg, 1989; Kanninen and Popelar, 1985; Suresh, 1991), a generally applicable fatigue law has not been established yet.

This phenomenological approach does not rely on the two background processes of fatigue cracking: (i) damage accumulation (also called cyclic damage), and (ii) bond breaking (named static damage as well), both of which are driven by genuine stresses and strains, which are the direct governing factors of fatigue (Noroozi et al., 2008; Ritchie, 1999; Sadananda et al., 1999; Vasudevan et al., 2001). This means that the pertinence of SIF to fatigue cracking is limited to represent adequately the strength of the near-tip stress–strain field, provided it is self-similar, the same way as it does under monotonic loading.

The sound approach to fatigue cracking analysis has two fundamental constituents. The first is to determine deformations, stresses and strains in the process zone near the crack tip under given loading patterns. The second is to characterise the intrinsic material...
susceptibility to fatigue — accumulation of damage and bond breaking at the crack tip, which are driven by genuine stress–strain trajectories. To crown it all, these two essentials — stress–strain state and damage–rupture — do influence one upon another.

Monitoring in situ of relevant mechanical variables in the crack tip vicinity is hardly feasible, whereas modelling opens a way to reveal them. Accounting there for two nonlinearities — the physical (material’s) and the geometrical (large deformations and strains) ones — is essential for realistic implications for cracking, as it has been repeatedly argued, e.g., by Gortmaker et al. (1981), Liu and Drugan (1993), McMeeking (1977), McMeeking and Parks (1979), Needleman and Tvergaard (1983), and Rice and Johnson (1977).

For growing cracks, the alteration of boundary conditions via crack advance is the third nonlinearity, which interacts with the former ones. This takes root from the dependence of plastic deformations on the boundary conditions succession as the kind of non-proportionality of boundary conditions, which consequences for the near-tip fields of growing cracks may be magnified by strongly nonproportional per se near-tip stressing. Since the pioneering analysis by Rice (1968), this effect has been confirmed repeatedly using various models of crack and constitutive behaviour of solids (see, e.g., Amazigo and Hutchinson, 1977; Cherepanov, 1974; Lam and McMeeking, 1984; Liu and Drugan, 1993).

Thus, the most precise revelation of the near-tip situation would be achieved taking into account all three mentioned nonlinearities. Obviously, ignoring any of them worsens the resolution of some facets of the crack-tip fields more or less substantially.

Numerous models have been developed up to date aiming to elucidate the near-tip deformation-, stress- and strain-fields under cyclic loading. Some of them offered closed-form solutions, such as the pioneering analysis by Rice (1967) and a series of posterior ones outlined elsewhere (Hertzberg, 1989; Kanninen and Popelar, 1985; Suresh, 1991). More realistic analyses of cracks under cyclic loading have been performed numerically using a variety of constitutive models of elastoplasticity. Stationary cracks have been addressed there, as well as growing ones employing usually rather arbitrary schemes of cutting the material beyond the tip with no relation to material damage or bond breaking (Ellyin and Wu, 1992; Fleck, 1986; McClung, 1991; McClung and Davidson, 1991; McClung and Sehitoglu, 1989; McClung et al., 1991; Wu and Ellyin, 1996). Most of them have not accounted for large near-tip deformations. Other simulations of cracks, although fulfilling the latter deficiency, have been either confined to monotonic loading (e.g., Liu and Drugan, 1993; McMeeking, 1977; McMeeking and Parks, 1979; Needleman and Tvergaard, 1983; Rice and Johnson, 1977) or presented reduced data concerning cracks under cyclic one (Gortmaker et al., 1981; Levkovitch et al., 2005; Roychowdhury and Dodds, 2003; Toribio and Kharin, 1998, 1999a,b, 2000, 2001, 2006, 2007; Toribio et al., 1999; Tvergaard, 2004, 2005, 2006).

A common shortcoming of the analyses, which addressed the near-tip situation with reference to fatigue, consists in the lack of realistic treatment of crack growth by means of material damage and bond breaking. Usual way to model crack growth by finite-size steps via finite element node release one-by-one at arbitrarily defined instance of each load cycle (Fleck, 1986; McClung, 1991; McClung and Sehitoglu, 1989; McClung et al., 1991; Roychowdhury and Dodds, 2003; Wu and Ellyin, 1996) is short of physical justification and has little, if any, resemblance with microstructure-conditioned phenomena of damage and rupture. To this end, even the very definition of crack advance, being strongly linked to the scale of observation, is ambiguous, since a variety of mechanisms, which involve imperfections of different scales, lead to the main flaw growth (Ritchie, 1999; Suresh, 1991). Their incorporation into quantitative description of crack propagation is obstructed by conceptual difficulties of bridging microstructural events of different scales, so that a success to date has been rather limited here.

Indeed, although the efforts involving damage mechanisms directly in the modelling of cracking have been undertaken since decades, usually by huge computational expenses, most of them have been limited to monotonic loading (to mention few, see, e.g., Gullerud et al., 2000; McMeeking, 1977; Miller et al., 1998; Moran et al., 1990). Efforts have been undertaken to incorporate explicit damage accumulation rules directly into modelling of fatigue cracking (Lynn and DuQuesnay, 2002; Nguyen et al., 2001; Stumpf and Szczuk, 2001), but these are still very few attempts in the early stage of development, where strong conceptual and technical difficulties can be anticipated on the way to implement damage mechanics in large-deformation formulations.

Owing to difficulties of involvement of damage and bond breaking mechanisms directly into analysis of fatigue cracks, the stress-, strain- or energy-based variables have been used as fatigue monitors (Chan and Lankford, 1983; Ellyin and Wu, 1992; Noroozi et al., 2008; Wu and Ellyin, 1996).

To this end, knowledge of the near-tip deformation-, stress- and strain-fields under cyclic loading is valuable for understanding fatigue, and it has been repeatedly requested (Noroozi et al., 2008; Sadananda et al., 1999; Sadananda and Vasudevan, 2003; Vasudevan et al., 2001) with the aim to gain insight into fatigue and to relate controllable mechanical variables with crack growth, thereby providing a framework for its characterisation and prediction.

Obviously, to obtain more sound implications for cracking, the best possible resolution of the involved crack-tip fields is desired. The accuracy of their evaluation relies on taking into account of all three commented nonlinearities. Among them, disregarding the one associated with crack extension obviously does not deteriorate the results with respect to the initiation of crack propagation. For growing cracks, disregard of this nonlinearity may have significance for the near-tip fields. However, would consequent inaccuracy be more or less substantial in comparison with faultiness from unrealistic modelling of crack advance by means of a node release can be verified only through more advanced analyses involving in a due manner all three nonlinearities concerned.

Meanwhile, among suggested conceptual mechanisms of FCG, the one of Laird and Smith, which was emphasised as appealing physical model for ductile materials (Ritchie, 1999; Suresh, 1991, p. 198), relies solely on the near-tip plastic deformations under cyclic loading with no bond breaking. Then, high-resolution large-deformation elastoplastic simulation of crack is the right way to visualise this mechanism of FCG, which hardly can elucidate the whole, but surely a part of the fatigue cracking phenomenon.


This paper aims to fulfil mentioned deficiencies of knowledge about the near-tip situation under cyclic loading. Large-deformation finite-element solutions have been generated to elucidate the peculiarities of the near-tip large deformation-, strain-, and stress-fields in elastoplastic solid. Consideration is given here to a two-dimensional ideal elastoplastic solid with a mode I plane strain crack under SSY conditions.

2. Basic modelling issues

Typical medium-strength steel was chosen as a prototype material. Although materials usually manifest strain-hardening elastoplastic behaviour, its idealisation with the elastic–perfectly-
plastic constitutive model can provide a reasonable first-order approximation. Limiting to this model, the key material characteristics were taken as follows: the Young modulus $E = 200$ GPa, the Poisson ratio $v = 0.3$ and the tensile yield stress $\sigma_y = 600$ MPa.

The constitutive equations for the rate/time-independent elastoplastic solid with von Mises yield surface and associated flow rule were employed.

