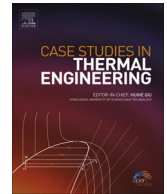




Contents lists available at ScienceDirect

Case Studies in Thermal Engineering

journal homepage: www.elsevier.com/locate/csite

Thermal analysis of convective fin with temperature-dependent thermal conductivity and heat generation

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ARTICLE INFO

Article history:

Received 5 January 2014

Received in revised form

3 May 2014

Accepted 14 May 2014

Available online 22 May 2014

Keywords:

Convective fin

Temperature-dependent thermal conductivity

Heat generation

Differential Transformation Method (DTM)

Thermal Analysis

ABSTRACT

In this study, a simple and highly accurate semi-analytical method called the Differential Transformation Method (DTM) is used for solving the nonlinear temperature distribution equation in a longitudinal fin with temperature dependent internal heat generation and thermal conductivity. The problem is solved for two main cases. In the first case, heat generation is assumed variable by fin temperature and in the second case, both thermal conductivity and heat generation vary with temperature. Results are presented for the temperature distribution for a range of values of parameters appeared in the mathematical formulation (e.g. N , ϵ_G , and G). Results reveal that DTM is very effective and convenient. Also, it is found that this method can achieve more suitable results compared to numerical methods.

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1. Introduction

Fins are the most effective instrument for increasing the rate of heat transfer. As we know, they increase the area of heat transfer and cause an increase in the transferred heat amount. A complete review on this topic is presented by Krause et al. [1]. Fins are widely used in many industrial applications such as air conditioning, refrigeration, automobile, chemical processing equipment and electrical chips. Although there are various types of the fins, but the rectangular fin is widely used among them, probably, due to simplicity of its design and its easy manufacturing process. For ordinary fins problem, the thermal conductivity assumes to be constant, but when temperature difference between the tip and base of the fin is large, the effect of the temperature on thermal conductivity must be considered. Also, it is very realistic that to consider the heat generation in the fin (due to electric current or etc.) as a function of temperature.

Aziz and Bouaziz [2] used the least squares method for predicting the performance of a longitudinal fin with temperature-dependent internal heat generation and thermal conductivity and they compared their results by Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM) and double series regular perturbation method and found that the least squares method is simpler than other applied methods. Razani and Ahmadi [3] considered circular fins with an arbitrary heat source distribution and a nonlinear temperature-dependent thermal conductivity and obtained the results for the optimum fin design. Unal [4] conducted an analytical study of a rectangular and longitudinal fin with temperature-dependent internal heat generation and temperature-dependent heat transfer coefficient. Another study about this issue

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(convective fin with both temperature dependent thermal conductivity and internal heat generation) was performed by Shouman [5]. Kundu [6] had solved a problem about thermal analysis and optimization of longitudinal and pin fins of uniform thickness subject to fully wet, partially wet and fully dry surface conditions. Domairry and Fazeli [7] solved the nonlinear straight fin differential equation by the Homotopy Analysis Method (HAM) to evaluate the temperature distribution and fin efficiency. Also, temperature distribution for annular fins with temperature-dependent thermal conductivity was studied by Ganji et al. [8] using HPM. The effects of temperature-dependent thermal conductivity of a moving fin with considering the radiation losses have been studied by Aziz and Khani [9]. Furthermore, Bouaziz and Aziz introduced a double optimal linearization method (DOLM) to get a simple and accurate solution for the temperature distribution in a straight rectangular convective–radiative fin with temperature-dependent thermal conductivity [10].

