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Comment

Comment on "Strong coupling in extended Hořava–Lifshitz gravity" [Phys. Lett. B 685 (2010) 197]

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ABSTRACT

We show that, contrary to the claim made in arXiv:0911.1299, the extended Hořava gravity model proposed in arXiv:0909.3525 does not suffer from a strong coupling problem. By studying the observational constraints on the model we determine the bounds on the scale of the ultraviolet modification for which the proposal yields a phenomenologically viable, renormalizable and weakly coupled model of quantum gravity.

Building on the seminal works by P. Hořava [1,2], we have recently proposed a power-counting renormalizable model for quantum gravity without Lorentz invariance [3]. Remarkably, the model is free of the pathologies [4–6,3,7] present in the original Hořava's proposal and associated with the additional mode of the gravitational excitations. This is achieved by providing the extra mode with a proper quadratic action around smooth backgrounds. We have argued in [3] that this property together with power-counting renormalizability of the theory ensures that the theory is weakly coupled all the way up to trans-Planckian energies.[1] We also argued in [3] that with appropriate choice of parameters the theory satisfies phenomenological constraints, and demonstrated this explicitly for the simplest tests provided by the gravitational potential between static sources (Newton's law) and homogeneous cosmology (Friedmann equation).

The consistency of the model presented in [3] has been recently questioned in [8], where it is claimed that the model suffers from the same kind of strong coupling problem as the previous versions of Hořava's proposal [6,7]. The aim of the present Letter is to show that this claim is unfounded. We support our arguments by considering a toy model where the absence of strong coupling is demonstrated both using the power counting and at the level of scattering amplitudes. We also analyze in more detail the observational bounds on the model and determine the window in the parameter space compatible with phenomenology and weak coupling.

1. Review of extended Hořava gravity

We start by describing briefly the model [3]. We consider the Arnowitt–Deser–Misner (ADM) decomposition for the metric,

$$ds^2 = \left(N^2 - N_i N^i\right) dt^2 - 2N_i\, dx^i\, dt - \gamma_{ij}\, dx^i\, dx^j. \tag{1}$$

This decomposition defines a foliation of space–time by 3-dimensional space-like surfaces thus splitting the coordinates into "space" and "time". We follow [2] and, unlike the case of general relativity (GR), consider this foliation structure as physical. This means that the group of invariance of the theory is not the full group of 4-dimensional diffeomorphisms, but only its subgroup consisting of foliation-preserving transformations

$$\mathbf{x} \mapsto \tilde{\mathbf{x}}(t, \mathbf{x}), \quad t \mapsto \tilde{t}(t). \tag{2}$$

The action of the model is taken in the form[2]

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[1] To be weakly coupled at *all* energies the model must fulfill the additional requirement that its marginal couplings do not develop Landau poles under the renormalization group flow. In other words, the theory must possess a weakly coupled UV fixed point. The Landau poles, if any, appear at exponentially high energies and are irrelevant for the purposes of the present Letter.

[2] The 3-dimensional indexes $i, j, \ldots$ are raised and lowered using γ_{ij}, and covariant derivatives are associated to γ_{ij}.

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$$S = \frac{M_P^2}{2} \int d^3x\, dt\, \sqrt{\gamma} N \big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}[\gamma_{ij}, a_i]\big), \tag{3}$$

where M_P is the Planck mass; K_{ij} is the extrinsic curvature tensor

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i), \tag{4}$$

with the trace K; γ is the determinant of the spatial metric γ_{ij}; λ is a dimensionless constant. The "potential" term $\mathcal{V}[\gamma_{ij}, a_i]$ in (3) depends on the 3-dimensional metric and the lapse N. The latter enters into the potential through the combination

$$a_i \equiv \frac{\partial_i N}{N}, \tag{5}$$

which is covariant under the symmetry (2). The dependence of the potential term on the gradients of the lapse is the key difference of the model (3) compared to the original Hořava's proposal [2].