A parallel-flanks slot with semicircular tip was taken as a model of an undeformed crack. Its width $b_0$ is the characteristic length scale in the model (Fig. 1a). This approach has been repeatedly substantiated and widely used (Gortemaker et al., 1981; Liu and Drugan, 1993; McMeeking, 1977; McMeeking and Parks, 1979; Needleman and Tvergaard, 1983; Toribio and Kharin, 1999b). Nevertheless, its pertinence to analysis of cracks is still sometimes questioned. Having no intention to repeat all the reasoning behind this modelling approach, it can be pointed out that cracks must blunt unavoidably either by dislocation emission from the tip in inherently ductile crystals, or by dislocation absorption at the tip in inherently brittle, but imperfect, i.e., real, ones. Despite the unlikelyhood of dislocation emission in this latter case, crack blunting by means of sinking-in of nearby dislocations lying in slip lines met by a growing crack since it has attained a certain mesoscopic size and its attractive forces on dislocations increased enough. Besides, crack blunting due to interaction with planar defects in crystals may be pointed out as another option (Miller et al., 1998). Although different crack tip shapes are possible from the point of view of plastic slip geometry (McClintock, 1971), the semicircular one seems to be an appropriate averaging. Finally, concerning the stress–strain states, cracks are distinguished from notches not by their widths $b_0$, nil or finite, but by a specific autonomous near-tip stress–strain field, which has the fixed self-similar shape and the potency established by a single parameter – SIF under SSY or $J$-integral under large scale yielding (McMeeking, 1977; McMeeking and Parks, 1979). As it has been proved, this kind of autonomous near-tip state is attained around a notch whenever its deformed width at the tip $b_t = b_0 + \delta$, where $\delta$ is the crack-tip opening displacement (CTOD), overcomes a certain level, and afterwards the difference between blunted notches and initially sharp cracks becomes negligible, as described elsewhere (McMeeking, 1977; McMeeking and Parks, 1979). According to the estimations by Kitagawa and Komeda (1986), McMeeking (1977), McMeeking and Parks (1979), this is achieved at $\delta/b_0 \geq 0.5–2$. Here, the value of $b_0 = 5$ μm was taken as a reasonable choice according to the data for medium-strength steels (Handerhan and Garrison, 1992).

Simulated load cases consisted of up to 10 cycles at different constant amplitudes and load ratios, and the effect of overload was addressed, too:

(I) $K_{\text{max}} = K_0$, $K_{\text{min}} = 0$ (i.e., $\Delta K = K_0$; $R = K_{\text{min}}/K_{\text{max}} = 0$);

(II) $K_{\text{max}} = 2K_0$, $K_{\text{min}} = 0$ ($\Delta K = 2K_0$; $R = 0$);

(III) $K_{\text{max}} = 2K_0$, $K_{\text{min}} = K_0$ ($\Delta K = K_0$; $R = 0.5$);

(IV) $K_{\text{max}} = K_0$, $K_{\text{min}} = 0$ ($\Delta K = K_0$; $R = 0$), with an overload to $K_{\text{app}} = 2K_0$ at the sixth cycle,

where the reference SIF value $K_0 = 30$ MPa√m was assigned. This renders the loading regimes I–IV under which fatigue cracking usually goes on in steels (Hertzberg, 1989; Suresh, 1991). In addition, with this $K_0$ and chosen initial crack width $b_0$, the textbook estimation of CTOD yields $\delta_t = k_2^2/\sigma_y = 1.5b_0$, which turns out to be high enough to come off the dependence of the crack-tip stress–strain state on the initial tip width, thereby approaching the $K$-driven autonomy proper to the crack-tip zone, as summarised in the previous paragraph.

To study the crack tip behaviour under $K$-dominated SSY, model design must ensure the $K$-controlled SSY at the superior load level $K = 2K_0$ by means of the choice of suitable dimensions of the test-piece: the characteristic size of the near-tip plastic zone must be there considerably smaller than the distance from the tip $2b_0$, over which the linear-elastic stress is dominated by the singular $K^{3/2}$-term of the whole-series solution (Gdoutos, 1993; Kaniinen and Popelar, 1985).

The first trial simulations were performed within a boundary layer formulation (Rice, 1967), which has been commonly used under monotonic loading (see, e.g., Liu and Drugan, 1993; McMeeking, 1977), and employed occasionally for cyclic one, too (Deshpande et al., 2001; Roychowdhury and Dodds, 2003). Then, $K$-controlled boundary conditions given by the known linear-elastic singular solution were imposed over a remote boundary of a circular domain of a sufficiently large radius $r_b = f_{\text{app}}$ (Fig. 1a). The requirements for $K$-dominated near-tip state were satisfied choosing $r_b = 1500b_0$.

However, ambiguities of this approach were met under cyclic loading as regards the choice between $K$-controlled displacement- or stress-imposed boundary conditions. Then, the use of entire specimen was considered a better way. Its characteristic dimension of the inelastic crack-tip region under plane strain was addressed, too: $K_{\text{app}}(t) = 0.014 << 1$ and much smaller ratios for $R_{\text{app}}/H$ and $R_{\text{app}}/W$.

Large-deformation elastoplastic solutions were generated using nonlinear finite element code with updated lagrangian formulation. Owing to the symmetry, the computations were performed for the quarter-panel shadowed in Fig. 1b. The mesh design (Fig. 2) followed the guidelines from the studies of large near-tip deformations under monotonic loading (Kitagawa and Komeda, 1986; Liu and Drugan, 1993; McMeeking, 1977; McMeeking and Parks, 1979; Needleman and Tvergaard, 1983). However, to postpone mesh degeneration, thereby enabling calculations for several

$$K = 1.158\sigma_{\text{app}}\sqrt{\pi a}.$$
load reversals, a better-adjusted refinement was required for the near-tip mesh, as well as for the load stepping in incremental analysis procedure. The majority of computations were performed with the mesh consisting of four-node quadrilaterals, where the average size of the smallest near-tip ones with the sides \( a_0 \) and \( b_0 \) was \( \sqrt{a_0 b_0} = 0.046b_0 \). To corroborate the mesh convergence of solutions, the meshes with the smallest elements being 4x4 division of the mentioned ones were tried, too.

3. Results

3.1. Crack-tip plastic zones

Crack-tip plastic zones have been the focus of interest as an indication of the material toughening capability, in particular, under cyclic loading (Ellyin and Wu, 1992; McClung, 1991; McClung and Davidson, 1991; McClung and Sehitoglu, 1989; Rice, 1967; Roychowdhury and Dodds, 2003; Wu and Ellyin, 1996). Although computations of plastic zone size and shape do not require large-deformation analysis, as far as the extent of plasticity is much farther than the domain affected by finite strains, the data about plastic zones under cyclic loading are presented here for the sake of completeness.

In published analyses of fatigue cracks, two plastic zones have been usually distinguished: the forward and the reversed ones formed, respectively, whilst approaching SIF maxima and minima along the loading routes (Ellyin and Wu, 1992; Hertzberg, 1989; McClung, 1991; McClung and Davidson, 1991; McClung and Sehitoglu, 1989; Rice and Johnson, 1977; Suresh, 1991). Under constant amplitude cycling, both zones appeared fairly stable during load cycling and crack propagation (McClung, 1991; McClung and Sehitoglu, 1989; Rice; 1967; Roychowdhury and Dodds, 2003; Suresh, 1991). At \( R > 0 \), the contours of the forward and reversed plastic zones, respectively, \( r_f(\theta) \) and \( r_f(\theta) \), where \( \theta \) is polar angle (Fig. 1), obeyed closely the \( (K/\sigma_f)^2 \)-scaling laws as follows:

\[
r_f = \frac{\sigma_f}{\sigma_y} \left( \frac{K_{\text{max}}}{\sigma_y} \right)^2 \quad \text{and} \quad \Delta r_f \approx \frac{1}{4} r_f(\Delta K) = \frac{1}{4} \left( \frac{\Delta K}{K_{\text{max}}} \right)^2 r_f(K_{\text{max}}),
\]

(3)

where the factor \( \varkappa \approx 0.1 \) for plane strain can be taken as an average of the published data. The ratio \( \Delta r_f/r_f \) used to be more or less stable around the magnitude 1/4 predicted by Rice (1967), varying from 0.1 to 0.3 according to modelling simulations and experimental measurements.

