



# Testing cosmic opacity from SNe Ia and Hubble parameter through three cosmological-model-independent methods

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## ARTICLE INFO

### Article history:

Received 2 October 2012

Received in revised form 29 November 2012

Accepted 9 December 2012

Available online 10 December 2012

Editor: S. Dodelson

### Keywords:

Cosmic opacity  
Hubble parameter  
Union2.1  
Constraint

## ABSTRACT

We use the newly published 28 observational Hubble parameter data ( $H(z)$ ) and current largest SNe Ia samples (Union2.1) to test whether the universe is transparent. Three cosmological-model-independent methods (nearby SNe Ia method, interpolation method and smoothing method) are proposed through comparing opacity-free distance modulus from Hubble parameter data and opacity-dependent distance modulus from SNe Ia. Two parameterizations,  $\tau(z) = 2\epsilon z$  and  $\tau(z) = (1+z)^{2\epsilon} - 1$  are adopted for the optical depth associated to the cosmic absorption. We find that the results are not sensitive to the methods and parameterizations. Our results support a transparent universe.

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## 1. Introduction

The unexpected dimming of Type Ia supernova (SNe Ia) is thought to be the evidence of acceleration of the universe [1]. In the frame of General Relativity (GR), the most famous explanation is the existence of dark energy with a negative pressure [2]. However, there are some issues on this plausible mechanism for observed SNe Ia dimming. The photon number conservation may be deviated. For example, it is due to the dust in our galaxy and oscillation of photons propagating in extragalactic magnetic fields into very light axions. This absorption, scattering or axion–photon mixing may lead to dimming [3]. Other mechanisms are widely proposed including modified gravity [4], dissipative processes [5], evolutionary effects in SNe Ia events [6], violation of cosmological principle [7] and so on. On the other hand, the deviation of photon number conservation is related to the correction of Tolman test [8] which is equivalent to measurements of the well-known distance-duality relation (DDR) [9]

$$\frac{D_L}{D_A}(1+z)^{-2} = 1, \quad (1)$$

where  $D_L$  is luminosity distance,  $D_A$  is angular diameter distance and  $z$  is redshift. The DDR is in fact a particular form of presenting the general theorem proved by Etherington known as the “reciprocity law” or “Etherington reciprocity theorem”. DDR holds for

general metric theories of gravity in any cosmic background and it is valid for any cosmological models based on the Riemannian geometry. It is independent of gravity equation and the universe components. However, DDR may be not valid in the case that photons do not travel along null geodesics or the cosmic opacity exists. Many efforts have been done to test DDR though astronomical observations [10]. Usually, they assume the form  $\frac{D_L}{D_A}(1+z)^{-2} = \eta(z)$ , where  $\eta(z) = 1 + \eta_0 z$  or  $\eta(z) = 1 + \eta_0 \frac{z}{1+z}$ . Compared to conservation of photon number, the assumptions that the mathematical tool used to describe the space–time of universe is Riemannian geometry and photon travels along null geodesic are more fundamental and unassailable, thus the deviation of DDR most possibly indicates cosmic absorption. In this case, the flux received by the observer will be reduced by a factor  $e^{-\tau(z)}$ , and observed luminosity distance can be obtained by [11]

$$D_{L,obs} = D_{L,true} e^{\tau(z)/2}, \quad (2)$$

where  $\tau(z)$  is the optical depth related to the cosmic absorption. The relation between  $\tau(z)$  and  $\eta(z)$  is  $e^{\tau(z)/2} = \eta(z)$  [12]. Following this assumption, Avgoustidis et al. [13] studied the difference between SNe Ia observations and Hubble parameter data.  $H(z)$  data are mainly obtained through the measurements of differential ages of red-envelope galaxies known as “differential age method”. The aging of stars can be regarded as an indicator of the aging of the universe. The spectra of stars can be converted to the information of their ages, as the evolutions of stars are well known. Since the stars cannot be observed one by one at cosmological scales, people usually take the spectra of galaxies which contain relatively