The existence of the preferred foliation structure reflects the non-relativistic nature of the model: space and time enter into the theory on different footings. This allows to introduce into the action terms with higher derivatives in spatial directions which improve the ultraviolet (UV) behavior of the graviton propagator [2]; at the same time the theory remains second order in time derivatives thus avoiding problems with unitarity. Out of the previous family of actions (3), one can construct power-counting renormalizable theories by considering the scaling transformations [2]

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}, \qquad t \mapsto b^{-3}t, \tag{6a}$$

$$N \mapsto N, \qquad N_i \mapsto b^2 N_i, \qquad \gamma_{ij} \mapsto \gamma_{ij}. \tag{6b}$$

Under this scaling, the kinetic part of the action (3) and the operators of dimension[3] 6 in $\mathcal{V}$ are left unchanged (they are marginal).[4] Operators of lower dimensions in $\mathcal{V}$ are relevant deformations. According to the standard arguments, considering operators up to dimension 6 in the potential gives rise to an action which is perturbatively renormalizable. Explicitly, the allowed potential term is

$$\begin{aligned}\mathcal{V} = {} & -\xi R - \alpha a_i a^i \\ & + M_P^{-2}\big(A_1 \Delta R + A_2 R_{ij}R^{ij} + A_3 a_i \Delta a^i + A_4 \big(a_i a^i\big)^2 \\ & + A_5 a_i a_j R^{ij} + \cdots\big) \\ & + M_P^{-4}\big(B_1 \Delta^2 R + B_2 R_{ij} R^{jk} R^i_k + B_3 a_i \Delta^2 a^i + B_4 \big(a_i a^i\big)^3 \\ & + B_5 a_i a^i a_j a_k R^{jk} + \cdots\big),\end{aligned} \tag{7}$$

where R_{ij}, R are the Ricci tensor and the scalar curvature constructed out of the metric γ_{ij}; $\Delta \equiv \gamma^{ij}\nabla_i\nabla_j$, and ξ, A_n, B_n are constants. The ellipses represent other possible operators of dimension 4 and 6 which can be constructed out of the metric γ_{ij} and are invariant under 3-dimensional diffeormorphisms.[5] In what follows we set $\xi = 1$, which can always be achieved by a suitable rescaling of time. At low energies the potential is dominated by the operators of the lowest dimension, namely, the terms in the first line of (7). This leads to the recovery in the infrared of the relativistic scaling dimension -1 for both space and time.

The explicit breaking of 4-dimensional diffeomorphisms down to the subgroup (2) gives rise to the presence of a new scalar gravitational degree of freedom [2,3,6]. Its properties at the quadratic level were analyzed in [3] where it was shown that the new mode is free of pathologies at all energies (it is neither a ghost nor a tachyon) in a wide range of parameters. The proper behavior of the mode at low energies is ensured by the following choice of the parameters λ and α (see Eqs. (3), (7))

$$0 < \frac{\lambda - 1}{3\lambda - 1}, \quad 0 < \alpha < 2. \tag{8}$$

The additional mode does not have a mass gap: at low energies it obeys a linear dispersion relation with a velocity generically different from that of gravitons (which is 1 in our choice of units). This signals the break down of Lorentz invariance down to arbitrary low energies. As we discuss below, this has phenomenological consequences that ultimately set bounds on the values of the parameters λ and α governing the low-energy physics of the model.

It is convenient to introduce the covariant form of the theory which we obtain using the method described in [6]. One encodes the foliation structure of space–time into a new Stückelberg field $\phi(t, \mathbf{x})$ by identifying the surfaces of the foliation with surfaces of constant ϕ,

$$\phi(t, \mathbf{x}) = \text{const}. \tag{9}$$

The invariance of the theory under reparameterization of the foliation surfaces translates into the invariance under reparamaterizations of ϕ,

$$\phi \mapsto f(\phi), \tag{10}$$

where f is an arbitrary monotonous function. The quantities appearing in the action (3) reduce to the standard geometrical objects (induced metric, extrinsic and intrinsic curvature) characterizing the embedding of the hypersurfaces defined by (9) in space–time. The central object in the construction of these quantities is the unit normal vector[6] u_μ. Explicitly,

$$u_\mu \equiv \frac{\nabla_\mu \phi}{\sqrt{\nabla_\nu \phi \nabla^\nu \phi}}. \tag{11}$$