In the present analysis, two definitions were employed to reveal plastic zones. The first one relied on the von Mises criterion that the equivalent stress \( \sigma_{eq} = \sigma_v \), as did, e.g., Fleck (1986). The second one used the equivalent plastic strain rate \( \dot{\varepsilon}_{pl} \) as a suitable detector. For rate-independent material, this latter may be defined through the incremental values of the equivalent plastic strain \( d\varepsilon_{eq} = \frac{1}{2} d\varepsilon_{pl} d\varepsilon_{pl}^{\text{T}} \), where \( d\varepsilon_{pl} \) are the components of the plastic strain increment tensor, and of any convenient time-like parameter, which characterises the advance of loading. To render \( \dot{\varepsilon}_{eq} \) positive on the forward \((dK > 0)\) and negative on the reversed \((dK < 0)\) phases of loading, it was taken for convenience

\[
\dot{\varepsilon}_{eq} = \frac{d\varepsilon_{eq}}{dK}.
\]

(4)

As far as this criterion turns out to be ambiguous when applied to finite-element solutions because zero-check is sensitive to otherwise insignificant numerical errors, it seemed reasonable to choose a tolerance level \( \varepsilon_0 > 0 \) and proceed with a looser criterion \( \varepsilon_{eq} > \varepsilon_0 \). An appropriate choice for \( \varepsilon_0 \) seemed to be the value that provided a consistent determination of the monotonic plastic zone by stress- and plastic strain rate-based criteria which both, naturally, must render the same with reasonable exactitude. Accordingly, \( \varepsilon_0 = 5 \times 10^{-4} \) was adopted. With these criteria, obviously, the stress-defined regions must be qualified as potentially plastic zones, since attaining the limit of elasticity neither means entry into the plastic regime nor can indicate plastic flow intensity, whereas with the plastic strain rate-based criterion the active plastic zones can be determined, where plasticity indeed is going on with appreciable intensity.

Calculated forward (at \( K \to K_{\text{max}} \)) and reversed (at \( K \to K_{\text{min}} \)) near-tip zones determined using adopted criteria acquired fairly stable self-similar shapes virtually insensitive to the cycle number \( N \) during constant amplitude loadings along each loading route from I to IV, including the post-overload cycling. They are displayed in Fig. 3 in the original configuration of the solid (rather small distinctions could be detected considering these plastic zones in original and deformed arrangements because finite deformations affect only the very close vicinity of the crack tip).

Concerning the stress-defined plastic zones, present calculations (Fig. 3, contour lines) corroborate in the main aspects the data from previous analyses starting from the superposition one due to Rice (1967), and so forth (Ellyin and Wu, 1992; McClung, 1991; McClung and Sehitoglu, 1989; Suresh, 1991):

(i) The “envelope” shapes and sizes of the monotonic (Fig. 3a, white line) and forward (Fig. 3a and c, dark lines) zones under constant amplitude cycling, as well as those for an overload peak, were virtually the same at the same SIF values, and agreed with the familiar monotonic load small-scale finite-element solutions, e.g., by Rice and Tracey (1973).

(ii) Both forward and reversed zones during constant amplitude loadings were fairly self-similar with the scales \( (K_{\text{max}}/\sigma_y)^2 \) and \( (\Delta K/\sigma_y)^2 \), respectively, and at given \( \Delta K \), the reversed zones were about 1/4 of the respective forward ones (Fig. 3a–d), as expressed by Eq. (3).
The post-overload von Mises zones \((\text{Fig. 3e and f})\) permuted, so that reversed zones acquired the shape and size of pre-overload forward ones and vice versa, which basically agrees with the superposition method of Rice (1967).

Regarding the von Mises zones, performed calculations revealed also differences with previous publications. As loading goes on, initially compact and rather convex regions, although preserving their "envelope" shapes, become eroded (dark lines in \text{Fig. 3a, c and f}): gulfs arise on their back flanks, and holes appear ahead of the crack tip. This erosion continues up to eventual partition of these zones. In addition, at given \(D_K\), these zones are somewhat sensible to the load ratio \(R\) (cf. \text{Fig. 3a vs. c, and Fig. 3b vs. d}).

Concerning the strain rate-defined plastic zones (grey-scale band plots in \text{Fig. 3}), it follows that plastic flow is negligible in wide peripherals of the forward von Mises regions during regular cycling, and of the reversed ones after an overload. Disregarding the into-depth plasticity spurts of the lowest-level band in \text{Fig. 3}, where the magnitude of \(\varepsilon_{eq}^p\) is the least significant, all zones of appreciable cyclic plasticity, both forward and reversed ones, generated under constant-amplitude loading regimes (including the post-overload cycling as well) are closely matching one another when scaled with \((\Delta K/\sigma_Y)^2\) as the size unit, whereas forward and reversed von Mises zones were markedly distinct. The depths of appreciable cyclic plasticity ahead of the tip, \(\Delta x_{CP}\), which are close to the \(x\)-dimensions of the smaller among previously considered high- and low-peak von Mises zones \(\Delta x_Y = \Delta x_Y(0 = 0)\), together with their expansions transversely to the crack, \(\Delta y_{CP}\), are as follows:

\[
\Delta x_{CP} \approx \Delta x_Y \approx 0.013(\Delta K/\sigma_Y)^2 \quad \text{and} \quad \Delta y_{CP} \approx 0.05(\Delta K/\sigma_Y)^2.
\]

This value of \(\Delta y_{CP}\) agrees with reported experimental measurements (see Hertzberg, 1989, p. 529).

Thus, the smaller ones among the forward and reversed von Mises zones represent the expansion of active cyclic plasticity in front of the crack tip under all considered loading routes, with or without an overload. So, active plastic zones are dependent mainly on \(D_K\), whereas the effects of other loading peculiarities are slight.

Despite rather small differences were detected with regard to the spreading of appreciable cyclic plasticity at high- and low-peak loads, this does not hold with regard to the distributions of plastic strain rate, \text{Fig. 3}, which are somewhat different between the loading routes, and notably different between forward and reversed zones under the same loading regime. \text{Fig. 4} represents the distributions of the equivalent plastic strain rate under different loading regimes ahead of the crack tip in material points identified with

\[\text{Fig. 3. Plastic zones determined using the stress-based von Mises (line contours) and plastic strain rate-based (grey-scale bands, arbitrary units) criteria in the tenth loading cycle (excepting the white line corresponding to initial monotonic loading up to the peak load of the first cycle): (a) and (b) – loading regime I, (c) and (d) – loading route III, (e) and (f) – post-overload cycling in the loading case IV. All plots are in the same scale; grids on the images have the dimensions } (0.1 \times 0.1)(\Delta K/\sigma_Y)^2 \text{ and numbers on the axes indicate their sizes in millimetres.} \]
their undeformed depths from the tip $X$ in $(\Delta K(\gamma_{\sigma})^2)$-units, where the fixed dimension $d_0$ of some arbitrary material element is also shown for reference. This confirms virtually equal depths of the forward and reversed active plastic zones presented by the first of Eq. (5), but the depths of appreciably high plastic strain rates in the forward plastic zones, as estimated from Fig. 4, are about twice as those in the reversed ones during constant amplitude cycling. The relation between the effective depths of forward and reversed cyclic plasticity, $X_{CP}$ and $X_{RCP}$, is roughly

$$x_{CPD} \approx \frac{1}{2} x_{CPF} = \frac{1}{2} \Delta x_{CP}$$ (6)

in contrast to the ratio of about $1/4$ proper to the spreads of forward and reversed plasticity according to the von Mises criterion, cf. Eq. (3). Apart from extensions of the forward and reversed plastic flows, their intensities are different, too, the forward one being notably higher along constant amplitude loading routes (Fig. 4). The influence of both $K_{max}$ and $R$, apart from $\Delta K$, on the effective depths and intensities of near-tip cyclic plasticity can be appreciated from Figs. 3 and 4. Within the same depths in $(\Delta K(\gamma_{\sigma})^2)$-units, higher $K_{max}$ intensifies forward plastic straining, whereas increase of $R$ reduces the reversed one. In addition, the role of overload is also visible: post-overload distributions of $\gamma_{\sigma}(X)$ in Fig. 4 are virtually symmetrical in forward and reversed active plastic zones, i.e., the effective expansions of forward and reversed cyclic plasticity become approximately the same, as well as their intensities do, too.

