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# Branching Fraction and *CP* Asymmetry Measurements in Inclusive $B \rightarrow X_s \ell^+ \ell^$ and $B \rightarrow X_s \gamma$ Decays from *BABAR*

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## Abstract

We present an update on total and partial branching fractions and on *CP* asymmetries in the semi-inclusive decay  $B \rightarrow X_s \ell^+ \ell^-$ . Further, we summarize our results on branching fractions and *CP* asymmetries for semi-inclusive and fully-inclusive  $B \rightarrow X_s \gamma$  decays. We present the first result on the *CP* asymmetry difference of charged and neutral  $B \rightarrow X_s \gamma$  decays yielding the first constraint on the ratio of Wilson coefficients  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$ .

## 1. Introduction

The decays  $B \to X_{s,d}\gamma$  and  $B \to X_{s,d}\ell^+\ell^-$  are flavor-changing neutral current (FCNC) processes that are forbidden in the Standard Model (SM) at tree level. However, they can proceed via penguin loops and box diagrams. Figure 1 shows the lowest-order diagrams for both processes. The effective Hamiltonian factorizes short-distance effects represented by perturbatively-calculable Wilson coefficients ( $C_i$ ) [1, 2] from long-distance effects specified by four-quark operators ( $O_i$ ):

$$H_{\rm eff} = \frac{G_F}{4\pi} \Sigma_i V_{\rm xb}^* V_{\rm xs,d} C_i(\mu) O_i. \tag{1}$$

Here,  $G_F$  is the Fermi constant,  $V_{xb}^*$  and  $V_{xs,d}$  are CKM elements (x = u, c, t) and  $\mu$  is the renormalization scale. The operators have to be calculated using nonperturbative methods, such as the heavy quark expansion [3, 4, 5, 6]. In  $B \rightarrow X_s \gamma$ , the dominant contribution arises from the magnetic dipole operator  $O_7$  with a top quark in the loop. Thus, the branching fraction depends on the Wilson coefficient  $C_7^{\text{eff}} = -0.304$  (NNLL) [7, 8]. Via operator mixing, the color-magnetic dipole operator  $O_8$  contributes in higher order with  $C_8^{\text{eff}} = -0.167$ (NNLL) [7, 8]. In  $B \rightarrow X_s \ell^+ \ell^-$ , the weak penguin





Figure 1: Lowest-order diagrams for  $B \to X_{s,d}\gamma$  (top) and  $B \to X_{s,d}\ell^+\ell^-$  (bottom).

and box diagrams contribute in addition. The vector part is represented by operator  $O_9$  with Wilson coefficient  $C_9^{\text{eff}} = 4.211$  (NNLL) [7, 8] while the axial-vector part is specified by operator  $O_{10}$  with Wilson coefficient  $C_{10}^{\text{eff}} = -4.103$  (NNLL) [7, 8]. Again, the top quark in the loop yields the most dominant contribution. New physics adds penguin and box diagrams with new particles modifying the SM values of the Wilson coefficients. In addition, scalar and pseudoscalar couplings may contribute introducing new Wilson coefficients  $C_S$  and  $C_P$ . Figure 2 shows examples of new physics processes involving a charged Higgs, a chargino and neutralinos [9, 10, 11, 12, 13, 14, 15]. These rare decays probe new physics at a scale of a few TeV.



Figure 2: New physics processes with a charged Higgs bosons (left), a chargino plus up-type squarks (middle) and neutralinos plus down-type squarks (right).