Note that u_μ is automatically invariant under the transformations (10). Other geometrical quantities associated to the foliation are constructed out of u_μ and its derivatives, see [6] for details. In this way one obtains the following covariant form of the action (3),

$$\begin{aligned}S = -\frac{M_P^2}{2} \int d^4x \sqrt{-g}\big\{ & {}^{(4)}R + (\lambda - 1)\big(\nabla_\mu u^\mu\big)^2 \\ & + \alpha u^\mu u^\nu \nabla_\mu u^\rho \nabla_\nu u_\rho \\ & + (\text{terms with higher derivatives})\big\}.\end{aligned} \tag{12}$$

This action describes a scalar–tensor theory of gravity invariant under 4-dimensional diffeomorphisms and the symmetry (10). Furthermore, after fixing the gauge $\phi = t$, it is equivalent to the non-covariant form (3). Thus the covariant (Stückelberg) formalism makes the presence of the extra scalar degree of freedom explicit.

The first line in (12) contains all the terms with up to two derivatives acting on u_μ and the metric; it describes the low-energy physics of the model. Note that this low-energy action is similar to a special case of the Einstein-aether theory (see [9] for

[3] We assign dimension -1 to the space coordinate. Then the dimension of time is -3, dimensions of the lapse and the 3-dimensional metric are zero, etc.

[4] This is true classically. At the quantum level one expects the coefficients in front of marginal operators to acquire logarithmic running under the renormalization group flow.

[5] Note that the operators with odd dimensions are forbidden by spatial parity. Similarly, the terms in the action with one time derivative of a_i are excluded by the time-reversal invariance. We stress that apart from these restrictions one must consider *all* operators of dimension up to 6 and compatible with the symmetries (2) to obtain a renormalizable action. Such operators are numerous and only a few of them are written explicitly in the above expression. The complete list of terms providing non-equivalent contributions *at the quadratic level* is given in [3].

[6] The Greek indices $\mu, \nu, \ldots$ are raised and lowered using the 4-dimensional metric $g_{\mu\nu}$, and the covariant derivatives with these indices are understood accordingly.

a recent review). The difference from the general Einstein-aether theory is that in our case the vector u_μ is, by its definition (11), hypersurface-orthogonal; i.e. it is characterized by a single scalar field.[7] Besides the low-energy part, the full action of the model also contains the terms with higher derivatives which we do not write explicitly. These terms arise from the second and third lines in the potential (7). Their important effect is to modify the dispersion relation of the modes at high energies,

$$E^2 = c^2 p^2 + \frac{p^4}{M_{*A}^2} + \frac{p^6}{M_{*B}^4}, \tag{13}$$

where E and p are the energy and momentum of the modes, and[8]

$$c^2 = \begin{cases} 1 & \text{for helicity-2 modes,} \\ c_s^2 \equiv \frac{\lambda-1}{\alpha} & \text{for scalar graviton.} \end{cases} \tag{14}$$

For simplicity, we assume $c_s \sim 1$ in what follows. In this case, one reads off the scales suppressing the higher derivative terms from (3), (7):

$$M_{*A},\ M_{*B} \sim \begin{cases} A_i^{-1/2} M_P,\ B_i^{-1/4} M_P & \text{for helicity-2 modes,} \\ \sqrt{\alpha} A_i^{-1/2} M_P,\ \alpha^{1/4} B_i^{-1/4} M_P & \text{for scalar graviton.} \end{cases} \tag{15}$$

As we now discuss, the presence of these higher-derivative terms is crucial to make the theory weakly coupled and renormalizable in the UV.

2. Would-be strong coupling and its resolution

In the covariant language, the issue raised in [8], can be understood as follows. Let us expand the low-energy action of the model (first line in (12)), around the background consisting of the Minkowski metric and the Stückelberg field linearly depending on time,

$$g_{\mu\nu}(t,\mathbf{x}) = \eta_{\mu\nu} + h_{\mu\nu}(t,\mathbf{x}), \qquad \phi(t,\mathbf{x}) = t + \chi(t,\mathbf{x}). \tag{16}$$

The result has the schematic form

$$S = M_P^2 \int d^4x \big[-h\Box h - \alpha(\partial_i \dot\chi)^2 + (\lambda-1)(\Delta\chi)^2 + (\lambda-1)\dot\chi(\Delta\chi)^2 + \cdots \big],$$

where, for the sake of the argument, we have written down only one of the interaction terms. The quadratic part of the perturbed action remains invariant under the relativistic scaling