3.2. Crack-tip deformations

For all load cases, deformations (displacements) near the crack tip proceeded similarly to those shown in Fig. 5. The crack profiles at low-peak loads acquired the keyhole shapes tapering in the wake behind the tip, somewhat similar to the observed in small-strain modelling by Fleck (1986), but here, in contrast, the crack width never and nowhere returned to its initial value, so that crack closure (meant as contact between its faces) have never been approached. These results agree also with small-deformation analysis by Ellyin and Wu (1992), as well as with similar finite-deformation modelling of a single loading cycle by Gortemaker et al. (1981), but they do disagree with numerous small-strain finite-element simulations where crack faces contact occurred upon unloading, however, when cracks were made growing by means of finite-element node release in each cycle as a species of “bond-breaking” (cf., e.g., Fleck, 1986; McClung and Sehitoglu, 1989; McClung et al., 1991; Roychowdhury and Dodds, 2003).

CTOD $\delta_t$ used to be the key characteristic of the near-tip fracture process zone (Kanninen and Popelar, 1985; McMeeking, 1977; McMeeking and Parks, 1979; Needleman and Tvergaard, 1983; Suresh, 1991). Although there are somewhat different “operational” CTOD definitions (Kanninen and Popelar, 1985; McMeeking, 1977; Suresh, 1991), the discrepancies between them under monotonic loading are minor. Under cyclic load, more hazy deformation patterns, such as shown in Fig. 5, add ambiguity in the matter. A definition of CTOD adopted by McMeeking (1977) as twice the $y$-displacement of the node $B_0$ (Fig. 5) was adopted here.

Cyclic trajectories $\delta_t = \delta_t(K,N)$ in Fig. 6 appear dependent on $K_{max}$ and $K_{min}$, and slightly influenced by $N$. Confirming $K$-dominance conditions, these plots for similar load cases I and II are fairly identical in corresponding dimensionless coordinates. Ratcheting of $\delta_t(K,N)$-trajectories vanishes with rising $N$, and they converge to fairly stationary loops.

At monotonic loading phases and during constant-amplitude cycling, CTOD paths in Fig. 6 agree with the results of other numerical and analytical models (Hertzberg, 1989; Kanninen and Popelar, 1985; McMeeking, 1977; Suresh, 1991). Numerical results for constant-amplitude loading routes may be approximated fairly well by simple formulae, respectively, for monotonic, cyclic reversed and forward loading phases:

$$\begin{align*}
\delta_{t\text{mon}}(K) & = 0.6 \frac{K^3}{E\gamma_N} \quad \text{(7a)} \\
\delta_{t\text{rev}}(K) & = \delta_{t\text{max}} \left[ 1 - \left(1 - \frac{K}{K_{\text{max}}} \right)^2 \right] \quad \text{(7b)} \\
\delta_{t\text{fwd}}(K) & = \delta_{t\text{min}} \left( 1 - \frac{K}{K_{\text{max}} - R} \right)^2 \left( 1 - \frac{R}{1 - R^2} \right) \quad \text{(7c)}
\end{align*}$$

where CTOD values at high- and low-peak loads are $\delta_{t\text{max}} = \delta_{t\text{max}}(K_{\text{max}})$ and $\delta_{t\text{min}} = \delta_{t\text{min}}(1 - (1 - R^2)^2)$.

For the load case IV, the bottom envelope of the $\delta_t(K)$-plot in Fig. 6c from the start of loading towards the absolute SIF maximum $K_{ov}$ is fairly the same as the monotonic-load trajectories shown in Fig. 6a and b and represented by expression (7a), being only interrupted by pre-overload constant amplitude load cycling. In comparison with the pre-overload ones, the post-overload $\delta_t(K)$-loops shift up-wards depending on the magnitude of $K_{ov}$, but preserving the shape and size of the former loops.

Thus, under constant amplitude loading regimes, whether or not interrupted by overload, the CTOD trajectories acquire fairly steady-state loop patterns. The shapes of these stationary loops are self-similar. Their sizes (Fig. 6) may be presented equivalently in terms solely of $\Delta K$ or of CTOD range $\Delta \delta_t = \delta_{t\text{max}} - \delta_{t\text{min}} \times \Delta K^2$, which can be inter-related according to Eqs. (7), and in agreement with other models (Suresh, 1991), as follows:

$$\Delta \delta_t = \delta_{t\text{max}}(1 - R^2)^2 \left( \frac{1}{2} - \frac{R}{1 - R^2} \right) = 0.6 \frac{\Delta K^2}{E\gamma_N}$$

(8)

The roles of $K_{\text{max}}$ (or $R$) and of an overload consist in establishing the locations of these loops in the $K - \delta_t$ plane, but their neither sizes nor shapes are affected noticeably.

Other feature of the crack-tip deformation revealed by performed simulations is the plastic crack advance $\Delta \alpha_p$, i.e., the irre-
versatile displacement of the crack tip apex point $A_0$ along the crack direction beyond the initial tip location, which is evident in Fig. 5. This way crack grows with no bond breaking, providing a visualisation of the Laird–Smith scheme of crack propagation by plastic blunting–resharpening, which was considered to be the intrinsic mechanism of the FCG in ductile materials (Ritchie, 1999; Suresh, 1991). It should be noticed that crack growth is sensible to an overload value for a variety of materials (Ritchie, 1999; Suresh, 1991). It is indiscernible. For the simulated load cases, $(da/dN)_p$ is of the order of $10^{-6} \text{ m/cycle}$, which is proper for the Paris regime in steels (Suresh, 1991). In spite of lack of representative amount of data, it may fit the numerical results with $m = 2.15$, which is a reasonable value for a variety of materials (Ritchie, 1999; Suresh, 1991). It should be noticed that crack growth is sensible to an overload which halts, and even back-downs, crack advance (Fig. 7d).

For all considered load cases, Fig. 7 displays $\Delta a_p$ in terms of a time-like parameter $t$, which choice for the rate-independent materials is a matter of convenience, and it was taken to render the sine-waveform of applied load path shown therein, too. The inferior straight-line bounds for the $\Delta a_p(t)$-patterns in Fig. 7 mark resulting crack advance. During constant-amplitude loading regimes, their slopes render the rates of plastic crack growth per cycle $(da/dN)_p$, which appears to be a function of $\Delta K$, whereas the role of $K$ is indiscernible. For the simulated load cases, $(da/dN)_p$ is of the order of $10^{-6} \text{ m/cycle}$, which is proper for the Paris regime in steels (Suresh, 1991). In spite of lack of representative amount of data, it may fit the numerical results with $m = 2.15$, which is a reasonable value for a variety of materials (Ritchie, 1999; Suresh, 1991). It should be noticed that crack growth is sensible to an overload which halts, and even back-downs, crack advance (Fig. 7d).

This way, calculated plastic crack growth by blunting-resharpening reproduces common experimental trends of FCG concerning the effects of $\Delta K$ and overload (Suresh, 1991).

### 3.3. Crack-tip stress–strain fields

The small-strain considerations provided rather insufficient resolution of the near-tip stress and strain distributions. This has been proved for monotonic loading (Kitagawa and Komeda, 1986; McMeeking, 1977; McMeeking and Parks, 1979; Needleman and Tvergaard, 1983; Rice and Johnson, 1977), supplying valuable data for interpretation of the fracture phenomena. In contrast, there has been a scarcity of analogous data for cracks under cyclic loading. An attempt to fill-in this deficiency is undertaken here.