## 2. Study of $B \to X_s \ell^+ \ell^-$

Using a semi-inclusive approach, we have updated the partial and total branching fraction measurements of  $B \to X_s \ell^+ \ell^-$  modes with the full BABAR data sample of  $471 \times 10^6 B\bar{B}$  events. We also perform the first measurement of direct CP asymmetry. For measuring partial and total branching fractions, we reconstruct 20 exclusive final states listed in Table 1. After accounting for  $K_L^0$  modes,  $K_S^0 \rightarrow \pi^0 \pi^0$  and  $\pi^0$  Dalitz decays, they represent 70% of the inclusive rate for hadronic masses  $m_{X_s}$  < 1.8 GeV. Using JETSET fragmentation and theory predictions, we extrapolate for the missing modes and those with  $m_{X_s} > 1.8$  GeV. We impose requirements on the beam-energy-substituted mass  $m_{ES} = \sqrt{E_{CM}^2 - p_B^{*2}} > 5.225$  GeV and on the energy difference  $-0.1 \ (0.05) < \Delta E = E_B^* - E_{CM}/2 < 0.05$  for  $X_s e^+e^- (X_s \mu \mu)$  modes where  $E_B^*$  and  $p_B^*$  are B momentum and B energy in the center-of-mass (CM) frame and  $E_{CM}$  is the total CM energy. We use no tagging of the  $\overline{B}$ decay.

To suppress  $e^+e^- \rightarrow q\bar{q}$  (q = u, d, s, c) events and  $B\bar{B}$  combinatorial background, we define boosted decision trees (BDT) for each  $q^2$  bin in  $e^+e^-$  and  $\mu^+\mu^-$  separately (see Table 2). From these BDTs, we determine a likelihood ratio  $(L_R)$  to separate signal from  $q\bar{q}$  and  $B\bar{B}$  backgrounds. We veto  $J/\psi$  and  $\psi(2S)$  mass regions and use them as control samples. Figures 3 and 4 show the  $m_{ES}$  and  $L_R$  distributions for  $e^+e^-$  modes in bin  $q_5$  and for  $\mu^+\mu^-$  modes in bin  $q_1$ , respectively.

We measure  $d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2$  in six bins of  $q^2 = m_{\ell\ell}^2$  and four bins of  $m_{X_s}$  defined in Table 2. We extract the signal in each bin from a two-dimensional fit to  $m_{ES}$  and  $L_R$ . Figure 5 shows the differential branching faction as a function of  $q^2$  (top) and  $m_{X_s}$  (bottom) [16].

Table 1: Exclusive modes used in the semi-inclusive  $B \to X_s \ell^+ \ell^-$  analysis.

Mode	Mode
$B^0 \rightarrow K^0_S \mu^+ \mu^-$	$B^+  ightarrow K^+ \mu^+ \mu^-$
$B^0 \rightarrow K^0_S e^+ e^-$	$B^+ \rightarrow K^+ e^+ e^-$
$B^0 \to K^{*0}(K^0_S \pi^0) \mu^+ \mu^-$	$B^+ \to K^{*+}(K^+\pi^0)\mu^+\mu^-$
$B^0 \to K^{*0}(K^+\pi^-)\mu^+\mu^-$	$B^+ \to K^{*+}(K^0_S \pi^+) \mu^+ \mu^-$
$B^0 \to K^{*0}(K^0_S \pi^0) e^+ e^-$	$B^+ \to K^{*+}(K^+\pi^0)e^+e^-$
$B^0 \to K^{*0}(K^+\pi^-)e^+e^-$	$B^+ \to K^{*+}(K^0_S \pi^+) e^+ e^-$
$B^0 \to K^0_S \pi^+ \pi^-) \mu^+ \mu^-$	$B^+ \rightarrow K^0_S \pi^+ \pi^0 \mu^+ \mu^-$
$B^0 \to K^+ \pi^- \pi^0 \mu^+ \mu^-$	$B^+ \rightarrow K^+ \pi^+ \pi^- \mu^+ \mu^-$
$B^0 \to K^0_S \pi^+ \pi^-) e^+ e^-$	$B^+ \to K^0_S \pi^+ \pi^0 e^+ e^-$
$B^0 \to K^+ \pi^- \pi^0 e^+ e^-$	$B^+ \rightarrow K^+ \pi^+ \pi^- e^+ e^-$

Table 2: Definition of the  $q^2$  bins.