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}, \qquad t \mapsto b^{-1} t, \tag{17a}$$

$$h_{\mu\nu} \mapsto b h_{\mu\nu}, \qquad \chi \mapsto \chi. \tag{17b}$$

The interaction terms for both fields have positive dimensions with respect to this scaling. This means that these interactions would become strong at a certain scale Λ *if no new physics appeared at a lower scale*. Under the assumption (motivated by phenomenological bounds) $|\lambda - 1| \sim \alpha \ll 1$, the covariant formalism with the action (12) allows to readily identify the scale Λ as

$$\Lambda = \sqrt{|\lambda - 1|} M_P \sim \sqrt{\alpha} M_P. \tag{18}$$

The scale (18) has been erroneously interpreted in [8] as the UV cutoff of the theory where the perturbative description breaks down. Actually, Λ is only the *cutoff of the low-energy approximation*. In the model described above, the would-be strong coupling is actually not present if the higher-derivative operators (which change the scaling dimensions of the interactions) enter into the game at energies lower than (18) (see the related discussion in [2]). For this to happen the energy scale of UV physics (15), which we collectively denote by M_*, must be smaller than Λ,[9]

$$M_* \lesssim \sqrt{\alpha} M_P. \tag{19}$$

Then, the power-counting analysis performed in the ADM frame (see (6)) shows that under the new scaling the interactions are at most marginal, meaning that there is no strong coupling at the scale Λ. We conclude that the correct interpretation of the scale (18) in the model at hand is that of the scale suppressing the higher-derivative operators.

Let us illustrate our point by a simple toy model. Consider a scalar theory with action

$$S = \alpha M_P^2 \int d^4x \left\{ \left(\varphi + \sum_{n\geqslant 2} a_n \varphi^n \right) \left[-\Box + \frac{\Delta^3}{M_*^4} \right] \varphi \right\}, \tag{20}$$

where the dimensionless coupling constants a_n are assumed to be somewhat smaller than 1. This scalar theory shares all the relevant properties with our actual gravity theory. At low momenta, $|\Delta| \ll M_*^2$, the higher derivative terms can be neglected and one obtains the following low-energy action

$$S_{low\ E} = -\alpha M_P^2 \int d^4x \left\{ \left(\varphi + \sum_{n\geqslant 2} a_n \varphi^n \right) \Box \varphi \right\}. \tag{21}$$

Clearly, the invariance of the quadratic part of the action with respect to the relativistic scaling transformations (17a) sets the scaling dimension of φ to be 1. The action contains irrelevant interactions under this scaling which naïvely become strong at the scale

$$\Lambda = \sqrt{\alpha} M_P. \tag{22}$$

However, this is not the case provided $M_* < \Lambda$. At momenta above M_* the quadratic action is dominated by the term with the highest number of spatial derivatives,

$$S^{(2)}_{high\ E} = \alpha M_P^2 \int d^4x \left\{ \varphi \left[-\partial_0^2 + \frac{\Delta^3}{M_*^4} \right] \varphi \right\}. \tag{23}$$

This is invariant under anisotropic scaling transformations (6a) with φ having scaling dimension zero. Consequently, all the interactions in the full action (20) become marginal at high energies, and the relative strength of the interaction terms with respect to the free part is always small.

It is instructive to see explicitly how the terms with higher derivatives prevent the theory from becoming strongly coupled in the language of scattering amplitudes. A well-known manifestation of the breakdown of perturbation theory is the saturation of unitarity bounds by tree-level amplitudes (see e.g. [10]). From the low-energy form of the action (20) one would conclude that tree-level unitarity is violated at the scale (22) and that perturbation

[7] In comparison with [9], we have absorbed one of the free parameters of the most generic action in a redefinition of time ($\xi = 1$).

[8] To be precise, the lower expressions holds in the "decoupling limit" when $\alpha, |\lambda - 1| \ll 1$. See [3] for the exact expression.