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uniform star population. Moreover,  $H(z)$  data can be obtained from the BAO scale as a standard ruler in the radial direction known as “Peak Method”. These methods are apparently independent of galaxy luminosity so that it will not be affected by cosmic opacity. However, SNe Ia observations are affected by many sources of opacity, such as the hosting galaxy, intervening galaxies, intergalactic medium, the Milky Way and exotic physics which affect photon conservation. Under the assumption  $D_L = D_A(1+z)^{2+\epsilon}$ , they investigated the cosmic opacity by confronting the standard luminosity distance in spatially flat  $\Lambda$ CDM model with the observed one from SNe Ia observations. Combining with the  $H(z)$  data, which is not affected by transparency but yields constraints on  $\Omega_m$ , and marginalizing over all other parameters except  $\epsilon$ , they got  $\epsilon = -0.04_{-0.07}^{+0.08}$  ( $2\sigma$ ). Noticing that their method depends on cosmological models, Holanda et al. [14] further proposed a model-independent estimate of  $D_L$  which are obtained from a numerical integration of  $H(z)$  data. They also explored the influence of different SNe Ia light-curve fitters (SALT2 and MLCS2K2) and found a significant conflict. Based on Holanda et al., we proposed three model-independent methods that are different from theirs to explore the cosmic opacity. They firstly got the luminosity distances from  $H(z)$  data (12 data) at corresponding redshifts (at  $H(z)$  data) and gave a polynomial fit based on these 12 luminosity distances data with their errors, then calculated the values at the redshifts corresponding to SNe Ia through this polynomial fit curve. On the contrary, we use SNe Ia data to get the luminosity distances at the redshifts corresponding to  $H(z)$  data through interpolation method, smoothing method and nearby SNe Ia method. Our data set contains 28 available  $H(z)$  data and the largest SNe Ia samples Union2.1 [15].

The Letter is organized as follows: In Section 2, we introduce the method of obtaining luminosity distance from  $H(z)$  data. In Section 3, we give the three methods that can convert SNe Ia luminosity distances to the luminosity distances at the redshifts corresponding to  $H(z)$  data. In Section 4, the results are performed. Finally, we make a conclusion in Section 5.

## 2. Luminosity distance from observational Hubble parameter data

In this section, we introduce the method proposed by Holanda et al. [14]. The expression of the Hubble parameter can be written in this form

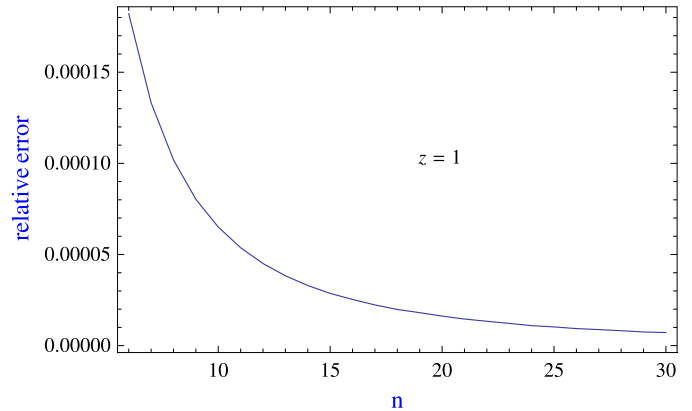
$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}, \quad (3)$$

which depends on the differential age as a function of redshift. Based on Jimenez et al. [16], Simon et al. [17] used the age of evolving galaxies and got nine  $H(z)$  data. Stern et al. [18] revised these data at 11 redshifts from the differential ages of red-envelope galaxies. Gaztañaga et al. [19] took the BAO scale as a standard ruler in the radial direction, obtained two data. Recently, Moresco et al. [20] obtained 8 data from the differential spectroscopic evolution of early-type galaxies as a function of redshift. Blake et al. [21] obtained 3 data through combining measurements of the baryon acoustic peak and Alcock–Paczynski distortion from galaxy clustering in the WiggleZ Dark Energy Survey. Zhang et al. [22] obtained another 4 data. Totally, we have 28 available data summarized in Table 1. Following Holanda et al. [14], we transform these 28  $H(z)$  data into luminosity distance. Using a usual simple trapezoidal rule, the comoving distance can be calculated by