$q^2$ bin	$q^2$ range [GeV <sup>2</sup> /c <sup>4</sup> ]	$m_{\ell\ell}$ range [GeV/c <sup>2</sup> ]
0	$1.0 < q^2 < 6.0$	$1.00 < m_{\ell\ell} < 2.45$
1	$0.1 < q^2 < 2.0$	$0.32 < m_{\ell\ell} < 1.41$
2	$2.0 < q^2 < 4.3$	$1.41 < m_{\ell\ell} < 2.07$
3	$4.3 < q^2 < 8.1$	$2.07 < m_{\ell\ell} <\!\!2.6$
4	$10.1 < q^2 < 12.9$	$3.18 < m_{\ell\ell} < 3.59$
5	$14.2 < q^2 < (m_B - m_K^*)^2$	$3.77 < m_{\ell\ell} < (m_B - m_K^*)$



Figure 3: Distributions of  $m_{ES}$  (left) and likelihood ratio (right) for  $B \rightarrow X_s e^+ e^-$  in  $q^2$  bin  $q_5$  showing data (points with error bars), the total fit (thick solid blue curves), signal component (red peaking curves), signal cross feed (cyan/lgrey curves),  $B\bar{B}$  background (magenta/dark grey smooth curve),  $e^+e^- \rightarrow q\bar{q}$  background (green/grey curves) and charmonium background (yellow/light grey curves).



Figure 4: Distributions of  $m_{ES}$  (left) and likelihood ratio (right) for  $B \rightarrow X_s \mu^+ \mu^-$  in  $q^2$  bin  $q_1$  showing data (points with error bars), the total fit (thick solid blue curves), signal component (red peaking curves), signal cross feed (cyan/lgrey curves),  $B\bar{B}$  background (magenta/dark grey smooth curve),  $e^+e^- \rightarrow q\bar{q}$  background (green/grey curves) and charmonium background (yellow/light grey curves)

Table 3 summarizes the differential branching fractions in the low and high  $q^2$  regions in comparison to the SM predictions [17, 18, 19, 18, 20, 21, 22, 23, 24, 25, 26, 27]. In both regions of  $q^2$ , the differential branching fraction is in good agreement with the SM prediction. These results supersede the previous *BABAR* measurements [28] and are in good agreement with the Belle results [29].



Figure 5: Differential branching fraction of  $B \to X_s e^+ e^-$  (blue points),  $B \to X_s \mu^+ \mu^-$  (black squares), and  $B \to X_s \ell^+ \ell^-$  (red triangles) versus  $q^2$  (top) and versus  $m_{X_s}$  (bottom) in comparison to the SM prediction (histogram). The grey-shaded bands show the  $J/\psi$  and  $\psi(2S)$  vetoed regions.

Table 3: The  $B \to X_s \ell^+ \ell^-$  branching fraction measurements in the low and high  $q^2$  regions [16] in comparison to the SM prediction.

Mode	$BABAR[10^{-6}]$	SM [10 <sup>-6</sup> ]
2		Shilio 1
$q^2[\text{GeV}^2/\text{c}^4]$	1 – 6	1-6
$B \to X_s \mu^+ \mu^-$	$0.66^{+0.82+0.30}_{-0.76-0.24}\pm0.07$	$1.59\pm0.11$
$B \rightarrow X_s e^+ e^-$	$1.93^{+0.47+0.21}_{-0.45-0.16}\pm0.18$	$1.64\pm0.11$
$B \to X_s \ell^+ \ell^-$	$1.60^{+0.41+0.17}_{-0.39-0.13}\pm0.07$	
$q^2$ [GeV <sup>2</sup> /c <sup>4</sup> ]	> 14.2	> 14.2
$B \to X_s \mu^+ \mu^-$	$0.60^{+0.31+0.05}_{-0.29-0.04}\pm0.00$	$0.25\substack{+0.07 \\ -0.06}$
$B \rightarrow X_s e^+ e^-$	$0.56^{+0.19+0.03}_{-0.18-0.03}\pm0.00$	
$B \to X_s \ell^+ \ell^-$	$0.57^{+0.16+0.03}_{-0.15-0.02}\pm0.00$	