Mustafa Inc [11] used HAM to obtain the efficiency of straight fin with temperature dependent thermal conductivity. The concept of Differential Transformation Method (DTM) was firstly introduced by Zhou [12] in 1986 which was used to solve both linear and nonlinear initial value problems in electric circuit analysis. This method can be applied directly for linear and nonlinear differential equations without requiring linearization, discretization or perturbation and this is the main benefit of this method. Ghafoori et al. [13] used the DTM for solving the nonlinear oscillation equation. Abdel-Halim Hassan [14] applied DTM for different systems of differential equations and he discussed the convergence of this method in several examples of linear and nonlinear systems of differential equations. Abazari and Abazari [15] applied the DTM and reduced differential transformation method (RDTM) for solving the generalized Hirota–Satsuma coupled KdV equation. They compared the results with the exact solution and they found that RDTM is more accurate than the classical DTM. Rashidi et al. [16] solved the problem of mixed convection about an inclined flat plate embedded in a porous medium by DTM; they applied the Pade approximation to increase the convergence of the solution. Abbasov et al. [17] employed DTM to obtain approximate solutions of the linear and nonlinear equations related to engineering problems and they showed that the numerical results are in good agreement with the analytical solutions. Balkaya et al. [18] applied the DTM to analyze the vibration of an elastic beam supported on elastic soil. Borhanifar et al. [19] employed the DTM on some PDEs and their coupled versions. Moradi and Ahmadikia [20] applied the DTM to solve the energy equation for a temperature-dependent thermal conductivity fin with three different profiles. Moradi [21] applied the DTM for thermal characteristics of straight rectangular fin for all types of heat transfer (convection and radiation) and compared its results by the numerical method with fourth order Runge–Kutta method using shooting method. Kundu et al. [22] applied the DTM for predicting fin performance of triangular and fully wet fins and they noticed that the fin performance of wet fins is almost independent on the relative humidity. Recently, Hatami and Ganji [23–26] and Hatami et al. [27] used analytical methods for solving the heat transfer through the porous fins with different geometries.

In the present letter, analytical solution of fin temperature distribution with temperature-dependent heat generation and thermal conductivity has been studied by the Differential Transformation Method. For this purpose, after a brief introduction for DTM and description of the problem, DTM is applied to find the approximate solution. Obtaining the analytical solution of the model and comparing with numerical results reveals the capability, effectiveness, simplicity and high accuracy of the presented method.

2. Differential transformation method principles

In this section the fundamental basic of the Differential Transformation Method is introduced. For understanding the method's concept, suppose that $x(t)$ is an analytic function in domain D , and $t=t_i$ represents any point in the domain. The function $x(t)$ is then represented by a one power series whose center is located at t_i . The Taylor series expansion function of $x(t)$ is in form of

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=t_i} \quad \forall t \in D \quad (1)$$

The Maclaurin series of $x(t)$ can be obtained by taking $t_i=0$ in Eq. (1) expressed as

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \quad \forall t \in D \quad (2)$$

As explained in [12], the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty} \frac{H^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \quad (3)$$

where $X(k)$ represents the transformed function and $x(t)$ is the original function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where H is a constant value. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H} \right)^k X(k) \quad (4)$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of the T -function $X(k)$, and its value is given by the sum of the T -function with $(t/H)k$ as its coefficient. In real applications, at the right choice of constant H , the larger values of argument k the discrete of spectrum reduces rapidly. The function $x(t)$ is expressed by a finite series and Eq. (4) can be written as

$$x(t) = \sum_{k=0}^n \left(\frac{t}{H}\right)^k X(k) \tag{5}$$

Some important mathematical operations performed by the Differential Transform Method are listed in Table 1.

3. Description of the problem

Consider a longitudinal fin with a constant rectangular profile, section area A , length L , perimeter P , thermal conductivity k , and heat generation q^* . Fin is attached to a surface with constant temperature T_b and losses heat to the surrounding medium with temperature T_∞ through a constant convective heat transfer coefficient h . In the problem we assume that the temperature variation in the transfer direction is negligible, so heat conduction occurs only in the longitudinal direction (x direction). A schematic of the geometry of described fin and other properties is shown in Fig. 1. For this problem, the governing differential equation and boundary condition can be written as [2]

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) + \frac{q^*}{k} = 0 \tag{6}$$

$$x = 0, \quad \frac{dT}{dx} = 0 \tag{7}$$

Table 1
Some fundamental operations of the differential transform method.

Origin function	Transformed function
$x(t) = \alpha f(x) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta G(k)$
$x(t) = \frac{d^m f(t)}{dt^m}$	$X(k) = \frac{(k+m)F(k+m)}{k!}$
$x(t) = f(t)g(t)$	$X(k) = \sum_{l=0}^k F(l)G(k-l)$
$x(t) = t^m$	$X(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k = m, \\ 0, & \text{if } k \neq m. \end{cases}$
$x(t) = \exp(t)$	$X(k) = \frac{1}{k!}$
$x(t) = \sin(\omega t + \alpha)$	$X(k) = \frac{\omega^k}{k!} \sin\left(\frac{k\pi}{2} + \alpha\right)$
$x(t) = \cos(\omega t + \alpha)$	$X(k) = \frac{\omega^k}{k!} \cos\left(\frac{k\pi}{2} + \alpha\right)$