$$r = c \int_0^z \frac{dz'}{H(z')} \approx \frac{c}{2} \sum_{i=1}^n (z_{i+1} - z_i) \left[ \frac{1}{H(z_{i+1})} + \frac{1}{H(z_i)} \right]. \quad (4)$$

**Table 1**  
Current published observational Hubble parameter data ( $\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ ).

$z$	$H(z)$	$\sigma_H$
0.07	69	19.6
0.09	69	12
0.12	68.6	26.2
0.17	83	8
0.179	75	4
0.199	75	5
0.2	72.9	29.6
0.24	79.69	3.32
0.27	77	14
0.28	88.8	36.6
0.352	83	14
0.4	95	17
0.43	86.45	3.27
0.44	82.6	7.8
0.48	97	62
0.593	104	13
0.6	87.9	6.1
0.68	92	8
0.73	97.3	7
0.781	105	12
0.875	125	17
0.88	90	40
0.9	117	23
1.037	154	20
1.3	168	17
1.43	177	18
1.53	140	14
1.75	202	40



**Fig. 1.** Relative error with respect to the data number at  $z = 1$ .  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ .

Since we use much more data than Holanda et al. (they used only 12 data), this trapezoidal rule will work much better. In Fig. 1, we show the relative error with respect to the data number at a characteristic redshift  $z = 1$ . We assume a standard  $\Lambda$ CDM model with  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ , and divide  $z = 1$  into different numbers of intervals averagely, then calculate the relative errors according to Eq. (4). We find that the relative errors decrease remarkably when the numbers of intervals increase from 12 to 28. With the standard error propagation formula, the error associated to the  $i$ th bin is given by

$$s_i = \frac{c}{2} (z_{i+1} - z_i) \left( \frac{\sigma_{H_{i+1}}^2}{H_{i+1}^4} + \frac{\sigma_{H_i}^2}{H_i^4} \right)^{1/2}, \quad (5)$$

where  $\sigma_{H_i}$  is the error of  $H(z)$  data. The error corresponding to certain redshift is the sum of  $s_i$ . The Hubble constant  $H_0 = 73.8 \pm 2.4 \text{ km/s/Mpc}$  [23] is used in our study. The 28 luminosity distance data from  $H(z)$  data are shown in Fig. 2, as well as the  $D_L$  from Union2.1 SNe Ia samples.

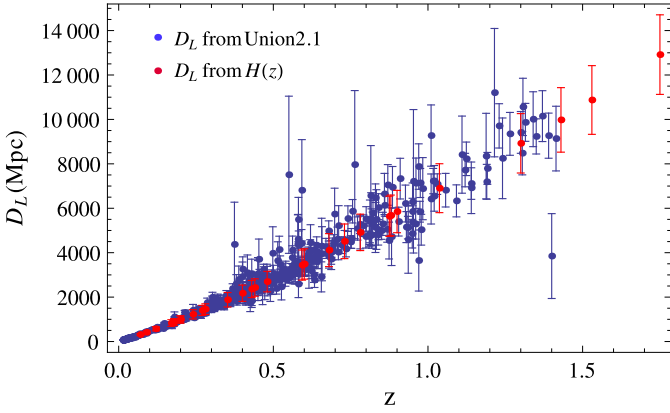


Fig. 2.  $D_L(z)$  obtained from measurements of the Hubble parameter data and the Union2.1 samples, respectively.