The direct CP asymmetry is defined by:

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(\bar{B} \to \bar{X}_s \ell^+ \ell^-) - \mathcal{B}(B \to X_s \ell^+ \ell^-)}{\mathcal{B}(\bar{B} \to \bar{X}_s \ell^+ \ell^-) + \mathcal{B}(B \to X_s \ell^+ \ell^-)}.$$
 (2)

We use 14 self-tagging modes consisting of all  $B^+$ modes and the  $B^0$  modes with decays to a  $K^+$  listed in Table 1 to measure  $\mathcal{R}_{CP}(B \to X_s \ell^+ \ell^-)$  in five  $q^2$  bins. Note that we have combined bins  $q_4$  and  $q_5$  due to low statistics. Figure 6 shows the *CP* asymmetry as a function of  $q^2$ . The SM prediction of the *CP* asymmetry in the entire  $q^2$  region is close to zero [30, 31, 32, 8]. In new physics models, however,  $\mathcal{R}_{CP}$  may be significantly enhanced [11, 33]. In the full range of  $q^2$  we measure  $\mathcal{R}_{CP} = 0.04 \pm 0.11 \pm 0.01$  [16], which is in good agreement with the SM prediction. The *CP* asymmetries in the five  $q^2$  bins are also consistent with zero.



Figure 6: The *CP* asymmetry as a function of  $q^2$ . The grey-shaded bands show the  $J/\psi$  and  $\psi(2S)$  vetoed regions.

## 3. Study of $B \rightarrow X_s \gamma$

In the SM, the  $B \rightarrow X_s \gamma$  branching fraction is calculated in next-to-next leading order (4 loops) yielding

$$\mathcal{B}(B \to X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$$
 (3)

for photon energies  $E_{\gamma} > 1.6 \text{ GeV} [34, 35].$ 

To extract the  $B \to X_s \gamma$  signal experimentally from  $e^+e^- \to B\bar{B}$  and  $e^+e^- \to q\bar{q}$  backgrounds, we use two very different strategies. The first strategy consists of a semi-inclusive approach in which we sum over 38 exclusive  $B \to X_s \gamma$  final states with  $1K^{\pm} (\leq 1K_s^0)$  or  $3K^{\pm}$ ,  $\leq 4\pi (\leq 2\pi^0)$ , and  $\leq 1\eta$ . We use no tagging of the other *B* meson. We need to model the missing modes. Due to large backgrounds, we select events with a minimum photon energy of  $E_{\gamma} > 1.9$  GeV and then extrapolate the branching fraction to photon energies  $E_{\gamma} > 1.6$  GeV. With this approach, we measure the branching fraction, *CP* asymmetry and the difference in *CP* asymmetries between charged and neutral *B* decays using  $471 \times 10^6 B\bar{B}$  events [36].

The second strategy is a fully inclusive approach. To suppress backgrounds from  $B\bar{B}$  and  $q\bar{q}$  decays, we impose stringent constraints on isolated photons to remove clusters that may have originated from  $\pi^0$  and  $\eta$  decays. We use a semileptonic tag of the other *B* meson and require a minimum photon energy of  $E_{\gamma} > 1.8$  GeV but impose no requirements on the hadronic mass system. Using  $383 \times 10^6 B\bar{B}$  events, we measure the  $B \to X_s \gamma$ branching fraction measurement and the *CP* asymmetry for  $B \to X_{s+d}\gamma$  [37, 38].

Table 4 summarizes our  $B \rightarrow X_s \gamma$  branching fraction measurements of the semi-inclusive and fully inclusive methods [36, 37, 38]. Figure 7 shows the BABAR results extrapolated to a minimum photon energy of 1.6 GeV in comparison to results from Belle [40, 41, 42], CLEO [43] and the SM prediction [34, 35]. Our results are in good agreement with those of the other experiments as well as the SM prediction.