[9] One may worry that the choice of M_* (and Λ) parametrically below M_P introduces a fine-tuning in the model. Let us emphasize that this is not the case: having M_* well below M_P is technically natural. From the point of view of the low-energy theory, the reason is that the cutoff is set by M_*, and not M_P. Thus, neither M_P nor M_* receive large corrections.

theory is no longer valid at higher energies. As we shall now discuss, this conclusion would be incorrect. This is essentially due to the peculiar kinematics of theories with anisotropic scaling, summarized by the dispersion relation (13), which makes the unitarity bound much milder at high energies as compared to the relativistic case.

To be concrete, we consider the s-channel scattering of two φ quanta with energy E_0 in the center of mass frame (which we assume to coincide with the preferred frame). The optical theorem yields for the s-wave amplitude[10] $\mathcal{M}(2 \to 2)$,

$$2\,\mathrm{Im}\,\mathcal{M}(2 \to 2) = \sum_n \left(\prod_{i=1}^n \int \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2\mathcal{E}(p_i)} \right) \left|\mathcal{M}(2 \to n)\right|^2 (2\pi)^4 \times \delta\Big(2E_0 - \sum \mathcal{E}(p_i)\Big) \delta^{(3)}\Big(\sum \mathbf{p}_i\Big), \tag{24}$$

where the sum runs over all possible final states. Note that this relation holds for arbitrary dispersion relation

$$E = \mathcal{E}(p). \tag{25}$$

Being a sum of positive numbers, the r.h.s. of (24) is larger than any of the summands. In particular,

$$2\,\mathrm{Im}\,\mathcal{M}(2 \to 2) \geqslant \int \frac{d^3 p}{(2\pi)^2} \frac{1}{4\mathcal{E}^2(p)} \left|\mathcal{M}(2 \to 2)\right|^2 \times \delta\big(2E_0 - 2\mathcal{E}(p)\big). \tag{26}$$

Performing the integrations and using $\mathrm{Im}\,\mathcal{M} \leqslant |\mathcal{M}|$ one obtains the bound on the absolute value of the amplitude,

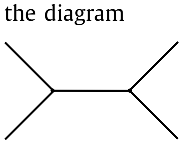
$$\left|\mathcal{M}(2 \to 2)\right| \leqslant 16\pi \mathcal{E}'(p_0) \frac{E_0^2}{p_0^2}, \tag{27}$$

where p_0 is related to the energy E_0 of the incoming particles by the dispersion relation (25). This bound simplifies when the dispersion relation is given by a power-law, $\mathcal{E}(p) = p^z / M_*^{z-1}$,

$$|\mathcal{M}| \leqslant 16\pi z [E_0 / M_*]^{3(z-1)/z}. \tag{28}$$

For relativistic particles, $z = 1$, this takes the familiar form $|\mathcal{M}| \leqslant 16\pi$. However, for the case of anisotropic scaling with $z > 1$ the bound (28) is less restrictive and allows the power-law growth of the amplitude with energy.

Let us check that in the model (20) the bound (27) is satisfied. The dispersion relation $\mathcal{E}(p) = p\sqrt{1 + (p/M_*)^4}$ interpolates between $z = 1$ and $z = 3$ at low and high energies respectively. The leading contribution to the tree-level amplitude comes from the diagram



which is estimated as

$$\mathcal{M}(2 \to 2) \sim \frac{E_0^2}{\alpha M_P^2}, \tag{29}$$

where each vertex contributes a factor $E_0^2 / \sqrt{\alpha} M_P$ and the propagator $1/E_0^2$. At low energies, $p \ll M_*$, the bound (27) reduces to the condition

$$\frac{p^2}{\alpha M_P^2} \lesssim 1.$$

Naïvely, this would imply the breakdown of tree-level unitarity at $\sqrt{\alpha} M_P$. However, if $\sqrt{\alpha} M_P \gtrsim M_*$, the low-energy approximation fails and the bound reads instead (for $p \gg M_*$)

$$M_*^2 \lesssim \alpha M_P^2, \tag{30}$$

which is indeed satisfied. Thus, we recover the same result that was derived from the scaling analysis (Eq. (19)): there are no signals that perturbation theory is breaking down at the scale $\sqrt{\alpha} M_P$.