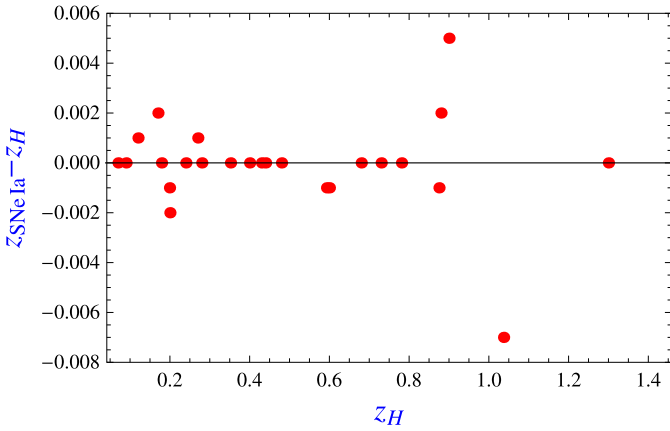


Fig. 3. Subtractions of redshifts between  $H(z)$  data and the associated SNe Ia.

### 3. Dealing with SNe Ia samples

In this section, we introduce three methods through which we can obtain the luminosity distance of one certain SNe Ia point at the same redshift of the corresponding  $H(z)$  data.

#### 3.1. Nearby SNe Ia method

Since the SNe Ia Samples are much larger than  $H(z)$  data, the nearby SNe Ia can be substituted for the one at the redshift corresponding to  $H(z)$  data. Points  $z_{\text{SNe Ia}} - z_H$  are centered around the line  $\Delta z = 0$ , as shown in Fig. 3 which plots the subtractions of redshifts between  $H(z)$  data and the associated SNe Ia. Similar with the DDR test [10], we have to choose a criterion based on the data. Our selection criterion is  $\Delta z = |z_H - z_{\text{SNe Ia}}| < 0.003$ . This selection criteria can be satisfied for most of the  $H(z)$  data except for the points at  $z = 0.9$  and  $z = 1.037$  ( $z = 1.43, 1.53, 1.75$  are obviously ruled out) and small enough to reduce the systematic errors and guarantee the accuracy.

#### 3.2. Interpolation method

In order to avoid any bias brought by redshift incoincidence between  $H(z)$  data and SNe Ia, as well as to ensure the integrity of the  $H(z)$  data, we can use the nearby SNe Ia points to obtain the luminosity distance of SNe Ia point at the same redshift of the corresponding  $H(z)$  data. This situation is similar with the cosmology-independent calibration of GRB relations directly from

SNe Ia [24]. When the linear interpolation is used, the distance modulus and the error can be calculated by

$$\mu = \mu_i + (\mu_{i+1} - \mu_i) \left[ \frac{z - z_i}{z_{i+1} - z_i} \right], \quad (6)$$

$$\sigma_\mu = \left[ \left( \frac{z_{i+1} - z}{z_{i+1} - z_i} \right)^2 \sigma_{\mu,i}^2 + \left( \frac{z - z_i}{z_{i+1} - z_i} \right)^2 \sigma_{\mu,i+1}^2 \right]^{1/2}, \quad (7)$$

where subscripts  $i$  and  $i + 1$  stand for the nearby data. We use the same method as used in [25] to obtain the best estimate for each SNe Ia which is weighted average of all available distance moduli at the same redshift. The derived distance modulus for each SNe Ia is

$$\mu = \left( \sum_i \mu_i / \sigma_{\mu_i}^2 \right) / \left( \sum_i \sigma_{\mu_i}^{-2} \right), \quad (8)$$

with its uncertainty  $\sigma_\mu = \left( \sum_i \sigma_{\mu_i}^{-2} \right)^{-1/2}$ .