Calculated near-tip cyclic stress and plastic strain fields in all considered load cases were fairly similar, and their contour-band plots are shown in Fig. 8 for the axial stress $\sigma_{yy}$, for the instantaneous equivalent plastic strain at specified point of the loading route, $\varepsilon_{eq}^p = \left( \frac{1}{2} \int \sigma_{yy}^p \, d\varepsilon_{yy}^p \right)^{1/2}$, and for the cumulative (accumulated) plastic strain attained along the loading history till specified instant, $\varepsilon_{cum}^p = \int \varepsilon_{eq}^p = \left( \frac{1}{2} \int \sigma_{yy}^p \, d\varepsilon_{yy}^p \right)^{1/2}$, which is used to be correlated with fatigue damage accumulation (Suresh, 1991). Extreme stresses at high- and low-peak loads, tensile $\sigma_{yy} > 0$ and compressive $\sigma_{yy} < 0$, respectively, are attained ahead of the tip. This is usual for near-tip stress fields affected by large geometry changes (Gortemaker et al., 1981; Kitagawa and Komeda, 1986; McMeeking, 1977; McMeeking and Parks, 1979; Needleman and Tvergaard, 1983). At each instant of the loading history, both measures of plastic strain rise monotonically as the crack tip contour is approached, which could be anticipated from the plastic strain rate behaviour in Figs. 3 and 4. The spatial distributions of $\varepsilon_{eq}^p$ and $\varepsilon_{cum}^p$ are otherwise somewhat different: the relief of $\varepsilon_{eq}^p$ (x, y) manifests preferable strain concentration just in front of the tip, $(\sigma_{yy}, (x, y)$ has two symmetric ridges emanating from the crack tip transversely to its plane $(\sigma_{xx}, \sigma_{xy}, \sigma_{yy})$. Note, that for plane-strain incompressible plasticity, extreme principal plastic strains $\varepsilon_{eq}^p (k = 1, 3)$ and are fairly proportional to equivalent plastic strain, e.g., the maximum one $\varepsilon_{eq}^p \approx \sqrt{2/3} \varepsilon_{eq}^p$, where the inexactness is proportional to the ratio of elastic and plastic portions of strains $\varepsilon_{eq}^p / \varepsilon_{eq}^p$ and $\varepsilon_{eq}^p / \varepsilon_{eq}^p \ll 1$ in the large-strain region.

Figs. 9 and 10 show the normal stress acting on the crack plane ahead of the tip, $\sigma_{yy}$. Its evolutions [Fig. 9] are presented in terms of the same time-like parameter $t$ rendering the sine-waveform of the applied load path. In spatial distributions, Fig. 10, the distance $X$ is normalised by the current crack tip width $b_0(t) = b_0 + \delta(t)$. Stress
fields in the close crack tip vicinity under constant amplitude loadings, including the post-overload cycling, manifested minute influence of $N$, similarly to what has already been pointed out for CTOD trajectories (Fig. 6).

In the close vicinity of the tip, at $X/b_t \lesssim (2–3)$, stress evolutions $\sigma_{yy}(t)$ in Fig. 9 get II-like shapes with approximately the same crack-tip stress ratio $R_{yy}(X) = \sigma_{yy}(X, K_{min})/\sigma_{yy}(X, K_{max}) \approx -1$ independent of applied load ratios $R$, whereas at farer locations from the tip the stress histories approach the sine-like patterns of applied load with $R_{yy}(X) \rightarrow R$. The near-tip stress distributions (Fig. 10) are approximately self-similar having $b_t(t)$ as the siltitude length-scale, fairly akin to the findings for monotonic loading (McMeeking, 1977). In particular, the post-overload phase, all high-peak cyclic stress distributions $\sigma_{yy}(X/b_t, K_{max})$, where $b_t = b_t^* + \delta_{max}$ nearly coincide one with another, and agree with the monotonic loading data (McMeeking, 1977). The applied load ratio $R$, i.e., $K_{max}$ provided $K_{max}$ remains the same, influences the low-peak stress profiles $\sigma_{yy}(X/b_t, K_{min})$, and it does mostly at $X/b_t \gtrsim 2$ (here $b_t = b_t^* + \delta_{min}$). The absolute tensile and compressive extremes of stress, which are attained at corresponding high- and low-peak loads, are, respectively, $\sigma_{yy}^{\text{max}} \approx 2.9\sigma_t$ and $\sigma_{yy}^{\text{min}} \approx -2.5\sigma_t$, and they occur around $X/b_t \approx (2–3)$, although apparently not in the same material point $X$. It follows from these data that the distributions of the near-tip stress range are dependent on both $K_{max}$ and $K_{min}$. $\Delta \sigma_{yy}(X, K_{max}, K_{min}) = \sigma_{yy}(X, K_{max}) - \sigma_{yy}(X, K_{min})$ becomes there as high as $(\Delta \sigma_{yy})_{\text{max}}(X) \approx 5\sigma_t$.

The effect of an overload on cyclic stresses in material points farer from the tip is approximately mirroring of asymmetric pre-overload stress evolution trajectories (Fig. 9c), as well as the spatial stress distributions for $K_{max}$ and $K_{min}$ (Fig. 10c), with corresponding revert of the value of $R_{CT}(X)$ from $R_{CT} \gg -1$ before overload to $R_{CT} \leq -1$ after this, i.e., that compressive stress near the tip increases following an overload. Correspondingly, the post-overload mean cyclic stress $\langle \sigma_{yy} \rangle_{\text{max}}$, which is the midway between $\sigma_{yy}(X, K_{min})$ and $\sigma_{yy}(X, K_{max})$, decreases. In locations closer to the tip, where stress cycling was before and continues to be after the overload nearly symmetrical, i.e., it maintains $R_{CT} \approx -1$, an overload reduces the local stress range $\Delta \sigma_{yy}$, so that its maximum $(\Delta \sigma_{yy})_{\text{max}}$ attains there about 70% of its pre-overload magnitude.

Concerning plastic strains, although the near-tip spatial distributions of $\hat{\varepsilon}_{xx}^{\text{eq}}$, as well as of $\hat{\varepsilon}_{yy}^{\text{cum}}$, were similar for all load cases (Fig. 8c–f), their evolutions in every fixed material point during load cycling, Figs. 11 and 12, depend substantially upon loading regimes. In the near-tip zone, $\hat{\varepsilon}_{xx}^{\text{eq}}$ climbs by alternating up- and down-ward steps corresponding to forward and reversed load excursions, respectively (Fig. 11). For $\hat{\varepsilon}_{yy}^{\text{cum}}$ both these steps, being otherwise identical to those of $\hat{\varepsilon}_{xx}^{\text{eq}}$, proceed up-wards (Fig. 12), this way reflecting that on the crack plane $\hat{\varepsilon}_{xy}^{\text{eq}} \approx (2/\sqrt{3}) \hat{\varepsilon}_{xx}^{\text{eq}}$ and $\hat{\varepsilon}_{yy}^{\text{cum}} \approx (2/\sqrt{3}) \hat{\varepsilon}_{xx}^{\text{cum}}$ (again, the inaccuracies of these equalities in the large-strain region are of the order of $\hat{\varepsilon}_{xx}^{\text{eq}}/\hat{\varepsilon}_{xx}^{\text{cum}} \ll 1$ vs. unity). The couples of mentioned steps, which correspond to forward and reversed phases of a cycle, are virtually identical between load cycles during constant amplitude loadings. As a result, respective rates of escalation in a cycle $d\hat{\varepsilon}_{xx}^{\text{eq}}/dN$ and $d\hat{\varepsilon}_{yy}^{\text{cum}}/dN$ are apparently steady dependent on the material point $X$ and on both loading parameters $\Delta K$ and $R$. These rates increase when approaching the crack tip in agreement with the trends of concentration of plastic strain rate $\hat{\varepsilon}_{xx}^{\text{eq}}$ in Fig. 4, as far as

$$
\frac{d\hat{\varepsilon}_{xx}^{\text{eq}}}{dN} \approx \int_{\text{cycle}} \frac{\hat{\varepsilon}_{xx}^{\text{eq}}}{dK} \, dK \quad \text{and} \quad \frac{d\hat{\varepsilon}_{yy}^{\text{cum}}}{dN} = \int_{\text{cycle}} \frac{\hat{\varepsilon}_{yy}^{\text{cum}}}{dK}
$$

according to the definition (4), as well as with the trends of concentration of $\hat{\varepsilon}_{xx}^{\text{eq}}$ and $\hat{\varepsilon}_{yy}^{\text{cum}}$ seen in Fig. 8c–f.