A few remarks are in order. The above arguments do not exclude the possibility that some marginal coupling of the theory develop a Landau pole when loop corrections are taken into account. If this turns out to be the case, the theory will become strongly coupled in the deep UV (thus spoiling the UV-completeness of the proposal). Even in this case, though, this would happen at an exponentially high energy. For example, in the case of the toy model (20) one can estimate this scale as

$$\Lambda_{Landau} \sim M_* \exp\left[\frac{1}{\beta (a_n(M_*))^\gamma} \right],$$

where β and γ are numerical coefficients of order 1. Certainly, the presence or absence of Landau poles in the extended Hořava model (3) is an important open issue requiring a detailed renormalization group analysis of the theory.

Another basic issue concerning the consistency of the theory (3) at the quantum level is to demonstrate the absence of anomalies in the symmetry (2). We hope to return to these issues in the future.

3. Observational bounds on the UV scale

The weak coupling condition (19) does not allow to take the parameters α, $|\lambda - 1|$ to zero. As already emphasized in [3], this implies that the model does not possess a GR limit in the IR, since inevitably a gapless scalar polarization persists down to the lowest energies. On the other hand, from the remarkable success of GR in the description of low-energy gravitational physics, one expects the phenomenological constraints to put upper bounds on the parameters α and $|\lambda - 1|$, and hence on M_*. Thus the real physical question is whether it is possible to comply with observations without lowering the scale Λ to an unacceptable level.

From the fact that the low-energy form of the action (12) corresponds to a special case of the Einstein-aether theory [9] one expects that the phenomenology of the two models may be similar. This expectation is supported by the results [3] for the weak gravitational field of static sources (at rest in the preferred frame) and for the expansion of the Universe. Similarly to the case of Einstein-aether theory, static sources in the model (3) give rise to a linear metric which has the same form as in GR with the Newton's constant[11]

$$G_N = \left(8\pi M_P^2 (1 - \alpha/2)\right)^{-1}. \tag{31}$$

Importantly, this implies that the PPN parameter γ^{PPN} has its GR value, $\gamma^{PPN} = 1$. The cosmological expansion in the model (3) is governed by the standard Friedmann equation with the effective gravitational constant $G_{cosm} \neq G_N$, which again coincides with the situation in the Einstein-aether theory. The phenomenological constraint $|G_{cosm}/G_N - 1| \lesssim 0.13$ [11] sets a mild bound [3]: $\alpha, |\lambda - 1| \lesssim 0.1$.

[10] We stick to the 'relativistic' normalization of the 1-particle states $|\mathbf{p}\rangle \equiv \sqrt{2\mathcal{E}(p)} a_{\mathbf{p}}^\dagger |0\rangle$. This choice leads to conventional expressions for the amplitudes at low energies where the relativistic dispersion relation is recovered.

[11] The expression (31) is obtained under the assumption that matter couples universally to the metric $g_{\mu\nu}$. A more general situation compatible with low-energy Lorentz invariance in the matter sector is coupling it to the universal effective metric $g_{\mu\nu}^{eff} = g_{\mu\nu} + \beta u_\mu u_\nu$, where β is a dimensionless parameter. This modification preserves the GR form of the weak gravitational field but changes the expression for the Newton's constant.

Following the guide of the Einstein-aether, one expects the most stringent constraints on the model (3) to come from the observational bound on the PPN parameter α_2^{PPN} which characterizes the preferred frame effects due to Lorentz violation (see [12] for the precise definition).[12] The detailed study of the PPN corrections in the model (3) will be reported elsewhere[13] [14]; here we only sketch the estimate for α_2^{PPN}. This parameterizes an angular dependent contribution to the Newtonian potential produced by a source moving with velocity v with respect to the preferred frame,

$$\Phi_N = -\frac{G_N m}{r}\left(1 + \frac{\alpha_2^{PPN}}{2} v^2 \sin^2\theta\right),$$

where m is the mass of the source, and θ is the angle between the radial vector and the velocity of the source with respect to the preferred frame, $\cos\theta = \hat{\mathbf{r}}\cdot\hat{\mathbf{v}}$. From the physical point of view, this contribution is due to the interaction via the Lorentz-violating scalar mode associated with the vector u_μ in the action (12) (cf. discussion in [15]). The result (31) for static gravitational field shows that for $\alpha \ll 1$ the scalar-exchange amplitude is suppressed by α. Thus we conclude that[14]

$$\alpha_2^{PPN} \sim \alpha. \tag{32}$$

The bound on α_2^{PPN} following from the observed alignment of the rotation axis of the Sun with the ecliptic [12] gives

$$\alpha,\ |\lambda - 1| \lesssim 4 \times 10^{-7},$$

where we again assume α and $|\lambda - 1|$ to be comparable. This translates into the bound on the suppression scale for higher-derivative operators

$$M_* \lesssim 10^{15}\ \text{GeV}. \tag{33}$$

To our knowledge, this is the strongest upper bound arising from gravitational physics.