#### 3.3. Smoothing method

We introduce a non-parametric method of smoothing supernova data over redshift using a Gaussian kernel in order to reconstruct luminosity distance [26]. This procedure was initially used in the analysis of the cosmic large scale structure [27]. Through this model-independent method, we can extract information of various cosmological parameters, such as Hubble parameter, the dark energy equation of state, the matter density. Wu and Yu [28] generalized this method to eliminate the impact of  $H_0$  and obtained the evolutionary curve of luminosity distance using SNe Ia Constitution set and Union2 set. In this Letter, we follow this generalized method to get the luminosity distance curve using Union2.1 set. Firstly, we obtain the variable  $\ln f(z) = \ln D_L(z) - \ln h$  through iterative method

$$\ln f(z)_n^s = \ln f(z)_{n-1}^s + N(z) \sum_i \left[ \ln f^{obs}(z_i) - \ln f(z)_{n-1}^s \right] \times \exp \left[ - \frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right], \quad (9)$$

where the reduced Hubble constant  $h = H_0/100$  and the normalization factor

$$N(z)^{-1} = \sum_i \exp \left[ - \frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right]. \quad (10)$$

The value of parameter  $\Delta$  was discussed by Shafieloo et al. [26]. The results are not sensitive to  $\Delta$ . Following Shafieloo et al. [26] and Wu and Yu [28],  $\Delta = 0.6$  is used in this Letter. Eq. (9) can give the smoothed luminosity distance at any redshift  $z$  after  $n$  iterations. When  $n = 1$

$$\begin{aligned} \ln f(z)_1^s &= \ln f(z)_0^s + N(z) \sum_i \left[ \ln f^{obs}(z_i) - \ln f(z)_0^s \right] \\ &\times \exp \left[ - \frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right] \\ &= \ln D_L(z)_0^s + N(z) \sum_i \left[ \ln f^{obs}(z_i) - \ln D_L(z)_0^s \right] \\ &\times \exp \left[ - \frac{\ln^2 \left( \frac{1+z}{1+z_i} \right)}{2\Delta^2} \right], \end{aligned} \quad (11)$$

where  $D_L(z)_0^s$  depends on the suggested background cosmological model. Following Wu and Yu [28], we adopt  $\omega$ CDM model

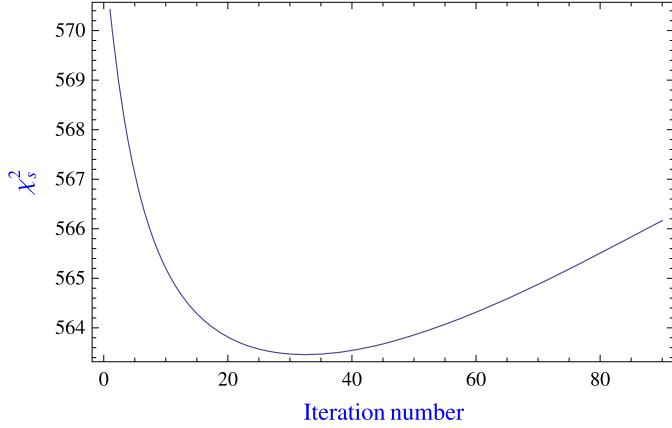


Fig. 4. Computed  $\chi_s^2$  for the reconstructed results at each iteration.

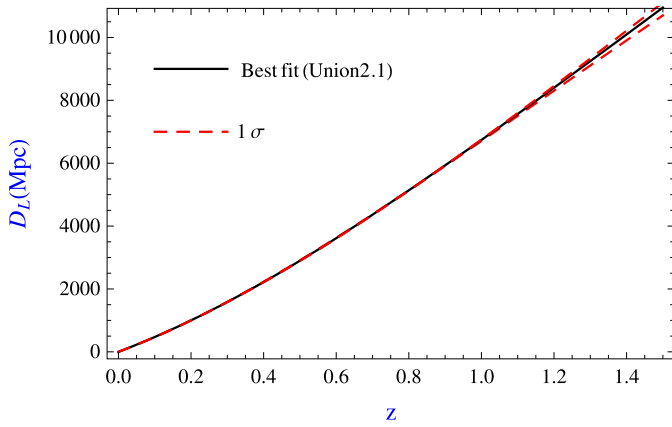


Fig. 5. Smoothed luminosity distance for Union2.1 samples.