For the semi-inclusive method, the direct *CP* asymmetry is defined by:

$$\mathcal{A}_{CP}(X_s\gamma) = \frac{\mathcal{B}(B \to X_s\gamma) - \mathcal{B}(B \to X_s\gamma)}{\mathcal{B}(\bar{B} \to \bar{X}_s\gamma) + \mathcal{B}(B \to X_s\gamma)}.$$
 (4)

The SM prediction yields  $-0.6\% < \mathcal{A}_{CP}(B \to X_s \gamma) < 2.8\%$  [45, 46]. Using 16 self-tagging exclusive modes and 471 × 10<sup>6</sup>  $B\bar{B}$  events, we measure  $\mathcal{A}_{CP}(B \to X_s \gamma) = (1.7 \pm 1.9_{stat} \pm 1.0_{sys})\%$  [47]. This supersedes the old BABAR measurement [48].We further measures the *CP* asymmetry difference between charged and neutral *B* decays:

$$\Delta \mathcal{A}_{CP} = \mathcal{A}_{CP}(B^+ \to X_s^+ \gamma) - \mathcal{A}_{CP}(B^0 \to X_s^0 \gamma), \quad (5)$$

Table 4: Our measurements of  $\mathcal{B}(B \to X_s \gamma)$  from the semiinclusive [36] and fully-inclusive [37] analyses and their extrapolations to  $E_{\gamma} > 1.6$  GeV. The first uncertainty is statistical, the second is systematic and the third is from model dependence and extrapolation to 1.6 GeV.

method	$E_{\gamma} >$	$\mathcal{B}(B\to X_s\gamma)[10^{-4}]$
semi-	1.9 GeV	$3.29 \pm 0.19 \pm 0.48$
exclusive	1.6 GeV	$3.52 \pm 0.20 \pm 0.51 \pm 0.04$
inclusive	1.8 GeV	$3.21 \pm 0.15 \pm 0.29 \pm 0.08$
	1.6 GeV	$3.31 \pm 0.16 \pm 0.30 \pm 0.10$



Figure 7: Summary of  $\mathcal{B}(B \to X_s \gamma)$  measurements from BABAR [36, 37, 38, 39], Belle [40, 41, 42], CLEO [43] and the HFAG average [44] in comparison to the SM prediction [34, 35] after extrapolation to  $E_{\gamma}^* > 1.6$  GeV.

which depends on the Wilson coefficients  $C_7^{\text{eff}}$  and  $C_8^{\text{eff}}$ :

$$\Delta \mathcal{A}_{CP} = 4\pi^2 \alpha_s \frac{\bar{\Lambda}_{78}}{m_b} Im \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}} \simeq 0.12 \frac{\bar{\Lambda}_{78}}{100 \text{ MeV}} Im \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}} \quad (6)$$

where the scale parameter  $\bar{\Lambda}_{78}$  is constrained by 17 MeV <  $\bar{\Lambda}_{78}$  < 190 MeV. In the SM,  $C_7^{\text{eff}}$  and  $C_8^{\text{eff}}$  are real so that  $\Delta \mathcal{A}_{CP}$  vanishes. However in new physics models, these Wilson coefficients may have imaginary parts yielding a non-vanishing  $\Delta \mathcal{A}_{CP}$ .

From a simultaneous fit to charged and neutral *B* decays, we measure  $\Delta \mathcal{R}_{CP}(B \rightarrow X_s \gamma) = (5.0 \pm 3.9_{stat} \pm 1.5_{sys})\%$  from which we set an upper and lower limit at 90% *CL* on  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$  [47]:

$$-1.64 < Im \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}} < 6.52 \text{ at } 90\% \text{ CL.}$$
(7)