The lower bound on M_* from purely gravitational physics is very mild. Direct tests of Newton's law at the distances ~ 10 μm imply [12]

$$M_* \gtrsim 0.1\ \text{eV}. \tag{34}$$

A stronger bound may be obtained under the additional assumption that M_* also sets the suppression scale for terms with higher powers of momentum p in the dispersion relations of the matter fields, specifically, of photons. Timing of active galactic nuclei [16] and gamma ray bursts [17] constrains the value of such terms. Note that odd powers of p can be forbidden (at least, in electrodynamics) by imposing parity. Then, the leading contribution to the dispersion relation has the form p^4/M_*^2 which yields [16,17]

$$M_* \gtrsim 10^{10}\text{–}10^{11}\ \text{GeV}. \tag{35}$$

Let us stress that, unlike the upper bound (18), this lower bound is model dependent: it relies on the assumption that the UV modification to the dispersion relation for photons appears at the same scale as that for scalar graviton. This need not hold in some formulations of the theory.

It is worth emphasizing the difference between the situation in the model (3) and that in Horava's original proposal, both in its projectable and non-projectable versions [2]. In both versions, the strong coupling scale calculated within the low-energy effective theory is so low that the introduction of any new physics at that scale is phenomenologically unacceptable. Indeed, as discussed in [6], in the non-projectable case the strong coupling scale for the additional mode is inversely proportional to the curvature radius of the background. It goes to zero for flat, cosmological and static backgrounds, invalidating the proposal.

In the projectable case it was shown [3,7] that the scalar graviton mode is unstable at large wavelengths. The requirement that the rate of the instability is smaller than the age of the Universe (in order not to spoil standard cosmology) gives the bound $|\lambda - 1|^{1/2} \lesssim H_0/M_*$, where H_0 is the present value of the Hubble parameter and M_* is the suppression scale of the higher-derivative operators [3,7]. On the other hand, M_* must be smaller than the strong coupling scale of the low-energy theory, which in this case is [7,14] $|\lambda - 1|^{3/4} M_P$. This gives the lower bound $M_* \lesssim (H_0^3 M_P^2)^{1/5} \simeq (100\ \text{m})^{-1}$. Comparing this with the experimental bound (34), one concludes that the projectable case in the weakly coupled regime is ruled out.

To sum up, we have shown that the claim [8] about the presence of strong coupling problem in the model (3) is unfounded. The absence of strong coupling is actually a built-in feature of the model. It suffices for the scale M_* suppressing the higher derivative operators to be slightly lower than the naive strong coupling scale calculated in the low-energy theory. The observational constraints on deviations of gravity from GR place an upper bound $M_* \lesssim 10^{15}$ GeV. Under the additional (model dependent) assumption that this scale is common for gravity and matter sectors one obtains a lower bound $M_* \gtrsim 10^{10}$–10^{11} GeV. Within this range, to the best of our knowledge, the model is compatible with the existing data. Thus, so far, the model is a phenomenologically viable candidate for a renormalizable quantum theory of gravity. Needless to say, whether the theory is truly renormalizable (anomaly free) and UV complete remains an important open issue. Another major question is the mechanism for the recovery of Lorentz invariance in the matter sector at low energies (see [18,19] for the detailed discussion of the problem).

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[13] The details of the PPN calculations in the model (3) are different from the Einstein-aether case due to the absence of the transverse vector mode.

[14] An explicit computation [14] yields $\alpha_2^{PPN} = \alpha(\alpha - \lambda + 1)/2(\lambda - 1) = \alpha(c_s^{-2} - 1)/2$, where c_s is defined in (14). This coincides with (32) when c_s is of order one.

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