Under constant amplitude loading, the effect of $\Delta K$ appears unambiguous: $d\hat{\varepsilon}_{xx}^{\text{eq}}/dN$ and $d\hat{\varepsilon}_{yy}^{\text{cum}}/dN$ always rise with $\Delta K$. The influence of $R$ is also evident, although its trend is less definite depending on material point location $X$, cf. Figs. 11a with 11c and Figs. 12a with 12c: closer to the tip, higher $R$ notably reduces both $d\hat{\varepsilon}_{xx}^{\text{eq}}/dN$ and $d\hat{\varepsilon}_{yy}^{\text{cum}}/dN$, whereas their sensitivity to $R$ apparently vanishes at locations farer from the tip.

An overload halts the ratchet-like increase per cycle of the instantaneous strain $\hat{\varepsilon}_{xx}^{\text{eq}}$ (and, equally, of the maximum tensile one $\hat{\varepsilon}_{xx}^{\text{cum}}$), and even some backward ratcheting is seen in Fig. 11d. On the contrary, in agreement with its definition as accumulation of positive-definite increments, the cumulative plastic strain $\hat{\varepsilon}_{yy}^{\text{cum}}$ continues rising after an overload with positive-definite velocity (10), although an overload can alter this cyclic climbing rate. This change looks rather vague, displaying dependence on a material point location $X$ (Fig. 12d).

4. Discussion

4.1. Significance of modelling limitations

The purpose of the current research was to provide high-resolution data about the plane-strain $K$-dominated crack-tip mechanics.
under cyclic loading attending to essential nonlinearities met in the crack tip vicinity. Then, implementation of the geometrically nonlinear formulation (large displacements and strains) was a must. Concerning the material’s nonlinearity, the results of elasto-plastic analyses are as good as employed constitutive models. To this end, the model of perfect plasticity is not the best one. Unfortunately, to overcome this shortcoming within the context of large strain elastoplasticity meets difficulties not only at the technical, but also at the conceptual level. The implementation of the plasticity effects potentially important under load reversals, such as the Bauschinger effect which involves the kinematic mode of hardening as the first one to start with, is considered to be rather problematic, if not controversial, and development of the kinematic hardening plasticity models turns out to be a non-trivial task (Sidoroff and Dogui, 2001; Svendesen, 2001; Wallin et al., 2003).

Nevertheless, as far as at large strains the strain-hardening capacity of material approaches saturation, the idealisation of material behaviour with the elastic–perfectly-plastic constitutive model can be an acceptable first-order approximation, provided the value of its key parameter $\sigma_Y$ corresponds not to the initial yield point, but to some “effective” yield stress as modified by hardening. This item has been checked to some extent in similar simulations performed using isotropic, kinematic and combined hardening rules (Toribio and Kharin, 1999a, 2000, 2001, 2006; Toribio et al., 1999). Apart from the enhanced propensity to bifurcating shear localisations, which is a known attribute of certain constitutive models, especially with anisotropy of any origin (Kuroda and Tvergaard, 2000; Needleman and Tvergaard, 1983), and the kinematic hardening being one of them, the effects of different constitutive laws on the near-tip displacements, stresses, and strains there were minor, limited to rather fine peculiarities of these fields.

Other modelling issue in the present paper, which may be limitation when applying the results to the interpretation of FCG, is that a nominally stationary crack, i.e., with no bond breaking, was considered. Nevertheless, with regard to the stress–strain variables in the crack tip vicinity, a series of published results for stationary and propagating cracks under cyclic loading (McClung, 1991; McClung and Davidson, 1991; Roychowdhury and Dodds, 2003), although performed mostly within small-strain formulation, pointed out minor differences between them. Therefore, generated results, evidently pertaining to the near-tip fields before the crack starts to grow, may be considered a reasonable first-order approximation for the propagation phase, too.

Moreover, although no bond breaking was involved in the present analysis, the crack here grew merely by large plastic deformations. This way, the crack in performed simulations turned out to be not all stationary.

4.2. Implications for FCG

A dearth of certitude about controllable mechanical variables, which govern FCG, is the problem of general concern in characterisation and prediction of fatigue cracking. Conclusions in favour of certain mechanistic criteria have been usually drawn from correlations between candidate variables and experimental data on FCG. The three primary checkpoints are there the ubiquitous experimental trends of crack growth – acceleration with $\Delta K$, the influence of $K_{\text{max}}$ (or $R$), and the effects of over- or underloads along...
the loading route, followed by the influences of variable amplitudes, block loads, etc.

The conventional approach, which has been the core one for decades, counts on $D_K$ as the sole intrinsic driving force for FCG, attributing the influence of $R$ (or $K_{max}$) to the extrinsic factor of peculiar alteration of boundary conditions of cracked solids under cyclic loadings – the crack closure at load descents, which yields the adjustment of the FCG driving force from a mere SIF amplitude $D_K$ to the effective cyclic SIF $D_K_{eff}$ to account solely for the loading route segments when the crack is still opened (Hertzberg, 1989; Suresh, 1991). To serve as rationale for the cited ever-present FCG trends, crack closure must count on equally ubiquitous origin, which is considered to be the near-tip plasticity, apart from its potential occasional sources – crack surface roughness, in-crack debris (oxides or other chemical depositions), etc.

On the other hand, the unified approach, which has been developed for about the past decade (Sadananda and Vasudevan, 2003; Sadananda et al., 1999; Vasudevan et al., 2001), involves both parameters of cyclic loading, $K_{max}$ and $ΔK$, as intrinsic driving forces, respectively, of rupture (bond breaking) and material degradation (damage accumulation) as two key constituents of fatigue. This originates from the presumption that stresses and strains, near-tip ones in the case of cracks, are genuine drivers of the fatigue phenomenon wherever it takes place.

Performed simulations evidence cyclic crack growth: Fig. 5 visualises the Laird–Smith mechanism of FCG, whereas Fig. 7 represents the effects of $ΔK$ and overload on this mode of FCG, which are consistent with textbook trends of fatigue cracking (Hertzberg, 1989; Suresh, 1991). Despite the crack upon unloading shrinks in a wake behind the tip, no signs of plasticity induced crack closure (PICC) have ever been observed. Moreover, Fig. 5 reveals that large near-tip strains and displacements do not produce filling-in the crack behind the tip by material plastically stretched transversely to the crack plane, as it was argued repeatedly as the mechanism of PICC upon unloading, e.g., on the basis of small-strain solutions (McClung et al., 1991, Fig. 8). On the contrary, calculated deformation trajectories visualise material transfer from the very tip locations, such as the vicinity of the material point $B_1$ in undeformed configuration in Fig. 5a, to those on the crack flanks well behind the tip, cf. the vicinity of the same point in Fig. 5d, which, being accompanied by material stretching along the crack faces, increments crack length.

Meanwhile, it has been argued that, to achieve fully developed effect of PICC, the growing crack must penetrate well deeply into the plastic zone (Fleck, 1986; Roychowdhury and Dodds, 2003). Nevertheless, closure has been usually detected since the very beginning of load cycling in relevant finite element simulations (Fleck, 1986; Roychowdhury and Dodds, 2003; Wu and Ellyin, 1991).
where crack increments were imposed in every cycle by node release. To this end, presented modelling manifests final plastic crack advance without closure up to about 1/8 of the cyclic plastic zone size $\Delta x_{CP} = \Delta x_N$. What might happen after substantially larger growth of a crack by plastic blunting-resharpening is hardly to foresee with certainty.