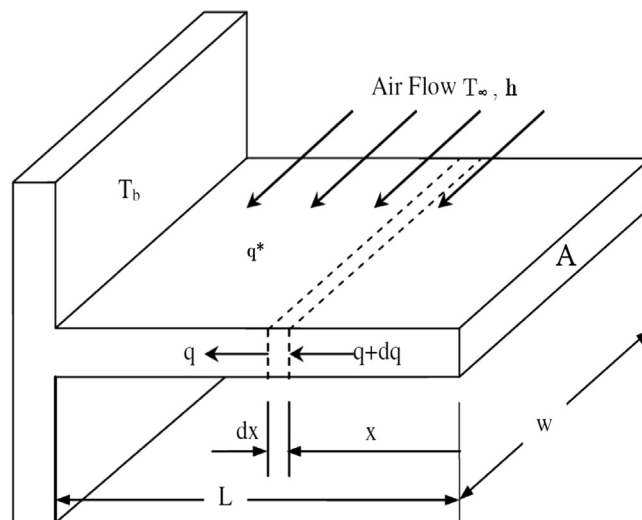


Fig. 1. Schematic of the fin geometry with the heat generation source.

$$x = L, \quad T = T_b \quad (8)$$

This problem is solved in two main cases using the DTM. In the following subsections, the governing equations for these two cases are introduced.

3.1. Fin with temperature dependent internal heat generation and constant thermal conductivity

In the first case, we assume that heat generation in the fin varies with temperature as Eq. (9) and the thermal conductivity is constant k_0

$$q^* = q_{\infty}^* (1 + \varepsilon(T - T_{\infty})) \quad (9)$$

where q_{∞}^* is the internal heat generation at temperature T_{∞} . With the introduction of the following dimensionless quantities:

$$\begin{aligned} \theta &= \frac{(T - T_{\infty})}{(T_b - T_{\infty})}, \quad X = \frac{x}{L}, \quad N^2 = \frac{hPL^2}{k_0A} \\ G &= \frac{q_{\infty}^* A}{hP(T_b - T_{\infty})}, \quad \varepsilon_G = \varepsilon(T_b - T_{\infty}) \end{aligned} \quad (10)$$

Eqs. (6)–(8) can be rewritten as

$$\frac{d^2\theta}{dX^2} - N^2\theta + N^2G(1 + \varepsilon_G\theta) = 0 \quad (11)$$

$$X = 0, \quad \frac{d\theta}{dX} = 0 \quad (12)$$

$$X = 1, \quad \theta = 1 \quad (13)$$

Now we apply DTM from Table 1 into Eq. (11) to find $\theta(x)$.

$$(k+1)(k+2)\theta(k+2) - N^2\theta(k) + N^2G(\delta(k) + \varepsilon_G\theta(k)) = 0 \quad (14)$$

Rearranging Eq. (14), a simple recurrence relation is obtained as follows:

$$\theta(k+2) = \frac{N^2\theta(k) - N^2G(\delta(k) + \varepsilon_G\theta(k))}{(k+1)(k+2)} \quad (15)$$

where

$$\delta(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases} \quad (16)$$

Similarly, the transformed form of boundary conditions can be written as

$$\theta(0) = a, \quad \theta(1) = 0 \quad (17)$$

By solving Eq. (15) and using boundary conditions (Eq. (17)), the DTM terms are obtained as

$$\begin{aligned} \theta(2) &= \frac{1}{2}N^2\theta(0) - \frac{1}{2}N^2G(1 + \varepsilon_G\theta(0)) \\ \theta(3) &= \frac{1}{6}N^2\theta(1) - \frac{1}{6}N^2G(1 + \varepsilon_G\theta(1)) \\ \theta(4) &= \frac{1}{12}N^2\theta(2) - \frac{1}{12}N^2G(1 + \varepsilon_G\theta(2)) \\ \theta(5) &= \frac{1}{20}N^2\theta(1) - \frac{1}{20}N^2G(1 + \varepsilon_G\theta(1)) \end{aligned} \quad (18)$$

Now by applying Eq. (4) into Eq. (18), a polynomial function will be obtained for a temperature whose coefficients are related to “a” due to Eq. (17). By applying the Eq. (13) as a boundary condition to this polynomial function, the constant parameter “a” will be obtained, so the temperature distribution equation will be estimated.