This is the first  $\Delta \mathcal{A}_{CP}$  measurements and the first constraint on  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$ . Figure 8 (top) shows the  $\Delta \chi^2$  of the fit as a function of  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$ . The shape of  $\Delta\chi^2$ as a function of  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$  is not parabolic indicating that the likelihood has a non-Gaussian shape. The reason is that  $\Delta\chi^2$  is determined from all possible values of  $\bar{\Lambda}_{78}$ . In the region ~ 0.2 <  $Im(C_8^{\text{eff}}/C_7^{\text{eff}}) < \sim$ 2.6 a change in  $Im(C_8^{\text{eff}}/C_7^{\text{eff}}) \Delta\chi^2$  can be compensated by a change in  $\bar{\Lambda}_{78}$  leaving  $\Delta\chi^2$  unchanged. For positive values larger (smaller) than 2.6 (0.2),  $\Delta\chi^2$  increases slowly (rapidly), since  $\bar{\Lambda}_{78}$  remains nearly constant at the minimum value (increases rapidly). For negative  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$  values,  $\bar{\Lambda}_{78}$  starts to decrease again, which leads to a change in the  $\Delta\chi^2$  shape. Figure 8 (bottom) shows  $\bar{\Lambda}_{78}$  as a function of  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$ .



Figure 8: The  $\Delta \chi^2$  function versus  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$  (top) and the dependence of  $\bar{\Lambda}_{78}$  on  $Im(C_8^{\text{eff}}/C_7^{\text{eff}})$  (bottom). The blue dark-shaded (orange light-shaded) regions show the 68% (90%) CL intervals.

In the fully-inclusive analysis, the  $B \rightarrow X_d$  decay cannot be separated from the  $B \rightarrow X_s$  decay and we measure:

$$\mathcal{A}_{CP}(X_{s+d}\gamma) = \frac{\mathcal{B}(\bar{B} \to \bar{X}_{s+d}\gamma) - \mathcal{B}(B \to X_{s+d}\gamma)}{\mathcal{B}(\bar{B} \to \bar{X}_{s+d}\gamma) + \mathcal{B}(B \to X_{s+d}\gamma)}.$$
 (8)

In the SM,  $\mathcal{A}_{CP}(B \to X_{s+d}\gamma)$  is zero [49]. From the charge of the *B* and  $\overline{B}$ , we determine the *CP* asymmetry. Using  $383 \times 10^6 B\overline{B}$  events, we measure  $\mathcal{A}_{CP}(B \to X_{s+d}\gamma) = (5.7 \pm 6.0 \pm 1.8)\%$ , which is consistent with the SM prediction [49]. Figure 9 shows a summary of all *CP* asymmetry measurements in comparison to the SM predictions.



Figure 9: Summary of  $\mathcal{A}_{CP}$  measurements for  $B \to X_s \gamma$  from semiinclusive analyses (*BABAR* [47], Belle [50]) and for  $B \to X_{s+d} \gamma$ from fully inclusive analyses (*BABAR* [37, 38, 39], CLEO [51]), Belle [52] and the HFAG average [44] in comparison to the SM prediction for  $B \to X_s \gamma$  [45, 46, 49].

### 4. Conclusion

We performed the first  $\mathcal{A}_{CP}$  measurement in five  $q^2$ bins in semi-inclusive  $B \rightarrow X_s \ell^+ \ell^-$  decays and updated the differential branching fraction. The  $B \rightarrow$  $X_s \ell^+ \ell^-$  partial branching fractions and *CP* asymmetries are in good agreement with the SM predictions. Our  $\mathcal{A}_{CP}$  measurement in the semi-inclusive  $B \to X_s \gamma$  decay is the most precise CP asymmetry measurement. The  $\Delta \mathcal{A}_{CP}(B \rightarrow X_s \gamma)$  result yields first constraint on  $Im(C_8^e/C_7^e)$ . The  $B \rightarrow X_s \gamma$  branching fractions and CP asymmetries are both in good agreement with the SM predictions. New progress on these inclusive decays will come from Belle II. For the  $B \rightarrow X_s \gamma$  and  $B \rightarrow X_s \ell^+ \ell^-$  semi-inclusive decays, we expect precision measurements. For the inclusive  $B \rightarrow X_s \gamma$  and  $B \to X_s \ell^+ \ell^-$  decays, we expect new possibilities by tagging the other  $\overline{B}$  meson via full B reconstruction.

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