Computations for considerably greater number of cycles do meet not only technical, but conceptual difficulties. Long-cycle loading routes can hardly be treated numerically, unless employing remeshing. Remeshing technologies are usually robust where the shape of a solid being remeshed is guided by some master surface, e.g., in common contact problems. But free surfaces, such as cracks, have no guidance, and thus, there exists no sound criterion to place the newly created mesh nodes, apart from some “reasonable” numerical attitude. This later, despite all precautions, can break against spurious mesh-dependence rooting in the potential non-uniqueness of corresponding field solutions, which is inherent to certain elastoplastic problems per se (Borja, 2002; Hill and Hutchinson, 1975; Kuroda and Tvergaard, 2000; Mathur et al., 1993; McClintock, 1971; Needleman, 1985; Rice and Tracey, 1973; Tvergaard et al., 1981). Then, even the relationship between solutions of the original continuum problem and of its finite-ele-

![Fig. 9. Evolutions of the axial stress ahead of the crack tip in material points $X/h_0 = 1.67$ (solid lines) and $10.9$ (dashed lines) under different loading regimes (shown by dotted lines in arbitrary units for reference): loading routs II (a), III (b) and IV (c).]
expensive) verifications of mesh (and remeshing) convergence are desired.

Although numerous publications have offered experimental and analytical support concerning PICC, it remains a controversial matter, which has both ardent supporters and committed opponents, cf., e.g., (Louat et al., 1993; McClung et al., 1991; Pippan and Riemelmoser, 1998; Sadananda et al., 1999; Suresh, 1991; Vasudevan et al., 2001). Its experimental manifestations measured ambiguities and have been repeatedly questioned (Sadananda et al., 1999; Vasudevan et al., 2001; Xu et al., 2000). Conclusions about the mechanisms of closure drawn from dislocation mechanics are contradictory, cf., e.g., (Louat et al., 1993) vs. (Deshpande et al., 2001; Pippan and Riemelmoser, 1998), as well as those from the continuum plasticity analyses, cf., e.g., present paper vs. (McClung et al., 1991). Not questioning a possibility of closure in general as a multi-origin phenomenon, PICC as its particular mechanism has raised quite strong controversy, up to the point that declarations to “believe that in the real world one always finds some degree of closure in fatigue” make this a matter of faith. Consequently, the role of PICC in FCG has been strongly questioned, too, offering explanations of the FCG trends (the effects of ΔK, R and overload) independent of the idea of closure as a standalone intrinsic factor (Noroozi et al., 2008; Sadananda et al., 1999; Vasudevan et al., 2001). Taking near-tip damage and bond breaking as the inherent factors of fatigue, the stress and strain fields have been emphasised instead as direct drivers of the fatigue process.

Anyway, despite no crack closure was observed, presented results on plastic crack growth by blunting-resharpening are consistent with some, but not all, of the main experimental trends of FCG: acceleration with ΔK and retardation by an overload, but without perceptible effect of the load ratio R, see Fig. 7. For this reason, as well as owing to apparent lack of opportunities to reproduce ample variability of the value of exponent m in Paris-like Eq. (9), the Laird–Smith-type mode of crack propagation can hardly be the whole mechanism of FCG, and fatigue must involve material damage and spitting driven by the stress–strain history in the process zone, whereas this plastic advance, being the clearly ubiquitous factor to FCG, can have there greater or less share.

Meanwhile, presented here near-tip cyclic stresses and strains not only manifest sensitivity to all three considered factors of fatigue cracking, i.e., ΔK, R (or Kmax), and overload (Figs. 9–12), but the affinities between the near-tip fields and experimental trends of FCG may be unveiled, too.

Starting with a fairly undisputable presumption that fatigue comprises two coupled processes – damage accumulation and rupture – which are both governed by relevant characteristics of the stress–strain fields (Ritchie, 1999; Sadananda and Vasudevan, 2003; Suresh, 1991; Vasudevan et al., 2001), the next scenario regarding FCG seems to be reasonable:

(i) According to common strain- or stress-life responses (Suresh, 1991), material damage goes forward being induced by cyclic stress or strain (“pure fatigue”), and its accumulation is used to be monitored either in terms of augmentation of cumulative plastic strain \( \varepsilon_{\text{cum}} \) (or absorbed plastic strain energy density \( W_p = \int \sigma_d \varepsilon_p \)) which, however, turns out to be equivalent to \( \varepsilon_{\text{cum}} \) for non-hardening Prandtl–Reuss-type elastoplastic solid, as far as \( W_p = \sigma_c \varepsilon_{\text{cum}} \) there, or to be associated with the range of relevant stress Δσ and its mean value \( \sigma_m \) (or stress ratio as the substitute of this latter) along with cycle number N.

(ii) Rupture (bond breaking) in damaged material occurs via some stress- or strain-controlled mechanism, which may be characterised by means of critical combination of damage and responsible stress–strain characteristics, such as principal stresses \( \sigma_k \) or total strains \( \varepsilon_{\text{cum}} = \varepsilon_{\text{p}}^{k} + \varepsilon_{\text{r}}^{k} (k = 1, 2, 3) \),

![Fig. 10. Distributions of the axial stress in front of the crack tip at the high- and low-peak loads at indicated cycles during loading regimes II (a), III (b) and I and IV (c); open symbols correspond to \( K_{\text{max}} \), and filled ones to \( K_{\text{min}} \); rhombi correspond to the earlier, and triangles to the later of the indicated loading cycles. Horizontal bars show the fixed material distance \( d_0/\ell_0 \) normalised by \( b \), corresponding to the high-peak load of the indicated loading route \( \ell_0 = I, II, III \) or IV.](image-url)
which may be resolved to define critical stress or strain, respectively, for stress- or strain-dominated rupture of a material weakened by accumulated damage.

(iii) Having recognised that rupture events are associated with certain microstructure-related scales, a consistent fracture criterion must involve a definite characteristic material distance $d_c$ over which damage-and-rupture criterion of the previous items (i) and (ii) must be surmounted (Hertzberg, 1989, Section 10.5.1; McMeeking, 1977), e.g., responsible stress or strain variable must exceed respective critical value over $d_c$ in accepted terms – line-averaged or in point-wise manner (cf., e.g., Taylor, 2008).

(iv) Finally, local rupture and crack increment by bond breaking $D_{ab}$ occur after a certain number of loading cycles $D_{Nb}$ have elapsed, provided the level of accumulated damage of the previous item (i) allows local stresses or strains to fulfill the rupture criterion, item (ii), over a microstructural distance $d_c$, item (iii). Consequently, the contribution of damage accumulation and bond breaking to the rate of FCG is as follows

$$\left(\frac{da}{dN}\right)_b = \frac{\Delta a_b}{\Delta N_b}. \quad (11)$$

Within this framework, the affinities between calculated near-tip stress–strain characteristics, considered within correspondent fixed material distance $d_c$ in the crack tip vicinity, and the experimental trends of the dependence of FCG rate $da/dN$ on $\Delta K$ and $R$ (evidently transformable into dependencies on $\Delta K$ and $K_{max}$), as well as on overload, can be pointed out as follows:

1. At fixed $R$, increase of $\Delta K$ accelerates the climbing rate per cycle of the accumulated plastic strain, $d\varepsilon_{cum}/dN$, Fig. 12a and b. Despite extreme values of local stresses under perfectly-plastic constitutive behaviour are not affected by the applied load levels $K_{max}$ and $K_{min}$, these latter broaden or reduce the length intervals where the highest stress range $\Delta \sigma_{yy}$ occurs according to $b(K_{max}, K_{min})$-scaled self-similarity of the near-tip stress fields (Fig. 10). Hence, the range of the principal stress beyond the tip $\Delta \sigma_{yy}$ over fixed critical distance $d_c$, does increase with $\Delta K$, too. As a result, admitting $\varepsilon_{cum}$ to be a measure or $\Delta \sigma_{yy}$ drive of damage, indicated behaviour of these variables implies the same: acceleration of damage accumulation and, consequently, faster FCG as well.