3.2. Fin with temperature dependent internal heat generation and temperature dependent thermal conductivity

In the second case, it is assumed that the thermal conductivity of fin is temperature-dependent as well as internal heat generation. If we consider it to vary linearly with temperature we have

$$k = k_0[1 + \beta(T - T_{\infty})] \quad (19)$$

The dimensionless form of Eq. (19) is

$$\frac{k}{k_0} = [1 + \epsilon_c \theta] \tag{20}$$

where

$$\epsilon_c = \beta(T_b - T_\infty) \tag{21}$$

Eq. (11) for this condition becomes

$$\frac{d}{dX} \left[(1 + \epsilon_c \theta) \frac{d\theta}{dX} \right] - N^2 \theta + N^2 G (1 + \epsilon_c \theta) = 0 \tag{22}$$

whose boundary conditions are given by Eqs. (12) and (13).

Now we must apply the DTM to this governing equation. By using Table 1 we have

$$\begin{aligned} & (k+2)(k+1)\theta(k+2) + \epsilon_c \sum_{m=0}^k \{ (k+1-m)\theta(k+1-m)(m+1)\theta(m+1) \} \\ & + \epsilon_c \sum_{m=0}^k \{ (k-m)\theta(k-m)(m+2)\theta(m+2) - N^2 \theta(k) \} \\ & + N^2 G (\delta(k) + \epsilon_c \theta(k)) = 0 \end{aligned} \tag{23}$$

Rearranging Eq. (22), a simple relation is obtained as follows:

$$\begin{aligned} \theta(k+2) = & \frac{-1}{(k+2)(k+1)} \epsilon_c \sum_{m=0}^k \{ (k+1-m)\theta(k+1-m)(m+1)\theta(m+1) \} \\ & + \epsilon_c \sum_{m=0}^k \{ (k-m)\theta(k-m)(m+2)\theta(m+2) - N^2 \theta(k) + N^2 G (\delta(k) + \epsilon_c \theta(k)) \} \end{aligned} \tag{24}$$

the boundary condition of this case is the same as the previous case Eq. (17). By solving Eq. (24) and using boundary conditions Eq. (17), the DTM terms for this case can be obtained as

$$\begin{aligned} \theta(2) &= \frac{N^2 \theta(0) - N^2 G \epsilon_c \theta(0) - N^2 G}{2(1 + \epsilon_c \theta(0))} \\ \theta(3) &= \epsilon_c \theta(2) \theta(1) + \frac{1}{6} N^2 \theta(1) - \frac{1}{6} N^2 G \epsilon_c \theta(1) \\ \theta(4) &= -\frac{1}{12} N^2 G \epsilon_c \theta(2) - \frac{3}{4} \epsilon_c \theta(3) \theta(1) - \frac{2}{3} \epsilon_c \theta^2(2) + \frac{1}{12} N^2 \theta(2) \end{aligned} \tag{25}$$

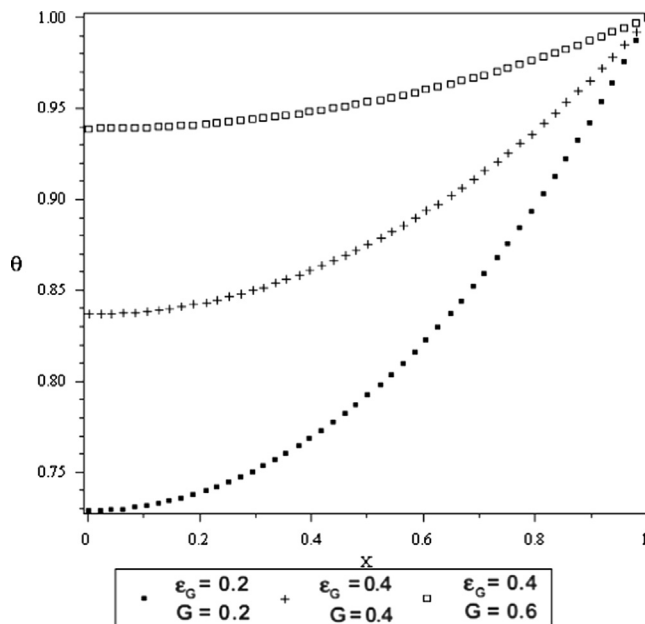


Fig. 2. Temperature distribution in the fin with temperature dependent internal heat generation and constant thermal conductivity for $N=1$.

Finally, by applying Eq. (4) into Eq. (25), a polynomial function will be obtained for a temperature whose coefficients are related to “ a ” due to Eq. (17). By applying Eq. (13) as a boundary condition to this polynomial function, the constant parameter “ a ” will be obtained, so the temperature distribution equation will be estimated.