with  $\omega = -0.9$  and  $\Omega_{m0} = 0.28$  as the background model. The relation between  $\ln f^{obs}(z_i)$  and  $\ln D_L^{obs}(z_i)$  which is obtained from observed SNe Ia is

$$\ln f^{obs}(z_i) = \frac{\ln 10}{5} [\mu^{obs}(z_i) - 42.38] = \ln D_L^{obs}(z_i) - \ln h, \quad (12)$$

$\mu^{obs}(z_i)$  is the observed distance modulus from SNe Ia. In order to know whether we get a best-fit value after some iterations, we calculate, after each iteration,  $\chi_s^2$

$$\chi_{s,n}^2 = \sum_i \frac{(\mu(z_i)_n - \mu^{obs}(z_i))^2}{\sigma_{\mu_{obs,i}}^2}. \quad (13)$$

The best-fit result is corresponding to the minimum value of  $\chi_{s,n}^2$ . The  $1\sigma$  corresponds to  $\Delta\chi_s^2 = 1$ . The results for Union2.1 samples are shown in Fig. 4 and Fig. 5, the minimum value  $n = 32$ .

#### 4. Constraints on cosmic opacity

The observed distance modulus can be expressed as [11]

$$\mu_{obs}(z) = \mu_{true}(z) + 2.5[\log_{10} e] \tau(z). \quad (14)$$

To examine the sensitivity of test results on the parametric form, we adopt two parameterizations,  $\tau(z) = 2\epsilon z$  and  $\tau(z) = (1+z)^{2\epsilon} - 1$  which are not strongly wavelength dependent on the optical band [29].  $\epsilon$  here describes the cosmic opacity. The former one is linear and it can be derived from the DDR parameterization  $D_L = D_A(1+z)^{2+\epsilon}$  for small  $\epsilon$  and redshift. To constrain the value of  $\epsilon$ , we use the usual maximum likelihood method of  $\chi^2$  fitting

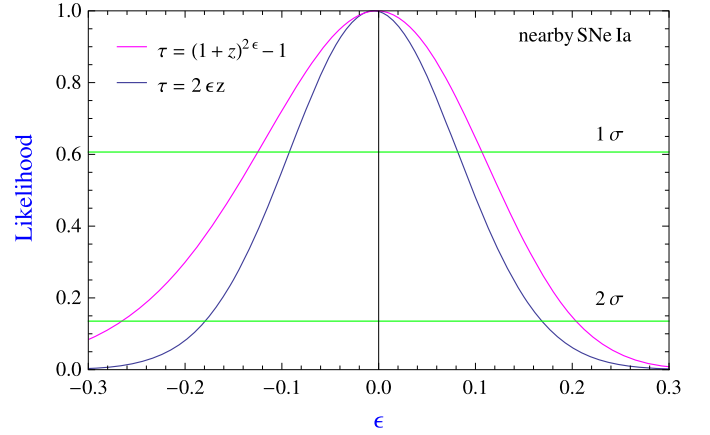


Fig. 6. Likelihood function for  $\epsilon$  using nearby SNe Ia method.