2. At fixed $\Delta K$, but rising $R$ (i.e., $K_{max}$), both $\varepsilon_{cum}$, Fig. 12a and c, and the mean principal stress ($\sigma_{yy}$)$_m$, cf. pre-overload plots in Fig. 10c with b, can be higher at some distances $d_c$ from the crack tip, and so, known strain- and stress-life trends of fatigue imply accelerated damage accumulation and, consequently, faster FCG. At the same time, however, with higher $R$ another damage drive, the local stress range $\Delta \sigma_y$ over material-dependent distance $d_c$, may diminish to the detriment of the affinity between near-tip cyclic stress field and FCG expectable from stress-life fatigue trends.
(3) On the other hand, with rising $K_{\text{max}}$, the near-tip normal stress $\sigma_{yy}$ and strain $\varepsilon_{\text{eq}}$ can attain higher values over definite material distance $d_c$ at high-peak load (Figs. 10a and c and 11a and b) and ease this way the fulfillment of whichever rupture criterion of critical stress or strain at lower level of accumulated damage, which implies that rupture event can occur earlier, having FCG acceleration as a consequence.

(4) Concerning the effects of overload on the near-tip cumulative plastic strain, stress range and ratio presumed, respectively, to be a measure of damage and drives of its accumulation, the influence of overload on $d\varepsilon_{\text{cum}}/dN$ is rather inexpressive, although its slight deceleration is detectable in certain near-tip locations, whereas post-overload reduction of the range of local stress and down-ward shift of its mean value are fairly clear in Figs. 9 and 10c. According to common strain- and stress-life trends of fatigue, this implies post-overload deceleration of damage accumulation, and consequently, delay of FCG.

(5) The consequences of overload are fairly straightforward for the mechanical variables potentially responsible for local stress- or strain-controlled rupture, i.e., for maximal stress $\sigma_{yy}$ or attained strains $\varepsilon_{\text{eq}}$ and $\varepsilon_{yy}$ over requested microstructural distance $d_c$, Figs. 9, 10 and 11d: overload reduces the former and halts climbing of the latter. This hampers fulfillment of local rupture criterion, and so, retards or halts FCG.

All these deductions about FCG from the data of present analysis of the near-tip stress–strain fields combined with stress- and strain-life fatigue trends agree with experimental tendencies of fatigue cracking (Hertzberg, 1989; Suresh, 1991), but are not here the consequences of crack closure, which merely have not occurred in simulated load cases. Thus, the effects of loading parameters on FCG, hitherto associated with PICC, can be neither necessarily reliant-on nor ascribable-to the closure behind the tip, but attributable to the stress–strain fields in front of it. Not questioning potential occurrence of crack closure proceeding from various origins, such as loading trajectory sensibility of plasticity, crack face roughness, oxidation, in-crack debris, etc., presented analysis points to cyclic stress–strain state ahead of the crack tip as the intrinsic governing factor of FCG, whereas crack closure may be accompanying extrinsic one, but not ubiquitous component of FCG.

Anyway, it follows that FCG might proceed by means of Laird–Smith-type deformational mechanism accompanied with damage accumulation and bond breaking, which render the resulting crack growth rate

$$
\frac{da}{dN} = \left( \frac{da}{dN} \right)_p + \left( \frac{da}{dN} \right)_b
$$

where the contribution of one or another mechanism could be more or less substantial.
Finally, two more items of the crack tip situation under cyclic loading may be briefly commented – cyclic plastic zones and CTOD. The former have been frequently focused as a factor responsible, at least partially, for FCG (Ellyin and Wu, 1992; Fleck, 1986; McClung, 1991; McClung and Sehitoglu, 1989; Rice, 1967; Roychowdhury and Dodds, 2003). However, resolution of plastic zones with no attention to how plasticity goes on, which the von Mises criterion does, has little relevance to fatigue cracking. In contrast, discriminating the intensities of plastic flow within these zones, e.g., in terms of the plastic strain rate \( \varepsilon_{pl} \) (Figs. 3, 4), indicates when, where and how appreciable cyclic plasticity proceeds. This can bring a better insight about the activity and extension of near-tip plasticity, e.g., in comparison with sizes of certain microstructural units, to make deductions about peculiarities of FCG in terms of relations “microstructure-properties”.

In the matter of cyclic CTOD, it continues to be one of the key parameters of the near-tip fracture processes. Its significance takes root in that the deformed tip geometry controls the near-tip physical slip-band patterns (McClintock, 1971 Rice and Johnson, 1977), as well as the spatial spread of the autonomous near-tip stress–strain fields of self-similar shapes (Figs. 3, 4, 9, 10) in relation with the sizes of relevant microstructural units, such as the critical distance \( d_c \) in the preceding discussion.

5. Conclusions

High resolution finite-element analysis of the near-tip fields of a plane-strain tensile crack in an ideal elastoplastic solid under cyclic loading was performed accounting for large crack tip geometry changes. The effects of load range, load ratio and overload on the near-tip plastic zones, crack-tip deformations, strains and stresses were addressed. More details about the near-tip situation were revealed beyond available small-strain modelling results.

The sizes and shapes of the forward and reversed plastic zones defined according to the von Mises stress-based criterion agreed with published data. Beyond that, the plastic strain rate-based criterion allowed a discrimination of the regions of appreciable plastic flow activity. All these zones matched closely one another, when scaled with \( (\Delta K/a)^2 \), during constant-amplitude loading regimes, including the post-overload cycling. However, at given \( \Delta K \), within approximately equal ranges of appreciable activity of cyclic plastic flow, its intensities were dependent on both \( K_{\text{max}} \) and \( K_{\text{min}} \), as well as sensitive to an overload.

Large-deformation trajectories of material elements near the crack tip displayed crack growth without bond breaking merely by means of material transfer from the tip apex vicinity onto the newly formed crack flanks, accompanied by rotation and stretching of material elements along the crack faces, thus contributing to crack length increase. No crack closure was manifested by the present large-deformation patterns in contrast with small-deformation analyses, which revealed only stretching of material transversely to the crack plane, so that deformed material filled-in the crack causing its closure upon unloading. Computed crack growth by cyclic plastic blunting-resharpening agreed with the real-world trends, such as acceleration with \( \Delta K \) and retardation by an overload. This way, the Laird–Smith mechanism, which is considered one of the basic intrinsic ways of fatigue cracking, was visualised.

Within a fairly undisputable framework that fatigue results from two processes – damage accumulation and rupture, which are both governed by relevant characteristics of the stress–strain fields – by way of correlation of the obtained stress–strain results with common experimental stress- and strain-life fatigue trends, the affinities were shown between computed near-tip stress–strain variables and the experimental trends of the fatigue crack growth rate, such as its dependence on \( \Delta K \) and \( K_{\text{max}} \) (or \( R \)) and retardation by an overload.

This way, obtained results imply that the stresses and strains ahead of the crack tip must rule fatigue cracking as far as they govern the key process constituents – damage accumulation and bond breaking. The effects of loading parameters on fatigue cracking, hitherto associated with crack closure, can be attributed solely to the stress–strain fields in front of the crack tip with no contribution of closure behind it. Although this closure can occur owing to loading route, crack face roughness, oxidation, in-crack debris or other extrinsic circumstances, its consequences for cracking must be delivered to the crack tip fracture process zone by respective alterations of the near-tip stress–strain field. Anyway, obtained results favour the approach that considers fatigue cracking to be a two-parameter process in terms of fracture mechanics variables \( \Delta K \) and \( K_{\text{max}} \), where cyclic stresses and strains ahead of the crack tip acquire the significance of the intrinsic factors of fatigue crack growth, which is the result of near-tip plastic deformation damage accumulation and rupture.

Acknowledgments

The authors wish to thank the support of this research by the following institutions: Spanish Ministry for Scientific and Technological Research (MCYT; Grant MAT2002-01831), Spanish Ministry for Education and Science (MEC; Grant BIA2005-08965) and Regional Government Junta de Castilla y León (JCyL; Grants SA078/04, SA067A05 and SA111A07).

References