4. Results and discussion

4.1. Case1: Fin with temperature dependent internal heat generation and constant thermal conductivity

Temperature distribution in case 1 (temperature dependent heat generation and constant thermal conductivity) is shown in Figs. 2–5. It is common that in fin design the N parameter is considered to be 1. Fig. 2 shows temperature distribution for this state and $\varepsilon_G=G=0.2$, $\varepsilon_G=G=0.4$ and $\varepsilon_G=G=0.6$. This choice of parameters represents a fin with moderate temperature dependent heat generation and the thermal conductivity variation of 20% between the base and the

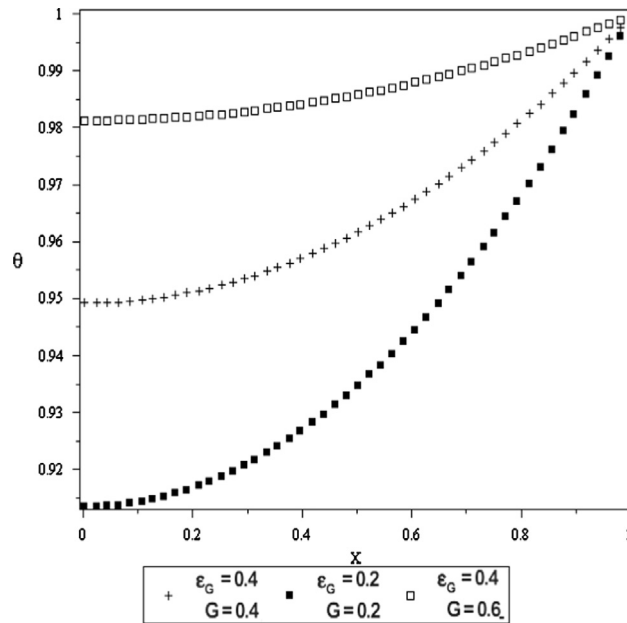


Fig. 3. Temperature distribution in the fin with temperature dependent internal heat generation and constant thermal conductivity for $N=0.5$.

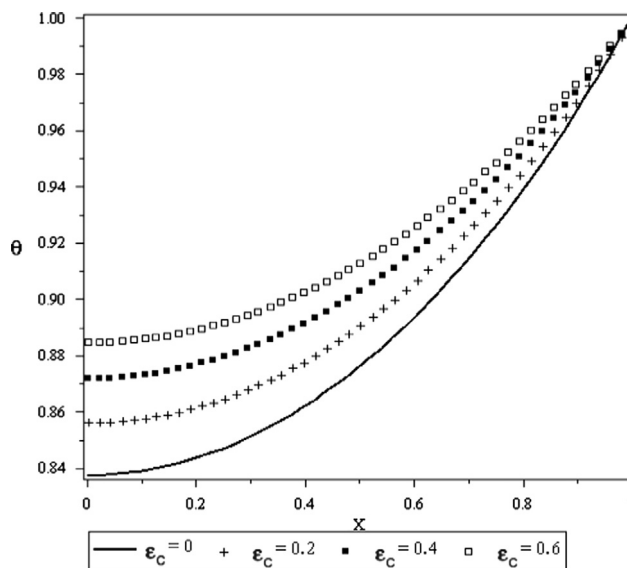


Fig. 4. Temperature distribution in the fin with temperature dependent internal heat generation and temperature dependent thermal conductivity for $N=1$, $G=0.4$, and $\varepsilon_c=0.4$.

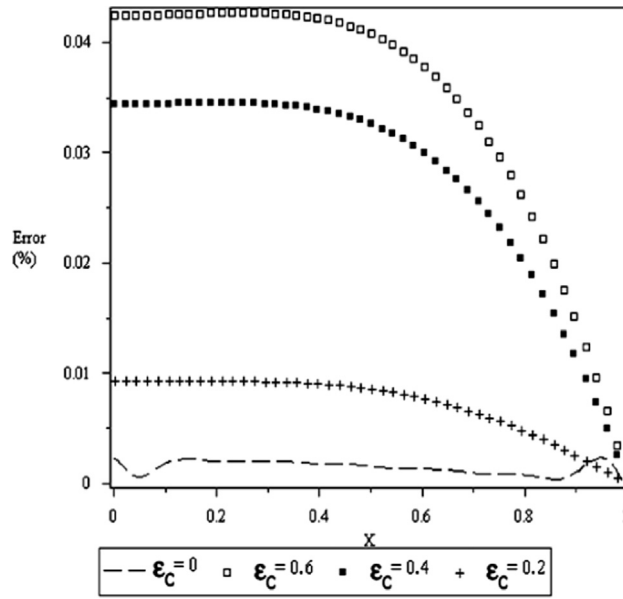


Fig. 5. Error of DTM in comparison by the numerical method for case 2 and $N=1$.