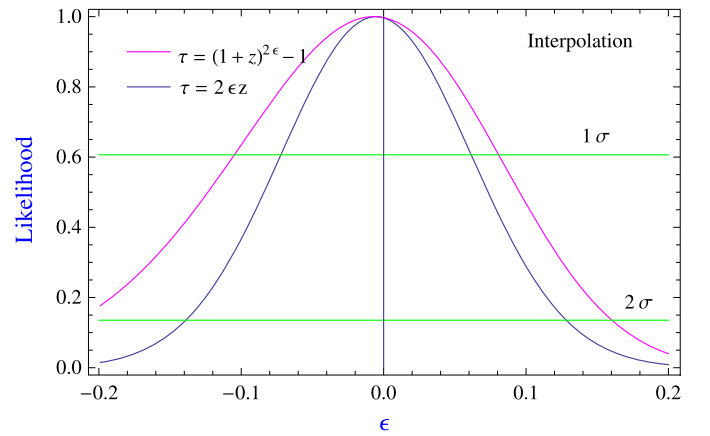


Fig. 7. Likelihood function for  $\epsilon$  using interpolation method.

$$\chi^2 = \sum_i \frac{(\mu_{obs}(i) - \mu_{true}(i) - 2.5[\log_{10} e] \tau(i))^2}{\sigma_{\mu_{obs}}^2 + \sigma_{\mu_{true}}^2}, \quad (15)$$

the subscript  $i$  stands for the data at the redshifts corresponding to  $H(z)$  data. Our results are shown in Fig. 6, Fig. 7 and Fig. 8, respectively. For smoothing method, we consider two cases:  $i_{max} = 25$  and  $i_{max} = 28$  (containing the data at  $z = 1.43, 1.53, 1.75$ ). From the likelihood of  $\epsilon$  using different methods, we can see current SNe Ia samples and  $H(z)$  data support a transparent universe. These results are slightly different from Holanda et al. [14] while their results seem a little prone to a non-transparent universe especially with MLCS2K2 compilation.

#### 5. Conclusion

Until now, modern cosmology has discovered many interesting phenomena behind which there exist underlying physical mechanisms. In principle, since the current astronomical observations are not precise enough to distinguish between different cosmological models, for example, various dark energy models are consistent with observations, we can explore all the possibilities among which the true one exists. Though the matter component in the universe is so diluted, photons will get through a huge space to observers, photon conservation can be violated by simple astrophysical effects or by exotic physics. Amongst the former, the attenuation is due to interstellar dust, gas, plasmas and so on. More exotic sources of photon conservation violation involve a coupling of photons to particles beyond the standard model of particle physics. Therefore, the concept of cosmic opacity should

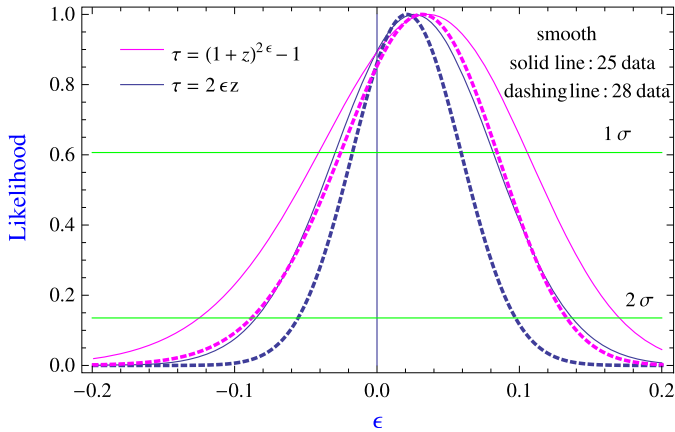


Fig. 8. Likelihood function for  $\epsilon$  using smoothing method.

be considered naturally. In this Letter, we use the current observational Hubble parameter data which is opacity-free and SNe Ia observations which depend on cosmic opacity to test whether the universe is transparent. The results from three model-independent methods converge to a point that the effects of cosmic opacity are vanished. For future study on this problem, we think the wavelength is a considerable factor and more independent methods of testing cosmic opacity will confirm the conclusion.

### Acknowledgements

This work was supported by the National Natural Science Foundation of China under the Distinguished Young Scholar Grant 10825313, the Ministry of Science and Technology National Basic Science Program (Project 973) under Grant No. 2012CB821804, the Fundamental Research Funds for the Central Universities and Scientific Research Foundation of Beijing Normal University.

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