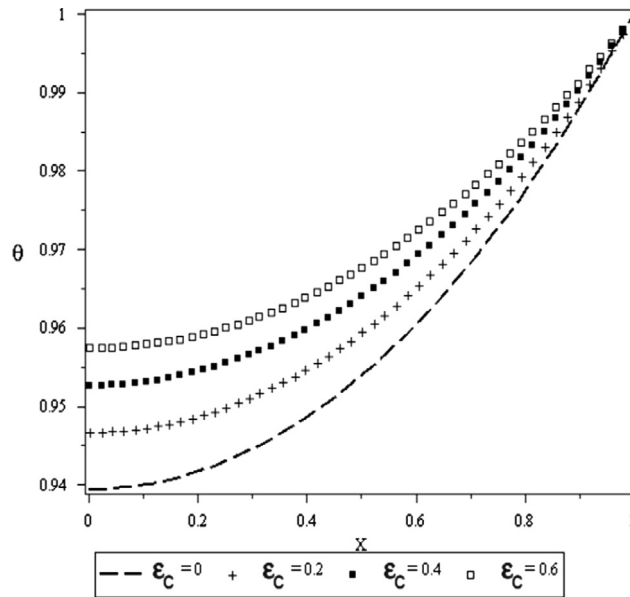


Fig. 6. Temperature distribution in the fin with temperature dependent internal heat generation and temperature dependent thermal conductivity for $N=0.5$, $G=0.4$, and $\epsilon_C=0.4$.

surrounding coolant temperatures that is often used in nuclear rods. As we see in the figure by increasing in ϵ_C and G temperature of the fin increased because of increasing in heat generation. By comparing the results with the numerical method, it was observed that the DTM has a good efficiency and accuracy. Fig. 3 shows a comparison result which pertain to $N=0.5$ (this choice is used in compact heat exchanger fin design), this figure illustrates that fin temperature in this condition is greater than $N=1$ state. The range of the calculated errors reveals that the DTM has a good agreement with numerical results.

4.2. Case 2: Fin with temperature dependent internal heat generation and temperature dependent thermal conductivity

Figs. 4–7 show the temperature distribution in case 2. As already mentioned, in case 2, thermal conductivity and heat generation are temperature dependent. Fig. 4 illustrates the temperature distribution with $N=1$, $\epsilon_C=G=0.4$ and ϵ_C increased from 0 to 0.6 with intervals 0.2. As seen in Fig. 4, when ϵ_C increases, the local fin temperature increases because the ability of

the fin to conduct heat increases. Fig. 5 shows the error of the DTM in comparison to the numerical method for $N=1$, $\varepsilon_G=G=0.4$ and a low maximum error in this figure emphasise on accuracy and efficiency of the Differential Transformation Method. As seen in this figure, maximum error for calculating the fin temperature is occurred at the fin tip as the base temperature is taken as a constant boundary value. In Fig. 6 the N parameter is decreased to 0.5 and temperature distribution is depicted when $G=\varepsilon_G=0.4$.

Finally, by a comparative assessment of figures introduced for this case and pervious case, it can be found that the local fin temperature increases as the parameters G , ε_G , and ε_C increase. The increase in parameter ε_G implies that the heat generation is increased and hence it causes to produce a higher temperature in the fin. An increase in ε_C means the thermal conductivity of the fin is increased and it makes more heat conducting through the fin and local temperature will increase.

5. Conclusion

In this paper, the Differential Transformation Method is applied to analyze the temperature distribution in a fin with temperature-dependent heat generation and thermal conductivity. The problem is solved for two main cases. In the first case just heat generation varies with temperature, and in the second case both heat generation and thermal conductivity are variable. In the problem, constants are chosen from real and industrial fins and DTM results are depicted and compared with the numerical one. Despite DTM simplicity, the solutions match with the numerical results within a maximum error of 0.05%. Obtaining the analytical DTM solution of the problem and comparing with numerical results reveals the facility, effectiveness, and high accuracy of this method.

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